

I. Eigenschappen van zuivere stoffen/Properties of pure substances

1 Algemeen/General

- $H \stackrel{\text{def}}{=} U + PV$, $h = u + Pv$
- $c_V \stackrel{\text{def}}{=} \left(\frac{\partial u}{\partial T} \right)_V$, $c_P \stackrel{\text{def}}{=} \left(\frac{\partial h}{\partial T} \right)_P$, $k \stackrel{\text{def}}{=} \frac{c_P}{c_V}$

2 Vaste stoffen en onsamendrukbare vloeistoffen/Solids and incompressible liquids

- $v = \text{constant}$
- $c_P = c_V = c$
- $\Delta u = c \Delta T$
- $\Delta h = c \Delta T + v \Delta P$
- $\Delta s = s_2 - s_1 = c \ln \frac{T_2}{T_1}$

3 Samengedrukte vloeistof/Compressed liquid

- $v = v_f(T)$, $u = u_f(T)$, $s = s_f(T)$, $h = h_f(T)$

4 Verzadigde vloeistof-damp mengsels/Saturated liquid-vapor mixtures

- $x \stackrel{\text{def}}{=} \frac{m_g}{m_f + m_g}$
- $v = x v_g + (1 - x) v_f = v_f + x(v_g - v_f) = v_f + x v_{fg}$
(idem voor u, h en s /same for u, h and s)
- Verzadigde vloeistof/Saturated liquid: $x = 0$,
Verzadigde damp/Saturated vapor: $x = 1$

5 Oververhitte damp/Superheated vapor

Tabel/Table A6 (water) en/and A13 (R134a)

6 Ideale gassen/Ideal gases

▪ Ideale gaswet/Ideal gas law:

1) $PV = n\bar{R}T$

met/with $\bar{R} = 8,314 \frac{\text{J}}{\text{mol K}}$ onafhankelijk van het gas/independent of the gas

2) $Pv = RT$

met/with $R = \frac{\bar{R}}{M}$ afhankelijk van het gas/dependent on the gas

▪ $c_p = c_v + R$

▪ u en h enkel functie van T / u and h only a function of T :

$c_v = \frac{du}{dT}$ en/and $c_p = \frac{dh}{dT}$

Polytrope toestandsveranderingen/Polytropic state changes

▪ $Pv^n = \text{constant}$

▪ $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{v_2}{v_1}\right)^{1-n}$ en/and $\frac{P_2}{P_1} = \left(\frac{v_2}{v_1}\right)^{-n}$

Toestandsveranderingen bij constante c_v en c_p /State changes with constant c_v and c_p

▪ $\Delta u = c_v \Delta T$
 $\Delta h = c_p \Delta T$

▪ $\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$

▪ Isentroop = polytroop met $n = k$ /
Isentrope = polytrope with $n = k$

II. Gesloten systemen/Closed Systems

1 Algemeen/General

- Behoud van massa/Conservation of mass: $\dot{m}_{in} = \dot{m}_{out} = 0$

- 1^{ste} hoofdwet (Behoud van Energie)/

1st law of thermodynamics (Conservation of energy):

$$\Delta E = Q - W$$

Q : warmte opgenomen door het systeem vanuit de omgeving/heat absorbed by the system from the environment

W : arbeid geleverd door het systeem op de omgeving/work done by the system on the surroundings

- 2^{de} hoofdwet (Entropiebalans)/

2nd law of thermodynamics (Entropy balance)

$$\Delta S = \sum_k \frac{Q_k}{T_k} + S_{gen} \quad (\text{met/with } S_{gen} \geq 0)$$

Q_k : warmte uitgewisseld bij temperatuur T_k /heat exchanged at temperature T_k

S_{gen} : gegenereerde entropie (altijd niet-negatief)/generated entropy (always non-negative)

2 Reversibele processen/Reversible processes

- Volumearbeid of expansiearbeid/Boundary work:

$$W_{1 \rightarrow 2} = \int_1^2 P dV$$

- Polytroop proces van ideaal gas/Polytropic process of an ideal gas:

$$W_{1 \rightarrow 2} = \frac{P_2 V_2 - P_1 V_1}{1 - n} \quad (n \neq 1)$$

$$W_{1 \rightarrow 2} = P_1 V_1 \ln \frac{V_2}{V_1} \quad (n = 1)$$

- Isotherm proces/Isothermal process: $Q = T\Delta S$

III. Stationaire open systemen/ Steady-state open systems

1 Algemeen/General

- Behoud van massa/Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

- 1^{ste} hoofdwet/1st law of thermodynamics:

