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MATH2501

WEEK 3

TUTOR:Edmond

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关于

HD EDUCATION

HD · EDUCATION（简称HD·EDU）成立于2018年1月，拥有学业辅导和职业规划两大核心业务。从创办伊始就秉承着 “让年轻人成为知识的生产者、传播者、受惠者” 的使命，坚持从留学生的角度出发，为他们量身制定属于他们的课程。“成为最受年轻人喜爱的教育品牌“一直是我们的不懈追求。

截止2020年，我们的Tutor人数已达1300人，业务范围涵盖了澳大利亚、新西兰、美国、英国4个国家的40多所高等校，为15万留学生提供了优质的学习辅导服务，成为澳大利亚华人留学生覆盖人数最多的在线教育学习平台。

HD·EDU的成长有你陪伴

课后，如果您有任何建议和意见，我们都非常欢迎您联系小助手分享您的想法，给予我们改进和提高的机会！

感谢您参与HD Education的辅导课程！

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TUTOR
Self-Introduction
自我介绍

TUTOR:Edmond

#

MATH2011分数：96/100

个人简介：大三ACTL/STAT在读，WAM DN

曾获荣誉（数学相关）：全国大学生数学竞赛与上海市大学生数学竞赛三等奖

教学经历（数学相关）：MATH1151、MATH1251、MATH1041、MATH2011、MATH2018/2019、MATH2501etc

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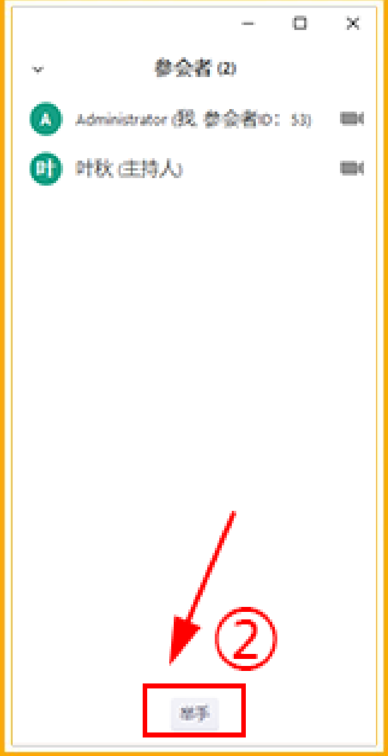
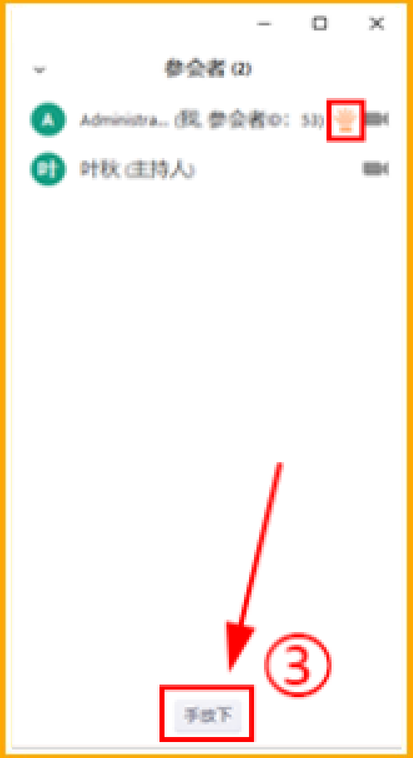
同学们有问题怎么办?





