In [1]:

```
from IPython.core.display import display, HTML
display(HTML("<style>.container { width:90% !important; }</style>"))
```

In [2]:

```
import json, sys
from urllib import request
from copy import copy

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import networkx as nx
```

Network Summary

Network source and preprocesing

In [3]:

```
CLIENT_ID = 56337756
APP_ID = 7379264
```

Load friends data from VK

Your token - sequence of symbols that goes after access_token= token = 'ad2d54a4cf4591c44a229d1b85d14665364670f02f4e4bf28dadc5d1e937643483f62bd7b8a80464f2ffa' # Suppose you want to get a friendlist of a particular user. # You also want hometown, sex and education to be contained in this list. # Finnaly, I suggest to put lang=en to aviod cyrillic issues uid = str(CLIENT_ID) url = u'https://api.vk.com/method/friends.get?fields=sex,city,education&uid=%s&lang=en&v=5.52&access_token=%s' %(uid ,token) res = request.urlopen(url).read() data = json.loads(res)

Drop deactivated accounts

my_friends_data = [u for u in data['response']['items'] if 'deactivated' not in u]# Your uid me = str(CLIENT_ID) # Using results of get.friends request datas = [] for i in range(0, len(my_friends_data)//100+1,1): uids = [] for u in my_friends_data: uids.append(str(u['id'])) uids = ','.join(uids[i*100:(i+1)*100]) # Mutual Friends Request url = u'https://api.vk.com/method/friends.getMutual?target_uids=%s&source_uid=%s&lang=en&v=5.52&access_token=%s' %(uids, me ,token) # Our Result res = request.urlopen(url).read() data = json.loads(res) datas += data['response'] mutual_frieds_data = datas

Add my info to the network. Maybe it will be usefull

my_data = {'id':CLIENT_ID, 'first_name':'Egor', 'last_name':'Dudyrev', 'sex': 2, 'city': {'id':1, 'title':'Moscow'}, 'university':128, 'university_name': 'НИУ ВШЭ (ГУ-ВШЭ)', 'faculty': 0, 'faculty_name':'', 'graduation':0, 'education_status':"Student (Master's)", }nodes_data = {u['id']: u for u in my_friends_data} nodes_data[CLIENT_ID] = my_dataconnections_data = {u['id']:u['common_friends'] for u in mutual_frieds_data} connections_data[CLIENT_ID] = [u['id'] for u in my_friends_data]

Saving data

with open('data/nodes_data.json','w') as f: json.dump(nodes_data, f) with open('data/connections_data.json','w') as f: json.dump(connections_data, f)

Load data

In [4]:

```
with open('data/nodes_data.json','r') as f:
    nodes_data = json.load(f)
with open('data/connections_data.json','r') as f:
    connections_data = json.load(f)
```

Drop My data as it's happend to be useless

In [5]:

```
nodes_data = {k:v for k,v in nodes_data.items() if v['id']!=CLIENT_ID}
connections_data = {k:[v_ for v_ in v if v_!=CLIENT_ID] for k,v in conn
ections_data.items() if k!=str(CLIENT_ID)}
```

Turn every node ID to string to prevent indexing mistakes

In [6]:

```
connections_data = {k: [v_ for v_ in v if str(v_) in connections_data.k
eys()]
                    for k,v in connections_data.items()}
nodes_data = {str(k):v for k,v in nodes_data.items()}
connections_data = {str(k):[str(v_) for v_ in v] for k,v in connections
_data.items()}
```

In [7]:

```
G = nx.from_dict_of_lists(connections_data)
```

Network Summary

Run Gephi to create understandable layout

Firstly, define position of nodes in the graph

In [8]:

```
nx.write_gml(G, 'network.gml')
```

Load gephi graph and add info to it

In [9]:

```
G = nx.Graph(nx.read_gml('network_prettify_3.gml',))
```

In [10]:

```
for k,d in nodes_data.items():
    for k_, v_ in d.items():
        if k_ in ['id', 'first_name', 'last_name', 'city', 'university_
name', 'faculty_name']:
            G.nodes[str(k)][k_] = v_
```

Getting *pos* feature based on Gephi layout

In [11]:

```
for n in G.nodes:
    G.nodes[n]['pos'] = (G.nodes[n]['graphics']['x'], G.nodes[n]['graph
ics']['y'])
    if 'city' in G.nodes[n]:
        G.nodes[n]['city'] = G.nodes[n]['city']['title']
```

Prepare graph for plotly

In [12]:

```
nodelist = list(G.nodes)

pos = {n: (G.nodes[n]['graphics']['x'], G.nodes[n]['graphics']['y']) fo
r n in nodelist}

dgrs = nx.degree(G)
sizes_by_degree = np.array([dgrs[n] for n in nodelist])
sizes_by_degree = np.log1p(sizes_by_degree/5)**3+7

colors_by_city = [{'Moscow':'orange', 'Izhevsk':'green'}.get(G.nodes[n]
['city'], 'grey')
        if 'city' in G.nodes[n] else 'grey' for n in nodelist]
```

In [13]:

```
for n, x in zip(nodelist, sizes_by_degree):
    G.nodes[n]['size_by_degree'] = x
```

In [14]:

```
import plotly.graph_objects as go
```

Node/Edge attributes

node attributes

In [15]:

```
G.nodes["779564"].keys()
```

Out[15]:

```
dict_keys(['graphics', 'id', 'first_name', 'last_name', 'c
ity', 'university_name', 'faculty_name', 'pos', 'size_by_d
egree'])
```

- id - ID пользователя ВК
- first_name - Имя
- last_name - Фамилия
- city - Город
- university_name - Университет
- faculty_name - Факультет
- pos - координата узла (создана в Gephi)
- graphics - данные от Gephi

edge attributes

In [16]:

```
G.edges[('779564','2661229')]
```

Out[16]:

```
{'id': 3288, 'value': 1.0}
```

- id - номер ребра
- value - вес (= 1)

Size, Order

In [17]:

```
print(f"Order of network: {len(G.nodes)}")
print(f"Size of network: {len(G.edges)}")
```

```
Order of network: 244
Size of network: 1657
```

