Интегралы и дифференциальные уравнения.

Отчёт по лекции и домашней работе от 02.11.2020

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Практика Т 5

№ 11.4.1

$z = e^{x^2+y^2}$ $x = a\cdot\cos t$

$y = a\cdot\sin t$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{dz}{dy}\cdot\frac{dy}{dt}$$

1 сп.

$$z = e^{x^2+y^2} = e^{(a\cdot\cos t)^2 + (a\cdot\sin t)^2} = e^{a^2(\cos t^2 + \sin t^2)} = e^{a^2}$$

здесь, x' и y''

то есть e^{a^2} - const

$$\frac{dz}{dt} = \left(e^{a^2}\right)'_t = 0$$

2 сп.

$$\frac{dz}{dx} = \left(e^{x^2+y^2}\right)'_x = e^{x^2+y^2}\cdot(x^2+y^2)_x = e^{x^2+y^2}\cdot 2x$$

$$\frac{dz}{dy} = 2y\cdot e^{x^2+y^2}$$

$$\frac{\partial x}{dt} = (a\cdot\cos t)'_t = -a\cdot\sin t$$

$$\frac{dy}{dt} = (a\cdot\sin t)'_t = a\cdot\cos t$$

Отсюда

$$\frac{dz}{dt} = 2x\cdot e^{x^2+y^2}\cdot(-a\sin t) + 2y\cdot e^{x^2+y^2}\cdot a\cdot\cos t =$$

(подставим)

$$= 2a\cdot\cos t\cdot e^{x^2+y^2}\cdot -a\sin t + 2a\sin t\cdot e^{x^2+y^2}\cdot a\cos t =$$

$$= 2a^2\cdot\sin t\cdot\cos t\cdot e^{x^2+y^2} - 2a^2\cdot\sin t\cdot\cos t\cdot e^{x^2+y^2}$$

№ 11.4.2

$z = x^5 + 2xy - y^3$ $x = \cos 2t$ $y = \arcsin t$

1) $\frac{\partial z}{\partial x}\ (x^5 + 2xy - y^3)'_x = 5x^4 + 2y$

$\frac{\partial z}{\partial y}\ (x^5 + 2xy - y^3)'_y = 2x - 3y^2$

2) $\frac{dx}{dt}\ (\cos 2t)'_t = -2\sin 2t$

$$\frac{\partial y}{\partial t} = (arctg\, t)'_t = \frac{1}{1+t^2}$$

3) $$\frac{dz}{dt} = (5x^4+2y)\cdot(-2\sin 2t) + (2x-3y^2)\cdot\frac{1}{1+t^2} =$$

$$= -2(5x^4+2y)\cdot\sin 2t + (2x-3y^2)\cdot\frac{1}{1+t^2}$$

11.4.3 $z = xy + xyV + y\nu^V$

$x = \sin t$

$y = \ln t$

$\nu = e^t$

$V = arctg\, t$

$$\frac{dz}{dt}$$

$z(x, y, \nu, V)$

$x(t)$ $y(t)$ $\nu(t)$ $V(t)$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial z}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial z}{\partial \nu}\cdot\frac{d\nu}{dt} + \frac{\partial z}{\partial V}\cdot\frac{dV}{dt}$$

$$\frac{\partial z}{\partial x} = y + yV$$

$$\frac{\partial z}{\partial y} = x + xV + \nu^V$$

$$\frac{\partial z}{\partial \nu} = yV$$

$$\frac{dz}{dV} = xy + y\nu^V$$

$$\frac{dx}{dt} = \cos t$$

$$\frac{dy}{dt} = \frac{1}{t}$$

$$\frac{d\nu}{dt} = e^t$$

$$\frac{dV}{dt} = \frac{1}{1+t^2}$$

Повт!

3) $\frac{dz}{dt} = (y + yv) \cdot \cos t + (x + xv + vr) \cdot \frac{1}{t} + yv \cdot e^t +$

$+ (xy + yv) \cdot \frac{1}{1+t^2} = y \cdot (1+v) \cdot \cos t + \frac{x + xv + vr}{t} +$

$+ yv \cdot e^t + \frac{(x + v) + y}{1+t^2}$

понял!