1.点【参会者】
2.点【举手】即可与老师实时互动
3.问题被解答了还可以【手放下】
4.点【聊天】/【Chat】就可以与老师实时文字互动哦

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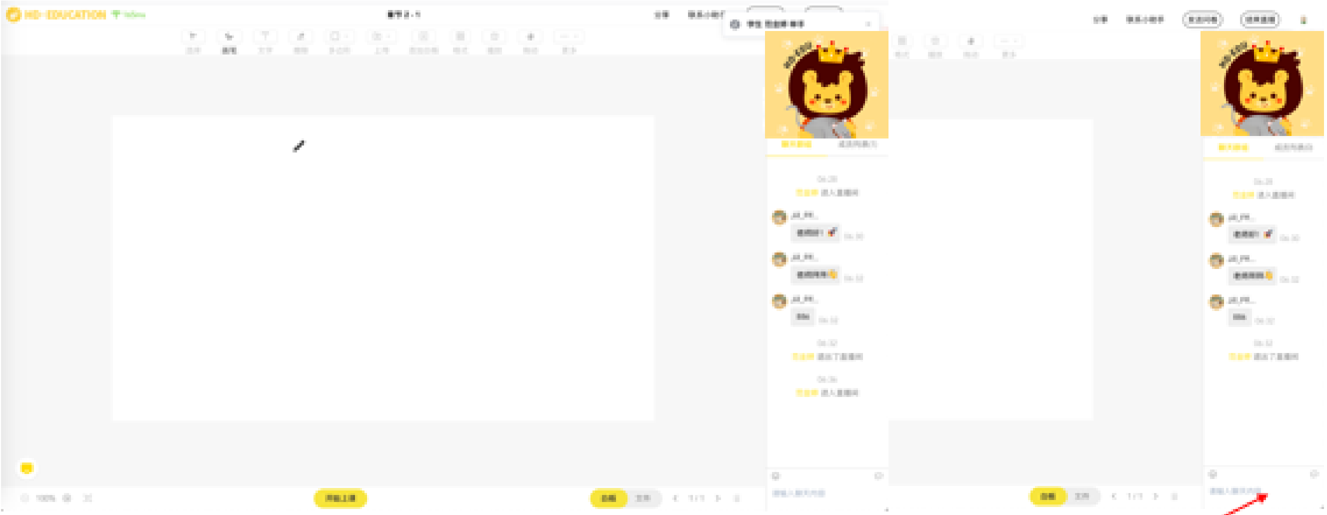
同学们有问题怎么办?

直播平台

互动方法

直播平台：举手+聊天室提问



点【参会者】再点【举手】，即可与老师实时互动！ 在此输入你想问的问题

问题被解答了还可以【手放下】

学科特点及学习方法

学科特点：

1. Lecture例题与Tutorial Question难度差异较大，切忌只做lecture例题
2. Lecture例题与Class Test考点不太一致，因为Lecture Slides上没有侧重点，需要系统地总结
3. Final题目与Tutorial上难度系数较大的题目类似但仍存在变化，主要陷阱比较多，计算量大

学习方法：

1. Lecture Slide上每一道例题要求掌握，前三周如果对知识点感到陌生，建议先花点时间复习大一数学课Notes
2. 强烈建议做一个错题集，将你觉得有意义或者是没有懂、容易错的题目收集到一起，方便考前的复习与回看
3. 及时跟上Lecture与Tutorial每周进度，避免考前突击，不然会很辛苦。

上节课作业回顾

内容回顾：Triple Integrals

本章节知识点

知识点1 Basis and Dimension (Coordinate Vector)

知识点2 Linear Map

知识点3 Past Paper 讲解

1.1.1 Definition of Coordinate Vector

重要程度：★★★
难易程度：★★★★★

$(V, +, \cdot, \mathbb{F})$ **vector space** $B = \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\} \subseteq V$ **basis**

$V = \operatorname{span} B$ $\mathbf{x} \in V \Rightarrow \mathbf{x} \in \operatorname{span} B \Rightarrow \mathbf{x} = \lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2 + \ldots + \lambda_n \mathbf{e}_n$

