KUMPULAN RUMUS MPS UTS SEMESTER 3

Politeknik Statistika STIS Angkatan 63

Simple Random Sampling	WR	WOR
All Posible Sample	N^n	C_n^N
Rata-rata	$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$	$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$
Varians	$s^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}[y_i - \bar{y}]^2$	$s^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}[y_i - \bar{y}]^2$
Varians rata-rata	$v(\bar{y}) = \frac{s^2}{n}$	$v(\bar{y}) = \frac{(N-n)}{N}\frac{s^2}{n}$
Total	$\hat{Y} = N\bar{y}$	
Varians Total	$v(\hat{Y}) = N^2[v(\bar{y})]$	
Proporsi	$p = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{a}{n}$ $y_i = 1\ atau\ 0, maka\ p = \frac{a}{n}$	
Total dengan Karakteristik	$\hat{Y} = Np$	
Varians Populasi	$v(p) = \frac{pq}{(n-1)}$	$v(p) = \frac{(N-n)}{N}\frac{pq}{(n-1)}$
Standar Error	$se(\hat{\theta}) = \sqrt{var(\hat{\theta})}$	$se(\hat{\theta}) = \sqrt{var(\hat{\theta})}$

Stratified Random Sampling	Strata ke-h	Populasi
Rata-rata	$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{h_i}$	$\bar{y}_{st} = \sum_{h=1}^{L} W_h\bar{y}_h$
Varians rata-rata	$v(\bar{y}) = [1-f_h]\frac{{s_h}^2}{n_h}$ $f_h = \frac{n_h}{N_h}$	$v[\bar{y}(st)] = \sum_{h=1}^{L} W(h)^2.v[\bar{y}(h)]$ $W_h = \frac{N_h}{N}$
Total	$\hat{Y} = N_h.\bar{y}_h$	$\hat{Y} = N.\bar{y}_{st} = \sum_{h=1}^{L}\hat{Y}_h$
Varians Total	$v(\hat{Y}_h) = N(h)^2.v(\bar{y}_h)$	$v(\hat{Y}_{st}) = N^2.v[\bar{y}_{st}] = \sum_{h=1}^{L} v(\hat{Y}_h)$
Proporsi	$p_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{h_i} = \frac{a_h}{n_h}$	$p_{st} = \sum_{h=1}^{L}\frac{N_h}{N}.p_h = \sum_{h=1}^{L} W_h.p_h$
Varians Proporsi	$v(p_h) = (1-f_h)\frac{p_h q_h}{(n_h-1)}$	$v(p_{st}) = \sum_{h=1}^{L} {W_h}^2.v(p_h)$
Standar Error	$se(\hat{\theta}) = \sqrt{var(\hat{\theta})}$	$se(\hat{\theta}) = \sqrt{var(\hat{\theta})}$
RSE	$RSE(\hat{\theta}) = \frac{se(\hat{\theta})}{\hat{\theta}}.100\%$	$RSE(\hat{\theta}) = \frac{se(\hat{\theta})}{\hat{\theta}}.100\%$
CI	$P(\hat{\theta} - d < \# < \hat{\theta} + d) = 1-\alpha$	$P(\hat{\theta} - d < \# < \hat{\theta} + d) = 1-\alpha$
Catatan Kaki :	$\# = nilai\ yg\ diestimasi(\bar{y}, \hat{Y})$ $\hat{\theta} =$ nilai estimator(μ, Y)	$d = se(\hat{\theta})Z_{\alpha/2}$

SIMPLE RANDOM SAMPLING (SAMPLE SIZE)

varians dari peneliti terdahulu diketahui (s^2)

WR	WOR
$n_0 = \frac{\left(Z_{\frac{\alpha}{2}}\right)^2.s^2}{d^2}$	$n = \frac{N\left(Z_{\frac{\alpha}{2}}\right)^2.s^2}{Nd^2 + \left(Z_{\frac{\alpha}{2}}\right)^2.s^2}$

Hubungan antara n dan n_0 $\quad n = \frac{n_0}{1+\frac{n_0}{N}}$

Presisi relative dan Koefisien Variasi

WR	WOR
$n_0 = \frac{\left(Z_{\frac{\alpha}{2}}\right)^2.C^2}{d'^2}$	$n = \frac{N\left(Z_{\frac{\alpha}{2}}\right)^2.C^2}{Nd'^2 + \left(Z_{\frac{\alpha}{2}}\right)^2.C^2}$;

