$= \ln|x^2+x+1| - 2\sqrt{3}\,\mathrm{arctg}\,\frac{\sqrt{3}(2x+1)}{3} + C$

$(12\,\mathrm{Oz})$

4-8 3 12

1) $\int \frac{7x+4}{(x-3)(x+2)}\,dx = \left[\begin{array}{l} (x-3)(x+2)=0 \\ x_1=3;\ x_2=-2 \\ \frac{7x+4}{(x-3)(x+2)} = \frac{A^{(x+2)}}{x-3} + \frac{B^{(x-3)}}{x+2} = \frac{A(x+2)+B(x-3)}{(x-3)(x+2)} \end{array}\right.$

$7x+4 = A(x+2) + B(x-3)$

1способ (метод неопр коэф)

$7x+4 = \underline{Ax} + \underline{\underline{2A}} + \underline{Bx} - \underline{\underline{3B}}$

$7x+4 = x(A+B) + (2A-3B)$

Повт СЛАУ

$\begin{cases} 7 = A+B \\ 4 = 2A-3B \end{cases}$ $\begin{cases} A = 7-B \\ 4 = 14-2B-3B \end{cases}$

$+C$

$\begin{cases} B = \boxed{2} \\ A = 7-2 = \boxed{5} \end{cases}$

2способ (метод частных значений)

$7x+4 = A(x+2) + B(x-3)$

$x_1 = 3$

$x_2 = -2$

$-14+4 = 0 + (-5B) \Rightarrow B = 2$

$21+4 = A\cdot 5 + \underset{\sim}{0}$

$A = 25:5 = 5$

$$\frac{[illegible]}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}\Bigg] =$$

$$= \int \frac{[illegible]}{x-1}dx + \int \frac{[illegible]}{x+2}dx = 3\ln|x-1| + 2\ln|x+2| + C$$

5) $$\int \frac{x^2+3x-2}{(x-1)(x+1)^2}dx = \Big[[illegible]\Big] = \int \frac{x^2+3x-2}{(x-1)(x+1)^2} =$$

$$\frac{x^2+3x-2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{(x+1)^2} + \frac{C}{x+1} =$$

$$\Rightarrow x^2+3x-2 = A(x+1)^2 + B(x-1) + C(x^2-1)$$

1 способ

$$x^2+3x-2 = Ax^2+2Ax+A + Bx - B + Cx^2 - C$$

$$x^2+3x-2 = x^2(A+C) + x(2A+B) + (A-B-C)$$

$$\begin{cases} A+C=1 \\ 2A+B=3 \\ A-B-C=-2 \end{cases} \quad \begin{cases} A=1-C \\ B=3-2A \\ A-B-C=-2 \end{cases} \quad \begin{cases} A=1-C \\ B=3-2+2C \\ A-B-C=-2 \end{cases}$$

$$\begin{cases} 1-C-(3+2C)-C=-2 \end{cases}$$

$$-4C=0$$

$$C=0$$

$$\begin{cases} C=0 \\ B=3 \\ A=1 \end{cases}$$

2 способ

$$x^2+3x-2 = A(x+1)^2 + B(x-1) + C(x^2-1)$$

$x=1$

$$1+3-2 = A\cdot 4 + B\cdot 0 + C\cdot 0$$

$$4 = A\cdot 4$$

$$A=1$$

$x=-1$

$$1-3-2 = A\cdot 0 + (-2B) + C\cdot 0$$

$$-6 = -2B$$

$$B=3$$

$x=0$; $A=1$; $B=3$

$$-2 = 1 - 3 - C$$

$$-2 = -2 - C$$

$$-C=0$$

$$C=0$$

$$\begin{cases} A=1 \\ B=3 \\ C=0 \end{cases}$$

$\frac{[illegible]}{(x+1)(x+1)^2} = \frac{1}{x+1} + \frac{-3}{(x+1)^2} + \frac{0}{x+1} = \frac{1}{x+1} - \frac{3}{(x+1)^2} \Big]$

$= \int \frac{1}{x+1}\,dx + \int \frac{3}{(x+1)^2}\,dx = \ln|x+1| + 3 \cdot \frac{-1}{(x+1)} + C =$