$\lambda_1, \lambda_2, \ldots, \lambda_n \in \mathbb{F}$

$$[\mathbf{x}]_B = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \vdots \\ \lambda_n \end{pmatrix}$$

coordinate vector of x with respect to B

Theorem: coordinate vector is unique

Proof:

$$[\mathbf{x}]_B = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_n \end{pmatrix} \Rightarrow \mathbf{x} = \mu_1 \mathbf{e}_1 + \mu_2 \mathbf{e}_2 + \ldots + \mu_n \mathbf{e}_n$$

by lin idep

$$\lambda_1 \mathbf{e}_1 + \lambda_2 \mathbf{e}_2 + \ldots + \lambda_n \mathbf{e}_n = \mu_1 \mathbf{e}_1 + \mu_2 \mathbf{e}_2 + \ldots + \mu_n \mathbf{e}_n$$

$$\Rightarrow (\lambda_1 - \mu_1)\mathbf{e}_1 + (\lambda_2 - \mu_2)\mathbf{e}_2 + \ldots + (\lambda_n - \mu_n)\mathbf{e}_n = \mathbf{0}$$

by the fact that B is linear independent

$$\Rightarrow \lambda_1 - \mu_1 = \lambda_2 - \mu_2 = \ldots = \lambda_n - \mu_n = 0$$

$$\Rightarrow \lambda_1 = \mu_1, \quad \lambda_2 = \mu_2, \quad \ldots, \quad \lambda_n = \mu_n$$

1.1.2 Finding Coordinate Vector

重要程度：★★★
难易程度：★★★★★

$\mathbf{e}_1 = (1, 2)$

$\mathbf{e}_2 = (-2, -1)$

$\mathbf{x} = (8, 13)$

$\{\mathbf{e}_1, \mathbf{e}_2\}$ **basis**

1.1.2 Finding Coordinate Vector

重要程度：★★★
难易程度：★★★★★

$$\mathbf{e}_1 = (2, -3, 0)$$

$$\mathbf{e}_2 = (-2, 3, 5)$$

$$\mathbf{x} = (2, -3, 1)$$

$$W = \operatorname{span}\{\mathbf{e}_1, \mathbf{e}_2\} \subseteq \mathbb{R}^3$$

$$\mathbf{x} \overset{?}{\in} W$$

1.1.2 Finding Coordinate Vector

重要程度：★★★
难易程度：★★★★★

$p_1 = 1 + 2x$

$p_2 = -2 - x$

$q = 8 + 13x$

$p_1, p_2, q \in P_1(\mathbb{R})$

2.1.1 Def. of Linear Map

$(V,+,\cdot,\mathbb{F})$ $(W,+,\cdot,\mathbb{F})$ **two vector spaces**

domain

$T: V \to W$ **linear map iff**

co-domain

(1) $T(\mathbf{x}+\mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y}) \quad \forall \mathbf{x},\mathbf{y} \in V$

(2) $T(\lambda \mathbf{x}) = \lambda T(\mathbf{x}) \quad \forall \lambda \in \mathbb{F}, \quad \forall \mathbf{x} \in V$

(1+2) $T(\lambda \mathbf{x} + \mu \mathbf{y}) = \lambda T(\mathbf{x}) + \mu T(\mathbf{y}) \quad \forall \lambda,\mu \in \mathbb{F}, \quad \forall \mathbf{x},\mathbf{y} \in V$

- 逆时针旋转90度：

$$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

- 逆时针旋转θ度[15]：

$$A = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

- 针对x轴反射：

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

- 在所有方向上放大2倍：

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

- 水平错切：

$$A = \begin{bmatrix} 1 & m \\ 0 & 1 \end{bmatrix}$$

- 挤压：

$$A = \begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}$$

- 向y轴投影：

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

2.1.2 Proof of Linear Map

重要程度：★★★
难易程度：★★★★★

$T : \mathbb{R}^2 \to \mathbb{R}^2$

$T(x_1, x_2) = (2x_1, x_1 - x_2)$

$T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$

$T(p)(x) = p(x-2)$

$T(p(x)) = p(x-2)$

2.1.2 Proof of Linear Map

重要程度：⭐⭐⭐
难易程度：⭐⭐⭐⭐⭐

$T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ $\quad T(p)(x) = (a+c) - (c+b)x + (a+b+c)x^2$ where $p(x) = a + bx + cx^2$

$$T\left(a + bx + cx^2\right) = (a+c) - (c+b)x + (a+b+c)x^2$$

2.1.2 Proof of Linear Map

重要程度：★★★
难易程度：★★★★★

Are the following functions linear? Prove your answers.

