Context-Free Path Querying with All-Path Semantics by Matrix Multiplication

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ABSTRACT

Context-Free Path Querying (CFPQ) allows one to use context-free grammars as path constraints in navigational graph queries. Many algorithms for CPFQ were proposed, but recently showed that the state-of-the-art CFPQ algorithms are still not performant enough for practical use. One promising way to achieve high-performance solutions for graph querying problems is to reduce them to linear algebra operations. Recently, there are two CFPQ solutions formulated in terms of linear algebra: the matrix-based CFPQ algorithm proposed by Azimov et al. (2018) and the Kronecker product-based CFPQ algorithm proposed by Orachev et al. (2020). However, the matrix-based algorithm still not support the most expressive all-path query semantics and cannot be truly compared with Kronecker product-based CFPQ algorithm. In this work, we introduce a new matrix-based CFPQ algorithm with all-path query semantics that allows us to extract all found paths for each pair of vertices. Also, we implement our algorithm by using appropriate high-performance libraries for linear algebra. Finally, we provide the comparison of the most performant linear algebra-based CFPQ algorithms.

1 INTRODUCTION

Formal language-constrained path querying [3] is a graph analysis problem in which formal languages are used as constraints for navigational path queries. In this problem a path in an edge-labeled graph is viewed as a word constructed by concatenation of edge labels. The formal languages are used to constrain the paths of interest: a query should find only paths labeled by words from the language. The most popular class of constraints used as navigational queries in graph databases are the regular ones. However, in some cases, regular languages are not expressive enough and context-free languages are used instead. The context-free path querying (CFPQ), can be used in many areas, for example, RDF analysis [22], static code analysis [15, 23], biological data analysis [17].

CFPQ have been studied a lot since the problem was first stated by Mihalis Yannakakis in 1990 [21]. Jelle Hellings investigates various aspects of CFPQ in [7–9] and formulates three possible querying semantics: *relational* that requires to find all vertex pairs reachable by some path of interest, *single-path* query semantics also requires to return the example of such path for all vertex pairs, and *all-path* query semantics that requires to return all such paths for all vertex pairs.

A number of CFPQ algorithms based on parsing techniques were proposed: (G)LL and (G)LR-based algorithms by Ciro M.

Medeiros et al. [12], Fred C. Santos et al. [16], Semyon Grigorev et al. [6], and Ekaterina Verbitskaia et al. [19]; CYK-based algorithm by Xiaowang Zhang et al. [22]; combinators-based approach to CFPQ by Ekaterina Verbitskaia et al. [20]. Yet recent research by Jochem Kuijpers et al. [11] shows that existing solutions are not applicable for real-world graph analysis because of significant running time and memory consumption.

One promising way to achieve high-performance solutions for graph querying problems is to reduce them to linear algebra operations. Rustam Azimov and Semyon Grigorev proposed a matrix-based algorithm for CFPQ with the relational query semantics in [2] and with the single-path query semantics in [18]. This algorithms provide a solution performant enough for real-world data analyses. However, in some cases, this is important to provide all founded paths and recently this matrix-based algorithm does not support the all-path query semantics. Another linear algebra-based CFPQ algorithm was proposed by Orachev et al. in [14]. This Kronecker product-based CFPQ algorithms creates more expressive and complex index that can be used for answering all three query semantics. Thus, the matrix-based CFPQ algorithm cannot be truly compared with the Kronecker product-based algorithm yet.

In this work we propose a matrix-based CFPQ algorithm for all-path query semantics and provide the comparison of the most performant linear algebra-based CFPQ algorithms.

To sum up, we make the following contributions in this paper.

(1) We modify the Azimov's matrix-based CFPQ algorithm and provide a matrix-based CFPQ algorithm for all-path query semantics. Our modification is still based on linear algebra, hence it still allows one to use high-performance libraries and utilize modern parallel hardware for CFPQ evaluation.
(2) We provide the comparison of the most performant linear algebra-based CFPQ algorithms. For this, we implement the proposed algorithm and compare it with other linear algebra-based implementations using real-world RDF dataset. We show that proposed algorithm is performant enough for real-world data analysis and comparable with other most performant CFPQ implementations.

2 PRELIMINARIES

In this section, we introduce common definitions in graph theory and formal language theory which are used in this paper. Also, we provide a brief description of CFPQ problems and AllPathIndex structure which is used as a base of our solution for all-path query semantics.