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_k \dot{m}_{in_k} \theta_k = \dot{Q}_{out} + \dot{W}_{out} + \sum_k \dot{m}_{out_k} \theta_k$$

$$\theta = h + \frac{v^2}{2} + gz$$

- 2^{de} hoofdwet/2nd law of thermodynamics:

$$\sum_k \frac{\dot{Q}_k}{T_k} + \sum_k \dot{m}_{in_k} s_k - \sum_k \dot{m}_{out_k} s_k + \dot{S}_{gen} = 0$$

(met/with $\dot{S}_{gen} \geq 0$)

2 Inwendig reversibele processen/ Internally reversible processes ($\dot{S}_{gen} = 0$)

- $w = - \int v dp + \frac{v_i^2 - v_e^2}{2} + g(z_i - z_e)$

- isotherm proces/isothermal process:

$$q = T\Delta s$$

- polytroop proces/polytropic process:

$$\int v dp = \begin{cases} \frac{n}{n-1} (P_2 v_2 - P_1 v_1) & (n \neq 1) \\ P_1 v_1 \ln\left(\frac{P_2}{P_1}\right) & n = 1 \end{cases}$$

IV. Systemen met onregelmatige stroming/ Unsteady-flow systems

- Behoud van massa/Conservation of mass:

$$m_{in} - m_{out} = \Delta m = m_2 - m_1$$

- 1^{ste} hoofdwet/1st law of thermodynamics:

$$\left[Q_{in} + W_{in} + \sum_k m_{in_k} \theta_k \right] - \left[Q_{out} + W_{out} + \sum_k m_{out_k} \theta_k \right] = m_2 e_2 - m_1 e_1$$

$$\theta = h + ke + pe, \quad e = u + ke + pe$$

- 2^{de} hoofdwet/2nd law of thermodynamics:

$$\sum_k \frac{Q_k}{T_k} + \sum_k m_{in_k} s_{in} - \sum_k m_{out_k} s_{out} + S_{gen} = (S_2 - S_1)_{cv}$$

V. Warmtetransport/Heat transfer

1 Conductie/Conduction

- Wet van Fourier/Fourier's law of conduction:

$$\dot{Q} = \frac{k \cdot A}{L} \Delta T$$

- Thermische weerstand/Thermal resistance:

$$R = \frac{L}{k \cdot A}$$

2 Convectie/Convection

- Wet van Newton voor afkoeling/Newton's law of cooling:

$$\dot{Q} = h_{conv} \cdot A \cdot \Delta T$$

- Thermische weerstand/Thermal resistance:

$$R = \frac{1}{h \cdot A}$$

3 Straling/Radiation

- Stefan-Boltzmann wet/Stefan-Boltzmann law:

$$\dot{Q} = \varepsilon \cdot \sigma \cdot A \cdot T^4$$

$$\sigma = 5,67 \cdot 10^{-8} \text{ W/m}^2 \text{K}^4$$

$$h_{rad} = \varepsilon \cdot \sigma (T^2 + T_{surr}^2)(T + T_{surr})$$

- Netto stralingswarmtetransport/Net radiation heat transfer:

$$\dot{Q} = \varepsilon \cdot \sigma \cdot A (T^4 - T_{surr}^4)$$

- Thermische weerstand/Thermal resistance:

$$R = \frac{1}{h \cdot A}$$

Formularium stromingsmechanica

Formulary fluid mechanics

1 Fluidum statica / Fluid statics

$$P = P_o + \rho gh = P_o + \rho gy \sin \theta$$

$$F_R = \int_A P dA = \int_A (P_o + \rho gy \sin \theta) dA = (P_o + \rho gy_C \sin \theta) A \text{ met/with } y_C = \frac{1}{A} \int_A y dA$$

$$y_P F_R = \int_A y P dA = P_o \int_A y dA + \rho g \sin \theta \int_A y^2 dA = P_o y_C A + \rho g \sin \theta I_{xx,O} \text{ met/with } I_{xx,O} = I_{xx,C} + y_C^2 A$$

$$F_B = \rho_f g V$$

2 Bernoulli

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 = \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2$$

