13.04.20 практика

№ 711.

1) $y = 3x^2$ $\quad y' = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$?

$\Delta y = f(x+\Delta x) - f(x)$

$\Delta y = 3\cdot(3+\Delta x)^2 - 3x^2 = 3(x^2 + 2x\cdot\Delta x + \Delta x^2) - 3x^2 =$

$= 3x^2 + 6x\Delta x + \Delta x^2 - 3x^2 = 3\Delta x\cdot(2x + \Delta x)$

$f'(x) = \lim\limits_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{3\cdot\Delta x(2x+\Delta x)}{\Delta x} =$

$= \lim\limits_{\Delta x \to 0} 3(2x+\Delta x) = 3\cdot(\lim\limits_{\Delta x \to 0}(2x) + \lim\limits_{\Delta x \to 0}\Delta x =$

$= 6x$

2) $y = \sin x$

$\Delta y = \sin(x+\Delta x) - \sin(x) = 2\cdot\sin\frac{x+\Delta x - x}{2}\cdot\cos\frac{x+\Delta x + x}{2} = 2\cdot\sin\frac{\Delta x}{2}\cdot\cos(x+\frac{\Delta x}{2})$

$y' = \lim\limits_{\Delta x \to 0}\frac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0}\frac{2\cdot\sin\frac{\Delta x}{2}\cdot\cos(x+\frac{\Delta x}{2})}{\Delta x} =$

$(\lim\limits_{\Delta x \to 0}\cos(x+\Delta x))\cdot\lim\limits_{\Delta x \to 0}\left(\frac{2\cdot\sin\frac{\Delta x}{2}}{\Delta x}\right) =$

$\lim 0/0 = 1$

$= \cos\lim\limits_{\Delta x}\cos x \cdot 1 = \cos x$

№ 716

1) $A(x) = \frac{9}{\sqrt[3]{x^2}} - 5^{x+1} = 9\cdot x^{-\frac{2}{3}} - 5^{x+1} =$

$= [(x^\alpha)' = \alpha\cdot x^{\alpha-1};\ (a^x)' = a^x\cdot\ln a;$

$A'(\varphi(x)) = A'_{\varphi(x)}\cdot\varphi';\ (U+V)' = U'+V']=$

$= (9\cdot x^{-2/3})'_x - (5^{x+1})'_x = 9\cdot(x^{-2/3})_x - 5^{x+1}(x+1)'_x\cdot$

$(x+1)' = 9\cdot(-\frac{2}{3})\cdot x^{-\frac{2}{3}-1} - 5^{x+1}\cdot\ln 5\,(x'+1') = -6\cdot x^{-\frac{5}{3}} -$

$- 5^{x+1}\cdot\ln 5\cdot 1 = -6\cdot x^{-\frac{5}{3}} - 5^{x+1}\cdot\ln 5$

2) $p(x) = (x^4 - x)(3\operatorname{tg}x - 1)$

$p'(x) = ((x^4 - x)(3\operatorname{tg}x - 1))' = (x^4 - x)' \cdot (3\operatorname{tg}x - 1) +$

$+ (x^4 - x) \cdot (3\operatorname{tg}x - 1)' = (4x^3 - 1)(3\operatorname{tg}x - 1) + (x^4 - x)\left(3 \cdot \frac{1}{\cos^2 x}\right) =$

№ 71.27

$p'(\varphi(x)) = p'_{\varphi(x)} \cdot \varphi'_x$

1) a) $y = \sin^2 x$

$y'_x = (\sin^2 x)'_x = (\sin x)^2)'_x =$

$= 2 \cdot (\sin x)'_x = +2 \cdot \cos x \sin x$

б) $y'_x = (\sin x)' = \cos x \cdot \cos x =$

$2 \sin \cdot \cos$

2) $y = \ln \operatorname{arctg}(3x)$

1- $\ln(?)' = \frac{1}{?}$

2- $\operatorname{arctg}?' = \frac{1}{1+?^2}$

3- $3x = 3$

$\frac{1}{\operatorname{arctg}(3x)} \cdot \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{(1+9x^2) \cdot \operatorname{arctg}(3x)}$

№ 7.1.58

$(\ln y)' = \frac{y'}{y} \Rightarrow y' = y \cdot (\ln y)'_x$

1) $y = x^{\sin x}$

$\ln y = \ln x^{\sin x}$ $\quad (\log_a b^c = c \cdot \log_a b)$

$\ln y = \sin x \cdot \ln x$

$(\ln y)' = (\sin x \cdot \ln x)'$

$$\frac{y'}{y} = (\sin x)^{\sin x} \cdot \ln x + \sin x \cdot (\ln x)'$$

$$= y' = y \cdot (\cos x \cdot \ln x + \frac{\sin x}{x}) = x^{\sin x} \cdot (\cos x \cdot \ln x + \frac{\sin x}{x})$$

$$y = \frac{(x-1)^3 \sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$$

$$\ln y = \ln \frac{(x-1)^3 \sqrt{x+2}}{\sqrt[3]{(x+1)^2}}$$

$$\ln y = \ln (x-1)^3 + \ln \sqrt{x+2} - \ln \sqrt[3]{(x+1)^2}$$

$$\ln y = 3 \cdot \ln (x-1) + \frac{1}{2} \cdot \ln (x+2) + \frac{2}{3} \ln (x+1)$$

$$(\ln y)' = (3 \ln (x-1) + \frac{1}{2} \cdot \ln (x+2) - \frac{2}{3} \cdot \ln (x+1))'$$

$$\frac{y'}{y} = 3 \cdot \frac{1}{x-1} \cdot (x-1)' + \frac{1}{2} \cdot \frac{1}{x+2} \cdot (x+2)' - \frac{2}{3} \cdot \frac{1}{x+1} \cdot (x+1)'$$

$$y' = y \cdot \left(\frac{3}{x-1} + \frac{1}{2(x+2)} - \frac{2}{3(x+1)} \right) =$$

$$= \frac{(x-1)^3 \cdot \sqrt{x+2}}{\sqrt[3]{(x+1)^2}} \cdot \left(\frac{3}{x-1} + \frac{1}{2(x+2)} - \frac{2}{3(x+1)} \right)$$