19.03.21

$$\int \sqrt[3]{x}\cdot\sqrt[3]{1+3\sqrt[3]{x^2}}\,dx = \int x^{\frac{1}{3}}\left(1+3x^{\frac{2}{3}}\right)^{\frac{1}{3}}dx =$$

$$= \left[\begin{array}{l} m, p, [illegible] \\ p=\frac{1}{3} \quad [illegible] \end{array}\right.$$

1) $p=\frac{1}{3}\ [illegible]$

2) $\frac{m+1}{n} = \frac{\frac{1}{3}+1}{\frac{2}{3}} = 2 \ [illegible]$

$1+3x^{\frac{2}{3}} = t^3 \ [illegible] \rightarrow$

$\rightarrow d(1+3x^{\frac{2}{3}}) = d(t^3) \rightarrow$

$x^{\frac{2}{3}} = \frac{t^3-1}{3}$ $\qquad (1+3x^{\frac{2}{3}})'dx = (t^3)'dt$

$x^{\frac{1}{3}} = \sqrt{\frac{t^3-1}{3}}$ $\qquad 2x^{-\frac{1}{3}}dx = 3t^2dt$

$\frac{1}{\sqrt{3}}\cdot\sqrt{t^3-1}$

$$dx = \frac{3}{2}t^2\,dt\cdot x^{1/3} = \frac{3}{2}t^2\cdot\frac{1}{\sqrt{3}}\cdot\sqrt{t^3-1}\,dt =$$

$$= \frac{\sqrt{3}}{2}t^2\cdot\sqrt{t^3-1}\,dt\Big] = \int\frac{1}{\sqrt{3}}\cdot\sqrt{t^3-1}\cdot\sqrt[3]{t^3}\cdot\frac{\sqrt{3}}{2}t^2\sqrt{t^3-1}\,dt =$$

$$= \frac{1}{2}\int t^3(t^3-1)\,dt = \frac{1}{2}\int [illegible]\,dt = \frac{1}{2}\left(\int t^6dt - \int t^3dt\right) =$$

$$= \frac{t^7}{14} - \frac{t^4}{8} = \frac{\left(\sqrt[3]{1+3\sqrt[3]{x^2}}\right)^7}{14} - \frac{\left(\sqrt[3]{1+3\sqrt[3]{x^2}}\right)^4}{8} + C =$$

$$= \frac{1}{14}\sqrt[3]{(1+3\sqrt[3]{x^2})^7} - \frac{1}{8}\sqrt[3]{(1+3\sqrt[3]{x^2})^4} + C$$

$$\int\frac{\sqrt[3]{x}\,dx}{\sqrt[3]{x^2}-\sqrt{x}} = \left[\begin{array}{l} q=2;\ r=3 \rightarrow k=6 \\ x=t^6;\ dx=d(t^6)=6t^5dt \end{array}\right] =$$

$$= \int\frac{t^2\cdot 6t^5\,dt}{t^4-t^3} = 6\int\frac{t^7\,dt}{t^4-t^3} = 6\int\frac{t^4\,dt}{t-1} =$$

$$\left[\begin{array}{l|l} t^4+0\cdot t^3+0\cdot t^2+0\cdot t & t-1 \\ \hline -\,t^4-t^3 & t^3+t^2+t+1 \\ \quad\ \ t^3 & \\ \quad -\,t^3-t^2 & \\ \qquad\ \ t^2 & \\ \qquad -\,t^2-t & \\ \qquad\quad\ \ t & \\ \qquad\quad -\,t-1 & \\ \qquad\qquad 1 & \end{array}\right] =$$

$$= 6\int\frac{(t^3+t^2+t+1)(t-1)+1}{(t-1)}\,dt = 6\int(t^3+t^2+t+1)\,dt + \int\frac{1}{t-1}\,dt$$