2.1 Basic Definitions of Graph Theory

In this paper, we use a labeled directed graph as a data model and define it as follows.

Definition 2.1. Labeled directed graph is a tuple $D = (V, E, \Sigma)$, where

- V is a finite set of vertices. For simplicity, we assume that the vertices are natural numbers ranging from 0 to $|V| - 1$,
- $E \subseteq V \times \Sigma \times V$ is a set of labeled edges,
- Σ is a set of edge labels.

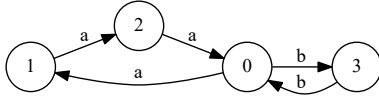An example of the labeled directed graph D_1 is presented in Figure 1. Here the set of labels $\Sigma = \{a, b\}$.

Figure 1: The input graph D_1.

Definition 2.2. The path π in the graph $D = (V, E, \Sigma)$ is a finite sequence of labeled edges $(e_1, ..., e_n)$, where $\forall i, 1 \le j \le n : e_i = (v_{i-1}, l_i, v_i) \in E$.

Definition 2.3. The word $l(\pi) \in \Sigma^*$ in the graph $D = (V, E, \Sigma)$ is the unique word $l_1...l_n$, obtained by concatenating the labels of the edges along the path $\pi = (e_1 = (v_0, l_1, v_1), \ldots, e_n = (v_{n-1}, l_n, v_n))$ in the graph D.

2.2 Basic Definitions of Formal Languages

We use context-free grammars as paths constraints, thus in this subsection we define context-free languages and grammars.

Definition 2.4. A context-free grammar G is a tuple (N, Σ, P, S), where

- N is a finite set of nonterminals
- Σ is a finite set of terminals, $N \cap \Sigma = \varnothing$
- P is a finite set of productions of the form $A \to \alpha$, where $A \in N$, $\alpha \in (N \cup \Sigma)^*$
- S is the start nonterminal

We use the conventional notation $A \xRightarrow[G]{*} w$ to denote, that a string $w \in \Sigma$ can be derived from a non-terminal A by some sequence of production rule applications from P in grammar G.

Definition 2.5. A context-free language is a language generated by a context-free grammar $G = (N, \Sigma, P, S)$:

$$L(G) = \{w \in \Sigma^* \mid S \xRightarrow[G]{*} w\}.$$

Definition 2.6. A context-free grammar $G = (N, \Sigma, P, S)$ is in *weak Chomsky normal form* (WCNF) if every production in P has one of the following forms:

- $A \to BC$, where $A, B, C \in N$
- $A \to a$, where $A \in N, a \in \Sigma$
- $A \to \varepsilon$, where $A \in N$

Note that weak Chomsky normal form differs from Chomsky normal form in the following:

- ε can be derived from any non-terminal;
- S can occur in the right-hand side of productions.

The matrix-based CFPQ algorithms process grammars only in weak Chomsky normal form, but every context-free grammar can be transformed into the equivalent grammar in this form.

Consider the context-free grammar $G_1 = (\{S\}, \{a, b\}, P, S)$, where P contains two rules: $S \to a\, S\, B$; $S \to a\, b$.

This grammar generates the context-free language:

$$L(G_1) = \{a^n b^n, n \in \mathbb{N}\}.$$

The following production rules of the grammar G_1^{wcnf} is a result of the transformation of G_1 to weak Chomsky normal form:

$$S \to A\,B \qquad S_1 \to S\,B \qquad B \to b$$
$$S \to A\,S_1 \qquad A \to a$$

2.3 Context-Free Path Querying

Definition 2.7. Let $D = (V, E, \Sigma)$ be a labeled graph, $G = (N, \Sigma, P, S)$ be a context free grammar. Then a *context-free relation* with grammar G on the labeled graph D is the relation $R_{G,D} \subseteq V \times V$:

$R_{G,D} = \{(v_0, v_n) \in V \times V \mid$

$\quad \exists \pi = (e_1 = (v_0, l_1, v_1), \ldots, e_n = (v_{n-1}, l_n, v_n)) \in \pi(D) :$

$\quad l(\pi) \in L(G)\}.$

For example, the vertex pair $(0, 0) \in R_{G_1, D_1}$, since there is a path in the labeled graph D_1 presented in Figure 1 from the vertex 0 to the vertex 0, whose labeling forms a word

$$w = aaaaaabbbbbb = a^6 b^6 \in L(G_1).$$

Finally, we can define context-free path querying problems.