a) $T:\mathbb{R}^2 \to \mathbb{R}^2$, $T(x_1, x_2) = (2x_1, x_1 - x_2)$;

b) $T:\mathbb{R}^2 \to \mathbb{R}^2$, $T(x_1, x_2) = (x_1 + 1, x_2)$;

c) $T:\mathbb{P}_2 \to \mathbb{P}_2$, $T(a + bt + ct^2) = (a + c) - (c + b)t + (a + b + c)t^2$;

d) $T:\mathbb{P} \to \mathbb{P}$, $T(p) = p + 2p' + 3p''$, where $\mathbb{P}$ is the vector space of polynomials (of any degree);

e) $T:\mathbb{P}_2 \to \mathbb{P}_2$, $T(p(t)) = p(t - 2)$;

f) $\det : M_{2,2} \to \mathbb{R}$, $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$;

g) $F : M_{2,2} \to M_{2,2}$, $F(X) = X^T$, the transpose of X.

2.1.2 Proof of Linear Map

重要程度：★★★
难易程度：★★★★★

Are the following functions linear? Prove your answers.

a) $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(x_1,x_2)=(2x_1,x_1-x_2)$;

b) $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(x_1,x_2)=(x_1+1,x_2)$;

c) $T:\mathbb{P}_2\to\mathbb{P}_2$, $T(a+bt+ct^2)=(a+c)-(c+b)t+(a+b+c)t^2$;

d) $T:\mathbb{P}\to\mathbb{P}$, $T(p)=p+2p'+3p''$, where $\mathbb{P}$ is the vector space of polynomials (of any degree);

e) $T:\mathbb{P}_2\to\mathbb{P}_2$, $T(p(t))=p(t-2)$;

f) $\det : M_{2,2}\to\mathbb{R}$, $\det\begin{pmatrix} a & b \\ c & d\end{pmatrix}=ad-bc$;

g) $F:M_{2,2}\to M_{2,2}$, $F(X)=X^T$, the transpose of X.

2.1.2 Proof of Linear Map

重要程度：★★★
难易程度：★★★★★

Are the following functions linear? Prove your answers.

a) $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(x_1,x_2)=(2x_1,x_1-x_2)$;

b) $T:\mathbb{R}^2\to\mathbb{R}^2$, $T(x_1,x_2)=(x_1+1,x_2)$;

c) $T:\mathbb{P}_2\to\mathbb{P}_2$, $T(a+bt+ct^2)=(a+c)-(c+b)t+(a+b+c)t^2$;

d) $T:\mathbb{P}\to\mathbb{P}$, $T(p)=p+2p'+3p''$, where $\mathbb{P}$ is the vector space of polynomials (of any degree);

e) $T:\mathbb{P}_2\to\mathbb{P}_2$, $T(p(t))=p(t-2)$;

f) $\det : M_{2,2}\to\mathbb{R}$, $\det\begin{pmatrix} a & b\\ c & d\end{pmatrix}=ad-bc$;

g) $F:M_{2,2}\to M_{2,2}$, $F(X)=X^T$, the transpose of X.

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

$T: V \to W$ linear map

$B = \{\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n\} \subseteq V$ basis

Theorem (linear map and basis):

linear map is completely and uniquely defined by it values on basis vectors

$$[\mathbf{x}]_B = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad \Rightarrow \quad T(\mathbf{x}) = x_1 T(\mathbf{e}_1) + x_2 T(\mathbf{e}_2) + \ldots + x_n T(\mathbf{e}_n)$$

Proof:

$$[\mathbf{x}]_B = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \quad \Rightarrow \quad \mathbf{x} = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \ldots + x_n \mathbf{e}_n$$

$$\Rightarrow \quad T(\mathbf{x}) = T\left(x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \ldots + x_n \mathbf{e}_n\right)$$

by linearity $\quad = x_1 T(\mathbf{e}_1) + x_2 T(\mathbf{e}_2) + \ldots + x_n T(\mathbf{e}_n)$

