UNIVERSITÀ DELLA CALABRIA

“Industrial Automation module”

“Design of a CNC machine simulation based on a reference generator and a feedback control system.”

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Introduction:

This project focuses on the design and implementation of a robust digital control system for DC motors within a CNC (Computer Numerical Control) machine, specifically a 2D laser cutting machine. The primary goal is to achieve precise and reliable control of the motor's position and velocity, crucial for accurate and efficient cutting operations on various materials, including wood.

The control system utilizes a cascade structure integrating Proportional (P) and Proportional-Integral (PI) controllers across three nested loops: position control (outer loop), velocity control (middle loop), and current control (inner loop). The position loop uses a proportional controller to convert position errors into velocity references. The velocity loop employs a PI controller to process these velocity errors, generating current references for the innermost loop. The current loop, also using a PI controller, regulates the motor current to ensure the appropriate voltage input.

To further enhance system performance, feedforward control is incorporated. This includes velocity and acceleration feedforward terms, which anticipate the motor's operational needs based on desired motion profiles, thereby improving response times and accuracy. The control design also compensates for disturbances and uncertainties in motor parameters, ensuring stability and robustness.

The system's effectiveness was validated through extensive simulations using MATLAB and Simulink. The results demonstrate the controller's ability to maintain a maximum position error of less than 2mm, even with a 10% uncertainty in motor parameters JJ (moment of inertia) and BB (damping coefficient). The simulations also confirm the system's stability and precise trajectory following for complex shapes, such as squares and circles, cut from a wooden slab.

This project underscores the significance of advanced control strategies in industrial automation, highlighting the potential of digital control techniques in enhancing the precision and efficiency of CNC machining operations. The successful implementation and validation of the control system contribute to the broader field of industrial automation, offering insights into effective control system design for high-precision applications.

A picture from a 2D CNC laser cutting machine:

The movement of the two axes (X and Y) of a 2D laser cutting machine is facilitated by two identical DC motors. These motors convert rotational motion into translational motion using a suitable mechanical setup. The motor specifications are as follows:

- Armature resistance ($RaRa$): 1Ω1Ω
- Armature inductance ($LaLa$): 1.0 mH1.0 mH
- Combined moment of inertia and damping coefficient (motor + load): J=0.1J=0.1, B=0.029B=0.029
- Electromotive force constant ($KeKe$) and torque constant ($KtKt$): 0.6 Volts sec/rad0.6 Volts sec/rad, 0.6 Nm/A0.6 Nm/A
- Maximum power: 30 KW30 KW
- Maximum armature current: 20 A20 A

- Maximum voltage: 1400 V1400 V

The mechanism that converts rotational motion to translational motion has a conversion factor of 1 m/rev=5001 m/rev=500.

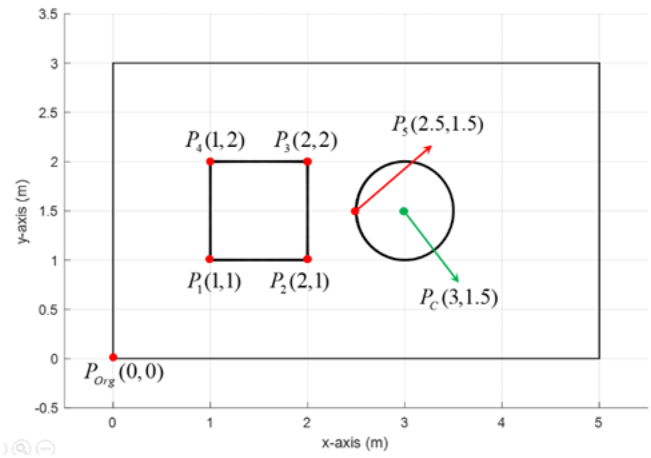
Additionally, for each motor, the system can measure armature current, rotational speed (rad/sec), and angular position (rad).

Project Objectives:

1. **Reference Signal Development**:
    - Create reference signals to enable the cutting machine to accurately cut a square and a circle from a rectangular slab measuring 5 meters in length and 3 meters in height.
    - The student is responsible for determining the size, placement, and orientation of the shapes.
    - The initial rest position of the machine is set at (0,0).
2. **Feedback Controller Design**:
    - Develop a feedback controller for each motor that ensures the position error does not exceed 2 mm.
    - This controller must maintain accuracy even when there is a 10% uncertainty in the parameters JJ (moment of inertia) and BB (damping coefficient) relative to their nominal values.