Gorgeous network layout

Code to plot graph using plotly

In [18]:

```
def plotly_graph(G, node_color, node_size, title=''):
    edge_x = []
    edge_y = []
    for edge in G.edges():
        x0, y0 = G.nodes[edge[0]]['pos']
        x1, y1 = G.nodes[edge[1]]['pos']
        edge_x.append(x0)
        edge_x.append(x1)
        edge_x.append(None)
        edge_y.append(y0)
        edge_y.append(y1)
        edge_y.append(None)

    edge_trace = go.Scatter(
        x=edge_x, y=edge_y,
        line=dict(width=0.5, color='#888'),
        hoverinfo='none',
        mode='lines')

    node_x = []
    node_y = []
    for node in G.nodes():
        x, y = G.nodes[node]['pos']
        node_x.append(x)
        node_y.append(y)

    node_trace = go.Scatter(
        x=node_x, y=node_y,
        mode='markers',
        hoverinfo='text',
        marker=dict(
            showscale=False,
            color=node_color,
            size=node_size,
            line_width=2
        )
    )

    node_text = [f"<br>".join([f"{k}: {v}" for k,v in G.nodes[n].items
()
                               if k not in ['graphics','pos']]) for n i
n G.nodes]

    node_trace.text = node_text

    fig = go.Figure(data=[edge_trace, node_trace],
             layout=go.Layout(
                title=title,
                titlefont_size=16,
                showlegend=False,
                hovermode='closest',
                margin=dict(b=20,l=5,r=5,t=40),
                xaxis=dict(showgrid=False, zeroline=False, showticklabe
ls=False),
                yaxis=dict(showgrid=False, zeroline=False, showticklabe
ls=False))
                )
    return fig
```

In [19]:

```
fig = plotly_graph(G, node_color=colors_by_city, node_size=sizes_by_deg
ree, title='Graph of friends by city (As example)')
```

fig.show()

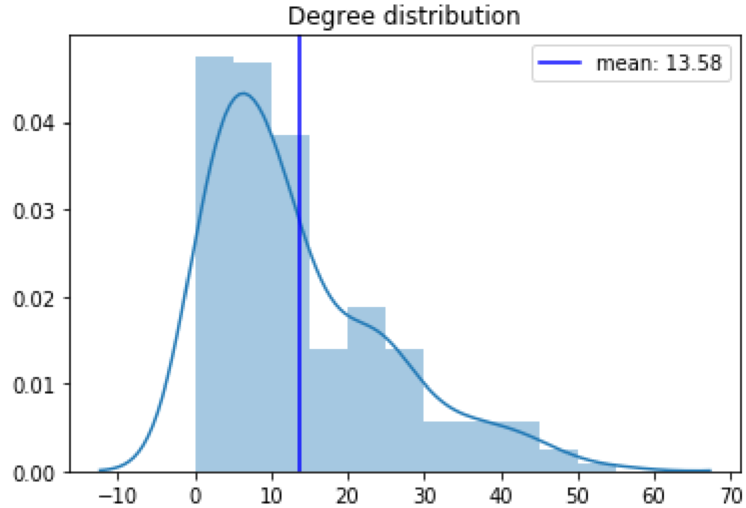
nodes_by_city

Degree distribution

Diameter, Clustering Coefficient

In [20]:

```
degrees = pd.Series(dict(nx.degree(G)))
sns.distplot(degrees)
plt.axvline(degrees.mean(), label=f"mean: {degrees.mean():.2f}", color=
'blue')
plt.legend()
plt.title('Degree distribution')
plt.show()
```



In [21]:

```
ccs_idxs = list(nx.connected_components(G))
ccs_idxs = sorted(ccs_idxs, key= lambda cc_idxs: -len(cc_idxs) )
ccs_Gs = [nx.subgraph(G, cc_idxs) for cc_idxs in ccs_idxs]
```

In [22]:

```
print(f"Number of connected components: {len(ccs_idxs)}")
print(f"Orders of connected components:", ', '.join([f"{len(cc_G.nodes
())}" for cc_G in ccs_Gs]))
print(f"Sizes of connected components:", ', '.join([f"{len(cc_G.edges
())}" for cc_G in ccs_Gs]))
print(f"Diameters of connected components:", ', '.join([f"{nx.diameter(
cc_G)}" for cc_G in ccs_Gs]))
```

```
Number of connected components: 14
Orders of connected components: 155, 42, 28, 6, 4, 1, 1,
1, 1, 1, 1, 1, 1, 1
Sizes of connected components: 1234, 318, 93, 7, 5, 0, 0,
0, 0, 0, 0, 0, 0, 0
Diameters of connected components: 7, 5, 6, 3, 2, 0, 0, 0,
0, 0, 0, 0, 0, 0
```

In [23]:

```
print(f"Mean Clustering Coefficient: {nx.cluster.average_clustering(G):
.4f}")
print(f'Clustering coefficients of Connected Components:',
      ', '.join([f"{nx.cluster.average_clustering(cc_G):.2f}" for cc_G
in ccs_Gs]))
```

```
Mean Clustering Coefficient: 0.5712
Clustering coefficients of Connected Components: 0.55, 0.6
8, 0.73, 0.17, 0.83, 0.00, 0.00, 0.00, 0.00, 0.00, 0.00,
0.00, 0.00, 0.00
```

In [24]:

```
import pandas as pd
```

In [25]:

```
ccs_ds = pd.DataFrame()
ccs_ds['idxs'] = ccs_idxs
ccs_ds['subgraph'] = ccs_Gs
ccs_ds['order'] = ccs_ds['subgraph'].apply(lambda G: len(G.nodes))
ccs_ds['size'] = ccs_ds['subgraph'].apply(lambda G: len(G.edges))
ccs_ds['diameter'] = ccs_ds['subgraph'].apply(lambda G: nx.diameter(G))
ccs_ds['clustering_coef'] = ccs_ds['subgraph'].apply(lambda G: nx.clust
er.average_clustering(G))
```

In [26]:

```
ccs_ds.head()
```

Out[26]:

	idxs	subgraph	order	size	diameter	clustering_coef
0	{416197567, 81158359, 17862450, 107025011, 900...	(779564, 1383897, 1950025, 2661229, 3217886, 4...	155	1234	7	0.554982
1	{178908792, 45968439, 87811969, 31162482, 2892...	(178908792, 45968439, 87811969, 31162482, 2892...	42	318	5	0.676960
2	{32582767, 268885442, 5191654, 49265157, 26099...	(32582767, 268885442, 5191654, 49265157, 26099...	28	93	6	0.734908
3	{43048325, 42367477, 44200088, 72420610, 88113...	(43048325, 42367477, 44200088, 88113309, 72420...	6	7	3	0.166667
4	{156070020, 94467180, 126150356, 135818294}	(156070020, 94467180, 126150356, 135818294)	4	5	2	0.833333