$= \ln|x+1| - \frac{3}{(x+1)} + C$

$7) \int \frac{x^5-1}{x^3+x^2+x}\,dx = \int \frac{x^5-1}{x(x^2+x+1)}\,dx = \Big[$

$$\begin{array}{r|l} x^5 + 0\cdot x^4 + 0\cdot x^3 + 0\cdot x^2 + 0\cdot x - 1 & x^3+x^2+x \\ \hline -\ x^5 + x^4 + x^3 & x^2-x \\ \hline -x^4 - x^3 + 0x^2 + 0\cdot x - 1 & \\ -\ -x^4 - x^3 - x^2 & \\ \hline \underbrace{+x^2 + 0\cdot x - 1}_{\text{остаток < делителя}} & \end{array}$$

$x^5 - 1 = (x^3+x^2+x)(x^2-x) + (x^2-1) \Big] =$

$= \int \frac{(x^3+x^2+x)(x^2-x)+(x^2-1)}{x(x^2+x+1)}\,dx = \int \frac{(x^3+x^2+x)(x^2-x)}{x(x^2+x+1)}\,dx + \int \frac{(x^2-1)}{x(x^2+x+1)}\,dx =$

$\int \left(\frac{x(x^2+x+1)(x^2-x)}{x(x^2+x+1)} + \frac{x^2-1}{x(x^2+x+1)} \right) dx =$

$= \int x^2\,dx - \int x\,dx + \int \frac{x^2-1}{x(x^2+x+1)}\,dx = \Big[[illegible]$

$\frac{x^2-1}{x(x^2+x+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+x+1}$

$x^2-1 = A(x^2+x+1) + (Bx+C)\cdot x$

I сп.

$x^2-1 = Ax^2 + Ax + A + Bx^2 + Cx$

$x^2-1 = (A+B)x^2 + (A+C)x + A$

$\begin{cases} A+B=1 \\ A+C=0 \\ A=-1 \end{cases} \rightarrow \begin{cases} B=2 \\ C=1 \\ A=-1 \end{cases}$

II сп.

$x^2-1 = A(x^2+x+1) + (Bx+C)x$

$x=0$

$-1 = A + 0$

$A=-1$

$x=1$

$0 = -1(3) + B + C$

$3 = B + C$

$x = -1$

$0 = -1 + B - C$

$B - C = 1$

$\begin{cases} B+C=3 \\ B-C=1 \end{cases}$

$\rightarrow \{ B-C=1 \quad [illegible]$

$\begin{cases} B=2 \\ [illegible] \\ C=1 \end{cases}$

$= \frac{-1}{x} + \frac{2x+1}{x^2+x+1}$

$= -\int \frac{dx}{x} + \int \frac{2x+1}{x^2+x+1}dx$

$= \int x^2dx - \int x\,dx + \int \frac{dx}{x} + \int \frac{2x+1}{x^2+x+1}dx =$

ДЗ

№8.3.8(б)

+ повт. реш.

Прилож. №8.3.5–8.3.8

$= \int x^2 dx - \int x\,dx - \int \frac{dx}{x} + \int \frac{2x+1}{x^2+x+1}dx =$

$\frac{x^3}{3} - \frac{x^2}{2} - \ln|x| + \ln(x^2+x+1) + C$

№8.3.9

$\int \frac{dx}{(x^2+1)^3} = \frac{2\cdot3-3}{2\cdot(2)}\cdot\int\frac{dx}{(x^2+1)^2} + \frac{x}{2(2)\cdot(x^2+1)^2} =$

$= \frac{x}{4(x^2+1)^2} + \frac34\int\frac{dx}{(x^2+1)^2} = \frac{x}{4(x^2+1)^2} + \frac34\cdot\left(\frac{x}{2(x^2+1)} + \frac12\cdot\int\frac{1}{x^2+1}dx\right) =$

$= \frac{x}{4(x^2+1)^2} + \frac34\cdot\left(\frac{x}{2(x^2+1)} + \frac12 \operatorname{arctg} x\right) + C =$

$= \frac38 \operatorname{arctg} x + \frac{3x}{8(x^2+1)} + \frac{x}{4(x^2+1)^2} + C$

№8.3.10

$\int\frac{dx}{(x^2-4x+29)^2} = \int\frac{dx}{((x-2)^2+25)^2} = \left[t=x-2;\ dt=dx\right] = \int\frac{dt}{(t^2+25)^2} =$

$= \frac{1}{50}\cdot\int\frac{dt}{t^2+25} + \frac{t}{50\cdot(t^2+25)} =$

$= \left[t=5z;\ dt=5dz\right] = \frac{1}{50}\cdot\int\frac{5}{25z^2+25}dz + \frac{t}{50\cdot(t^2+25)} =$