Definition 2.8. Context-free path querying problem with relational query semantics is the problem of finding context-free relation $R_{G,D}$ for a given directed labeled graph D and a context-free grammar G.

In other words, the result of context-free path query evaluation is a set of vertex pairs such that there is a path between them that forms a word from the language generated by the given context-free grammar.

Using this definition we can also define context-free path querying problems with single-path and all-path query semantics.

Definition 2.9. Context-free path querying problem with single-path query semantics for a given directed labeled graph D and a context-free grammar G is the problem of finding context-free relation $R_{G,D}$ and finding for each vertex pair $(v_0, v_n) \in R_{G,D}$ the one example of path π between these vertices such that $l(\pi) \in L(G)$.

Definition 2.10. Context-free path querying problem with all-path query semantics for a given directed labeled graph D and a context-free grammar G is the problem of finding context-free relation $R_{G,D}$ and finding for each vertex pair $(v_0, v_n) \in R_{G,D}$ all paths π between these vertices such that $l(\pi) \in L(G)$.

2.4 Matrix-Based Algorithm

Our algorithm is based on the Azimov's CFPQ algorithm [2] which is based on matrix operations. This algorithm reduce CFPQ to operations over Boolean matrices and as a result allows one to use high-performance linear algebra libraries and utilize modern parallel hardware for CFPQ.

Note, that the algorithm computes not only the context-free relation $R_{G,D}$ but also a set of context-free relations $R_{G_A,D} \subseteq V \times V$ for every $A \in N$ where $G_A = (N, \Sigma, P, A)$. Thus it provides information about paths which form words derivable from any nonterminal $A \in N$.

We use the idea similar to one that was used for the CFPQ with single-path query semantics in [18]. We store additional information in matrices to be able to restore all paths which form words derivable from any nonterminal in the given grammar.

In order to do this, we introduce the

$$AllPathIndex = (left, right, middles)$$

— the elements of matrices which describe the found paths as concatenations of two smaller paths and help to restore each path after the index creation. Here $left$ and $right$ stand for the indexes of starting and ending vertices in the founded path, $middles$ — the set of indexes of intermediate vertices used in the concatenation of two smaller paths. When we do not find the path for some vertex pair i, j, we use the $AllPathIndex = \bot = (0, 0, \emptyset)$.

Additionally, we will use the notation of *proper matrix* which means that for every element of the matrix with indexes i, j it either $AllPathIndex = (i, j, \_)$ or $\bot$.

For proper matrices we use a binary operation $\otimes$ defined for AllPathIndexes AP_1, AP_2 which are not equal to $\bot$ and with $AP_1.right = AP_2.left$ as

$$AP_1 \otimes AP_2 = (AP_1.left, AP_2.right, \{AP_1.right\}).$$

And if at least one operand is equal to $\bot$ then $AP_1 \otimes AP_2 = \bot$.

For proper matrices we also use a binary operation $\oplus$ defined for AllPathIndexes AP_1, AP_2 which are not equal to $\bot$ with $AP_1.left = AP_2.left$ and $AP_1.right = AP_2.right$ as

$$AP_1 \oplus AP_2 = (AP_1.left, AP_1.right, AP_1.middles \cup AP_2.middles).$$

If only one operand is equal to $\bot$ then $AP_1 \oplus AP_2$ equal to another operand. If both operands are equal to $\bot$ then $AP_1 \oplus AP_2 = \bot$.

Using $\otimes$ as multiplication of AllPathIndexes, and $\oplus$ as an addition, we can define a *matrix multiplication*, $a \odot b = c$, where a and b are matrices of a suitable size, that have AllPathIndexes as elements, as $c_{i,j} = \bigoplus_{k=1}^{n} a_{i,k} \otimes b_{k,j}$.

Also, we use the element-wise + operation on matrices a and b with the same size: $a + b = c$, where $c_{i,j} = a_{i,j} \oplus b_{i,j}$.

3 MATRIX-BASED CFPQ ALGORITHM FOR ALL-PATH QUERY SEMANTICS

In this section, we propose the matrix-based algorithm for CFPQ w.r.t. the all-path query semantics (see Listing 1). This algorithm is a modification of Azimov's matrix-based algorithm for CFPQ and it constructs the set of matrices T with AllPathIndexes as elements.