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

Example 1 $T : \mathbb{R}^2 \to \mathbb{R}^2$ **linear map**

$$T(1,0) = (3,-2) \quad \text{and} \quad T(0,1) = (4,7)$$

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2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

- $T : \mathbb{R}^2 \to \mathbb{R}^2$ **linear map**

$$T(4,7) = (3,-1) \quad \text{and} \quad T(3,5) = (-2,7)$$

$$T\left(x_1, x_2\right) = ?$$

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

- $T:\mathbb{R}^2\to\mathbb{R}^2$ **linear map**

$$T(4,7)=(3,-1) \quad \text{and} \quad T(3,5)=(-2,7)$$

$$T\left(x_1,x_2\right)=?$$

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Find a formula for $T(x_1, x_2)$, given that

a) $T(1,0) = (3,4)$ and $T(0,1) = (4,9)$;

b) $T(4,7) = (3,4)$ and $T(3,5) = (4,9)$;

c) $T(5,7) = (3,4)$ and $T(2,7) = (2,5)$.

2.2 Linear Map and Basis

重要程度：⭐⭐⭐
难易程度：⭐⭐⭐⭐⭐

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Find a formula for $T(x_1, x_2)$, given that

a) $T(1, 0) = (3, 4)$ and $T(0, 1) = (4, 9)$;

b) $T(4, 7) = (3, 4)$ and $T(3, 5) = (4, 9)$;

c) $T(5, 7) = (3, 4)$ and $T(2, 7) = (2, 5)$.

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

3. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Find a formula for $T(x_1, x_2)$, given that

a) $T(1,0) = (3,4)$ and $T(0,1) = (4,9)$;

b) $T(4,7) = (3,4)$ and $T(3,5) = (4,9)$;

c) $T(5,7) = (3,4)$ and $T(2,7) = (2,5)$.

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

$$T : P_1(\mathbb{R}) \to P_1(\mathbb{R}) \qquad T(p_1) = e_1 \quad \text{and} \quad T(p_2) = e_2$$

where $\quad p_1(x) = 5 + 3x \quad \text{and} \quad p_2(x) = 2 + 7x$

$$e_1(x) = 1 \quad \text{and} \quad e_2(x) = x$$

Find $\quad T(q), \quad q = 1 + x$

2.2 Linear Map and Basis

重要程度：★★★
难易程度：★★★★★

$$T : P_1(\mathbb{R}) \to P_1(\mathbb{R})$$

$$T(e_1)(x) = 3 + 4x \quad \text{and} \quad T(e_2)(x) = 4 + 9x$$

standard basis

$$e_1(x) = 1 \quad \text{and} \quad e_2(x) = x$$

Find

$$T(q), \quad q(x) = -1 + x$$

2.3 Matrix of Linear Map

重要程度：★★★
难易程度：★★★★★

Matrix of Linear Map的实质：Linear Map and Basis的延伸

问题：Find matrix of linear map w.r.t basis A to basis B

case I: standard to standard
case II: standard to non-standard
case III: non-standard to standard
case IV: non-standard to non-standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target
step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$
step 3: matrix multiplication

2.3 Matrix of Linear Map

重要程度：★★★
难易程度：★★★★★

case I: standard to standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

$T(x_1, x_2) = (3x_1, x_1 + 2x_2)$ $\quad T: \mathbb{R}^2 \to \mathbb{R}^2$ $\quad S = \{\mathbf{e}_1, \mathbf{e}_2\}$

2.3 Matrix of Linear Map

重要程度：★★★
难易程度：★★★★★

case II: standard to non-standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

standard basis

$T(x_1, x_2) = (3x_1, x_1 + 2x_2)$ $\quad T : \mathbb{R}^2 \to \mathbb{R}^2$ $\quad B = \{\mathbf{e}_1, \mathbf{e}_2\}$ $\quad C = \left\{ \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \end{pmatrix} \right\} = \{\mathbf{w}_1, \mathbf{w}_2\}$