$d' = \frac{d}{\bar{x}}$

$C = \frac{\hat{S}}{\bar{x}}$

Dengan Proporsi

WR	WOR
$n_0 = \frac{\left(Z_{\frac{\alpha}{2}}\right)^2.PQ}{d^2}$	$n = \frac{N\left(Z_{\frac{\alpha}{2}}\right)^2.PQ}{Nd^2 + \left(Z_{\frac{\alpha}{2}}\right)^2.PQ}$

STRATIFIED RANDOM SAMPLING

ALOKASI SAMA

$n = \frac{L\sum_{h=1}^{L} N_h^2 S_h^2}{N^2D^2 + \sum_{h=1}^{L} N_h S_h^2}$ $\quad dimana\ n_h = \frac{n}{L}$ $\quad dan\ D = \frac{d}{Z_{\frac{\alpha}{2}}}$

(WOR)	(WR)
$v(\bar{y}st) = \left(\frac{L}{n}\sum_{h=1}^{L} W_h^2(1-f_h)s_h^2\right)$	$v(\bar{y}st) = \left(\frac{L}{n}\sum_{h=1}^{L} W_h^2 s_h^2\right)$

ALOKASI SEBANDING

$n = \frac{N\sum_{h=1}^{L} N_h S_h^2}{N^2D^2 + \sum_{h=1}^{L} N_h S_h^2}$ $\quad dimana\ n_h = \frac{N_h}{N}.n$ $\quad dan\ D = \frac{d}{Z_{\frac{\alpha}{2}}}$

(WOR)	(WR)
$v(\bar{y}st) = \left(\frac{(1-f)}{n}\sum_{h=1}^{L} W_h s_h^2\right)$	$v(\bar{y}st) = \left(\frac{1}{n}\sum_{h=1}^{L} W_h s_h^2\right)$

ALOKASI NEYMAN

$n = \frac{(\sum_{h=1}^{L} N_h S_h)^2}{N^2D^2 + \sum_{h=1}^{L} N_h S_h^2}$ $\quad dimana\ n_h = \frac{N_h S_h}{\sum_{h=1}^{L} N_h S_h}.n$

(WOR)	(WR)
$v(\bar{y}st) = \left(\frac{1}{N^2 n}\sum_{h=1}^{L}(1-f_h)N_h S_h\right).(\sum_{h=1}^{L} N_h S_h)$	$v(\bar{y}st) = \frac{1}{N^2 n}(\sum_{h=1}^{L} N_h S_h\ )^2$

ALOKASI OPTIMUM

$n_h = \frac{N_h\frac{S_h}{\sqrt{C_h}}}{\sum_{h=1}^{L} N_h\frac{S_h}{\sqrt{C_h}}}.n$ $\quad dan \quad$ $C = C_0 + \sum_{h=1}^{L} C_h n_h$

Fixed Variance Minimum Cost	Fixed Cost Minimum Varians
$n = \frac{\left(\sum_{h=1}^{L} N_h S_h\sqrt{c_h}\right)\left(\sum_{h=1}^{L} N_h S_h / \sqrt{c_h}\right)}{N^2D^2 + \sum_{h=1}^{L} N_h S_h^2}$	$n = \frac{\left(\sum_{h=1}^{L} N_h S_h / \sqrt{c_h}\right)}{\left(\sum_{h=1}^{L} N_h S_h\sqrt{c_h}\right)}(C - C_0)$
(WOR) $v(\bar{y}st) = \frac{1}{N^2 n}\left(\sum_{h=1}^{L}(1-f_h)N_h s_h / \sqrt{C_h}\right)\left(\sum_{h=1}^{L} N_h S_h / \sqrt{C_h}\right)$	(WR) $v(\bar{y}st) = \frac{1}{N^2 n}\left(\sum_{h=1}^{L} N_h S_h\ /\ \sqrt{c_h}\right)^2$

