Complex and Social Networks: Lab session 5

Finding community structure

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TASK 1

Introduction

In this session we will run and compare different community finding algorithms. We will use implementations from the igraph package.

Given an undirected graph, we want to analyze, for each available community finding algorithm the value achieved by the output partition for each of the following criteria: 'Triangle Partition Ratio' (TPT), 'expansion', 'conductance' and 'modularity'.

- Triangle partition ratio: fraction of nodes in C that belong to a triad

$$TPT = \frac{|\{u : u \in C \text{ and } \{(w,v) \in E : w,v \in C, (u,w),(u,v) \in E\} \neq \emptyset\}|}{n_c}$$

- Expansion: number of edges per node leaving the cluster

$$\frac{f_c}{n_c}$$

- Conductance: fraction of total edge volume that points outside the cluster

$$\frac{f_c}{2m_c + f_c}$$

- Modularity: difference between numebr of edges in C and the expected nr. of edges $E[mc]$ of a random graph with the same degree distribution

$$\frac{1}{4m}\left(m_c - E\left[m_c\right]\right)$$

Where:

$$f_c = C = |\{(u,v)|u \in C, v \notin C\}|$$

$$m_c = C = |\{(u, v)|u, v \in C\}|$$

and n_c is just the number of nodes.

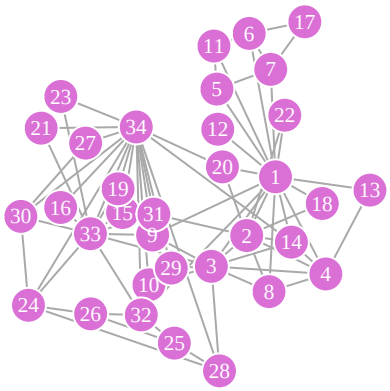
To calculate TPT of a community division we decided to calculate a weighted average of the TPT evaluated on each cluster. For the other metrics this is not necessary.

Test on the Zachary Karate Graph

This is a social network of friendships between 34 members of a karate club at a US university in the 1970s.

[See W. W. Zachary, An information flow model for conflict and fission in small groups, Journal of Anthropological Research 33, 452-473 (1977)]

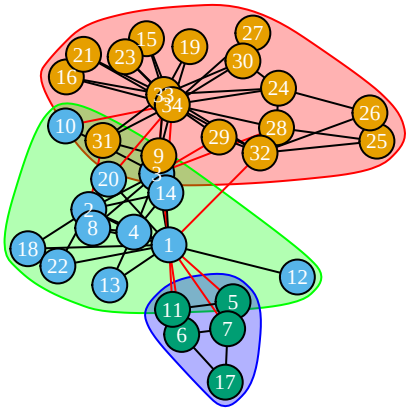
This network became a popular example of community structure.



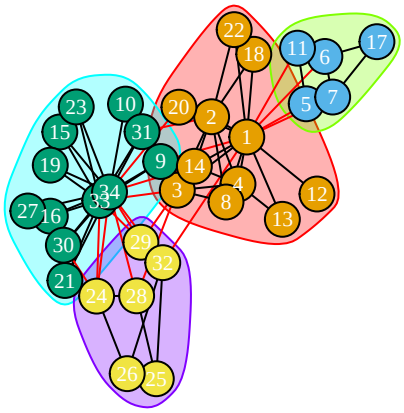
	Ncluster	TPT	Expansion	Conductance	Modularity
Edge Betweennes	5	8.500000	0.7058824	0.1818182	0.4012985
Fastgreedy	3	12.029412	0.5588235	0.1386861	0.3806706
Label propagation	2	21.500000	0.2941176	0.0684932	0.3717949
Leading eigenvector	4	6.941177	0.7647059	0.2000000	0.3934089
Multilevel	4	10.941176	0.6176471	0.1555556	0.4188034
Optimal	4	10.676471	0.6176471	0.1555556	0.4197896
Spinglass	4	10.676471	0.6176471	0.1555556	0.4197896
Walktrap	5	4.117647	0.9411765	0.2580645	0.3532216
Infomap	3	16.205882	0.4117647	0.0985915	0.4020381

The infomap algorithm seems to work very well: with only three clusters we got quite high TPT and

modularity (as we want), and low conductance and expansion (as we want).