Let $G = (N, \Sigma, P, S)$ be the input context-free grammar, $D = (V, E, \Sigma)$ be the input graph. The result of the algorithm is a set of matrices T which stores information about all paths in the graph D that form a word derivable from some nonterminal of the context-free grammar G. Note that in line 4 we add the special value n to the $T_{k,l}^{A_i}.middles$ to specify that this path is a single-edge path or an empty path π_ε.

After constructing a set of matrices T or so-called *index*, we can construct a set of all paths π between specified vertex pair (i, j) and a non-terminal A such that $A \xRightarrow[G]{*} l(\pi)$. The index T already stores data about all paths derivable from each nonterminal. However, the set of such paths can be infinite. From a

Listing 1 CFPQ algorithm for all-path query semantics

1: **function** ALLPATHCFPQ(
        $D = (V, E, \Sigma)$,
        $G = (N, \Sigma, P, S)$)   ▷ Grammar in WCNF
2:     $n \leftarrow |V|$
3:     $T \leftarrow \{T^A \mid A \in N, T^A \text{ is a matrix } n \times n, T_{i,j}^A \leftarrow \bot\}$
4:     **for all** $(i, x, j) \in E, A \mid A \rightarrow x \in P$ **do** $T_{i,j}^A \leftarrow (i, j, \{n\})$
5:     **for all** $A \mid A \rightarrow \varepsilon \in P$ **do** $T_{i,i}^A \leftarrow (i, i, \{n\})$
6:     **while** any matrix in T is changing **do**
7:         **for all** $A \rightarrow BC \in P$ where T^B or T^C are changed **do**
8:             $T^A \leftarrow T^A + (T^B \odot T^C)$
9:     **return** T

practical perspective, it is necessary to use lazy evaluation or limit the resulting set of paths in some other way. For example, one can try to query some fixed number of paths or query paths of fixed maximum length.

We propose the algorithm (see Listing 2) for extracting these paths. Our algorithm returns a set with the empty path π_ε only if $i = j$ and $A \rightarrow \varepsilon \in P$. If the AllPathIndex for given i, j, A is equal to $\bot$ then our algorithm returns $\emptyset$ since such paths do not exist. Note that in line 19 we use the operation $\cdot$ which naturally generalizes the path concatenation operation by constructing all possible concatenations of path pairs from the given two sets. It is assumed that the sets are computed lazily, so as to ensure the termination in case of an infinite number of paths.

Listing 2 All paths extraction algorithm

1: **function** EXTRACTALLPATHS($i, j, A, T = \{T^{A_i}\}, G = (N, \Sigma, P, S)$)
2:     $index \leftarrow T_{i,j}^A$
3:     **if** $index = \bot$ **then**
4:         **return** $\emptyset$ ▷ Such paths do not exist
5:     $n \leftarrow$ size of the square matrix T^A
6:     $resultPaths \leftarrow \emptyset$
7:     **for all** $middle \in index.middles$ **do**
8:         **if** $middle = n$ **then**   ▷ Add single-edge or empty paths
9:             **for all** $x \mid A \rightarrow x \in P$ **do**
10:                 **if** $(i, x, j) \in E$ **then**
11:                     $resultPaths \leftarrow resultPaths \cup \{((i, x, j))\}$
12:             **if** $(i = j) \wedge (A \rightarrow \varepsilon \in P)$ **then**
13:                 $resultPaths \leftarrow resultPaths \cup \{\pi_\varepsilon\}$
14:         **else**▷ Add to result the concatenated paths from i to $middle$ and from $middle$ to j
15:             **for all** $A \rightarrow BC \in P$ **do**
16:                 $index_B \leftarrow T_{i,middle}^B$
17:                 $index_C \leftarrow T_{middle,j}^C$
18:                 **if** $(index_B \neq \bot) \wedge (index_C \neq \bot)$ **then**
19:                     $lPaths \leftarrow$ EXTRACTALLPATHS($i, middle, B, T, G$)
20:                     $rPaths \leftarrow$ EXTRACTALLPATHS($middle, j, C, T, G$)
21:                     $resultPaths \leftarrow resultPaths \cup lPaths \cdot rPaths$
22:     **return** $resultPaths$

3.1 Correctness

The following correctness theorem holds.