$z = 3^{x^2} \cdot \operatorname{arctg} y$

$x = \frac{v}{V}$

$y = v \cdot V$

$\frac{dz}{dv}$?

$\frac{dz}{dV}$?

$\frac{dz}{dx} = (3^{x^2} \cdot \operatorname{arctg} y)'_x = 3^{x^2} \cdot \ln 3 \cdot 2x \cdot \operatorname{arctg} y =$

$= 2x \cdot 3^{x^2} \cdot \ln 3 \cdot \operatorname{arctg} y$

$\frac{dz}{dy} = (3^{x^2} \cdot \operatorname{arctg} y)'_y = \frac{1}{1+y^2} \cdot 3^{x^2}$

$\frac{dx}{dv} = \left(\frac{v}{V}\right)'_v = \frac{1}{V}$

$\frac{dx}{dV} = \left(\frac{v}{V}\right)'_V = v \cdot -1 \cdot \frac{1}{V^2} = -\frac{v}{V^2}$

$\frac{dy}{dv} = (v \cdot V)' = V$

$\frac{dy}{dV} = (v \cdot V)' = v$

$\frac{dz}{dv} = \frac{dz}{dx} \cdot \frac{dx}{dv} + \frac{dz}{dy} \cdot \frac{dy}{dv} = (2x \cdot 3^{x^2} \cdot \ln 3 \cdot \operatorname{arctg} y) \cdot \frac{1}{V}$

$+ \left(\frac{1}{1+y^2} \cdot 3^{x^2}\right) \cdot V = 3^{(v/V)^2} \cdot 2 \cdot \frac{v}{V} \cdot \ln 3 \cdot \operatorname{arctg}(vV) \cdot \frac{1}{V} + 3^{(v/V)^2} \cdot$

$\cdot \frac{1}{1+(vV)^2} \cdot V$

$\frac{dz}{dV} = (2x \cdot 3^{x^2} \cdot \ln 3 \cdot \operatorname{arctg} y)\left(-\frac{v}{V^2}\right) + \left(\frac{1}{1+y^2} \cdot 3^{x^2}\right) \cdot$

$\cdot v = -3^{(v/V)^2} \cdot \left(2 \cdot \frac{v^2}{V^3} \cdot \ln 3 \cdot \operatorname{arctg}(vV) - \frac{v}{1+(vV)^2}\right)$

$$z=\frac{x^2}{y} \qquad x=v-2V \qquad y=2v+V$$

1) $$dz=\frac{\partial z}{\partial x}\cdot dx+\frac{\partial z}{\partial y}\cdot dy$$

$$dx=\frac{\partial x}{\partial v}\cdot dv+\frac{\partial x}{\partial V}\cdot dV$$

$$dy=\frac{\partial y}{\partial v}\cdot dv+\frac{\partial y}{\partial V}\cdot dV$$

2) $$\frac{\partial z}{\partial x}=\frac{1}{y}\cdot 2x=\frac{2x}{y}$$

$$\frac{\partial z}{\partial y}=x^2\cdot(-1)\cdot y^{-1-1}=-\frac{x^2}{y^2}$$

3) $$\left.\begin{aligned}\frac{\partial x}{\partial v}&=(v-2V)'=1\\ \frac{\partial x}{\partial V}&=(v-2V)'=-2\end{aligned}\right\}\ dx=1dv-2dV$$

4) $$\left.\begin{aligned}\frac{\partial y}{\partial v}&=(2v+V)'=2\\ \frac{\partial y}{\partial V}&=(2v+V)'=1\end{aligned}\right\}\ dy=2dv+2dV$$

5) Подставим результаты действ 1, 3, 4

$$dz=\underbrace{\frac{2x}{y}}_{\frac{2x}{2y}}\cdot\underbrace{(dv-2dV)}_{3y}-\underbrace{\frac{x^2}{y^2}}_{\frac{x}{2y}}\underbrace{(2dv+dV)}_{4y}=$$

$$=\frac{2x}{y}\cdot dv-\frac{2x}{y}\cdot 2dV-\frac{x^2}{y^2}\cdot 2dv-\frac{x^2}{y^2}dV=$$

$$=dv\left(\frac{2x}{y}-\frac{2x^2}{y}\right)+\left(-\frac{4x}{y}dV-\frac{x^2}{y^2}dV\right)=$$