Structural Analysis

Degree/Closeness/Betweennes centralities

Top nodes interpretation

In [27]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

In [28]:

```
def get_degrees(G):
    return pd.Series(dict(nx.degree(G))).sort_values(ascending=False)

def get_degree_centrality(G):
    return pd.Series(dict(nx.degree_centrality(G))).sort_values(ascendi
ng=False)

def get_closeness_centr(G):
    return pd.Series(dict(nx.closeness_centrality(G))).sort_values(asce
nding=False)

def get_betweeness_centr(G):
    return pd.Series(nx.betweenness_centrality(G)).sort_values(ascendin
g=False)
```

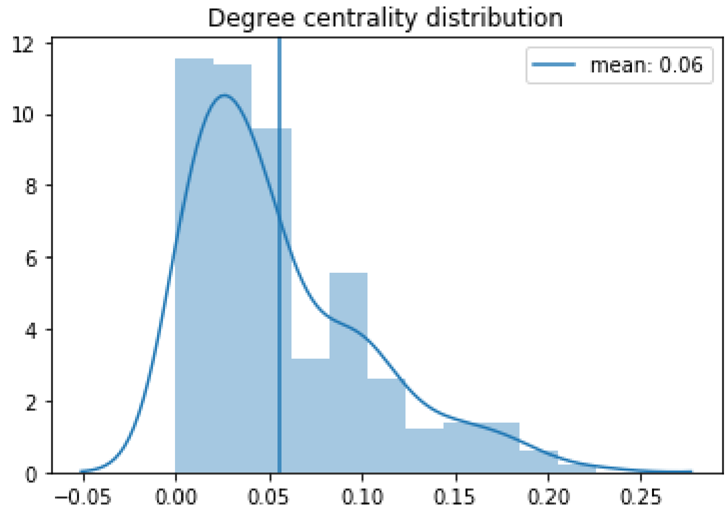
Degree

In [29]:

```
en_degrees = get_degrees(G)
en_degree_centr = get_degree_centrality(G)
```

In [30]:

```
sns.distplot(en_degree_centr)
plt.title('Degree centrality distribution')
plt.axvline(en_degree_centr.mean(), label=f"mean: {en_degree_centr.mean
():.2f}")
plt.legend()
plt.show()
```



In [31]:

```
for k,v in en_degree_centr.iteritems():
    G.nodes[k]['degree_centr'] = v
```

In [32]:

```
idx = 2
G.nodes[en_degree_centr.index[idx]], en_degree_centr.values[idx]*len(G.
nodes), nx.degree(G)[en_degree_centr.index[idx]]
```

Out[32]:

```
({'graphics': {'x': -560.88116,
   'y': -441.51453,
   'z': 0.0,
   'w': 10.0,
   'h': 10.0,
   'd': 10.0,
   'fill': '#94cc7d'},
  'id': 52128315,
  'first_name': 'Anastasia',
  'last_name': 'Nelidova',
  'city': 'Izhevsk',
  'university_name': 'ИжГТУ им. М. Т. Калашникова (бывш. И
МИ)',
  'faculty_name': 'Приборостроительный факультет',
  'pos': (-560.88116, -441.51453),
  'size_by_degree': 19.842590514079138,
  'degree_centr': 0.1934156378600823},
 47.193415637860085,
 47)
```

Nodes interpretation:

- 1) (55 mutual friends): Andrey Maximenko: One of Scout leaders from Izhevsk
- 2) (49 mutual friends): Tatyana Volnova: Other, more formal, Scout leader from Izhevsk
- 3) (47 mutual friends): Anastasia Nelidova: Connected with previous two

All are in the biggest connected component

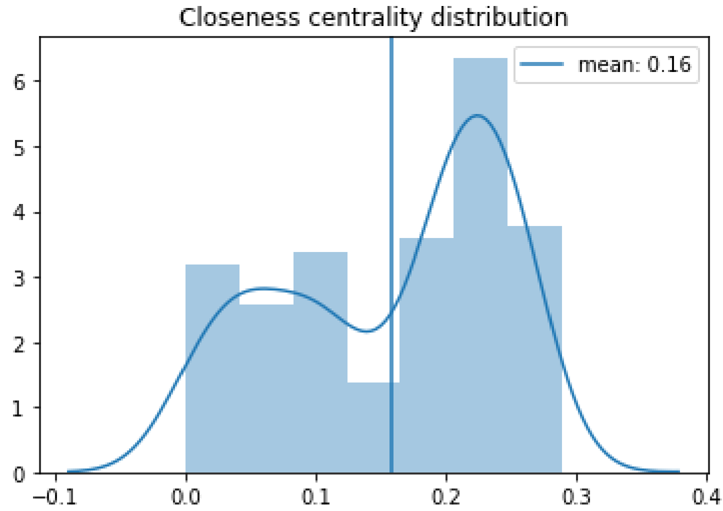
Closeness

In [33]:

```
en_closeness_centr = get_closeness_centr(G)
```

In [34]:

```
sns.distplot(en_closeness_centr)
plt.title('Closeness centrality distribution')
plt.axvline(en_closeness_centr.mean(), label=f"mean: {en_closeness_cent
r.mean():.2f}")
plt.legend()
plt.show()
```



In [35]:

```
for k,v in en_closeness_centr.iteritems():
    G.nodes[k]['closeness_centr'] = v
```

In [36]:

```
idx = 2
G.nodes[en_closeness_centr.index[idx]], np.mean(list(dict(nx.shortest_p
aths.shortest_path_length(G, en_closeness_centr.index[idx])).values()))
```

Out[36]:

```
({'graphics': {'x': -560.88116,
   'y': -441.51453,
   'z': 0.0,
   'w': 10.0,
   'h': 10.0,
   'd': 10.0,
   'fill': '#94cc7d'},
  'id': 52128315,
  'first_name': 'Anastasia',
  'last_name': 'Nelidova',
  'city': 'Izhevsk',
  'university_name': 'ИжГТУ им. М. Т. Калашникова (бывш. И
МИ)',
  'faculty_name': 'Приборостроительный факультет',
  'pos': (-560.88116, -441.51453),
  'size_by_degree': 19.842590514079138,
  'degree_centr': 0.1934156378600823,
  'closeness_centr': 0.2741480556711518},
 2.296774193548387)
```

Nodes interpretation:

- 1) (2.18 handshakes) Natalya Bazhenova - A Scout leader who now works in my school
- 2) (2.23 handshakes) Irina "Metel" - A Scout leader who lives close to my school
- 3) (2.3 handshakes) Anastasia Nelidova - Connected with previous two

