second solution to (18.147) as a linear combination of (18.148) and (18.149) given by

$$U(a,c;x) \equiv \frac{\pi}{sin\pi c}\left[\frac{M(a,c;x)}{\Gamma(a-c+1)\Gamma(c)} - x^{1-c}\frac{M(a-c+1,2-c;x)}{\Gamma(a)\Gamma(2-c)}\right].$$

This has a wll behaved limit asa c approaches an integer

18.11.1 Properties of confluent hypergeometric functions

The properties of confluent hypergeometric functions can be derived from those of ordinary hypergeometri functions by letting $x \to x/b$ and taking the limit $b \to \infty$, in the same way as both the equation and its solution wew derived.A general procedure of this sort is called *confluence* process.

Special cases

The general nature of the confluent hypergeometric equation allows one to write a large number of elementary functions in terms of the confluent hypergeometric functions *M(A,C;X)*. Once again, such identifications can be made from the series expansions (18.148) directly, or by transformations of the confluent hypergeometric quation into a more familiar equation for which the solutions are already known.Some particular examples of well known secial cases of the confluent hypergeometric function ar as follows:

$$M(a,a;x) = e^x, \qquad M(1,2;2X) = \frac{e^x sinhx}{x},$$

$$M(-n,1;x) = L_n(x), \qquad M(-n,m+1;x) = \frac{n!m!}{(n+m)!}L_n^m(x),$$

$$M(-n,\frac{1}{2};x^2) = \frac{(-1)^n n!}{(2n)!}H_{2n}(x) \qquad M(-n,\frac{3}{2};x^2), = \frac{(-1)^n n!}{2(2n+1)!}\frac{H_{2n+1}(x)}{x},$$

$$M(v+\frac{1}{2},2v+1;2ix) = v!e^{ix}(\frac{x}{2})^{-v}J_v(x), \qquad M(\frac{1}{2},\frac{3}{2};-x^2) = \frac{\sqrt{\pi}}{2x}erf(x)$$

where n and m are integers L_n^m is an associatef Legendre polynomial, $H_n(x)$ is a Hermite polynomial, $J_v(x)$ is a Bessel function and erf(x) is the error function discussed in section 18.12.4

Integral representation

Using te integral reprsentation (18.144) of the ordinary hypergeometric function, exchangin a and b and carring out the process of confluece gives

$$M(a,c,x) = \frac{\Gamma(c)}{\Gamma(a)\gamma(c-a)}\int_0^1 e^{tx}ta-1(1-t)^{c-a-1}dt,$$