Chapter One: the design of cut

The design process begins with determining the main points on a rectangular piece of wood. Initially, the laser cutting head starts at its rest position, (0,0). The first movement sequence can be described as moving from (0,0) to (1,1) without activating the laser. Once at point (1,1), the laser is activated to cut a square with 1-meter sides, moving sequentially through points (1,1) → (2,1) → (2,2) → (1,2) → (1,1). After completing the square, the laser is turned off, and the head moves to point (2.5,1.5). The laser is then activated to cut a circle, starting and ending at point (2.5,1.5). Finally, the laser is turned off, and the head returns to the rest position (0,0).



The trajectory is determined using polynomial interpolation between adjacent points, employing a fifth-degree polynomial to generate the reference. This method estimates values between known data points by drawing straight lines and considering the function as the combination of these lines. The polynomial is defined as:

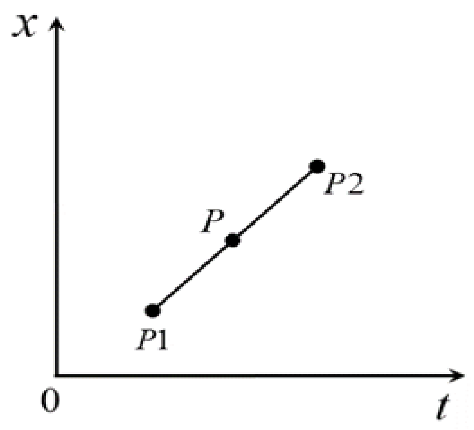
$$x(t) = at^5 + bt^4 + ct^3 + dt^2 + et + f$$

Considering a generic point P between two points $P1$ and $P2$ with coordinates ($X1$,$Y1$) and ($X2$,$Y2$), respectively, the point P can be calculated as:

$P{=}P1{+}\lambda(P2{-}P1)$ for $0{\le}\lambda{\le}1$.

By varying λ, the value of point P moves along the line formed by $P1$ and $P2$.

Generic Point P located between the points $P1$ and $P2$:

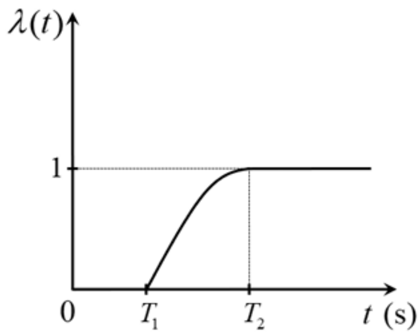


The time to travel from $P1$ to $P2$ is obtained by making λ a function of time t:

$\lambda(t)=t-T1/T2-T1$

The time axis can be connected with the dimensionless parameter σ through the relation:

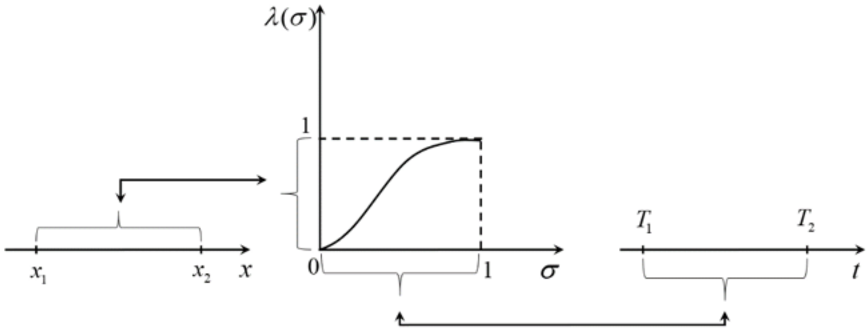
$\sigma=t-T1/T2-T1$



The time T1 corresponds to the moment in which we are at the P1 while in T2 corresponds to the moment in which we arrive at P2. The graph presents a smooth shape because a polynomial of degree 5 is used.

