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Вариант № 25

№ 1.1

$$\int \frac{5\,x\,dx}{(35-6x^2)^8} = \left[\begin{array}{l} t = 35-6x^2; \\ dt = (35-6x^2)'_x = -12x\,dx \end{array}\right] = -\frac{5}{12}\int\frac{dt}{t^8} =$$

$$= -\frac{5}{12}\cdot -\frac{1}{7t^7} + C = \frac{5}{84t^7} + C = \frac{5}{84(35-6x^2)^7} + C$$

№ 1.2

$$\int 3\cdot(4{,}5\cdot x^8 - 0{,}5\cdot x^5)\cdot \ln(x^3)\cdot dx = 1{,}5\int x^5(9x^3-1)\cdot \ln(x^3)\,dx =$$

$$= 4{,}5\int x^5\cdot(9x^3-1)\cdot \ln x\cdot dx = \left[\begin{array}{ll} u = \ln x & \to u' = \frac{1}{x} \\ v' = x^5(9x^3-1) & v = x^9 - \frac{x^6}{6} \end{array}\right] =$$

$$= \left(x^9 - \frac{x^6}{6}\right)\ln x - \int\left(x^8 - \frac{x^5}{6}\right)dx = \left(x^9 - \frac{x^6}{6}\right)\ln x - \int x^8\,dx + \int\frac{x^5}{6}\,dx =$$

$$= \left(x^9 - \frac{x^6}{6}\right)\ln x - \frac{x^9}{9} + \frac{x^6}{36} + C$$

№ 1.3.

$$\int \frac{x - \frac{17}{5}}{x^3-2x^2+6x-12} = \int \frac{x-\frac{17}{5}}{x^2(x-2)+6(x-2)} = \int\frac{x-\frac{17}{5}}{(x^2+6)(x-2)} = \left[\frac{Ax+B}{x^2+6} + \frac{C}{x-2}\right] =$$

$$= \Big[\; x - \frac{17}{5} = (Ax+B)(x-2) + C(x^2+6) \to x - \frac{17}{5} = Ax^2 - 2Ax + Bx - 2B + Cx^2 \ldots$$

$$\begin{cases} A + C = 0 \\ B - 2A = 1 \\ \ldots = -\frac{17}{5} \end{cases} \to \begin{cases} B = 2A+1 \\ C = -A \\ \ldots = -\frac{17}{5} \end{cases}$$

$$\int \frac{5\,x\,dx}{(35-6x^2)^5} = \left[\begin{array}{l} t = 35-6x^2 \\ dt = (35-6x^2)'_x = -12x\,dx \end{array} \right] = -\frac{5}{12}\int \frac{dt}{t^5}$$

$$= -\frac{5}{12}\cdot -\frac{1}{4t^4} + C = \frac{5}{64t^4} + C = \frac{5}{84(35-6x^2)^4} + C$$

№ 1.2

$$\int 3\cdot(4{,}5\cdot x^8 - 0{,}5\cdot x^5)\cdot \ln(x^3)\cdot dx = 1{,}5\int x^5(9x^3-1)\cdot \ln(x^3)dx =$$

$$= 4{,}5\int x^5\cdot(9x^3-1)\cdot \ln x\cdot dx = \left[\begin{array}{ll} u=\ln x \rightarrow & u'=\frac{1}{x} \\ v'=x^5(9x^3-1) & v=x^9-\frac{x^6}{6} \end{array} \right] =$$

$$= \left(x^9-\frac{x^6}{6}\right)\ln x - \int\left(x^8-\frac{x^5}{6}\right)dx = \left(x^9-\frac{x^6}{6}\right)\ln x - \int x^8 dx + \int \frac{x^5}{6}dx =$$

$$= \left(x^9-\frac{x^6}{6}\right)\ln x - \frac{x^9}{9} + \frac{x^6}{36} + C$$

№ 1.3

$$\int \frac{x-\frac{17}{5}}{x^3-2x^2+6x-12} = \int \frac{x-\frac{17}{5}}{x^2(x-2)+6(x-2)} = \int \frac{x-\frac{17}{5}}{(x^2+6)(x-2)} = \left[\frac{Ax+B}{x^2+6} + \frac{C}{x-2} \right] =$$