$= \frac{1}{250}\cdot\int\frac{dz}{z^2+1} + \frac{t}{50\cdot(t^2+25)} = \frac{1}{250}\cdot\operatorname{arctg} z + \frac{x-2}{50(x^2-4x+29)} + C =$

$= \frac{1}{250}\cdot\operatorname{arctg}\left(\frac{x-2}{5}\right) + \frac{x-2}{50(x^2-4x+29)} + C$

№8.3.11

$\int\frac{3x-2}{(x^2+6x+10)^2}dx = \left[A=3;\ B=-2;\ p=6;\ q=10;\ n=2\right] =$

$= \frac32\int\frac{2x+6}{(x^2+6x+10)^2}dx + \left(-2-\frac{18}{2}\right)\cdot\int\frac{dx}{(x^2+6x+10)^2} = \left[t=x^2+6x+10;\ y=x+3;\ a=\sqrt{10-9}=1\right] =$

$= \frac32\int\frac{dt}{t^2} - 11\cdot\int\frac{dy}{(y^2+1)^2} = -\frac32\cdot\frac{1}{x^2+6x+10} - 11\cdot\left(\frac{y}{2(y^2+1)} + \frac12\int\frac{dy}{y^2+1}\right) =$

$= -\frac{3}{2(x^2+6x+10)} - \frac{11y}{2(y^2+1)} - \frac{11\operatorname{arctg}(x+3)}{2} + C$

№8.2.13

$$\int \frac{2x-3}{(x-5)(x+2)}dx = \int \frac{A}{(x-5)}dx + \int \frac{B}{x+2}dx$$

$$2x-3 = A(x+2) + B(x-5)$$

Способ 1

$$2x-3 = Ax+2A + Bx-5B$$
$$2x-3 = x(A+B) + (2A-5B)$$

$$\begin{cases} A+B=2 \\ 2A-5B=-3 \end{cases} \quad \begin{cases} A=2-B \\ 4-2B-5B=-3 \end{cases} \quad \begin{cases} B=1 \\ A=1 \end{cases}$$

Способ 2

$$x_1=-2 \to -7 = -7B \Rightarrow B=1$$
$$x_2=5 \to 7 = 7A \Rightarrow A=1$$
$$\begin{cases} B=1 \\ A=1 \end{cases}$$

$$\int \frac{1}{(x-5)}dx + \int \frac{1}{x+2}dx = \ln|x-5| + \ln|x+2| + C$$

№8.2.14

$$\int \frac{x-2}{x^2-6x+5}dx = \int \frac{x-2}{(x-1)(x-5)}dx = \int \frac{A}{(x-1)}dx + \int \frac{B}{x-5}dx =$$

$$x-2 = A(x-1) + B(x-5)$$

1 способ

$$x-2 = Ax-A + Bx-5B$$
$$x-2 = x(A+B) - (A+5B)$$

$$\begin{cases} A+B=1 \\ A+5B=2 \end{cases} \quad \begin{cases} A=1-B \\ A+5B=2 \end{cases} \quad \begin{cases} B=\frac{1}{4} \\ A=\frac{3}{4} \end{cases}$$

2 способ

$$x_1=1 \to -1 = 0 - 4B \to B=\frac{1}{4}$$
$$x_2=5 \to 3 = 4A + 0 \to A=\frac{3}{4}$$

$$= \frac{3}{4}\int \frac{dx}{x-5} + \frac{1}{4}\int \frac{dx}{x-1} = \frac{3}{4}\ln|x-5| + \frac{1}{4}\ln|x-1| + C$$

№8.2.15

$$\int \frac{dx}{x^4+x^2} = \int \frac{dx}{x^2(x^2+1)} = \int \frac{x^2+1-x^2}{x^2(x^2+1)}dx =$$

$$= \int \frac{1}{x^2}dx - \int \frac{1}{x^2+1}dx = -\frac{1}{x} - \operatorname{arctg} x + C$$