$$= -4\int \frac{(t+1)(t-1)-1}{(t+1)}\,dt =$$

$$\cancel{\left(\int (t+1)\,dt - \int \frac{1}{t+1}\,dt\right) = 2(t+1)^2 =}$$

$$= 2\sqrt{x} + 4\sqrt[4]{x} + 4\ln\left|\sqrt[4]{x}+1\right| + C$$

$$\int \frac{dx}{\sqrt{(2x+1)^2} - \sqrt{2x+1}} = \left[\;\right] = \left[\;\right.$$

$$2x+1 = t^6 \to x = \frac{t^6-1}{2} \to dx = d\left(\frac{t^6-1}{2}\right) = \frac{1}{2}(t^6-1)' = 3\,t^5\,dt \;\Big]$$

$$t = \sqrt[6]{2x+1}$$

$$= 3\int \frac{t^5\,dt}{\sqrt{t^{12}} - \sqrt{t^6}} = 3\int \frac{t^5\,dt}{t^4 - t^3} = 3\int \frac{t^2\,dt}{t-1} =$$

$$= 3\int (t+1)\,dt + 3\int \frac{dt}{t-1} = (3/2)\cdot t^2 + 3t + 3\ln|t-1| + C$$

$$\cancel{\frac{3}{2}x^{\frac{1}{3}} + 3x^{\frac{1}{6}} + 3\ln\left|x^{\frac{1}{6}} - 1\right| + C}$$

$$= \frac{3}{2}\cdot(2x+1)^{\frac{1}{3}} + 3\cdot(2x+1)^{\frac{1}{6}} + 3\ln\left|(2x+1)^{\frac{1}{6}} - 1\right| + C$$

Д.З.

2.4.6 → 2.4.7, 2.4.14, 2.4.11

pdf почта

Пис Д3 Часть 2 можно решать → 20ти

$= 4\,\mathrm{arctg}\,t - 2\,\mathrm{arctg}\,t - \ln(t \ldots$ $+ \ln(\sqrt[6]{2x+1} - 1) + C$

$= 2\,\mathrm{arctg}\sqrt[6]{2x+1} - \ln(\sqrt[6]{2x+1} + 1)$

№ 4.6

$$\int \frac{dx}{1+\sqrt[3]{x+1}} = \left[\begin{matrix} n=3;\ \text{НОК}=3;\ t=\sqrt[3]{x+1} \\ x+1=t^3,\ x=t^3-1,\ dx=3t^2\,dt \end{matrix}\right] =$$

$$= \int \frac{3t^2\,dt}{t+1} = 3\int \frac{t^2-1+1}{t+1}\,dt = 3\left(\int \frac{(t+1)(t-1)}{t+1} + \int \frac{dt}{t+1}\right) =$$

$$= 3\int (t-1)\,dt + 3\int \frac{dt}{t+1} = 3\frac{t^2}{2} - 3t + 3\ln|t+1| + C =$$

$$= \frac{3\sqrt[3]{(x+1)^2}}{2} - 3\sqrt[3]{x+1} + 3\ln|\sqrt[3]{x+1} + 1| + C$$

№ 4.8

$$\int \frac{\sqrt{x}}{x^2\cdot\sqrt{x-1}}\,dx = \int x^{1/2}\cdot x^{-2}\cdot\left(\sqrt{x-1}\right)^{-1} dx = \int x^{-\frac{3}{2}}\cdot\sqrt{(x-1)^{-1}}\,dx$$

$$= \int x^{-\frac{3}{2}}\cdot(x-1)^{-\frac{1}{2}}\,dx = \left[\begin{matrix} m=-3/2;\ n=1;\ p=-1/2 \\ 1)\ p \notin \mathbb{Z} \\ 2)\ \dfrac{-\frac{3}{2}+1}{1} = -\frac{1}{2} \notin \mathbb{Z} \\ 3)\ \underbrace{\dfrac{-\frac{3}{2}+1}{1} + \frac{1}{2} = -\frac{2}{2} = -1 \in \mathbb{Z}} \rightarrow \end{matrix}\right.$$

$$\rightarrow -1\cdot x^{-1} + 1 = t^2 \rightarrow t = \sqrt{\frac{x-1}{x}} \rightarrow x = -\frac{1}{t^2-1}$$

