Интегралы и дифференциальные уравнения

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ИВТ 1 группа/1 подгруппа

5) $\lim\limits_{x\to1}\left(\frac{1}{1-x}-\frac{1}{1-x^3}\right)=[0\cdot0]=$

$=\lim\limits_{x\to1}\left(\frac{1}{(1-x)(1+x+x^2)}-\frac{1}{(1-x)(1+x)}\right)=\lim\limits_{x\to1}\left(\frac{1+x-(1+x+x^2)}{(1+x)(1-x)(1+x+x^2)}\right)=$

$=\lim\limits_{x\to1}\left(\frac{x^2}{(1+x)(1-x)(1+x+x^2)}\right)=\frac{1}{2\cdot0\cdot3}=\frac{1}{0}=\infty$

10 Часть 4

– Частные приращ –

$\Delta_x z=f(x+\Delta x;y)-f(x;y)$

$\Delta_y z=f(x;y+\Delta y)-f(x;y)$

– Полн. приращ –

$\Delta z=f(x+\Delta x;y+\Delta y)-f(x;y)$

$\Delta z\neq\Delta_x z+\Delta_y z$ (в общ. случ).

– Частная –

$z'_x=\lim\limits_{\Delta x\to0}\frac{\Delta_x z}{\Delta x};\ \frac{\partial z}{\partial x};\ \frac{\partial f}{\partial x}(x;y);\ \frac{\partial}{\partial x}z;$

$\frac{\partial}{\partial x}f;\ \frac{\partial}{\partial x}f(x;y)$

$z'_y=\lim\limits_{\Delta y\to0}\frac{\Delta_y z}{\Delta y};\ \frac{\partial z}{\partial y};\ \frac{\partial f}{\partial y}(x;y)\ \dots$

№ 11.3.1

$z=xy^2-\frac{x}{y}$; $\Delta_x z$?; $\Delta_y z$?; Δz – ?

$M_0(3;-2)$; $\Delta x=0{,}1$; $\Delta y=-0{,}05$

1) $M_0(3;-2)$

Пусть $x_0=3$; $y_0=-2$

$\Downarrow$

$x=x_0+\Delta x=3{,}1 \Rightarrow M_1(3{,}1;-2{,}05)$

$y=y_0+\Delta y=-2{,}05$

2) $z(M_0) = z(3;-2) = z = xy^2 - \frac{x}{y} = 3\cdot(-2)^2 - \frac{3}{2} =$

$= 3\cdot 4 + \frac{3}{2} = 13{,}5$

$z(x_0+\Delta x; y_0) = z(3{,}1; -2) = 3{,}1\cdot 4 - \frac{3{,}1}{-2} =$

$= 12{,}4 + 1{,}55 = 13{,}95$

$z(x_0; y_0+\Delta y) = z(3; -2{,}05) = 3\cdot 4{,}2025 + \frac{3}{2{,}05} =$

$= 12{,}6075 + \frac{3}{2{,}05} \approx 12{,}6075 + 1{,}4634 \approx 14{,}0709 \approx$

$\approx 14{,}07$

$z(M_1) = z(3{,}1; -2{,}05) = 3{,}1\cdot 4{,}2025 + \frac{3{,}1}{2{,}05} \approx 13{,}0278 +$

$+ 1{,}5122 \approx 14{,}5400$

3) $\Delta_x z = z(x_0+\Delta x; y_0) - z(x_0; y_0) = 13{,}95 - 13{,}5 =$

$= 0{,}45$

$\Delta_y z = z(x_0; y_0+\Delta y) - z(x_0; y_0) = 14{,}07 - 13{,}5 =$

$= 0{,}57$

4) $\Delta z = z(x_0+\Delta x; y_0+\Delta y) - z(x_0; y_0) = 14{,}54 - 13{,}5 =$

$= 1{,}04$

11.3.2

$z = x^2 y$

$M_0(1;2)$

1) $\Delta x = 0{,}1$

$\Delta y = -0{,}2$

$x = 1{,}1$

$y = 1{,}8$

2) $z(x_0; y_0) = 1\cdot 2 = 2;$

$z(x_0+\Delta x; y_0) = 1{,}21\cdot 2 = 2{,}42$

$z(x_0; y_0+\Delta y) = 1\cdot 1{,}8 = 1{,}8$

$z(x_0+\Delta x; y_0+\Delta y) = 1{,}21\cdot 1{,}8 = 2{,}178$

3) $\Delta_x z = z(x_0+\Delta x; y_0) - z(x_0; y_0) = 0{,}42$

$\Delta_y z = 1{,}8 - 2 = -0{,}2$

$\Delta z = 2{,}175 - 2 = 0{,}175$

Дифференциал ф.

$$dz = \underbrace{z'_x dx}_{d_x z} + \underbrace{z'_y dy}_{d_y z}$$

полн дифф

частные дифференциалы

$dx = \Delta x$

$dy = \Delta y$

$z'_x = f'_x(x_0, y)$ or $f'_x(x_0; y_0)$

$z'_y = f'_y(x_0, y)$ or $f'_y(x_0; y_0)$

$$f(x_0+\Delta x; y_0+\Delta y) \approx f(x_0; y_0) + f'_x(x_0; y_0)\cdot\Delta x + f'_y(x_0; y_0)\cdot\Delta y$$

