№ 6.4 46

1) $\lim\limits_{x\to\infty}\left(1+\frac{k}{x}\right)^x, \; k\in R$

$$\lim_{x\to\infty}\left(1+\frac{k}{x}\right)^x = \left[\left(1+\frac{k}{\infty}\right)^\infty = (1)^\infty\right] = \lim_{x\to\infty}\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^x =$$

$$= \lim_{x\to\infty}\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^x = \lim_{x\to\infty}\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^{\frac{x}{k}\cdot k} =$$

$$= \lim_{x\to\infty}\left(\left(1+\frac{1}{\left(\frac{x}{k}\right)}\right)^{\frac{x}{k}}\right)^k = \lim_{x\to\infty}{}^k\left(1+\frac{1}{\frac{x}{k}}\right)^{\frac{x}{k}} = \left[y=\frac{x}{k},\; x\to\infty,\; k\in R \Rightarrow y\to\infty\right]$$

$$= \lim_{y\to\infty}{}^k\left(1+\frac{1}{y}\right)^y = e^k$$

2) $\lim\limits_{x\to 0}\sqrt[x]{1+5x} = \lim\limits_{x\to 0}(1+5x)^{1/x} = \left[(1^\infty)\right] = \left[\lim\limits_{x\to 0}(1+x)^{\frac{1}{x}} = e\right] =$

$$\left[y=5x,\; x\to 0 \Rightarrow y\to 0;\; x=\frac{y}{5}\right] = \lim_{y\to 0}(1+y)^{\frac{1}{y/5}} = \lim_{y\to 0}(1+y)^{\frac{1}{y}\cdot 5} =$$

$$= \left(\lim_{y\to 0}(1+y)^{1/y}\right)^5 = e^5$$

3) $\lim\limits_{x\to\infty}\left(\frac{x+3}{x-2}\right)^x = \left[\left(\frac{\infty}{\infty}\right)^\infty\right] = \lim\limits_{x\to\infty}\left(\frac{x(1+\frac{3}{x})}{x(1-\frac{2}{x})}\right)^x =$

$= \lim\limits_{x\to\infty}\left(\frac{1+\frac{3}{x}}{1-\frac{2}{x}}\right)^x = \lim\limits_{x\to\infty}\frac{(1+3/x)^x}{(1-2/x)^x} =$

$\frac{\lim\limits_{x\to\infty}(1+3/x)^x}{\lim\limits_{x\to\infty}(1-2/x)^x} = \frac{e^3}{e^{-2}} = e^5$

4) $\lim\limits_{x\to 0}\frac{e^{2x}-1}{7x} = \left[\frac{e^{2\cdot 0}-1}{7\cdot 0} = \frac{e^0-1}{0} = \frac{1-1}{0} = \frac{0}{0}\right] =$

$= \left[\lim\limits_{x\to 0}\frac{e^{2x}-1}{7x};\ y=2x \Rightarrow x=\frac{y}{2};\ x\to 0 \Rightarrow y\to 0\right] = \lim\limits_{y\to 0}\frac{e^y-1}{7\cdot\frac{y}{2}} =$

$= \lim\limits_{y\to 0}\frac{2}{7}\cdot\frac{e^y-1}{y} = \frac{2}{7}$

№6.4.48

$$\lim_{x \to 0} \sqrt[2x]{1+3x} = [1] =$$

$$= \begin{bmatrix} 3x = y \\ x = \frac{y}{3} \end{bmatrix} =$$

$$= \lim_{x \to 0} (1+y)^{\frac{3}{2y}} = \frac{3}{2} e$$

№6.4.49

$$\lim_{x \to \infty} \left(\frac{x-5}{x+4} \right)^{x} = \left[\left(\frac{\infty}{\infty} \right)^{\infty} \right] = \lim_{x \to \infty} \left(\frac{1 - \frac{5}{x}}{1 + \frac{4}{x}} \right)^{x} =$$

$$= \frac{e^{5}}{e^{4}} = e^{-9}$$

№6.4.50

$$\lim_{x \to 0} \left(\frac{3 + 5x}{3 + 2x} \right)^{\frac{1}{x}} = \left[\left(\frac{3}{3} \right)^{\infty} \right] =$$

$$= \lim_{x \to 0} \left(\frac{1 + \frac{5}{3}x}{1 + \frac{2}{3}x} \right)^{1/x} = \frac{e^{\frac{5}{3}}}{e^{2/2}} = e^{3/3} = e^{1}$$

№6.451

$$\lim_{x\to 2} \frac{e^x - e^2}{x-2} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 2} \frac{e^2(e^{x-2} - 1)}{e\cdot(x-2)} = e^2 \cdot 1$$

№6.452

$$\lim_{x\to\infty} \left(\frac{5-x}{6-x}\right)^{x+2} = \lim_{x\to\infty} \left(\frac{-x(1-\frac{5}{x})}{-x(1-\frac{6}{x})}\right)^{x+2} =$$

$$\lim_{x\to\infty} \left(\frac{1+\frac{-5}{x}}{1+\frac{-6}{x}}\right)^{x} \cdot \lim_{x\to\infty} \left(\frac{1+\frac{-5}{x}}{1+\frac{-6}{x}}\right)^{2} =$$

$$= \frac{-5}{-6} \cdot 1 = \frac{5}{6}$$

№6453

$$\lim_{x\to e} \frac{\ln x - 1}{x-e} = \frac{\ln(\frac{x}{e})}{x-e} t = \frac{\log_e x}{x-e} =$$

$$= \frac{e}{e-e} = \frac{1}{0} = \infty$$

№6.4

$$\lim_{x\to 0}$$

№6.4

$$\lim_{x\to}$$

$$= \lim_{x\to}$$

№6.4.54

$$\lim_{x\to 0}(1-\sin x)^{\frac{1}{\sin x}} = e$$

№6.4.55

$$\lim_{x\to\infty} x\left[\ln(x+3)-\ln x\right] =$$

$$= \lim_{x\to\infty} x\cdot \ln\left(1+\frac{3}{x}\right) = \lim_{x\to\infty} x\cdot \ln 1\cdot \ln\frac{3}{x} = 0$$