3 Momentum analyse / Momentum analysis

$$\frac{dN_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho \eta dV + \int_{CS} \rho \eta \vec{V} \cdot d\vec{A}$$

Behoud van massa / Conservation of mass:

$$\frac{d}{dt} \int_{CV} \rho dV + \int_{CS} \rho \vec{V} \cdot d\vec{A} = 0$$

Behoud van momentum / Conservation of momentum:

$$\begin{aligned} \sum \vec{F} &= \frac{d}{dt} \int_{CV} \rho \vec{V} dV + \int_{CS} \rho \vec{V} \vec{V} \cdot d\vec{A} \\ \sum \vec{F} &= \frac{d}{dt} \int_{CV} \rho \vec{V} dV + \sum_{out} \beta \dot{m} \vec{V}_{avg} - \sum_{in} \beta \dot{m} \vec{V}_{avg} \end{aligned}$$

4 Inwendige stroming / Internal flow

Navier-Stokes:

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) \end{aligned}$$

Reynoldsgetal / Reynolds number:

$$Re = \frac{VD}{\nu} = \frac{\rho VD}{\mu}$$

Stroming in buis / Pipe flow:

$$u(r) = -\frac{R^2}{4\mu} \left(\frac{dP}{dx} \right) \left(1 - \frac{r^2}{R^2} \right)$$
$$Q = -\frac{\pi R^4}{8\mu} \frac{\partial P}{\partial x}$$

Plaatstroming / Plate flow:

$$u(x) = \frac{1}{2\mu} \frac{dP}{dx} y^2 + \left[\frac{U_p}{a} - \frac{1}{2\mu} \frac{dP}{dx} a \right] y$$

Major/Minor losses:

$$\Delta P = f \frac{L}{D} \frac{\rho V_{avg}^2}{2} + \sum_i K_i \frac{\rho V_{avg}^2}{2}$$

Laminaire stroming / laminar flow:

$$f = \frac{64}{Re}$$

Turbulente stroming / turbulent flow:

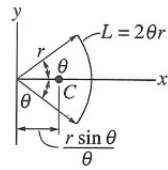
$$\frac{1}{\sqrt{f}} \cong -2.0 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

Krachtwerking / fluid forces:

$$C_D = \frac{F_D}{(\rho V^2/2)A}$$
$$C_L = \frac{F_L}{(\rho V^2/2)A}$$

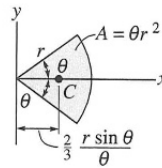
Geometrische eigenschappen van lijn- en oppervlakte-elementen