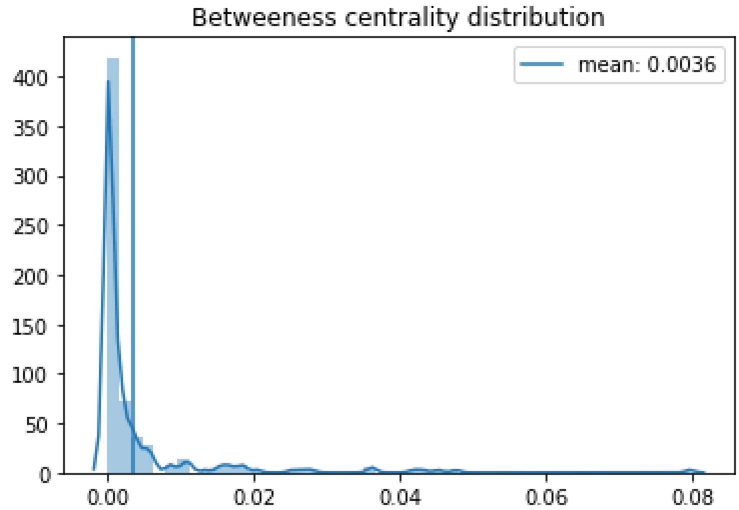
Betweeness

In [37]:

```
en_betweeness_centr = get_betweeness_centr(G)
```

In [38]:

```
sns.distplot(en_betweeness_centr)
plt.title('Betweeness centrality distribution')
plt.axvline(en_betweeness_centr.mean(), label=f"mean: {en_betweeness_ce
ntr.mean():.4f}")
plt.legend()
plt.show()
```



In [39]:

```
for k,v in en_betweeness_centr.iteritems():
    G.nodes[k]['betweeness_centr'] = v
```

In [40]:

```
idx = 2
G.nodes[en_betweeness_centr.index[idx]], en_betweeness_centr.values[idx
]
```

Out[40]:

```
({'graphics': {'x': 35.87362,
   'y': -92.172775,
   'z': 0.0,
   'w': 10.0,
   'h': 10.0,
   'd': 10.0,
   'fill': '#94cc7d'},
  'id': 75721602,
  'first_name': 'Grisha',
  'last_name': 'Mukhachev',
  'city': 'Uva',
  'university_name': '',
  'faculty_name': '',
  'pos': (35.87362, -92.172775),
  'size_by_degree': 9.101745386768128,
  'degree_centr': 0.05349794238683128,
  'closeness_centr': 0.18073464410912973,
  'betweeness_centr': 0.045267795430744114},
 0.045267795430744114)
```

Nodes interpretation:

- 1) (8% smallest paths) - Svyatoslav Medvedev - played in a school orchestra, knows some of the family
- 2) (4.8% smallest paths) - Vladimir Reznikov - played in a school orchestra, has a blog
- 3) (4.5% smallest paths) - Grisha Mukhachev - family, knows Svyatoslav Medvedev

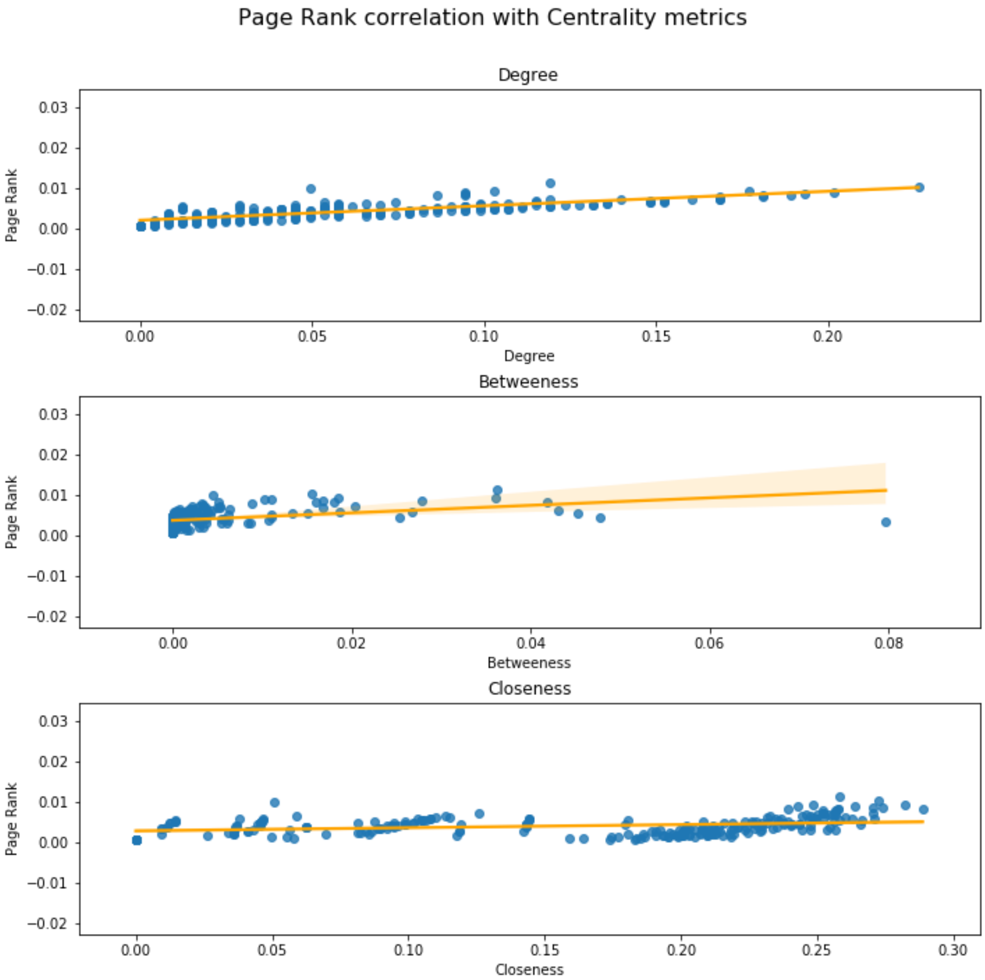
Page-Rank. Comparison with centralities

In [41]:

```
pr = pd.Series(nx.pagerank(G, alpha=0.85)).sort_values(ascending=False)
```

In [42]:

```
plt.figure(figsize=(10,10))
for idx, centr in enumerate([('Degree', en_degree_centr), ('Betweeness'
, en_betweeness_centr), ('Closeness', en_closeness_centr)]):
    plt.subplot(3,1,idx+1)
    title, centr = centr
    sns.regplot(x=centr.sort_index(), y=pr.sort_index(), line_kws={'col
or':'orange'})
    plt.title(title)
    plt.xlabel(title)
    plt.ylabel('Page Rank')
plt.tight_layout()
plt.suptitle('Page Rank correlation with Centrality metrics', size=16)
plt.subplots_adjust(top=0.9)
plt.show()
```



Page Rank highly correlates with Degree Centrality. It is expected since Page Rank favors nodes which have a lot of links, i.e. higher degree.