$$\left[\quad x-\frac{17}{5} = (Ax+B)(x-2) + C(x^2+6) \rightarrow x-\frac{17}{5} = Ax^2 - 2Ax + Bx - 2B + Cx^2 + 6C \right.$$

$$\begin{cases} A+C=0 \\ B-2A=1 \\ 6C-2B=-\frac{17}{5} \end{cases} \rightarrow \begin{cases} B=2A+1 \\ C=-A \\ -6A-4A-2=-\frac{17}{5} \end{cases}$$

$$-10A = -\frac{7}{5}$$

$$\left. \begin{cases} A=\frac{7}{50} \\ C=-\frac{7}{50} \\ B=\frac{32}{25} \end{cases} \right] =$$

$$=\int\left(\frac{(7/50)x+32/25}{x^2+6}+\frac{-7/50}{x-2}\right)dx=\frac{7}{50}\int\frac{x\,dx}{x^2+6}+\frac{32}{25}\int\frac{dx}{x^2+6}-\frac{7}{50}\int\frac{dx}{x-2}=$$

$$=\begin{bmatrix} t=x^2+6 \\ dt=2x\,dx \end{bmatrix}=\frac{7}{100}\int\frac{dt}{t}+\frac{32}{25}\cdot\frac{\sqrt6}{6}\,\mathrm{arctg}\,\frac{\sqrt6 x}{6}-\frac{7}{50}\ln|x-2|+C=$$

$$=\frac{7\ln|x^2+6|}{100}+\frac{16\sqrt6}{75}\,\mathrm{arctg}\,\frac{\sqrt6 x}{6}-\frac{7}{50}\ln|x-2|+C$$

№ 4

$$\int\frac{2\,dx}{25+\sqrt{26x+30}}=\begin{bmatrix} 26x+30=t^2;\ dx=\frac{t\,dt}{13} \\ x=\frac{t^2-30}{26};\ t=\sqrt{26x+30} \end{bmatrix}=$$

$$=\frac{2}{13}\int\frac{t\,dt}{25+t}=\left[\begin{array}{r|l} -\,t & t+25 \\ \underline{t+25} & 1 \\ -25 & \end{array}\ \rightarrow\ 1-\frac{25}{t+25}\right]=$$

$$=\frac{2}{13}\left(\int dt-25\int\frac{dt}{t+25}\right)=\frac{2}{13}\left(t-25\ln|t+25|\right)+C=$$

$$=\frac{2\sqrt{26x+30}-50\ln|\sqrt{26x+30}+25|}{13}+C$$

№ 5

$$\int\frac{3}{11\cdot\cos^2(x)+19\sin^2x}dx=\begin{bmatrix} t=\mathrm{tg}x;\quad dx=\frac{dt}{t^2+1};\ \cos x=\frac{1}{\sqrt{1+t^2}} \\ x=\mathrm{arctg}x;\ \sin x=\frac{t}{\sqrt{1+t^2}} \end{bmatrix}=$$

$$= \frac{2}{13}\int \frac{t\,dt}{25+t} = \left[\begin{array}{r|l} t & t+25 \\ -\;t+25 & 1 \\ \hline -25 & \end{array} \;\rightarrow\; 1 - \frac{25}{t+25} \right] =$$

$$= \frac{2}{13}\cdot\left(\int dt - 25\int\frac{dt}{t+25}\right) = \frac{2}{13}\left(t - 25\ln|t+25|\right) + C =$$

$$= \frac{2\sqrt{26x+30} - 50\ln\left|\sqrt{26x+30}+25\right|}{13} + C$$

№ 1.5

$$\int \frac{3}{11\cdot\cos^2(x) + 19\sin^2 x}\,dx = \left[\begin{array}{l} t = tg\,x; \quad dx = \frac{dt}{t^2+1};\ \cos x = \frac{1}{\sqrt{1+t^2}} \\ x = arctg\,x; \quad \sin x = \frac{t}{\sqrt{1+t^2}} \end{array}\right] =$$

$$= \int \frac{3\,dt}{(t^2+1)\cdot\left(19\frac{t^2}{t^2+1} + 11\cdot\frac{1}{t^2+1}\right)} = \int\frac{3\,dt}{19\cdot t^2+11} = \frac{3\sqrt{209}}{209}\cdot arctg\,\frac{\sqrt{209}\,t}{11} + C =$$