$$=\frac{x}{y}\cdot 2\cdot\left(1-\frac{x}{y}\right)dv-\frac{x}{y}\left(4+\frac{x}{y}\right)dV=\frac{v-2V}{(2v+V)^2}\left(2(v+3V)dv-\right.$$

$$\left.-(9v+2V)dV\right)$$

дата

ФИО

консп

Решения контрол

Домашняя работа

№ 11.4.4

$z = x^2 + y^2 + xy$

$x = a \cdot \sin t$

$y = a \cdot \cos t$

$$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$$

1) $\frac{dz}{dx} = (x^2 + y^2 + xy)'_x = 2x + y$

$\frac{dz}{dy} = (x^2 + y^2 + xy)'_y = 2y + x$

2) $\frac{dx}{dt} = (a \cdot \sin t)'_t = a\cos t$

$\frac{dy}{dt} = (a \cdot \cos t)' = -a\sin t$

3) $\frac{dz}{dt} = (2a\sin t + a\cos t) \cdot a\cos t + (2a\cos t + a\sin t) \cdot (-a\sin t)$

$= a^2\cos t \cdot (2\sin t + \cos t) - a^2\sin t\,(2\cos t + \sin t)$

№ 11.4.5

$z = \cos(2t + 4x^2 - y)$

$x = \frac{1}{t}$

$y = \frac{\sqrt{t}}{\ln t}$

1) $\frac{dz}{dx} = (\cos(2t + 4x^2 - y))'_x = 8x \cdot -\sin(2t + 4x^2 - y) =$

$= -8x\sin(2t + 4x^2 - y)$

$\frac{dz}{dy} = \sin(2t + 4x^2 - y)$

2) $\frac{dx}{dt} = \left(\frac{1}{t}\right)'_t = -\frac{1}{t^2}$

$$\frac{dy}{dt}=\left(\frac{\sqrt{t}}{\ln t}\right)'_t=\frac{(\sqrt{t})'\cdot\ln t-(\ln t)'\cdot\sqrt{t}}{(\ln t)^2}=$$

$$=\frac{\frac{1}{2\sqrt{t}}\cdot\ln t-\sqrt{t}\cdot\frac{1}{t}}{(\ln t)^2}$$

3) $$\frac{dz}{dt}=\left(\frac{8}{t}\cdot\sin\left(2t+4\cdot\frac{64}{t^2}-\frac{\sqrt{t}}{\ln t}\right)\right)\cdot-\frac{1}{t^2}$$

$$+\sin\left(2t+4\cdot\frac{64}{t^2}-\frac{\sqrt{t}}{\ln t}\right)\cdot\frac{\frac{1}{2\sqrt{t}}\cdot\ln t-\sqrt{t}\cdot\frac{1}{t}}{(\ln t)^2}=$$

$$=\left(\frac{8}{t^3}+\frac{\frac{1}{2\sqrt{t}}\cdot\ln t-\sqrt{t}\cdot\frac{1}{t}}{(\ln t)^2}\right)\cdot\sin\left(2t+\frac{256}{t^2}-\frac{\sqrt{t}}{\ln t}\right)$$

№ 11.4.6

$z=x^2y^3u$,

$x=t$

$y=t^2$

$u=\sin t$

1) $$\frac{dz}{dx}=(x^2y^3u)'_x=2xy^3u$$

2) $$\frac{dz}{dy}=(x^2y^3u)'_y=3y^2x^2u$$

$$\frac{dz}{du}=(x^2y^3u)'_u=x^2y^3$$

2) $$\frac{dx}{dt}=(t)'_t=1$$

$$\frac{dy}{dt}=(t^2)'_t=2t$$

$$\frac{du}{dt}=\cos t$$

3) $$\frac{dz}{dt}=2xy^3u+3y^2x^2u\cdot 2t+x^2y^3\cdot\cos t=$$

$$=2\cdot t\cdot(t^2)^3\cdot\sin t+3(t^2)^2\cdot t^2\cdot\sin t\cdot 2t+t^2\cdot(t^2)^3\cdot\cos t=$$