$$dx = -2\frac{t\,dt}{(t^2-1)^2} \Big] =$$

$$= \int \left(-\frac{1}{t^2-1}\right)^{-\frac{3}{2}} \cdot \left(-\frac{1}{t^2-1} - 1\right)^{-\frac{3}{2}} \cdot \frac{-2t\,dt}{(t^2-1)^2} =$$

$$= \int (1-t^2)^{\frac{3}{2}} \cdot \left(\frac{t^2}{1-t^2}\right)^{-\frac{3}{2}} \cdot 2t\,dt = \int \frac{2t}{(1-t^2)^{\frac{3}{2}}} \cdot \frac{(1-t^2)^{\frac{3}{2}}}{t}\,dt =$$

$$= \int 2\,dt = 2t + C = 2\sqrt{\frac{x-1}{x}} + C$$

c5.4.10

$$\int \sqrt{x}\,(1+\sqrt[3]{x})^4\,dx = \int x^{\frac{1}{2}}(1+x^{\frac{1}{3}})^4\,dx =$$

$$= \left[\begin{array}{l} m=\frac{1}{2};\ n=\frac{1}{3};\ p=4 \to \underline{p \in Z} \to \text{НОК}(2,3)=6 \to \\ \to x=t^6 \to t=\sqrt[6]{x} \to dx=6t^5\,dt \end{array}\right] =$$

$$= \int t^3 \cdot (1+t^2)^4 \cdot 6t^5\,dt = 6\int t^8 \cdot (1+t^2)^4\,dt = 6\cdot\int t^8 \cdot (t^8+4t^6+6t^4+4t^2+1)\,dt$$

$$= 6\cdot\int (t^{16}+4t^{14}+6t^{12}+4t^{10}+t^8)\,dt = 6\cdot\frac{t^{17}}{17} + 24\frac{t^{15}}{15} + 36\frac{t^{13}}{13} +$$

$$+ 24\frac{t^{11}}{11} + 6\frac{t^9}{9} + C = \frac{6\sqrt[6]{x^{17}}}{17} + \frac{8\sqrt{x^5}}{5} + \frac{36\sqrt[6]{x^{13}}}{13} + \frac{24\sqrt[6]{x^{11}}}{11} + \frac{2\sqrt{x^3}}{3} + C$$

c5.4.11

$$\int \frac{dx}{x^4\sqrt{x^2+1}} = \int x^{-4}(x^2+1)^{-\frac{1}{2}}\,dx = \left[\begin{array}{l} m=-4;\ n=2;\ p=-\frac{1}{2} \\ 1)\ p \notin Z \\ 2)\ \frac{-4+1}{2} = -\frac{3}{2} \notin Z \\ 3)\ \underline{-\frac{3}{2}-\frac{1}{2} = -2 \in Z} \to t^2 = 1\cdot x^{-2}+1 \\ x = \sqrt{\frac{1}{t^2-1}} \\ dx = d\left(\sqrt{\frac{1}{t^2-1}}\right) = \end{array}\right.$$

$$\left. = -\frac{t\,dt}{(t^2-1)^{\frac{3}{2}}}\right] = \int (t^2-1)^2 \left(\frac{1}{t^2-1} \cdot t^2\right)^{-\frac{1}{2}} \cdot \frac{-t\,dt}{(t^2-1)^{\frac{3}{2}}} = \int (t^2-1)^{2-\frac{3}{2}} \cdot (-t) \cdot \frac{\sqrt{t^2-1}}{t^2}\,dt =$$

$$= -\int t^2\,dt + \int dt = -\frac{1}{3}t^3 + t + C = \left[t = \frac{\sqrt{x^2+1}}{x}\right] = -\frac{1}{3}\left(\sqrt{\frac{x^2+1}{x}}\right)^3 +$$

$$+ \frac{\sqrt{x^2+1}}{x} + C = \frac{\sqrt{x^2+1}\,(2x^2-1)}{3x^3} + C$$