In [43]:

```
idx = 2
G.nodes[pr.index[idx]], pr.values[idx]
```

Out[43]:

```
({'graphics': {'x': 103.67067,
   'y': -479.10315,
   'z': 0.0,
   'w': 10.0,
   'h': 10.0,
   'd': 10.0,
   'fill': '#94cc7d'},
  'id': 269933041,
  'first_name': 'Viktor',
  'last_name': 'Galushko',
  'city': 'Moscow',
  'university_name': 'УРИО (бывш. УРАО)',
  'faculty_name': '',
  'pos': (103.67067, -479.10315),
  'size_by_degree': 8.83275828030883,
  'degree_centr': 0.04938271604938272,
  'closeness_centr': 0.05084745762711864,
  'betweeness_centr': 0.00448366946683445},
 0.010139115687574l2)
```

Nodes interpretation:

- 1) (1.13%) - Maxim Sterkhov - played in a school orchestra
- 2) (1.03%) - Andrey Maximenko - Scout leader
- 3) (1.01%) - Viktor Galushko - Head of Bauman Physical education class

Assortative Mixing according to node attributes

In [44]:

```
pd.Series({attr: nx.assortativity.attribute_assortativity_coefficient(G
, attr) for attr in ['city', 'university_name', 'faculty_name']})
```

Out[44]:

```
city               0.210493
university_name    0.103921
faculty_name       0.078636
dtype: float64
```

Node structural equivalence/similarity

In [45]:

```
import scipy.spatial as spt
```

In [46]:

```
def plotDist(A, figsize=(10,10)):
    f, ax = plt.subplots(2, 2, figsize=figsize)
    ax[0, 0].imshow(A, cmap = 'Greys', interpolation = 'None')
    ax[0, 0].set_title('Adjacency Matrix')

    D = np.corrcoef(A)
    ax[1, 0].imshow(D, cmap = 'Greys', interpolation = 'None')
    ax[1, 0].set_title('Correlation coeff.')

    dVec = spt.distance.pdist(A, metric = 'euclidean')
    D = spt.distance.squareform(dVec)
    ax[0, 1].imshow(D, cmap = 'Greys', interpolation = 'None')
    ax[0, 1].set_title('Euclidean Dist.')

    dVec = spt.distance.pdist(A, metric = 'cosine')
    D = spt.distance.squareform(dVec)
    ax[1, 1].imshow(D, cmap = 'Greys', interpolation = 'None')
    ax[1, 1].set_title('Cosine Dist.')
```

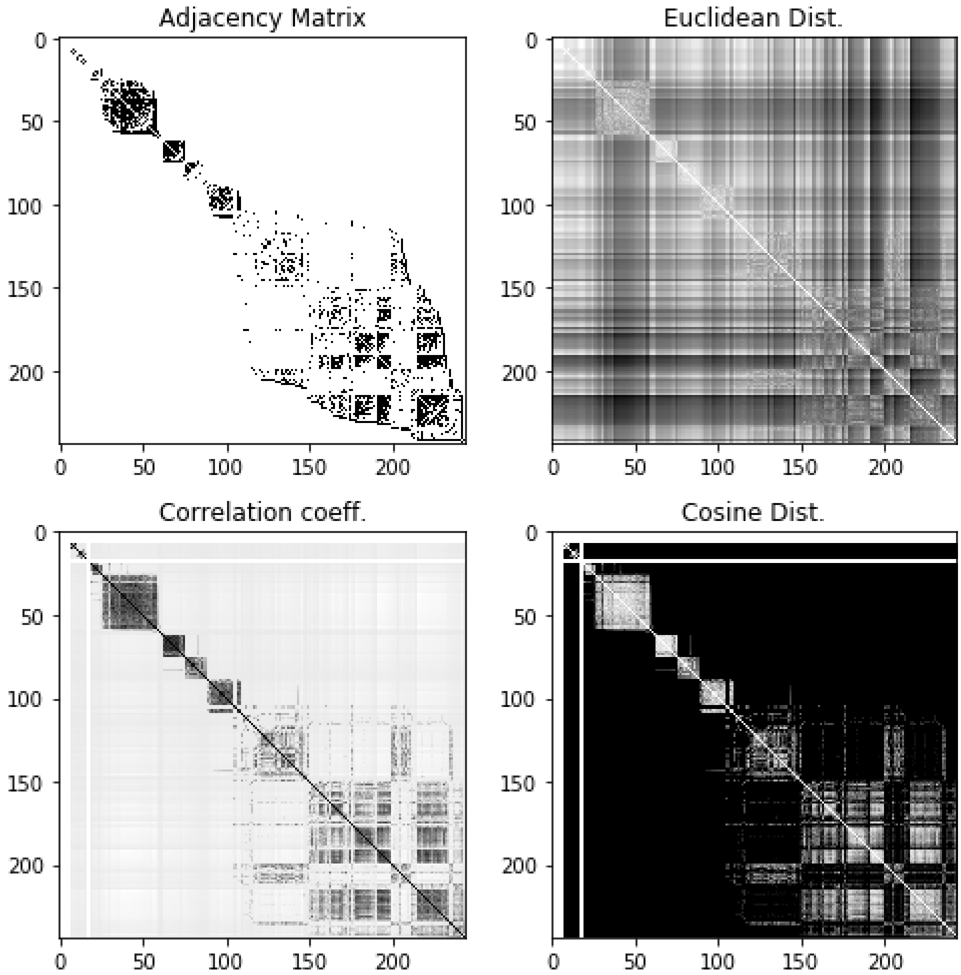
In [47]:

```
cm = list(nx.utils.reverse_cuthill_mckee_ordering(G))
A = nx.adjacency_matrix(G, nodelist=cm).todense()
```

In [48]:

```
plotDist(A, figsize=(7,7))
plt.tight_layout()
plt.show()
```

```
/root/anaconda3/lib/python3.7/site-packages/numpy/lib/func
tion_base.py:2534: RuntimeWarning:

invalid value encountered in true_divide

/root/anaconda3/lib/python3.7/site-packages/numpy/lib/func
tion_base.py:2535: RuntimeWarning:

invalid value encountered in true_divide
```