$$=2t^7\cdot\sin t+6t^7\cdot\sin t+t^8\cdot\cos t=$$

$$=t^7(2\sin t+6\sin t+t\cos t)=t^7(8\sin t+t\cos t)$$

№ 11.4.7

$z=e^{xy}\ln(x+y)$,

$x=t^3$

$y=1-t^3$

1) $$\frac{dz}{dx}=(e^{xy}\cdot\ln(x+y))'_x=ye^{xy}\cdot\ln(x+y)+e^{xy}\cdot\frac{1}{x+y}$$

$$\frac{dz}{dy} = xe^{xy} \cdot \ln(x+y) + e^{xy} \cdot \frac{1}{x+y}$$

$$2)\ \frac{dx}{dt} = (t^3)'_t = 3t^2$$

$$\frac{dy}{dt} = (1 - t^3)' = -3t^2$$

$$3)\ \frac{dz}{dt} = \left(ye^{xy} \cdot \ln(x+y) + e^{xy} \cdot \frac{1}{x+y}\right) \cdot 3t^2 - \left(xe^{xy} \cdot \ln(x+y) + e^{xy} \cdot \frac{1}{x+y}\right) \cdot -3t^2 = 3t^2\left(\left(e^{xy} \cdot \ln(x+y)\right)(y-x)\right)$$

№11.48

$z = xy \operatorname{arctg}(xy)$; $x = t^2+1$; $y = t^3$

$$1)\ \frac{dz}{dx} = (xy \operatorname{arctg}(xy))'_x = y \cdot \left(x' \cdot \operatorname{arctg}(xy) + x \cdot \frac{1}{1+x^2y^2}\right)$$

$$= y \operatorname{arctg}(xy) + \frac{xy}{1+x^2y^2}$$

$$\frac{dz}{dy} = (xy \cdot \operatorname{arctg}(xy))'_y = x \cdot \operatorname{arctg}(xy) + \frac{xy}{1+x^2y^2}$$

$$2)\ \frac{dy}{dt} = 3t^2$$

$$\frac{dx}{dt} = 2t$$

$$3)\ \frac{dz}{dt} \cancel{= t\left((x+y) \cdot \operatorname{arctg}(xy) + 2\frac{xy}{1+x^2y^2}\right)}$$

$$3)\ \frac{dz}{dt} = t\left(2\left(y \cdot \operatorname{arctg}(xy) + \frac{xy}{1+x^2y^2}\right) + 3t\left(x \operatorname{arctg}(xy) + \frac{xy}{1+x^2y^2}\right)\right)$$

$$= t\left(2\left(t^3 \cdot \operatorname{arctg}(t^5+t^3) + \frac{t^5+t^3}{1+[illegible]}\right) + 3t\left((t^2+1) \cdot \operatorname{arctg}(t^5+t^3) + \frac{t^5+t^3}{[illegible]}\right)\right)$$

№11.49

$z = e^{2x-3y}$

$x = \operatorname{tg} t$

$y = t^2 - t$

$$1)\ \frac{dz}{dy} = (e^{2x-3y})'_y = -3 \cdot e^{2x-3y}$$

$$\frac{dz}{dx} = (e^{2x-3y})'_x = 2 \cdot e^{2x-3y}$$

$$2)\ \frac{dx}{dt} = (\operatorname{tg} t)'_t = \frac{1}{\cos^2 t}$$

$$\frac{dy}{dt} = 2t - 1$$

$$3)\ \frac{dz}{dt} = 2e^{2x-3y} \cdot \frac{1}{\cos^2 t} - 3 \cdot e^{2x-3y}(2t-1) =$$

$$= e^{2x-3y}\left(\frac{2}{\cos^2 t} - 6t + 3\right) = e^{2\operatorname{tg} t - 3t^2 + 3t}\left(\frac{2}{\cos^2 t} - 6t + 3\right)$$

№ 1.4.10

$z = x^y$

$x = \ln t$

$y = \sin t$

1) $\frac{dz}{dx} = (x^y)'_x = y \cdot x^{y-1}$

$\frac{dz}{dy} = x^y \cdot \ln x$

2) $\frac{dx}{dt} = \frac{1}{t}$

$\frac{dy}{dt} = +\cos t$

$\frac{dz}{dt} = \left(\sin t \cdot (\ln t)^{\sin t - 1}\right) \cdot \frac{1}{t} + \left((\ln t)^{\sin t} \cdot \ln(\ln t)\right) \cdot$

$\cdot \cos t$