Prognostic modelling, Ex. 1

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1) Classification of eq. of motions

$$H(p_1, p_2, q_1, q_2) = \frac{1}{2m}(p_1^2 + p_2^2) + \frac{1}{2\gamma}(q_1 - q_2)^2$$

with: $-\frac{\partial H}{\partial q} = \dot{p}$, $\frac{\partial H}{\partial p} = \dot{q}$

$$\frac{\partial H}{\partial q_1} = \frac{1}{\gamma}(q_1 - q_2) = -\dot{p}_1 \qquad -\frac{\partial H}{\partial q_2} = \frac{1}{\gamma}(q_1 - q_2) = \dot{p}_2$$

$$\frac{\partial H}{\partial p_1} = \frac{p_1}{m} = \dot{q}_1 \iff p_1 = m\dot{q}_1 \Rightarrow \dot{p}_1 = m\ddot{q}_1$$

$$\frac{\partial H}{\partial p_2} = \frac{p_2}{m} = \dot{q}_2 \iff p_2 = m\dot{q}_2 \Rightarrow \dot{p}_2 = m\ddot{q}_2$$

it follows:

$$\frac{1}{\gamma}(q_1 - q_2) = -m\ddot{q}_1$$

$$\iff \ddot{q}_1 = -\frac{1}{m\gamma}(q_1 - q_2) \qquad \mathrm{I}$$

$$\frac{1}{\gamma}(q_1 - q_2) = m\ddot{q}_2$$

$$\iff \ddot{q}_2 = \frac{1}{m\gamma}(q_1 - q_2) \qquad \mathrm{II}$$

Classification: linear, inhomogeneous

i) Independent variables: I: t, q_2 ; II: t, q_1

ii) dependent variables: I: q_1 ; II: q_2

iii) Highest degree of differentiation: 2

iv) Combination of integrals and differentials: No

v) Grade of stochastics: No randomness (deterministic)