2.3 Matrix of Linear Map

重要程度：⭐⭐⭐
难易程度：⭐⭐⭐⭐⭐

case II: standard to non-standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

step 3: matrix multiplication

e.g. Given that $T: R^2 \to R^2$ has matrix $A = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ w.r.t the standard basis$\{(1,0), (0,1)\}$, find the matrix of T w.r.t the basis $\{(1,3), (3,7)\}$

2.3 Matrix of Linear Map

重要程度：★★★
难易程度：★★★★★

case III: non-standard to standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

step 3: matrix multiplication

$T(x_1, x_2) = (3x_1, x_1 + 2x_2)$ $T: \mathbb{R}^2 \to \mathbb{R}^2$ $B = \left\{ \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \end{pmatrix} \right\} = \{\mathbf{v}_1, \mathbf{v}_2\}$ $C = \{\mathbf{e}_1, \mathbf{e}_2\}$

2.3 Matrix of Linear Map

重要程度：★★★
难易程度：★★★★★

case III: non-standard to standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

step 3: matrix multiplication

If $T : \mathbb{R}^2 \to \mathbb{R}^2$ has matrix

$$A = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$$

with respect to the basis $B = \{(1,3),(3,7)\}$ of $\mathbb{R}^2$, find the matrix of T with respect to the standard basis.

2.3 Matrix of Linear Map

重要程度：★★★
难易程度：★★★★★

case IV: non-standard to non-standard

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

step 3: matrix multiplication

Example 4

$$T(x_1, x_2) = (3x_1, x_1 + 2x_2) \qquad T : \mathbb{R}^2 \to \mathbb{R}^2 \qquad B = C = \left\{ \begin{pmatrix} 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 6 \end{pmatrix} \right\} = \{\mathbf{u}_1, \mathbf{u}_2\}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \lambda_1 \mathbf{u}_1 + \lambda_2 \mathbf{u}_2 \qquad \Rightarrow \qquad T(x_1, x_2) = \lambda_1 T(\mathbf{u}_1) + \lambda_2 T(\mathbf{u}_2)$$

$$T(\mathbf{u}_1) = T(1,5) = \begin{pmatrix} 3 \\ 11 \end{pmatrix} = a_{11}\mathbf{u}_1 + a_{21}\mathbf{u}_2$$

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in M_{2,2}(\mathbb{R})$$

$$T(\mathbf{u}_2) = T(1,6) = \begin{pmatrix} 3 \\ 13 \end{pmatrix} = a_{12}\mathbf{u}_1 + a_{22}\mathbf{u}_2$$

$$U = (\mathbf{u}_1 \quad \mathbf{u}_2) \qquad U = \begin{pmatrix} 1 & 1 \\ 5 & 6 \end{pmatrix} \quad \Rightarrow \quad U^{-1} = \ldots = \begin{pmatrix} 6 & -1 \\ -5 & 1 \end{pmatrix}$$

$$A = U^{-1} \cdot (T(\mathbf{u}_1) \quad T(\mathbf{u}_2)) = \begin{pmatrix} 6 & -1 \\ -5 & 1 \end{pmatrix} \begin{pmatrix} 3 & 3 \\ 11 & 13 \end{pmatrix} = \begin{pmatrix} 7 & 5 \\ -4 & -2 \end{pmatrix}$$

Let $\mathbf{v}_1 = (1, -2, 0)$, $\mathbf{v}_2 = (0, -1, 1)$, $\mathbf{v}_3 = (1, 0, -1)$; suppose that $T : \mathbb{R}^3 \to \mathbb{R}^3$ is a linear transformation and that

$$T(\mathbf{v}_1) = 5\mathbf{v}_1 + \mathbf{v}_2 \ , \quad T(\mathbf{v}_2) = 5\mathbf{v}_2 + \mathbf{v}_3 \ , \quad T(\mathbf{v}_3) = 5\mathbf{v}_3 \ .$$

a) Write down the matrix of T with respect to the basis consisting of the vectors $\mathbf{v}_3$, $\mathbf{v}_2$, $\mathbf{v}_1$, in that order.

