

27325

$$\lim_{x \rightarrow 0} (\cos 2x)^{\frac{1}{x^2}} = [1^\infty]$$

$$y = (\cos 2x)^{\frac{1}{x^2}}$$

$$\ln y = \ln (\cos 2x)^{\frac{1}{x^2}} = \frac{1}{x^2} \cdot \ln (\cos 2x)$$

$$\lim_{x \rightarrow 0} (\ln y) = \ln \left(\lim_{x \rightarrow 0} y^x \right) = 0$$
$$\lim_{x \rightarrow 0} y = e^0$$

$$\lim_{x \rightarrow 0} y = 1$$

$$\lim_{x \rightarrow 0} (\ln \cdot \cos 2x / 2) = \left[\frac{0}{0} \right] = \left(\left(\frac{1}{\cos 2x} \cdot (-2 \cdot \sin(2x)) \right) \cdot 2x \right)$$

$$= \lim_{x \rightarrow 0} \left(-2 \frac{\sin 2x}{\cos 2x} \cdot \frac{1}{2x} \right) = \lim_{x \rightarrow 0} \left(-\frac{\sin 2x}{x \cos 2x} \right) = \left[\frac{0}{0} \right] =$$

$$\lim_{x \rightarrow 0} \left(\frac{2}{\cos(2 \cdot x)} \right) = -\frac{2}{1} = -2$$

$$\lim_{x \rightarrow 0} (\ln y) = -2$$

$$\lim_{x \rightarrow 0} y = e^{-2} = \frac{1}{e^2}$$

$$\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)^{x^2} = [\infty^0]$$

$$y = \left(\frac{1}{x} \right)^{x^2} = \frac{1}{x^{x^2}} = \frac{1}{e^{x^2 \ln x}}$$

$$\ln y = \ln \frac{1}{e^{x^2 \ln x}} = -x^2 \ln x$$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \ln \left(\frac{1}{x} \right)^{x^2} = \lim_{x \rightarrow 0} x^2 \ln \frac{1}{x} = 0$$

$$= \lim_{x \rightarrow 0} -x^2 \ln x = 0$$

$$\lim_{x \rightarrow 0} \ln y = 0$$

$$\lim_{x \rightarrow 0} y = \lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{x^2} = 1 \quad \{e^0\}$$

~ 7.3.2*

$$\lim_{x \rightarrow 0} x \frac{1}{1 + \ln x}$$

$$\lim_{x \rightarrow 0} (\ln y) = \lim_{x \rightarrow 0} \frac{1}{1 + \ln x} \cdot \ln x = \lim_{x \rightarrow 0} \frac{\ln x}{1 + \ln x} = \left[\frac{\infty}{\infty} \right]$$

$$\lim_{x \rightarrow 0} \frac{\left(\frac{1}{x}\right) \cdot (1 + \ln x) - \left(\frac{1}{x}\right) \cdot \ln x}{(1 + \ln x)^2} =$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x} + \ln x / x - \ln x / x}{(1 + \ln x)^2} = \lim_{x \rightarrow 0} \frac{\frac{1}{x} (\cancel{1 + \ln x} - \ln x)}{(1 + \ln x)^2}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{1}{x} \cdot \frac{1}{(1 + \ln x)^2}}{\lim_{x \rightarrow 0} x (1 + \ln x)^2} = \frac{1}{0} = \infty$$