$$= \frac{3\sqrt{209}}{209}\cdot arctg\,\frac{\sqrt{209}\cdot tg\,x}{11} + C$$

№ 1.6

$$\int x \cdot \arcsin(-10x) \cdot dx = \int -x \arcsin(10x)\, dx =$$

$$= \left[\begin{array}{ll} u = \arcsin 10x & \rightarrow u' = \frac{10}{\sqrt{-100 \cdot x^2 + 1}} \\ v' = -x dx & v = -\frac{x^2}{2} \end{array}\right] = -\frac{x^2 \cdot \arcsin(10x)}{2} + \int \frac{5x^2 dx}{\sqrt{1-100x^2+1}} =$$

$$= \left[\begin{array}{ll} x = \frac{\sin t}{10} & t = \arcsin(10x) \\ dx = \frac{\cos t}{10} dt & \end{array}\right] = \left[5 \int \frac{\sin^2 t \cdot \frac{\cos t}{100}}{\cos t} dt = \frac{1}{20} \int \sin^2 t\, dt\right] =$$

$$= -\frac{x^2 \cdot \arcsin 10x}{2} + \frac{1}{20} \int \sin^2 t\, dt = -\frac{x^2 \arcsin 10x}{2} + \frac{1}{40} \cdot$$

$$\cdot \int (1 - \cos 2t)\, dt = -\frac{x^2 \arcsin 10x}{2} + \frac{1}{40} \cdot \left(\frac{t}{2} - \frac{\sin 2t}{4}\right) + C =$$

$$= -\frac{x^2 \arcsin 10x}{2} + \frac{\arcsin 10x}{80} - \frac{2 \sin \arcsin 10x \cdot \cos \arcsin 10x}{160} + C =$$

$$= \frac{-40x^2 \arcsin 10x + \arcsin 10x}{80} - \frac{20x \cdot 10x \sqrt{1-100x^2}}{160} + C =$$

$$= \frac{-40x^2 \arcsin 10x + \arcsin 10x}{80} - \frac{200x^2 \sqrt{1-100x^2}}{160} + C$$

№ 1.7

$$\int_{-4}^{9} \frac{dx}{\sqrt{5-4x-x^2}} = \int_{-4}^{9} \frac{dx}{\sqrt{9-(x+2)^2}} = \int_{-4}^{9} \frac{d(x+2)}{\sqrt{3^2-(x+2)^2}} = \arcsin \frac{x+2}{3} \Big|_{-4}^{9} =$$

$$= \arcsin \frac{11}{3} + \arcsin \frac{2}{3}$$

№ 1.7

$$\int_{-4}^{9} \frac{dx}{\sqrt{5-4x-x^2}} = \int_{-4}^{9} \frac{dx}{\sqrt{9-(x+2)^2}} = \int_{-4}^{9} \frac{d(x+2)}{\sqrt{3^2-(x+2)^2}} = \arcsin \frac{x+2}{3} \Big|_{-4}^{9} =$$

$$= \arcsin \frac{11}{3} + \arcsin \frac{2}{3}$$

№ 2.1

$$y' = -2y \Rightarrow \left[y' = \frac{dy}{dx} \right] \Rightarrow \frac{dy}{dx} = -2y \Rightarrow \frac{dy}{y} = -2dx \Rightarrow$$

$$\Rightarrow \int \frac{dy}{y} = -2\int dx \rightarrow \ln|y| = -2x + C \rightarrow \left[\ln y = x \rightarrow y = e^x \right] \Rightarrow$$

$$\Rightarrow |y| = e^{-2x} \cdot e^C \Rightarrow \left[C - const \Rightarrow e^C - const \Rightarrow \text{можно принять } e^C = C \right] \rightarrow$$

$$\rightarrow y = C \cdot e^{-2x}$$

$$\begin{cases} y(0) = C \cdot e^{-2\cdot 0} = C \cdot e^0 = C \\ y(0) = 26 \text{ , по условию} \end{cases} \Rightarrow C = 26 \Rightarrow$$

Частное решение имеет вид:

$$y = 26 \cdot e^{-2x}$$

№ 2.2

$$x \cdot y' = 2 \cdot \sqrt{27x^2 + y^2} + y$$