The closest random graph model similar to my SN

Erdos-Renyi Graph model

$$p = \frac{N_{edges}}{N_{possible\ edges}}$$
$$N_{possible\ edges} = \frac{N_{nodes}(N_{nodes} - 1)}{2}$$

In [49]:

```
n = len(G.nodes)
p = len(G.edges)/(n*(n-1)/2)
n, p
```

Out[49]:

```
(244, 0.055892869189772654)
```

In [50]:

```
er_G = nx.erdos_renyi_graph(n, p)
er_degree_centr = get_degree_centrality(er_G)
er_closeness_centr = get_closeness_centr(er_G)
er_betweeness_centr = get_betweeness_centr(er_G)
```

Preferential attachment

$$N_{edges} = m(N_{nodes} - m)$$
$$m^2 - N_{nodes}m + N_{edges} = 0$$
$$m = \frac{1}{2}\left(N_{nodes} \pm \sqrt{N_{nodes}^2 - 4N_{edges}}\right)$$

In [51]:

```
n = len(G.nodes)
m = (n-np.sqrt(n**2-4*len(G.edges)))/2
m = np.round(m).astype(int)
n, m
```

Out[51]:

```
(244, 7)
```

In [52]:

```
ba_G = nx.barabasi_albert_graph(n, m)
ba_degree_centr = get_degree_centrality(ba_G)
ba_closeness_centr = get_closeness_centr(ba_G)
ba_betweeness_centr = get_betweeness_centr(ba_G)
```

In [53]:

```
len(ba_G.edges()), len(G.edges)
```

Out[53]:

```
(1659, 1657)
```

Small World

$$N_{edges} = \frac{N_{nodes} * k}{2}$$
$$k = \frac{2N_{edges}}{N_{nodes}}$$

In [54]:

```
n = len(G.nodes)
k = np.round(2*len(G.edges)/len(G.nodes)).astype(int)
n, k
```

Out[54]:

```
(244, 14)
```

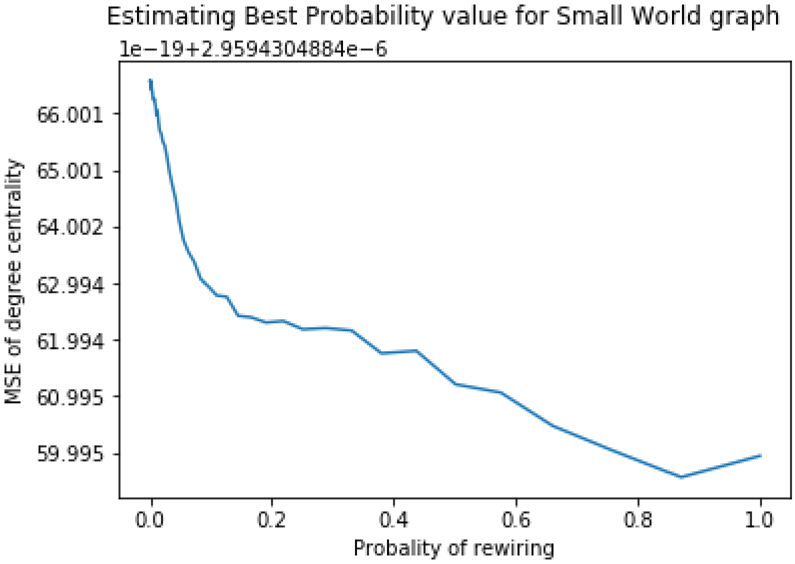
Choosing Probability of rewiring

In [55]:

```
ps = np.logspace(-6, 0, 101)
errors = []
for p in ps:
    centrs = [get_degree_centrality(nx.watts_strogatz_graph(n,k,p)) for
i in range(10)]

    errors.append(np.mean([(centr.mean()-en_degree_centr.mean())**2 for
centr in centrs]))
```

In [56]:

```
plt.plot(ps, errors)
plt.xlabel('Probality of rewiring')
plt.ylabel('MSE of degree centrality')
plt.suptitle('Estimating Best Probability value for Small World graph')
plt.show()
```



In [57]:

```
p = ps[np.argmin(errors)]
p
```

Out[57]:

0.8709635899560796

In [58]:

```
ws_G = nx.watts_strogatz_graph(n, k, p)
ws_degree_centr = get_degree_centrality(ws_G)
ws_closeness_centr = get_closeness_centr(ws_G)
ws_betweeness_centr = get_betweeness_centr(ws_G)
```