重要程度：★★★
难易程度：★★★★★

2.3 Matrix of Linear Map OTH.

重要程度：★★★
难易程度：★★★★★

方法：step 1: 根据题意写出 $T(A) = AM$ and $T(B) = BT_1$, where M indicates the map under basis A, note that A, M, B is given, and T_1 is the target

step 2: $T_1 = B^{-1}T(B) = B^{-1}T(AA^{-1}B) = B^{-1}AMA^{-1}B$

step 3: matrix multiplication

A function $T : \mathbb{P}_2 \to \mathbb{P}_2$ is defined by $T(p(t)) = tp'(t)$.

a) Show that T is linear.

b) Find the matrix of T with respect to the basis $B = \{1, 1 + t, t^2\}$ of $\mathbb{P}_2$.

2.4 Matrix Multiplication Linear Map

重要程度：

难易程度：

$A \in M_{mn}(\mathbb{R})$ $\quad T_A : \mathbb{R}^n \to \mathbb{R}^m$ $\quad T_A(\mathbf{x}) = A \cdot \mathbf{x}, \quad \forall \mathbf{x} \in \mathbb{R}^n$

Theorem (matrix multiplication linear map): T_A **is a linear map**

Proof by (1+2)

by distributive and associative props of matrix multiplication

$$T_A(\lambda\mathbf{x} + \mu\mathbf{y}) = A\left(\lambda\mathbf{x} + \mu\mathbf{y}\right) = \lambda(A\mathbf{x}) + \mu(A\mathbf{y}) = \lambda T_A(\mathbf{x}) + \mu T_A(\mathbf{y})$$

Example $\quad T : \mathbb{R}^2 \to \mathbb{R}^2 \qquad T(x_1, x_2) = (2x_1, x_1 - x_2)$

$$(2x_1, x_1 - x_2) = \begin{pmatrix} 2x_1 \\ x_1 - x_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \end{pmatrix} \in M_{2,2}(\mathbb{R}) \qquad T(x_1, x_2) = T_A(\mathbf{x}), \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

by Matrix Multiplication Linear Map Theorem the map T_A **is linear**

2.5 Vector Subspaces associated with Linear Maps

重要程度：☆☆☆
难易程度：☆☆☆☆☆

$T : V \to W$ linear map

$$\operatorname{null} T = \{\mathbf{x} \in V : \quad T(\mathbf{x}) = \mathbf{0}\} \subseteq V$$

Theorem (null space of linear map): null space is a subspace of V

Proof (by Subspace Theorem):

(1) $\mathbf{0} \in \operatorname{null} T \qquad \mathbf{x} \in V \qquad T(\mathbf{0}) = T(0 \cdot \mathbf{x}) = 0 \cdot T(\mathbf{x}) = \mathbf{0}$

(2) $\mathbf{x}, \mathbf{y} \in \operatorname{null} T \quad \Rightarrow \quad T(\mathbf{x}) = T(\mathbf{y}) = \mathbf{0}$

$$T(\mathbf{x} + \mathbf{y}) = T(\mathbf{x}) + T(\mathbf{y}) = \mathbf{0} + \mathbf{0} = \mathbf{0} \quad \Rightarrow \quad \mathbf{x} + \mathbf{y} \in \operatorname{null} T$$

(3) $\mathbf{x} \in \operatorname{null} T \quad \Rightarrow \quad T(\mathbf{x}) = \mathbf{0}$

$$T(\lambda \mathbf{x}) = \lambda T(\mathbf{x}) = \lambda \cdot \mathbf{0} = \mathbf{0} \quad \Rightarrow \quad \lambda \mathbf{x} \in \operatorname{null} T, \quad \forall \lambda \in \mathbb{F}$$

2.5 Image of Linear Map

重要程度：★★★
难易程度：★★★★★

Example 1

$$V = \left\{ p \in P_2(\mathbb{R}) : \quad p(2) = 0 \quad \text{and} \quad p(-1) = 0 \right\} \subseteq P_2(\mathbb{R})$$ **show that V is subspace**