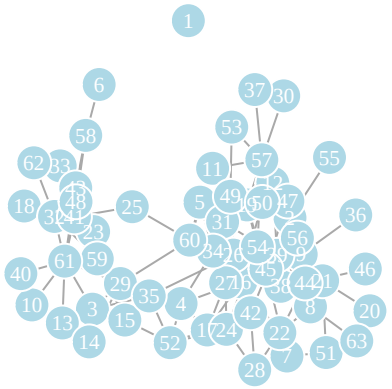
With four cluster instead we have three algoritms that hold similar results, for example:



Dolphin social network

All members of a school were assumed associated. Half-weight index (HWI) was used to quantify the frequency of association among individuals. Data source

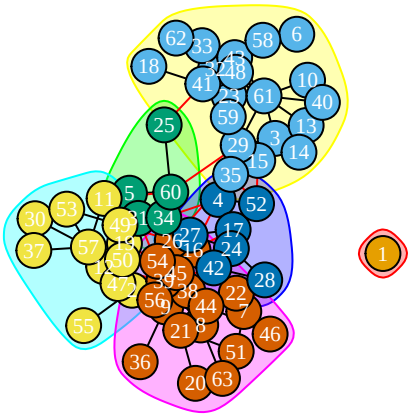
Lusseau, David, et al. “The bottlenose dolphin community of Doubtful Sound features a large proportion of long-lasting associations.” Behavioral Ecology and Sociobiology 54.4 (2003): 396-405.



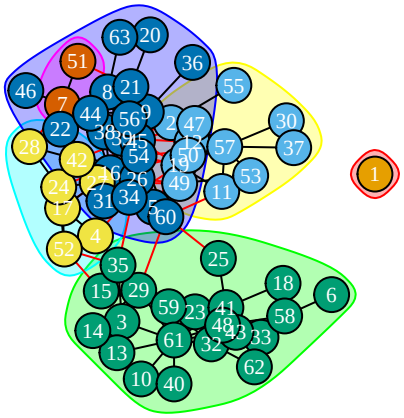
	Ncluster	TPT	Expansion	Conductance	Modularity
Edge Betweennes	6	23.14286	0.5079365	0.1118881	0.5193821
Fastgreedy	5	28.04762	0.4444444	0.0965517	0.4923263
Label propagation	5	24.20635	0.4920635	0.1080139	0.5097504
Leading eigenvector	6	14.98413	0.7301587	0.1691176	0.4911989
Multilevel	6	21.30159	0.5714286	0.1276596	0.5277283
Optimal	6	20.07937	0.6031746	0.1357143	0.5285194
Walktrap	5	27.19048	0.4444444	0.0965517	0.4888454
Infomap	7	16.98413	0.6825397	0.1563636	0.5189470

Spinglass is omitted because the algorithm does not work with unconnected graphs.

Multilevel community detection algorithm



Edge betweenness community detection algorithm

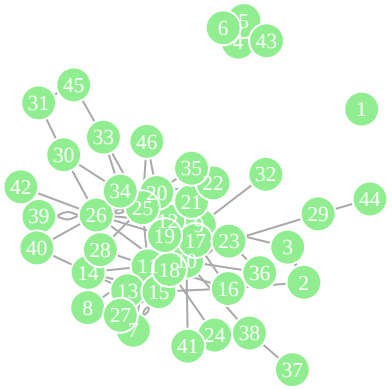


Both multilevel and edge betweenness seems to be good choice for our dolphin school.

Tortoises

A bipartite network was first constructed based on burrow use - an edge connecting a tortoise node to a burrow node indicated burrow use by the individual. Social networks of desert tortoises were then constructed by the bipartite network into a single-mode projection of tortoise nodes.

Sah, Pratha, et al. "Inferring social structure and its drivers from refuge use in the desert tortoise, a relatively solitary species." Behavioral Ecology and Sociobiology 70.8 (2016): 1277-1289.

