Example 11.11 Let us compare, on the basis of avarage response time, the performance of two identical servers each with its won seperate queue, to the case when there is only a single queue to hold customers for both servers.The systems to compare are illustrated in figure 11.17. We shall also check to see how these two possibilities compare to a single processor working twice as fast

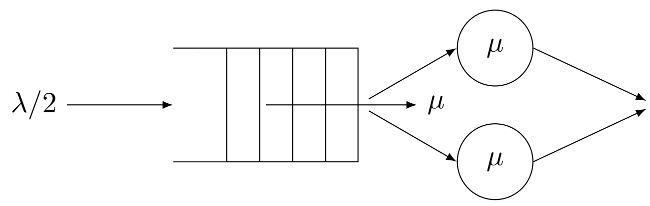

Figure 1: Two configurations.

In the fist case, we have two independent $M/M/1$ queues, each with arrival rate $\lambda/2$ and service rate μ.It follows that $\rho = (\lambda/2) = \lambda/(2\mu)$.The mean number in each $M/M/1$ queue is given by $\rho/(1-\rho)$ so that the mean number of customers in the first scenario is given as

$$L_1 = E[N_1] = 2 \times \frac{\rho}{1-\rho} = \frac{2\rho}{1-\rho}.$$

The avarage response time can now be found by using Little's law.We have

$$E[R_1] = \frac{1}{\lambda} E[N_1] = \frac{1}{\lambda} \frac{2\rho}{(1-\rho)} = \frac{2}{2\mu - \lambda}$$

Now consider the second scenario in witch the system may be represented as an $M/M/2$ queue.To find $E[R_2]$, we first must find $E[N_2](= L_2)$.The mean number of customers in an $M/M/c$ queue with arrival rate λ and service rate per server is given by

$$E[N_2] = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^c \lambda\mu}{(c-1)!(c\mu - \lambda^2)} \rho_0 with \frac{\lambda}{c\mu} or \lambda/\mu = c\rho$$

With $c = 2$, we obtain

$$L_2 = E[N_2] = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2 \lambda\mu}{(2\mu - \lambda)^2} \rho 0 = \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^2 (\lambda/\mu)}{(1/\mu^2)(2\mu - \lambda^2)} \rho_0$$

$$= \frac{\lambda}{\mu} + \frac{(\lambda/\mu)^3}{(2 - \lambda/\mu)^2} \rho_0$$

$$= 2\rho + \frac{(2\rho)^3}{(2 - 2\rho)^2} \rho_0$$