2.5 Image of Linear Map

重要程度：★★★
难易程度：★★★★★

$T: V \to W$

linear map

$$\operatorname{im} T = \left\{ \mathbf{x} \in W : \quad T(\mathbf{a}) = \mathbf{x}, \quad \mathbf{a} \in V \right\} \subseteq W$$

Theorem (image of linear map): image of linear map is a subspace of W

Proof (by Subspace Theorem):

see "Nullspace of Linear Map" for details

(1) $T(\mathbf{0}) = \ldots = \mathbf{0} \quad \Rightarrow \quad \mathbf{0} \in \operatorname{im} T$

(2) $\mathbf{x}, \mathbf{y} \in \operatorname{im} T \quad \Rightarrow \quad \mathbf{x} = T(\mathbf{a}) \quad \text{and} \quad \mathbf{y} = T(\mathbf{b}) \quad \mathbf{a}, \mathbf{b} \in V$

$$\mathbf{x} + \mathbf{y} = T(\mathbf{a}) + T(\mathbf{b}) = T(\mathbf{a} + \mathbf{b}) \quad \Rightarrow \quad \mathbf{x} + \mathbf{y} \in \operatorname{im} T$$

(3) $\mathbf{x} \in \operatorname{im} T \quad \Rightarrow \quad \mathbf{x} = T(\mathbf{a}), \quad \mathbf{a} \in V$

$$\lambda \mathbf{x} = \lambda T(\mathbf{a}) = T(\lambda \mathbf{a}) \quad \Rightarrow \quad \lambda \mathbf{x} \in \operatorname{im} T$$

2.5 Image of Linear Map

重要程度：★★★
难易程度：★★★★★

What they mean by the transformation T is the transformation which is induced by multiplication by A. You can verify that matrix multiplication is in fact a linear mapping, and in our particular case we have the linear mapping $T: \mathbf{x} \mapsto A\mathbf{x}$.

The image is then defined as the set of all outputs of the linear mapping. That is

$$\operatorname{Im}(T) = \{\mathbf{y} \in \mathbb{R}^4 \mid \mathbf{y} = A\mathbf{x} \text{ such that } \mathbf{x} \in \mathbb{R}^5\}$$

If you play around with the mapping a little bit then you should find that the image is in fact a very familiar subspace associated with the matrix A (take a look at how the mapping T acts on the standard basis).

The kernel is correspondingly defined as the set of all inputs which are taken to zero.

$$\ker(T) = \{\mathbf{x} \in \mathbb{R}^5 \mid A\mathbf{x} = \mathbf{0}\}$$

Again, there is a familiar subspace of the matrix A associated with the kernel, look carefully at the definition and you should be able to figure out what it is.

2.5 Image of Linear Map

重要程度：★★★
难易程度：★★★★★

10. Find bases for the kernels and images of the following linear transformations. Hence obtain the nullity and rank of each transformation.

a) $T : \mathbb{R}^4 \to \mathbb{R}^3$, $T(\mathbf{x}) = \begin{pmatrix} 1 & -1 & 1 & 2 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 6 \end{pmatrix} \mathbf{x}$;

b) $T : \mathbb{R}^4 \to \mathbb{R}^3$, $T(\mathbf{x}) = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 5 & 9 & -8 & 1 \\ 3 & 8 & 3 & -2 \end{pmatrix} \mathbf{x}$;

c) $T : \mathbb{P}_4 \to \mathbb{R}^2$, $T(p) = \big(p(0), p'(0)\big)$;

d) $F : M_{3,3} \to M_{3,3}$, $F(X) = X - X^T$ where X^T denotes the transpose of X.

2.5 Image of Linear Map

重要程度：⭐⭐⭐
难易程度：⭐⭐⭐⭐⭐

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2.5 Image of Linear Map

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重难点总结

重难点总结

Coordinate Vector, Linear Map

HD·EDUCATION

下节课预告

下节课预告

WEEK 4： Scalar Product

课程结束后，如果您对课程或者服务的任何建议和意见

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