In [59]:

```
len(ws_G.edges()), len(G.edges)
```

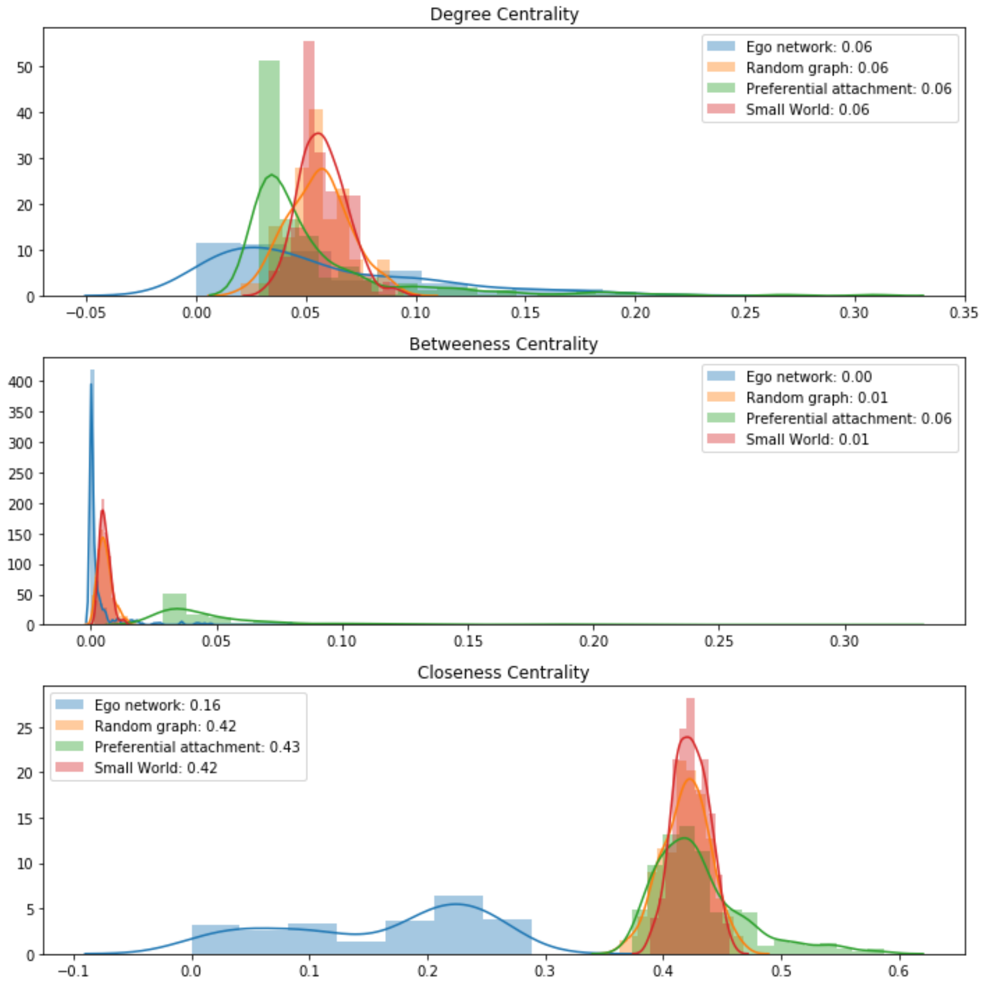
Out[59]:

```
(1708, 1657)
```

Combining Random Graph models

In [60]:

```
plt.figure(figsize=(10,10))
for idx, data in enumerate([(en_degree_centr, er_degree_centr, ba_degre
e_centr, ws_degree_centr, 'Degree Centrality'),
                             (en_betweeness_centr, er_betweeness_centr,
ba_degree_centr, ws_betweeness_centr,  'Betweeness Centrality'),
                             (en_closeness_centr, er_closeness_centr, b
a_closeness_centr, ws_closeness_centr, 'Closeness Centrality' )
                            ]):
    plt.subplot(3, 1, idx+1)
    for idx_, label in enumerate(['Ego network', 'Random graph', 'Prefe
rential attachment', 'Small World',]):
        if data[idx_].var()>1e-10:
            sns.distplot(data[idx_], label=f'{label}: {data[idx_].mean
():.2f}', kde=True)
        else:
            sns.distplot(data[idx_], label=f'{label}: {data[idx_].mean
():.2f}', kde=False)
    plt.title(data[-1])
    plt.legend()


plt.tight_layout()
plt.show()
```



Choose best Random Graph model

In [61]:

```
rg_ds = pd.DataFrame(index=['Ego','Random graph', 'Preferential attachm
ent', 'Small World'])
rg_ds['Degree'] = [x.median() for x in [en_degree_centr, er_degree_cent
r, ba_degree_centr, ws_degree_centr]]
rg_ds['Betweeness'] = [x.median() for x in [en_betweeness_centr, er_bet
weeness_centr, ba_betweeness_centr, ws_betweeness_centr]]
rg_ds['Closeness'] = [x.median() for x in [en_closeness_centr, er_close
ness_centr, ba_closeness_centr, ws_closeness_centr]]
rg_ds
```

Out[61]:

	Degree	Betweeness	Closeness
Ego	0.041152	0.000317	0.193645
Random graph	0.057613	0.005274	0.419689
Preferential attachment	0.041152	0.001881	0.423345
Small World	0.057613	0.005254	0.423345

In [62]:

```
rg_ds = (rg_ds-rg_ds.loc['Ego']).abs().rank()
rg_ds['Mean Rank'] = rg_ds.rank().mean(1)
rg_ds
```

Out[62]:

	Degree	Betweeness	Closeness	Mean Rank
Ego	1.5	1.0	1.0	1.166667
Random graph	3.5	4.0	2.0	3.166667
Preferential attachment	1.5	2.0	3.5	2.333333
Small World	3.5	3.0	3.5	3.333333

No Random Graph is ideal for my network. In general Preferential attachment model gives better results.

Community Detection

Clique search

In [63]:

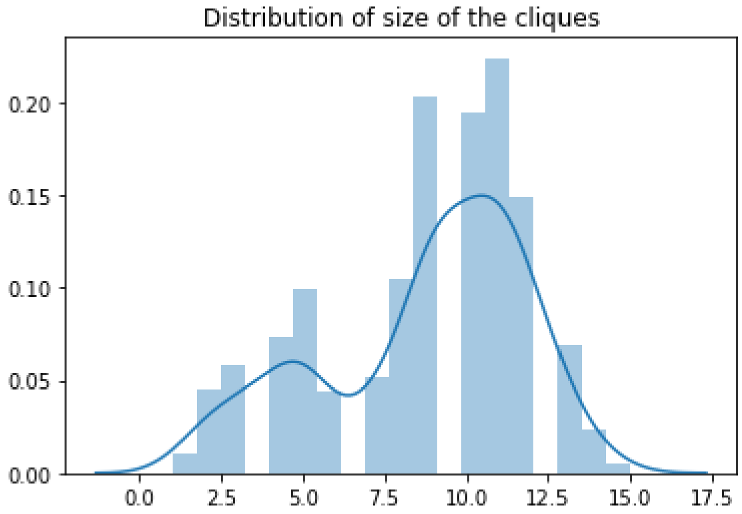
```
cliques = list(nx.find_cliques(G))
```

In [64]:

```
cliques = sorted(cliques, key=lambda x: -len(x))
```

In [65]:

```
sns.distplot([len(x) for x in cliques])
plt.title('Distribution of size of the cliques')
plt.show()
```



People are actively communicate in groups of 5 and 10 persons. Which is coherent with the basic theory of amount of people in small in medium size teams.

Best results of various community detection algorithms

In [66]:

```
def get_community_colors(com):
    #put all one-node communities to a single community
    one_node_comms = [c for c in com if len(c)==1]
    one_node_comms = frozenset([y for x in one_node_comms for y in x])
    com = [c for c in com if len(c)>1]+[one_node_comms]
    com = sorted(com, key=lambda x: -len(x))

    # get color index for every node
    colors_ = {x:idx for idx,c in enumerate(com) for x in c}
    colors_ = {x: colors_.get(x, len(set(colors_.values()))) for x in n
odelist}

    # assign rgb color to node base on community
    palette = sns.color_palette(n_colors=len(set(colors_.values())))
    palette = ['rgb('+','.join([f"{int(x*255)}" for x in c])+')' for c
in palette]
    colors_ = [palette[colors_[n]] for n in nodelist]
    return com, colors_
```

k-clique

In [67]:

```
com_kclique = list(nx.community.k_clique_communities(G, 3))
com_kclique, colors_ = get_community_colors(com_kclique)
```

plotly_graph(G, colors_, sizes_by_degree, title='K-clique community detection')

kclique

In [68]:

```
d = {n:idx for idx, c in enumerate(com_kclique) for n in c}
for n in G.nodes:
    G.nodes[n]['k-clique'] = d.get(n, max(d.values())+1)
```