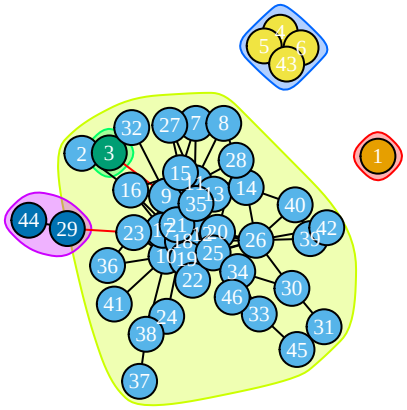


	Ncluster	TPT	Expansion	Conductance	Modularity
Edge Betweennes	15	19.565217	0.7173913	0.1843575	0.3062478
Fastgreedy	7	6.565217	0.8695652	0.2325581	0.4022339
Label propagation	5	64.782609	0.0652174	0.0143541	0.1221520
Leading eigenvector	7	5.108696	1.0000000	0.2771084	0.3584906
Multilevel	7	7.434783	0.8260870	0.2183908	0.4272428
Optimal	7	6.521739	0.8478261	0.2254335	0.4311588
Walktrap	9	23.804348	0.5434783	0.1336898	0.3528836
Infomap	9	15.391304	0.6739130	0.1712707	0.3921324

This time we see that, given the difficult disconnetted graph, the different community algorithms performs really differently.

The only algorithm that find a small number of cluster is the Label propagation algorithm.

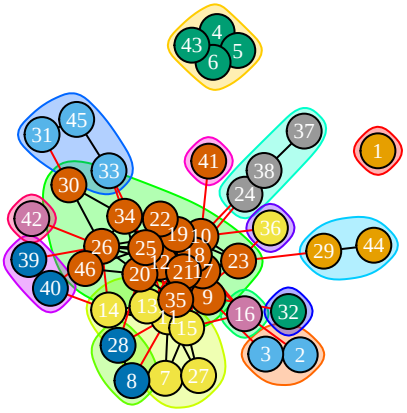
Label propagation community detection algorithm



Unfortunatly, a glance is enough to say that this division is not satisfyng. The algorithm mainly divid in different clusters the disconnected components.

On the other extreme, the edge betweenness algorithm finds 15 clusters.

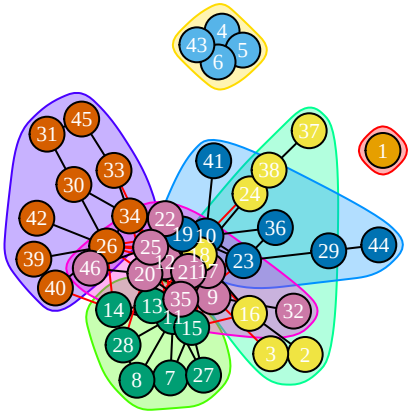
Label propagation community detection algorithm



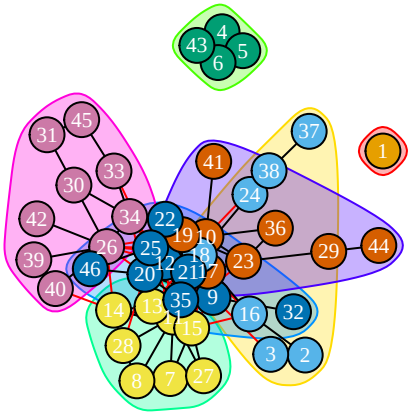
Does this makes any sense?

We can try to see partitions computed by other algorithms (but we still have to question ourselves if it makes sense to us having 7/9 cluster on only 46 individuals). This two algorithms return an almost identical result, but the second one takes a little bit of computational time more.

Multilevel community detection algorithm

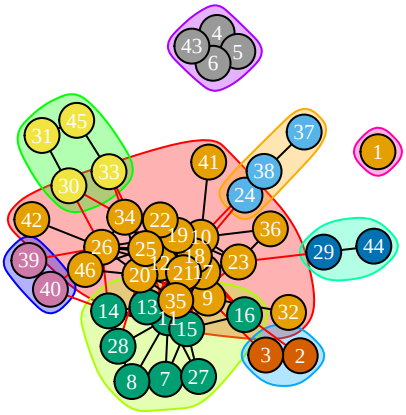


Optimal community detection algorithm

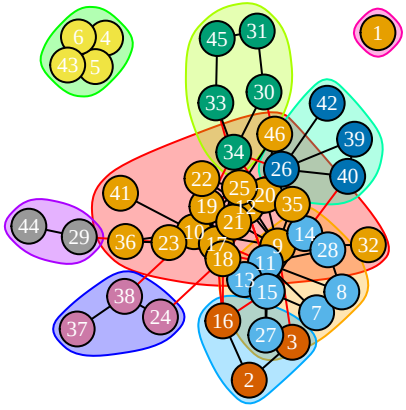


With 9 clusters, we have again two algorithms that finds similar result. The first one is a little bit better on the TPT metric, the other one is slightly better in all the other metrics.