Systematic Sampling

Varians penduga rata-rata	I	$V(\bar{y}_{sy}) = \frac{1}{k}\sum_{i=1}^{k}(\bar{y} - \bar{Y})$
	II	$V(\bar{y}_{sy}) = \frac{N-1}{N}S^2 - \frac{k(n-1)}{N}S^2_{wsy}$ Keterangan : $S^2_{wsy} = \frac{1}{k(n-1)}\sum_{i}^{k}\sum_{j}^{n}(y_{ij} - \bar{y}_i)^2$ $S^2 = \frac{1}{N-1}\sum_{i}^{k}\sum_{j}^{n}(y_{ij} - \bar{Y})^2$
	III	$V(\bar{y}_{sy}) = \frac{s^2}{n}\frac{N-1}{N}[1 + (n-1)\rho]$ $\rho = \frac{2}{kn(n-1)}\sum_{i=1}^{k}\sum_{j<j'}^{n}(y_{ij} - \bar{Y})(y_{ij'} - \bar{Y})\frac{N}{(N-1)S^2}$
Rata-rata sampel untuk sampel sistematik ke-i		$\bar{y}_i = \frac{1}{n}\sum y_{ij}$
Paired Selection Mode	n genap	$v(\bar{y}) = \frac{1-f}{n^2}\sum_{i=1}^{\frac{n}{2}}(y_{2i} - y_{2i-1})^2$
	n ganjil	$v(\bar{y}) = \frac{1-f}{n(2m)}\sum_{i=1}^{\frac{n}{2}}(y_{2i} - y_{2i-1})^2$ $m = \frac{n+1}{2}$
Successive Difference Model		$v(\bar{y}) = \frac{1-f}{2n(n-1)}\sum_{i=1}^{n-1}(y_{i+1} - y_i)^2$

Stratified PPS

Probability selection unit ke-I pada strata ke-h	$P_{hi} = \frac{X_{hi}}{X_h}$
Fraksi sampling unit ke-I strata ke-h	$f_{hi} = \pi_{hi} = p_{hi} \cdot n_h = \frac{X_{hi}}{X_h} \cdot n_h$
Estimasi total karakteristik strata ke-h	$\hat{Y}_h = \sum_{i=1}^{n_h}\frac{y_{hi}}{\pi_{hi}} = \frac{1}{n_h}\sum_{i=1}^{n_h}\frac{y_{hi}}{p_{hi}}$
Estimasi varians total karakteristik strata ke-h	$v(\hat{Y}_h) = \frac{1}{n_h(n_h-1)}\sum_{i=1}^{n_h}\left(\frac{y_{hi}}{p_{hi}} - \hat{Y}_h\right)^2$
Estimasi total karakteristik populasi	$\hat{Y}_{st} = \sum_{h=1}^{L}\hat{Y}_h = \sum_{h=1}^{L}\sum_{i=1}^{n_h}\frac{y_{hi}}{\pi_{hi}} = \sum_{h=1}^{L}\frac{1}{n_h}\sum_{i=1}^{n_h}\frac{y_{hi}}{p_{hi}}$
Estimasi varians total karakteristik populasi	$v(\hat{Y})_{st} = \sum_{h=1}^{L}v(\hat{Y}_h) = \sum_{h=1}^{L}\frac{1}{n_h(n_h-1)}\sum_{h=1}^{n_h}\left(\frac{y_{hi}}{p_{hi}} - \hat{Y}_h\right)^2$
Estimasi rata-rata karakteristik populasi	$\bar{y}_{st} = \frac{\hat{Y}}{N}$
Estimasi varians rata-rata karakteristik populasi	$v(\bar{y})_{st} = \frac{1}{N^2}v(\hat{Y})$

PPS Sampling

Cara Pemilihan Sampel

- Kumulatif $AR \le X$
- Systematic
  Linear : $AR_1 \le I$
  Sirkular : $AR_1 \le X$
- Lahiri : $AR \le N, AR' \le x_i\ max$

PPS WOR Des Raj (Orderd)

Peluang terpilih unit i	$\hat{Z}_i = \sum_{k=1}^{i-1}y_k + \left(1 - \sum_{k=1}^{i-1}p_k\right)\frac{y_i}{p_i}$	Estimasi rata-rata dan varians rata-rata $\bar{z} = \frac{\hat{Z}}{N}$ $v(\bar{z}) = \frac{v(\hat{Z})}{N^2}$
Estimasi total dan varians total	$\hat{Z} = \frac{1}{n}\sum_{i=1}^{n}\hat{Z}_i; \quad v(\hat{Z}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}(\hat{Z}_i - \hat{Z})^2$	

$\hat{Z}$ bisa disebut $\hat{Y}_D$ (tergantung dosen)