It can be shown that by taking into considerations of the relationships between $\lambda(\sigma)$ and position ($x1$,$x2$), as well as σ and time ($T1$,$T2$) and implementing these analytically, based on the polynomial and time relationships, we derive:

$$\lambda(\sigma)=a\sigma5+b\sigma4+c\sigma3+d\sigma2+e\sigma+f$$



To calculate the coordinates of all points between $P1$ and $P2$:

$$X = X1 + \lambda(X2 - X1)$$

$$Y = Y1 + \lambda(Y2 - Y1)$$

The polynomial coefficients a,b,c,d,e are determined by evaluating the position, velocity, and acceleration polynomials at specific points:

$$\lambda(0)=0,\ \lambda(1)=1$$

$$\lambda'(0)=0,\ \lambda'(1)=0$$

$$\lambda''(0)=0,\ \lambda''(1)=0$$

Solving this system of linear equations, we get:

$$a=6,\ b=-15,\ c=10,\ d=0,\ e=0,\ f=0$$

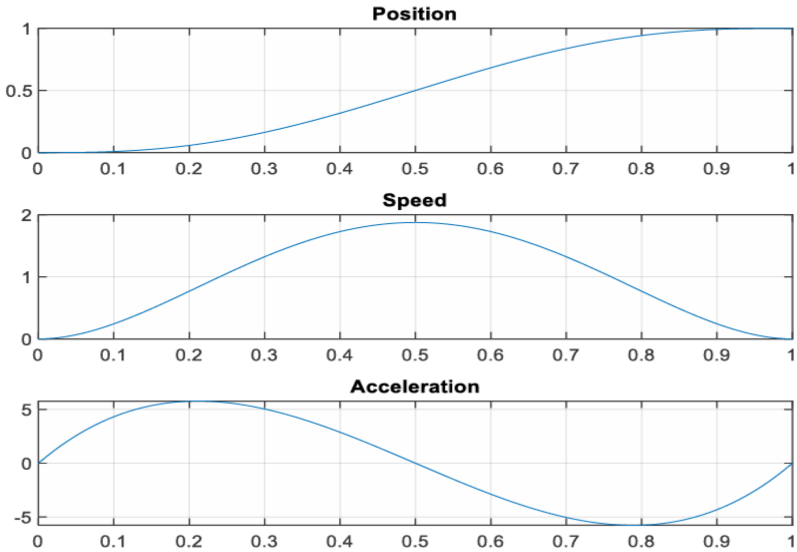
Thus, the polynomial for $\lambda(\sigma)$ is:

$$\lambda(\sigma) = 6\sigma^5 - 15\sigma^4 + 10\sigma^3$$
$$\lambda'(\sigma) = 30\sigma^4 - 60\sigma^3 + 30\sigma^2$$
$$\lambda''(\sigma) = 120\sigma^3 - 180\sigma^2 + 60\sigma$$

The same method is used to generate the circular trajectory with the parametric equations:

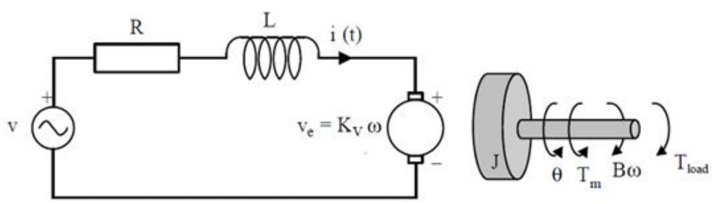
$$X = X_C + R\cos(\theta)$$
$$Y = Y_C + R\sin(\theta)$$
for $0 \leq \theta \leq 2\pi$.

In the picture below, we can see the position, velocity, and acceleration for the trajectory corresponding to the first movement of the cutting machine head from (0,0) to (1,1).



Chapter Two, DC motor system modeling:

To develop an effective control scheme, we begin with the mathematical model of the DC motor. The machine head's movement to the target cutting position relies on a motor with these characteristics for each axis. DC motors transform electrical energy into mechanical energy through rotary motion, which is produced by a magnetic field:



$L(L_a)$: armature inductance
$R(R_a)$: armature resistanse
J: load and armature inertia
$B_\omega(B)$: motor friction coefficient (e.g brushes)
θ: rotor position (rad)
$V(V_a)$: applied voltage (armature voltage)(v)
$i(t)(i_a)$: armature current
v_e: back emf (V)
T_m: generated torque (Nm)
T_{load}: Disturbance
$K_v(K_m)$: Torque coefficient and electromotive force