Plaats van het zwaartepunt



Segment van een cirkelboog

Plaats van het zwaartepunt

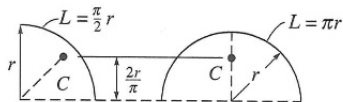


Segment van een cirkelvlak

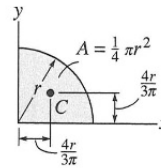
Traagheidsmoment van oppervlak

$$I_x = \frac{1}{4} r^4 (\theta - \frac{1}{2} \sin 2\theta)$$

$$I_y = \frac{1}{4} r^4 (\theta + \frac{1}{2} \sin 2\theta)$$



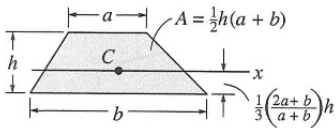
Kwart en halve cirkelboog



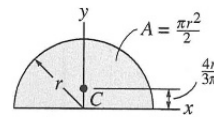
Kwart cirkelvlak

$$I_x = \frac{1}{16} \pi r^4$$

$$I_y = \frac{1}{16} \pi r^4$$



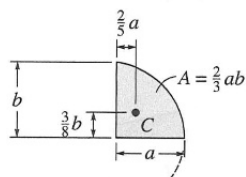
Het vlak van een trapezoïde



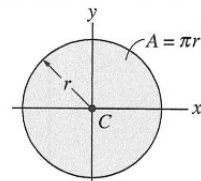
Half cirkelvlak

$$I_x = \frac{1}{8} \pi r^4$$

$$I_y = \frac{1}{8} \pi r^4$$



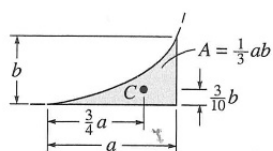
Half paraboolvlak



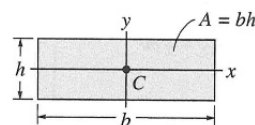
Cirkelvlak

$$I_x = \frac{1}{4} \pi r^4$$

$$I_y = \frac{1}{4} \pi r^4$$



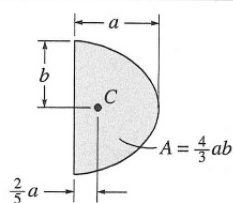
Exparaboolvlak



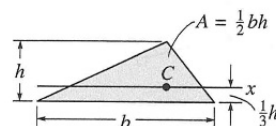
Rechthoekig vlak

$$I_x = \frac{1}{12} b h^3$$

$$I_y = \frac{1}{12} h b^3$$



Paraboolvlak



Driehoekig vlak

$$I_x = \frac{1}{36} b h^3$$

Integralen

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{dx}{a+bx} = \frac{1}{b} \ln(a+bx) + C$$

$$\int \frac{dx}{a+bx^2} = \frac{1}{2\sqrt{-ba}} \ln \left[\frac{a+x\sqrt{-ab}}{a-x\sqrt{-ab}} \right] + C, \quad ab < 0$$

$$\int \frac{x dx}{a+bx^2} = \frac{1}{2b} \ln(bx^2+a) + C,$$

$$\int \frac{x^2 dx}{a+bx^2} = \frac{x}{b} - \frac{a}{b\sqrt{ab}} \tan^{-1} \frac{x\sqrt{ab}}{a} + C, \quad ab > 0$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \left[\frac{a+x}{a-x} \right] + C, \quad a^2 > x^2$$

$$\int \sqrt{a+bx} dx = \frac{2}{3b} \sqrt{(a+bx)^3} + C$$

$$\int x\sqrt{a+bx} dx = \frac{-2(2a-3bx)\sqrt{(a+bx)^3}}{15b^2} + C$$

$$\int x^2\sqrt{a+bx} dx = \frac{2(8a^2-12abx+15b^2x^2)\sqrt{(a+bx)^3}}{105b^3} + C$$

$$\int \sqrt{a^2-x^2} dx = \frac{1}{2} \left[x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right] + C, \quad a > 0$$

$$\int x\sqrt{a^2-x^2} dx = -\frac{1}{3} \sqrt{(a^2-x^2)^3} + C$$

$$\int x^2\sqrt{a^2-x^2} dx = -\frac{x}{4} \sqrt{(a^2-x^2)^3} + \frac{a^2}{8} \left(x\sqrt{a^2-x^2} + a^2 \sin^{-1} \frac{x}{a} \right) + C, \quad a > 0$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{1}{2} \left[x\sqrt{x^2 \pm a^2} \pm a^2 \ln(x + \sqrt{x^2 \pm a^2}) \right] + C$$

$$\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} \sqrt{(x^2 \pm a^2)^3} + C$$

$$\int x^2\sqrt{x^2 \pm a^2} dx = \frac{x}{4} \sqrt{(x^2 \pm a^2)^3} \mp \frac{a^2}{8} x\sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln(x + \sqrt{x^2 \pm a^2}) + C$$

$$\int \frac{dx}{\sqrt{a+bx}} = \frac{2\sqrt{a+bx}}{b} + C$$

$$\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2} + C$$

$$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \ln \left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}} \right] + C, \quad c > 0$$

$$= \frac{1}{\sqrt{-c}} \sin^{-1} \left(\frac{-2cx-b}{\sqrt{b^2-4ac}} \right) + C, \quad c < 0$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int x \cos(ax) dx = \frac{1}{a^2} \cos(ax) + \frac{x}{a} \sin(ax) + C$$

$$\int x^2 \cos(ax) dx = \frac{2x}{a^2} \cos(ax) + \frac{a^2 x^2 - 2}{a^3} \sin(ax) + C$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

$$\int x e^{ax} dx = \frac{e^{ax}}{a^2} (ax-1) + C$$

$$\int \sinh x dx = \cosh x + C$$

$$\int \cosh x dx = \sinh x + C$$