PPS WR *(Hansen Hurwutz Estimator)*

Peluang terpilihnya unit ke-i	$p_i = \frac{X_i}{\sum_{i=1}^{N}X_i} = \frac{X_i}{X}$	Estimasi total unit ke-i	$\hat{Y}_i = \frac{y_i}{p_i}$
Fraksi sampling	$f = \pi_i = p_i \cdot n = \frac{X_i}{X}n$	Sampling *weight*	$w = \frac{1}{f} = \frac{X}{nX_i}$
Estimator total pps & Estimator rata-rata	$\hat{Y}_{pps} = \sum_{i=1}^{n}w \cdot y_i = \sum_{i=1}^{n}\frac{X}{nX_i} \cdot y_i = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i}$ $\bar{y}_{pps} = \frac{\hat{Y}_{pps}}{N}$		
Varians total	$V(\hat{Y}_{pps}) = \frac{1}{n}\sum_{i=1}^{N}p_i\left(\frac{y_i}{p_i} - Y\right)^2$ $v(\hat{Y}_{pps}) = \frac{s^2}{n} = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{Y}_{pps}\right)^2$		
Varians Rata-rata	$v(\bar{y}_{pps}) = \frac{1}{N^2}V(\hat{Y}_{pps}); \quad \bar{y} = \frac{\hat{Y}}{N}$ $v(\bar{y}_{pps}) = \frac{1}{N^2}v(\hat{Y}_{pps})$		

PPS WR dan WOR *(Horvitz Thompson Estimator)*

WOR	$\pi_i = p_i + \sum_{j\neq i}^{N}\frac{p_j p_i}{(1-p_j)} = p_i\left[1 + \sum_{j\neq i}^{n}\frac{p_j}{(1-p_i)}\right]$ $\pi_{ij} = \frac{p_i p_j}{(1-p_i)} + \frac{p_j p_i}{(1-p_j)} = p_i p_j\left[\frac{1}{(1-p_i)} + \frac{1}{(1-p_j)}\right]$
WR	$\pi_i = 1 - (1-p_i)^n$ $\pi_{ij} = \pi_i + \pi_j - \left[1 - (1-p_i-p_j)^n\right]$
Estimator total karakteristik	$\hat{Y}_{HT} = \sum_{i=1}^{v}\frac{y_i}{\pi_i}$
Estimasi varians total karakteristik	$v(\hat{Y}_{HT}) = \sum_{i=1}^{v}\left(\frac{1-\pi_i}{\pi_i^2}\right)y_i^2 + \sum_{i=1}^{v}\sum_{j\neq i}\left(\frac{\pi_{ij} - \pi_i\pi_j}{\pi_i\pi_j}\right)\frac{y_i y_j}{\pi_{ij}}$ $= \sum_{i=1}^{v}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{v}\sum_{j>i}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_i y_j$ v : jumlah unit yang berbeda dalam sampel

Relative Effiency: $RE = \frac{v(\hat{Y}_{pps})}{v_{pps}(\hat{Y}_{srs})} \times 100\%, \quad dimana \quad v_{pps}(\hat{Y}_{srs}) = \frac{1}{n^2}\left[N\sum_{i=1}^{n}\frac{y_i^2}{p_i} - n\hat{Y}_{pps}^2\right] + \frac{1}{n}v(\hat{Y}_{pps})$

PPS WOR Murthy's Method

Untuk n=2 (sampel terdiri dari unit I dan unit j)

$P(s\|i) = \frac{p_j}{1-p_i}$	$P(s\|j) = \frac{p_i}{1-p_j}$	$P(s) = \frac{p_i p_j(2-p_i-p_j)}{(1-p_i)(1-p_j)}$
Est. Total	$\hat{Y}_M = \frac{1}{(2-p_i-p_j)}\left[(1-p_j)\frac{y_i}{p_i} + (1-p_i)\frac{y_j}{p_j}\right]$	
Var. Total	$v(\hat{Y}_M) = \frac{(1-p_i)(1-p_j)(1-p_i-p_j)}{(2-p_i-p_j)^2}\left(\frac{y_i}{p_i} - \frac{y_j}{p_j}\right)^2$	

Untuk kasus umum (n > 2)

Probability Selection Terpilihnya sampel ke-i	$\frac{p_i}{(1 - \sum_{i=1}^{i-1}p_i}$
$\hat{Y}_M = \frac{1}{P(s)}\sum_{i=1}^{n}P(s\|i)y_i$	
$v(\hat{Y}_M) = \frac{1}{\{P(s)\}^2}\left[\sum_{i=1}^{n}\sum_{j>i}^{n}[P(s).P(s\|ij) - P(s\|i).P(s\|j)\}.p_i p_j.\left(\frac{y_i}{p_i} - \frac{y_j}{p_j}\right)^2\right]$	

PPS WOR RHC

Est Total	$\widehat{Y} = \sum_{g=1}^{n}X_g\frac{y_g}{x_g}$
Est Varians	$v(\hat{Y}_{RHC}) = \frac{(\sum_{g=1}^{n}{N_g}^2 - N)}{(N^2 - \sum_{g=1}^{n}{N_g}^2)}\sum_{g=1}^{n}X_g\left(\frac{y_g}{x_g} - \hat{Y}_{RHC}\right)$