Modularity based

In [69]:

```
com_mod = nx.algorithms.community.modularity_max.greedy_modularity_comm
unities(G)
com_mod, colors_ = get_community_colors(com_mod)
```

plotly_graph(G, colors_, sizes_by_degree, title='Modularity based community detection')

mod_comunity

In [70]:

```
d = {n:idx for idx, c in enumerate(com_mod) for n in c}
for n in G.nodes:
    G.nodes[n]['Modularity based community'] = d.get(n, max(d.values())
+1)
```

Girvan-Newman

In [71]:

```
from tqdm.notebook import tqdm
```

In [72]:

```
from itertools import islice
```

In [73]:

```
com_gns = [c for c in islice(nx.algorithms.community.centrality.girvan_
newman(G), 5)]
```

In [74]:

```
com_gn = com_gns[3]
```

In [75]:

```
com_gn, colors_ = get_community_colors(com_gn)
```

plotly_graph(G, colors_, sizes_by_degree, title='Girvan-Newman community detection')

girvan_newman

In [76]:

```
d = {n:idx for idx, c in enumerate(com_gn) for n in c}
for n in G.nodes:
    G.nodes[n]['Girvan-Newman community'] = d.get(n, max(d.values())+1)
```

In [77]:

```
d_ = {0: 'Scouts',
 1: 'School',
 2: 'BMSTU',
 3: 'Family',
 4: 'BMSTU Orchestra',
 5: 'BMSTU Physical Education',
 6: 'No mutual friends',
 7: 'HSE',
 8: 'Summer Camp',
 9: 'Summer Camp 2'}
for n in G.nodes:
    G.nodes[n]['Community'] = d_[d[n]]
```

Incremental algorithms presented during classes

In [78]:

```
def MCL(A, tol, p, alpha):
    step = 1
    col_sums = A.sum(axis = 0)
    T = A / col_sums[np.newaxis, :]
    M = T
    M = np.nan_to_num(M)
    while(1):
        step += 1
        # Expancion step:
        M1 = np.linalg.matrix_power(M, p)
        # Inflation step:
        M1 = np.power(M1, alpha)
        col_sums = M1.sum(axis = 0)
        M1 = M1 / col_sums[np.newaxis, :]
        M1 = np.nan_to_num(M1)
        M1[M1<=tol] = 0
        if np.linalg.norm(M - M1) == 0:
            return M1
        else:
            M = M1.copy()
```

In [79]:

```
def MCL_communities(A, tol, p, alpha, cm):
    M = MCL(np.array(A), tol, p, alpha)

    com_mcl = []
    for idx, n in enumerate(cm):
        if M[idx].sum()>0:
            com = set([n]+[cm[idx_] for idx_,v in enumerate(M[idx]) if
v==1])
            com_mcl.append(frozenset(com))
    return com_mcl
```

In [80]:

```
cm = list(nx.utils.reverse_cuthill_mckee_ordering(G))
A = nx.adjacency_matrix(G, nodelist=cm).todense()
```

Empiricaly find the best tolerance value

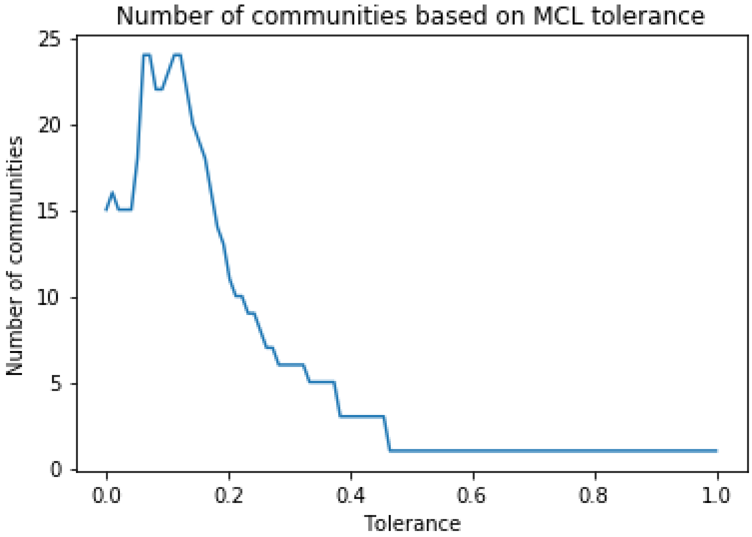
In [81]:

```
tols = list(np.linspace(0.00001, 1, 100))
com_lens = []
for t in tqdm(tols):
    com_mcl = MCL_communities(A, t, 2, 2, cm)
    com_mcl, colors_ = get_community_colors(com_mcl)
    com_lens.append(len(com_mcl))
com_lens = np.array(com_lens)
```

```
/root/anaconda3/lib/python3.7/site-packages/ipykernel_laun
cher.py:4: RuntimeWarning:

invalid value encountered in true_divide

/root/anaconda3/lib/python3.7/site-packages/ipykernel_laun
cher.py:14: RuntimeWarning:

invalid value encountered in true_divide
```

In [82]:

```
plt.plot(tols, com_lens)
plt.title('Number of communities based on MCL tolerance')
plt.xlabel('Tolerance')
plt.ylabel('Number of communities')
plt.show()
```



In [83]:

```
t = tols[np.argmax(com_lens==3)]
```

In [84]:

```
com_mcl = MCL_communities(A, t, 2, 2, cm)
com_mcl, colors_ = get_community_colors(com_mcl)
```

```
/root/anaconda3/lib/python3.7/site-packages/ipykernel_laun
cher.py:4: RuntimeWarning:

invalid value encountered in true_divide

/root/anaconda3/lib/python3.7/site-packages/ipykernel_laun
cher.py:14: RuntimeWarning:

invalid value encountered in true_divide
```

plotly_graph(G, colors_, sizes_by_degree, title='MCL community detection')

In [85]:

```
d = {n:idx for idx, c in enumerate(com_mcl) for n in c}
for n in G.nodes:
    G.nodes[n]['MCL community'] = d.get(n, max(d.values())+1)
```

Save model for Gephi visualization

In [86]:

```
G_fin = G.copy()
for n in G_fin.nodes:
    del G_fin.nodes[n]['pos']
```

In [87]:

```
nx.write_gexf(G_fin, 'graph_final.gexf')
```