№ 6.4.38

$$\lim_{x\to 0} \frac{\sin^2 3x}{\sin^2 2x} = \lim_{x\to 0} \frac{\left(\frac{\sin^2 3x}{x^2}\right)}{\left(\frac{\sin^2 2x}{x^2}\right)} = \frac{9}{4} = 2\frac{1}{4}$$

№ 6.4.39

$$\lim_{x\to 0} \frac{\frac{\sin 2x}{\cos 2x}}{\sin 5x} = \frac{\frac{\sin 2x}{\cos 2x \cdot x}}{\sin 5x / x} = \frac{\frac{2}{1}}{5} = 2/5$$

№ 6.4.40

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right] = \left[\sin^2 x + \cos^2 x = 1\right] =$$

$$= \left[\sin x = \sqrt{1-\cos x}\right] = \left[\sin^2 = \sqrt{1-\cos x}\right]$$

$$\lim_{x\to 0} \frac{\sin x^2}{x^2} = 1$$

№ 6.4.41

$$\lim_{x\to 0} x \cdot \mathrm{ctg} = [0 \cdot -] = \lim_{x\to 0} x \cdot \frac{\cos x}{\sin x} =$$

$$\lim_{x\to 0} \frac{x \cos x}{\sin x} = \lim_{x\to 0} 1^{-1} \cdot 1 = 1$$

№ 6.4.42

$$\lim_{x\to 0} \frac{\operatorname{arctg} 2x}{x} = \left[\begin{matrix} t = \operatorname{arctg} 2x \\ x = \frac{1}{2}\operatorname{tg} t \end{matrix} \right] =$$

$x \to 0 \Rightarrow t \to 0$

$$\lim_{t\to 0} \frac{\cancel{\sin} 2t}{\operatorname{tg} t} = \lim_{t\to 0} \frac{2t \cdot \cos t}{\sin t} = \cancel{1}\ 2$$

№ 6.4.43

$$\lim_{x\to 0} \frac{\cos 5x - \cos 3x}{x^2} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{-2 \cdot \sin 4x \cdot \sin x}{x^2} = \lim_{x\to 0} \frac{-4 \sin 2x \cdot \sin x}{x^2} =$$

$$= \lim_{x\to 0} \frac{-8 \sin x \cdot \cos x \cdot \sin x}{x^2} = -8 \cdot 1 \cdot 1 = -8$$

№ 6.4.44

$$\lim_{x\to 1} \frac{\sin 6\pi x}{\sin \pi x} = \left[\frac{0}{0}\right] = \lim_{x\to 1} \frac{2 \sin 3\pi x \cdot \cos 3\pi x}{\sin \pi x} =$$

$$\lim_{x\to 1} \frac{2 \cdot \frac{\sin 3\pi x}{x} \cdot \cos 3\pi x}{\frac{\sin \pi x}{x}} = \lim_{x\to 1} \frac{2 \cdot 3\pi \cdot \cos 3\pi x}{\pi} = 6 \cdot -1 = -6$$

№ 6.4.45

$$\lim_{x\to\frac{\pi}{2}} \frac{\sin 2x}{tg\,4x} = \left[\frac{0}{0}\right] = \lim_{x\to\frac{\pi}{2}} \frac{2\sin x\cdot\cos x}{\frac{\sin 4x}{\cos 4x}} =$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{2\sin x\cos x}{\left(\frac{2\sin 2x\ \cos 2x}{\cos 4x}\right)} = \lim_{x\to\frac{\pi}{2}} \frac{2\sin x\cos x}{\left(\frac{4\sin x\cos x\cdot\cos 2x}{\cos 4x}\right)} =$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{\left(\frac{2\cos 2x}{\cos 4x}\right)} = \left[\frac{1}{\left(\frac{-2}{1}\right)}\right] = \frac{1}{-2} = -(1/2)$$

№ 6.4.40 по записи

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \left[\frac{0}{0}\right] = \left[\begin{array}{l} 1-\cos\alpha = 2\cdot\sin^2\frac{\alpha}{2} \\ \frac{1-\cos\alpha}{2} = \sin^2\frac{\alpha}{2}\end{array}\right. =$$

$$= \lim_{x\to 0} \frac{2\cdot\sin^2\frac{x}{2}}{x^2} = \lim_{x\to 0}\left(2\cdot\frac{\sin\frac{x}{2}}{x}\cdot\frac{\sin\frac{x}{2}}{x}\right) =$$

$$= \lim_{x\to 0}\left(2\cdot\frac{\sin x/2}{2\cdot x/2}\cdot\frac{\sin x/2}{2\cdot x/2}\right) = \frac{1}{2}\lim_{x\to 0}\left(\frac{\sin x/2}{x/2}\right)^2 =$$

$$= \frac{1}{2}\cdot 1 = 1/2$$