Electric model:

$$L_a \frac{di_a}{dt} = V_a - R_a i_a - K_m \omega$$

Mechanical model:

$$J \frac{d\omega}{dt} = K_m i_a - B\omega - T_{load}$$

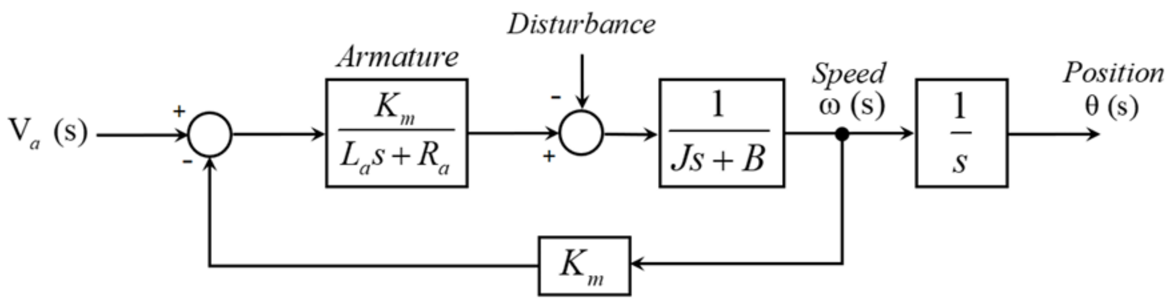
Where:

$$\frac{d\omega}{dt} = \omega$$

By taking these all into consideration and taking Laplace Transformation, we can get into the transfer function as:

$$G(s) = \frac{\theta(s)}{V_a} = \frac{K_m}{(L_a s + R_a)(Js + B) + K_m^2}$$

Then the block diagram of a permanent DC motor can be obtained:



The previously analyzed DC motor can be represented by two main components: (1) The electrical part, which takes voltage as input and produces torque as output; and (2) The mechanical part, which takes torque as input and produces angular velocity as output. The poles of the electrical and mechanical first-order subsystems can be calculated as:

$$P_e = -\frac{R_a}{L_a}$$

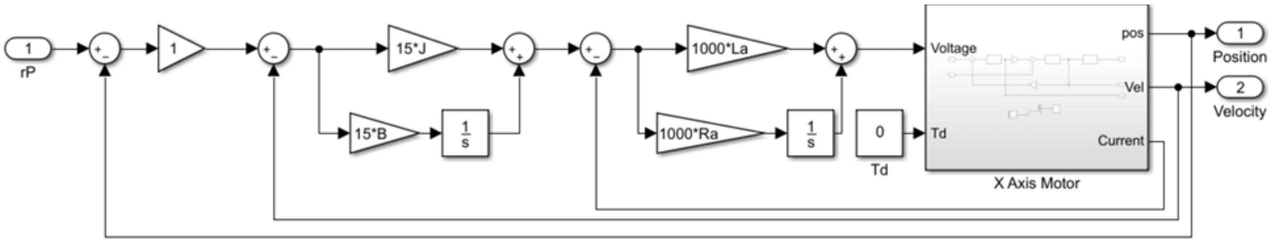
$$P_m = -\frac{B}{J}$$

Since all the poles have negative real parts, it can be proven that the systems is in asymptotically stability condition.

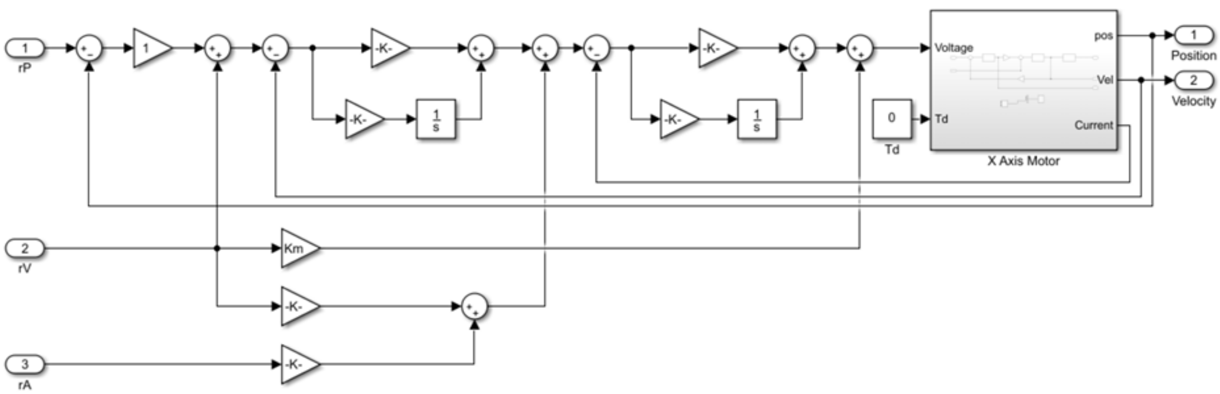
Chapter Three, The control scheme of DC Engine:

In this project, a cascade structure is used to compensate for the back electromotive force. The structure includes: (1) a P (proportional) regulator with unity gain for the position loop. (2) a PI (Proportional-Integral) regulator for the speed loop to compensate for disturbing torque, with Kp=15J and Ki=15B. (3) another PI regulator for the current loop with $Kp1$=1000$La1$ and $Ki1$=11000$Ra1$. The analysis is conducted from the outermost loop to the innermost loop (outside → inside), assuming ideal conditions for the inner loops. The control output of each outer loop serves as the reference input for the next inner loop. Figure 13 illustrates the implemented cascade structure, which is identical for both axes of the cutting machine.

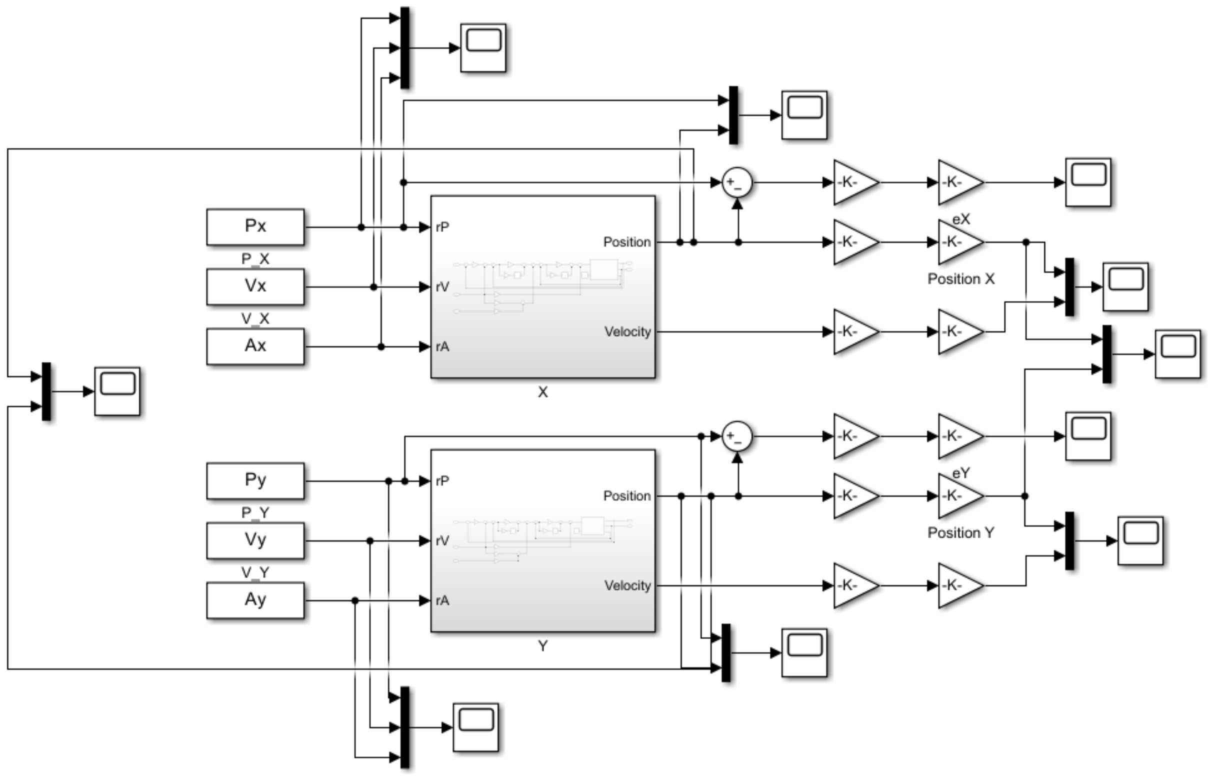
Here you can find the Cascade scheme implemented for the DC motor (X-axis):