8.3.16

$$\int \frac{x^5+x^4-8}{x^3-4x}dx = \Big[$$

$$\begin{array}{l|l} x^5+x^4+0\cdot x^3+0\cdot x^2+0\cdot x-8 & x^3-4x \\ -\;x^5-4x^3 & \overline{x^2+x+4} \\ \quad x^4+4x^3+0\cdot x^2+0\cdot x-8 & \\ \quad -\;x^4-4x^2 & \\ \qquad 4x^3+4x^2+0\cdot x-8 & \\ \qquad -\;4x^3-16x & \\ \qquad\quad 4x^2+16x-8 & \end{array}$$

$$\Big]=$$

$$= \int\left((x^2+x+4) + \frac{4x^2+16x-8}{x^3-4x}\right)dx =$$

$$= \int x^2dx + \int xdx + 4\int dx + \int\frac{4x^2}{x^3-4x}dx + \int\frac{16x}{x^3-4x}dx - 8\int\frac{dx}{x^3-4x} =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x + \int\frac{2x}{x^2-4}dx + \int\frac{16}{x+4}dx - 8\int\frac{dx}{x^3-4x} + C =$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x \begin{bmatrix} t=x^2-4 \Rightarrow dt=2xdx \\ z=x+4 \Rightarrow dz=dx \end{bmatrix} = \frac{x^3}{3}+\frac{x^2}{2}+4x+\ln|x^2-4|+16\ln|x+4|$$

$$-8\int\frac{1}{(\frac{4}{x^2}-1)x^3}dx = \left[v=\frac{4}{x^2}-1 \to dv=-\frac{8}{x^3}dx\right] = \frac{x^3}{3}+\frac{x^2}{2}+4x+\ln|x^2-4|+16\ln|x+4|+$$

$$+\int\frac{1}{v}dv = \frac{x^3}{3}+\frac{x^2}{2}+4x+\ln|x^2-4|+16\ln|x+4|+\ln\left|\frac{4}{x^2}-1\right|+C$$

8.3.17

$$\int\frac{dx}{x^3-8} = \int\frac{1}{(x-2)(x^2+2x+4)}dx = \int\left(\frac{1}{12(x-2)} - \frac{x+4}{12(x^2+2x+4)}\right)dx =$$

$$= \frac{1}{12}\int\frac{1}{x-2}dx - \frac{1}{12}\int\frac{x+4}{x^2+2x+4}dx =$$

$$= \frac{1}{12}\cdot\left(\int\frac{1}{x-2}dx - \int\frac{x+4}{x^2+2x+4}dx\right) = \left[t=x-2 \Rightarrow dt=dx\right] =$$

$$= \frac{1}{12}\cdot\left(\ln|x-2| - \int\frac{x+1}{x^2+2x+4}dx + 3\int\frac{dx}{x^2+2x+4}\right) = \left[t=x^2+2x+4 \Rightarrow dt=(2x+2)dx\right] =$$

$$= \frac{1}{12}\cdot\left(\ln|x-2| - \frac{1}{2}\ln(x^2+2x+4) + 3\int\frac{dx}{(x+1)^2+3}\right) = \left[z=\frac{x+1}{\sqrt{3}} \Rightarrow dz=\frac{1}{\sqrt{3}}dx\right] =$$

$$= \frac{1}{12}\left(\ln|x-2| - \frac{1}{2}\ln(x^2+2x+4) + 3\cdot\int\frac{\sqrt{3}}{3z^2+3}dz\right) + C =$$

$$= \frac{1}{12}\left(\ln(x-2) - \frac{1}{2}\ln(x^2+2x+4) + \sqrt{3}\cdot arctg\frac{(x+1)\sqrt{3}}{3}\right) + C$$

8.3.18

$$\int\frac{7x^3-10x^2+50x-77}{(x^2+9)(x^2+x-2)}dx = \int\frac{7x^3-10x^2+50x-77}{(x-1)(x-2)(x^2+9)}dx =$$

$$= \int\frac{x-2}{x^2+9}dx + \int\frac{7}{x+2}dx - \int\frac{1}{x-1}dx = \int\frac{x\,dx}{x^2+9} - \int\frac{2dx}{x^2+9} + 7\ln|x+2| - \ln|x-$$

$$= \begin{bmatrix} t=x^2+9 \to dt=2xdx \\ v=\frac{x}{3} \to dv=\frac{1}{3}dx \end{bmatrix} = \frac{1}{2}\ln(x^2+9) - \frac{2}{3}\int\frac{1}{v^2+1}dv + 7\ln|x+2| - \ln|x-$$

$$= \frac{1}{2}\ln(x^2+9) - \frac{2}{3}\cdot arctg\frac{x}{3} + 7\ln|x+2| - \ln|x-1| + C$$