Линеаризация функции

$z = f(x, y)$ в окр. $M_0(x_0; y_0)$

№ 11.3.9

$$z = \frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}$$

z'_x − ?

z'_y − ?

$$z'_x = \left(\frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}\right)'_x = \frac{1}{y^3}\cdot(x)' + y\cdot(x^{-3})'_x - \frac{1}{6y}\cdot(x^{-2})'_x$$

$$= \frac{1}{y^3}\cdot 1 + y\cdot -3\cdot x^{-4} - \frac{1}{6y}\cdot(-2)\cdot x^{-2-1} = \frac{1}{y^3} - \frac{3y}{x^4} + \frac{1}{3x^3y}$$

$$z'_y = \left(\frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}\right)'_y = x\cdot(y^{-3})'_y + \frac{1}{x^3}(y)'_y - \frac{1}{6x^2}\cdot(y^{-1})'_y =$$

$$= x\cdot -3\cdot y^{-4} + \frac{1}{x^3}\cdot 1 - \frac{1}{6x^2}\cdot -1\cdot y^{-2} = -\frac{3x}{y^4} + \frac{1}{x^3} + \frac{1}{6x^2y^2}$$

11.3.10

$$z = \frac{x^2 - 2xy}{y^2 + 2xy + 1} \qquad z'_x,\ z'_y - ?$$

$$z'_x = \frac{(x^2-2xy)'\cdot(y^2+2xy+1) - (x^2-2xy)\cdot(y^2+2xy+1)'}{(y^2+2xy+1)^2}$$

$$= \frac{(2x-2y)(y^2+2xy+1) - (x^2-2xy)\cdot 2y}{(y^2+2xy+1)^2}$$

$$z'_y = \frac{(x^2-2yx)'_y\,(y^2+2xy+1)-(x^2-2xy)(y^2+2xy+1)'_y}{(y^2+2xy+1)^2} =$$

$$= \frac{-2x\,(y^2+2xy+1)-(x^2-2xy)(2y+2x)}{(y^2+2xy+1)^2}$$

№ 11.3.16

$$z = \cos\frac{x^2+y^2}{x^3+y^3};\ z'_x,\ z'_y,\ d_x z,\ d_y z,\ dz\ -?$$

$$1)\ z'_x = \left(\cos\frac{x^2+y^2}{x^3+y^3}\right)' = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\left(\frac{x^2+y^2}{x^3+y^3}\right)' =$$

$$= -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{(x^2+y^2)'_x(x^3+y^3)-(x^3+y^3)'_x(x^2+y^2)}{(x^3+y^3)^2} =$$

$$= -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{2x(x^3+y^3)-3x^2(x^2+y^2)}{(x^3+y^3)^2} =$$

$$= \frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}$$

$$2)\ z'_y = \left(\cos\frac{x^2+y^2}{x^3+y^3}\right)' = \ldots = \frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\cdot$$

$$\cdot\sin\frac{x^2+y^2}{x^3+y^3}$$

$$3)\ \cancel{z'_x}\ d_x z = z'_x\,dx = \frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\,dx$$

$$d_y z = z'_y\,dy = \frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\,dy$$

$$dz = d_x z + d_y z = \frac{3x^2(x^2+y^2)-2x(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\,dx +$$

$$+ \frac{3y^2(x^2+y^2)-2y(x^3+y^3)}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\,dy =$$

$$= \frac{1}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}\Big((3x^2(x^2+y^2)-2x(x^3+y^3))dx +$$

$$+ (3y^2(x^2+y^2)-2y(x^3+y^3))dy\Big)$$

№ 11.3.17

$u = \frac{x}{\sqrt{y^2+z^2}}$ $\qquad du - ?$

$du = u'_x\,dx + u'_y\,dy + u'_z\,dz$

$u'_x = \left(\frac{x}{\sqrt{y^2+z^2}}\right)'_x = \frac{1}{\sqrt{y^2+z^2}}\cdot(x)' = \frac{1}{\sqrt{y^2+z^2}}$

$u'_y = \left(\frac{x}{\sqrt{y^2+z^2}}\right)'_y = x\cdot\left(-\frac{1}{2}\right)\cdot(y^2+z^2)^{-\frac{1}{2}-1}\cdot$

$\cdot(y^2+z^2)' = x\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{\sqrt{(y^2+z^2)^3}}\cdot 2y = \frac{-xy}{\sqrt{(y^2+z^2)^3}}$

$u'_z = \left(\frac{x}{\sqrt{(y^2+z^2)^3}}\right)'_z = \ldots = -\frac{xz}{\sqrt{y^2+z^2}}$

$du = \frac{1}{\sqrt{y^2+z^2}}\,dx + \frac{-xy}{\sqrt{(y^2+z^2)^3}}\,dy + \frac{-xz}{\sqrt{(y^2+z^2)^3}}\,dz =$

$= \frac{dx}{\sqrt{y^2+z^2}} - \frac{xy\,dy + xz\,dz}{\sqrt{(y^2+z^2)^3}}$

Д/З - отчёт

- Для себя → перечит + потрен ([illegible])

- Тема 5 и 6 - консп теории