To compensate for the Known Disturbances and to Reduce Steady-State Error we have added the feedforward gains to our model:

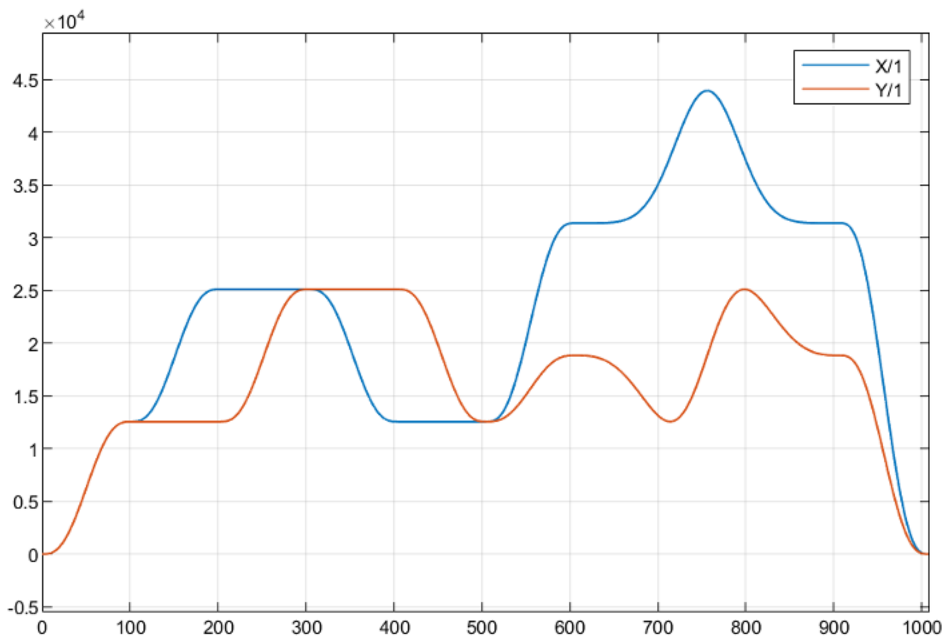


The general scheme of 2D CNC cutting machine can be shown as follow:



Chapter Four, Results from the plots:

The following image describes the behavior of the position along the X and Y axis:



The final trajectory result is depicted. The periods when the machine's laser is off are highlighted in green. The geometric shapes cut out by the laser are shown in black. The square has a side length of 1m, and the circle, with a radius of 0.5m, is centered at $PC(3,1.5)$.