Walktrap community detection algorithm



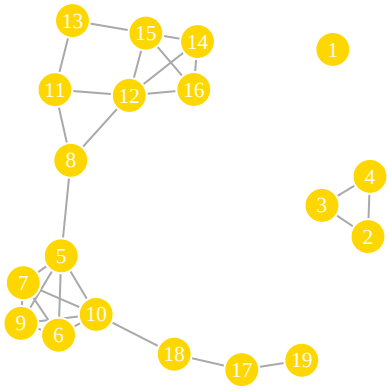
Infomap community detection algorithm



Weaver aves

A network edge was drawn between individuals that used the same nest chambers either for roosting or nest-building at any given time within a series of observations at the same colony in the same year, either together in the nest chamber at the same time or at different times.

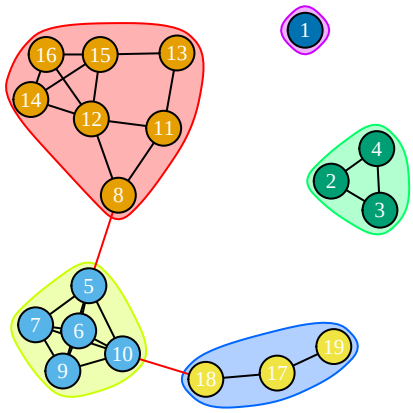
van Dijk, Rene E., et al., "Cooperative investment in public goods is kin directed in communal nests of social birds." Ecology letters 17.9 (2014): 1141-1148.



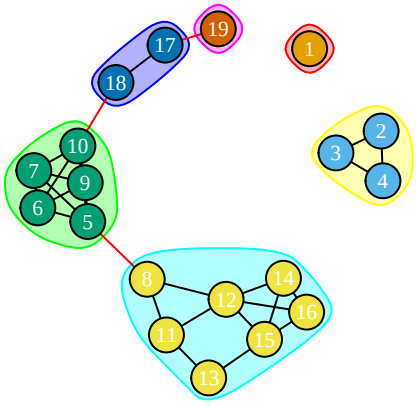
	Ncluster	TPT	Expansion	Conductance	Modularity
Edge Betweennes	5	4.631579	0.1052632	0.0370370	0.5860969
Fastgreedy	5	4.578947	0.1578947	0.0566038	0.5465561
Label propagation	5	4.631579	0.1052632	0.0370370	0.5860969
Leading eigenvector	6	4.631579	0.1578947	0.0566038	0.5529337
Multilevel	5	4.631579	0.1052632	0.0370370	0.5860969
Optimal	5	4.631579	0.1052632	0.0370370	0.5860969
Walktrap	5	4.631579	0.1052632	0.0370370	0.5860969
Infomap	5	4.631579	0.1052632	0.0370370	0.5860969

This time we have a more "easy" graph to partition (communities are easy to see even just by looking at it), and results algotrithms seems indeed more consistent. All the algorithm finds the similiar solution, with the exception that one of them divides one of the communities in two.