THEOREM 1. *Let $G = (N, \Sigma, P, S)$ be the input context-free grammar, $D = (V, E, \Sigma)$ be the input graph, and T be a set of matrices returned by the algorithm in Listing 1. Then for any i, j and for any non-terminal $A \in N$, index $= T_{i,j}^A$ and index $= (i, j, middles) \neq \bot$ iff $(i, j) \in R_{G_A, D}$ and there is a path π from vertex i to j such that $l(\pi) \in G_A = (N, \Sigma, P, A)$.*

PROOF SKETCH. At each iteration of the main cycle in the lines 6-8 of the algorithm, the new paths corresponding to nonterminals $A \in N$ are considered using the rules $A \rightarrow BC \in P$. These new paths are obtained by the concatenation of two smaller paths corresponding to the nonterminals B and C. At the initialization step of the algorithm in lines 3-5, we consider all single-edge or empty paths corresponding to the derivation tree of height 1. Thus, it can be shown that at iteration l of the main cycle we consider all paths π such that there is a derivation tree of the height $h \leq l + 1$ for the string $l(\pi)$ and a context-free grammar G_A. Therefore, the theorem can be proved using the induction on the height of such derivation trees.

$\square$

Now, using the theorem 1 and induction on the length of the path, it can be easily shown that the following theorem holds.

THEOREM 2. *Let $G = (N, \Sigma, P, S)$ be the input context-free grammar, $D = (V, E, \Sigma)$ be the input graph, and T be a set of matrices returned by the algorithm in Listing 1. Then for any i, j and for any non-terminal $A \in N$ such that index $= T_{i,j}^A$ and index $= (i, j, middles) \neq \bot$, the algorithm in Listing 2 for these parameters will return a set of all paths π from vertex i to j such that $l(\pi) \in G_A = (N, \Sigma, P, A)$.*

We can, therefore, determine whether $(i, j) \in R_{G,D}$ by asking whether $T_{i,j}^S = \bot$. Also, we can extract all paths which forms a word from the context-free language $L(G)$ by using our algorithm in Listing 2. Thus, we show how the context-free path query evaluation w.r.t. the all-path query semantics can be solved in terms of matrix operations.

3.2 An Example

In this section, we provide a step-by-step demonstration of the proposed algorithms.

We run the query on a graph D_1, presented in Figure 1. We provide a step-by-step demonstration of the work of algorithm in Listing 1 with the given graph D and grammar G_1^{wcnf} from section 2. After the matrix initialization in lines **3-5** of this algorithm, we have a set of matrices $T^{(1)}$, presented in Figure 2.

$$T^{(1),A} = \begin{pmatrix} \bot & (0,1,\{4\}) & \bot & \bot \\ \bot & \bot & (1,2,\{4\}) & \bot \\ (2,0,\{4\}) & \bot & \bot & \bot \\ \bot & \bot & \bot & \bot \end{pmatrix}$$

$$T^{(1),B} = \begin{pmatrix} \bot & \bot & \bot & (0,3,\{4\}) \\ \bot & \bot & \bot & \bot \\ \bot & \bot & \bot & \bot \\ (3,0,\{4\}) & \bot & \bot & \bot \end{pmatrix}$$

Figure 2: The initial matrices for the example query. The PathIndexes $T_{i,j}^{(1),S_1}$ and $T_{i,j}^{(1),S}$ are equal to $\bot$ for every i, j.

After the initialization, the only matrices which will be updated are T^{S_1} and T^S. These matrices obtained after the first loop iteration is shown in Figure 3.

When the algorithm at some iteration finds new paths for some non-terminal in the graph D_1, then it adds corresponding AllPathIndexes to the matrix for this non-terminal. For example, after the first loop iteration, AllPathIndex $(2, 3, \{0\})$ is added to the matrix T^S. This AllPathIndex is added to the element with a row index $i = 2$ and a column index $j = 3$. This means, that there is a path π from the vertex 2 to the vertex 3, such that

$$T^{(2),S} = \begin{pmatrix} \bot & \bot & \bot & \bot \\ \bot & \bot & \bot & \bot \\ \bot & \bot & \bot & (2,3,\{0\}) \\ \bot & \bot & \bot & \bot \end{pmatrix}$$

Figure 3: The first iteration of computing the transitive closure for the example query. The PathIndexes $T_{i,j}^{(1),S_1}$ are equal to $\bot$ for every i, j.

$S \underset{G_1^{\text{wcnf}}}{\overset{*}{\Longrightarrow}} l(\pi)$ and this path obtained by concatenation of two smaller paths via vertex 0.