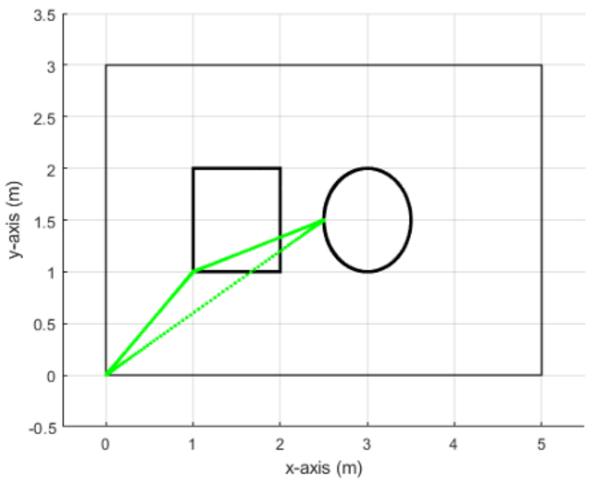
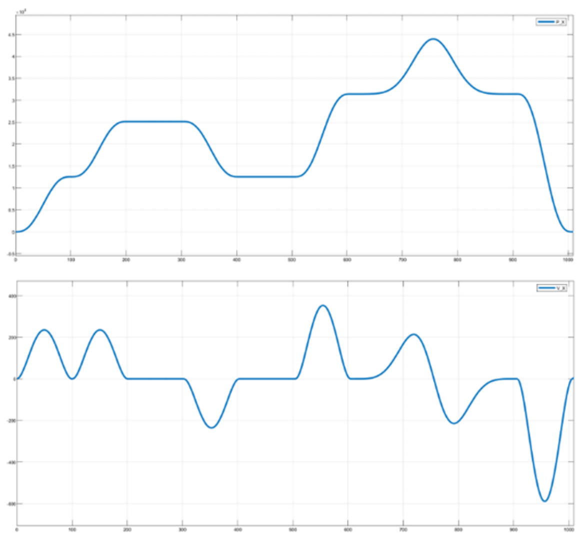
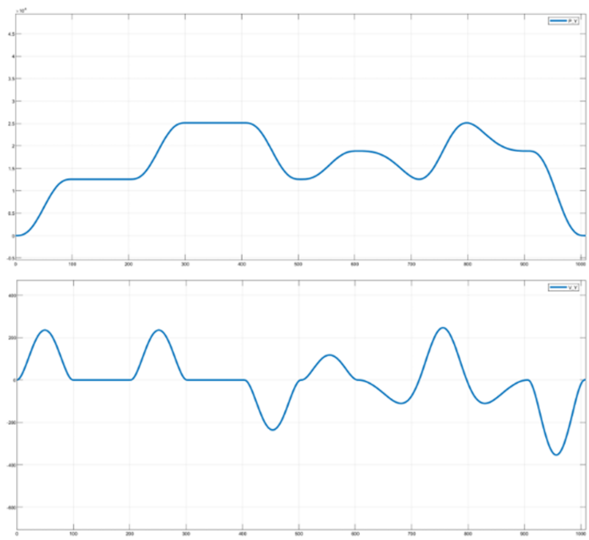


Figure (a) illustrates the relationship between the position and speed of the machine head along the X-axis, with position shown at the top and velocity at the bottom. Figure (b) presents similar results for the Y-axis. When the position graph's slopes are negative, the velocity values are also negative; when there is no displacement, the velocity is zero; and when the slopes are positive, the velocity values are positive. This is because velocity is the derivative of position.

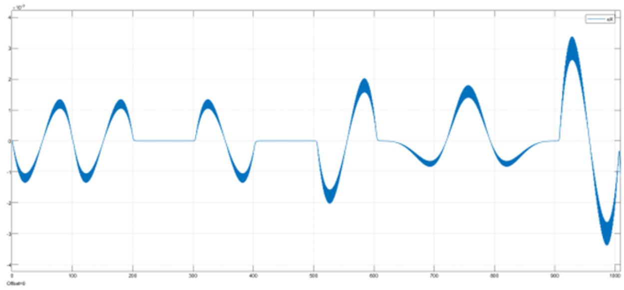


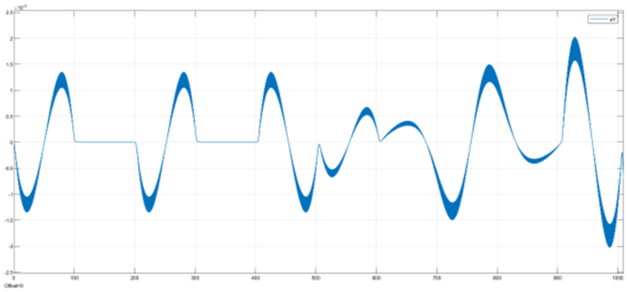
(a)



(b)

Finally, Figures below show the error in the position on the X-axis and the Y-axis.





Chapter Five, Conclusion:

In this project, a two-dimensional cutting machine was designed to cut a square with a side length of 1m and a circle with a radius of 0.5m from a 5m by 3m sheet of wood. Matlab R2023a software, along with SIMULINK, was utilized for this purpose. The operation of the reference generator was validated by plotting the position, speed, and acceleration of the cutting machine head along its trajectory. A functional schematic for the X-axis and Y-axis DC motors was implemented, and the system controller was designed using a cascade structure to compensate for the back electromotive force. The graphs in the final section demonstrate that: (1) The machine head starts from and returns to the rest position (0,0)Org after making the cuts; (2) There is a proper relationship between the position and velocity of the system's axes; (3) The feedback controller for each motor ensures a maximum position error of less than 2mm, even with a 10% uncertainty in parameters J and B relative to their nominal values.