Infomap community detection algorithm



Leading eigenvector community detection algorithm



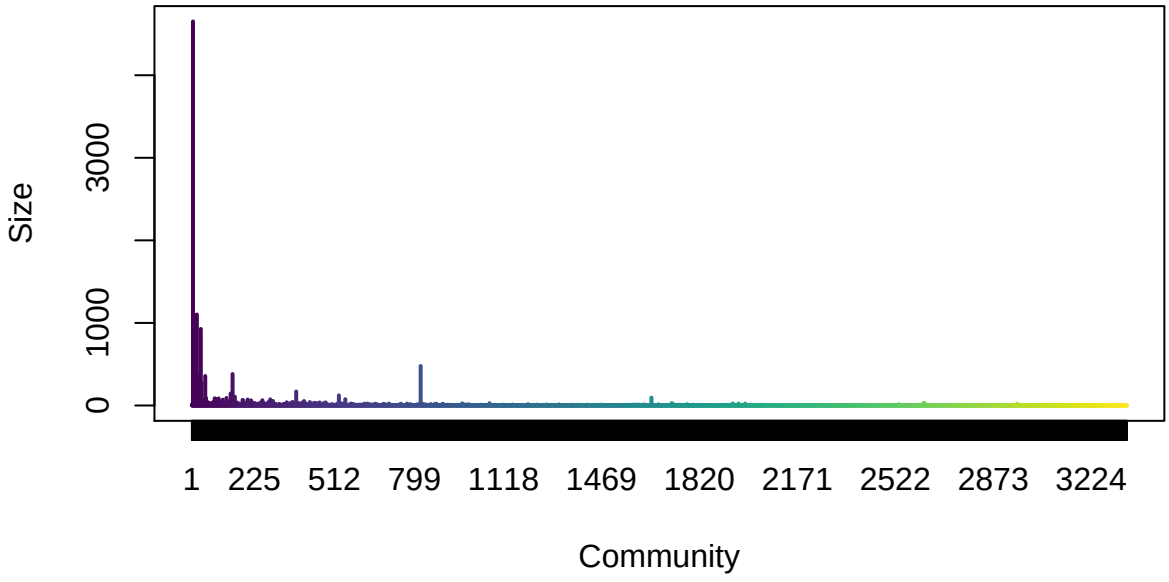
TASK 2

Introduction

In this section, we are going to analyze the resultant communities of applying one community detection algorithm to a huge network, the Wikipedia network. Since this network is a directed graph, only two algorithms can handle it, which are `edge.betweenness.community`, `label.propagation.community` and `walktrap.community` (the other ones crashed). Only the `walktrap.community` algorithm finish its execution in a reasonable time, hence it is the one we used.

Analysis of the communities

The algorithm groups the nodes in 3352 communities. In the following figure, you can see the distribution of the nodes along the communities.



As you can observe, nodes are not very well distributed. We have only 15 communities that have more than 100 nodes, while 1528 of them have less than 3 (only 16 have only one single node). However, we are going to take a look at the labels of the nodes of some of these communities.

We selected randomly 10 communities:

1. "Kd-trie", "Prefix hash tree", "Trie", "Burstsort", "Hash trie" and "Acyclic deterministic finite automaton".
2. "Etherloop" and "High Speed Voice and Data Link".
3. "Fleet Satellite Communications System" and "UHF Follow-On System".
4. "Loop Maintenance Operations System" and "Craft access system".
5. "Sheikh Ahmad Dede" and "Heaven & Earth (TV series)".
6. "Comedy of errors", "Havoc, Inc", "Farce" and "Body swap".
7. "Family Ties (Farscape episode)", "Meltdown (Farscape episode)" and "A Human Reaction".

8. "Penrose method", "One man, one vote" and "Plural voting".
9. "Porto Alegre Manifesto" and "Armand Mattelart".
10. "Elizabeth Moberly" and "Environmental factor".

Doing a little bit of research in Wikipedia, we found a common point between nodes of each community:

1. Tries.
2. Voice and data transmision.
3. Satellite communication systems of the U.S. Navy.
4. -
5. TV show and a man who appeared in.
6. Literature.
7. Episodes of the Farscape TV show.
8. Voting.
9. A manifesto and a man who signed it.
10. Cancer.

In point 4, we could not find any entrance in Wikipedia for "Craft access system". Moreover, in point 10, we have "Elizabeth Moberly": a British research psychologist and theologian who became involved in cancer research, and "Environmental factor", where the main result of it is cancer. The relation is almost inexistent.

We also had a look to the largest community (4651 nodes) to see if there is a relation between its nodes. We selected randomly 10 elements of the community, which are: "Bairstow's method", "Boolean algebra (structure)", "Ideal (set theory)", "Pappus graph", "Parity (mathematics)", "Semi-continuity", "Elongated triangular cupola", "Methods of contour integration", "Jensen's Device", "Domain (ring theory)". Again, doing some research in Wikipedia, we notice that all nodes are related with mathematics.

Conclusions

To sum up, community detection algorithms (`walktrap.community` to be concrete) group nodes into communities correctly, but probably not the best way. I.e., the communities it creates are good because nodes within it are related, but they could be inside one where they fit better. Furthermore, we are sure that most of the small communities can be in a bigger one with a more general topic, for instance, the TV shows of points 5 and 7.