The calculation of the index T is completed after k iterations, when a fixpoint is reached: $T^{(k)} = T^{(k-1)}$. For the example query, $k = 14$ since $T_{14} = T_{13}$. The resulted matrix for non-terminal S is presented in Figure 4.

$$T^{(14),S} = \begin{pmatrix} (0,0,\{1\}) & \bot & \bot & (0,3,\{1\}) \\ (1,0,\{2\}) & \bot & \bot & (1,3,\{2\}) \\ (2,0,\{0\}) & \bot & \bot & (2,3,\{0\}) \\ \bot & \bot & \bot & \bot \end{pmatrix}$$

Figure 4: The final matrix for non-terminal S after computing the index.

Now, after constructing the index, we can construct the context-free relation

$$R_{G_1^{\text{wcnf}},D_1} = \{(0,0), (0,3), (1,0), (1,3), (2,0), (2,3)\}.$$

In the relation $R_{G_1^{\text{wcnf}},D_1}$, we have all vertex pairs corresponding to paths, whose labeling is in the language $L(G_1^{\text{wcnf}}) = \{a^n b^n \mid n \geq 1\}$. Using the algorithm in Listing 2 we can restore paths for each vertex pair from the context-free relation. For example, given $i = j = 0$, non-terminal S, set of resulted matrices T, and context-free grammar G_1^{wcnf}, the algorithm in Listing 2 returns an infinite set of all paths from vertex 0 to vertex 0 whose labeling form words from the following set $\{a^6 b^6, a^{12} b^{12}, a^{18} b^{18}, \ldots\}$. Following the path corresponding to the word $a^{6m} b^{6m}$, we will go through the cycle with a labels $2m$ times and through the cycle with b labels $3m$ times for all $m \geq 1$.

4 EVALUATION

The goal of this evaluation is to investigate the applicability of the proposed matrix-based algorithm to CFPQ with all-path query semantics and to provide the comparison of the most performant linear algebra-based CFPQ algorithms. We will compare the following CFPQ implementations:

- *MtxSingle* — the implementation from [18] of the matrix-based CFPQ algorithm for the single-path query semantics,
- *Tns* — the implementation from [14] of the Kronecker product-based CFPQ algorithm for all three query semantics including the all-path query semantics,
- *MtxAll* — the implementation of the proposed matrix-based CFPQ algorithm for all-path query semantics which utilizes SuiteSparse[1] [4] implementation of GraphBLAS API for matrix manipulations.

All implementations utilize CPU and represent matrices in sparse format. First, we measured the execution time and required

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[1]SuiteSparse is a sparse matrix software which includes GraphBLAS API implementation. Project web page: http://faculty.cse.tamu.edu/davis/suitesparse.html. Access date: 14.01.2021.

Table 1: RDFs properties

Graph	#V	#E	Queries
pathways	6238	18 598	g_1, g_2
gohierarchy	45 007	980 218	g_1, g_2
enzyme	48 815	109 695	g_1, g_2
eclass_514en	239 111	523 727	g_1, g_2
go	272 770	534 311	g_1, g_2
geospecies	450 609	2 311 461	g_1, g_2, geo
taxonomy	5 728 398	14 922 125	g_1, g_2

memory of the index creation. Then we compared the practical applicability of the paths extraction for both implementations *MtxAll* and *Tns* of the CFPQ with all-path query semantics.

For evaluation, we used a PC with Ubuntu 18.04 installed. It has Intel core i7-6700 CPU, 3.4GHz, and DDR4 64Gb RAM. We only measure the execution time of the algorithms themselves, thus we assume an input graph is loaded into RAM in the form of its adjacency matrix in the sparse format.

4.1 Dataset Description

We use the graphs and respective queries from the CFPQ_Data dataset[2] provided in [18]. This dataset contains the real-world RDFs with properties presented in table 1, and queries g_1, g_2, geo which are different variations of the *same-generation query* [1] — an important example of real-world queries that are context-free but not regular.

4.2 Evaluation Results

The results of the index creation for all three implementations are presented in Table 2. We can see that the running time of both CPU and GPGPU versions for the relational query semantics is small even for graphs with a big number of vertices and edges. The relatively small number of edges of interest may be the reason for such behavior. We believe it is necessary to extend the dataset with new queries that involve more different types of edges. Also, we can see, that $RG_C USPrel$ implementation which uses CUSP requires more memory. As we can see, the matrix-based algorithm for relational query semantics implemented for RedisGraph is more than 1000 times faster than the one based on annotated grammar implemented for Neo4j [17] and uses more than 4 times less memory. We can conclude that the matrix-based algorithm is more performant than other CFPQ algorithms for query evaluation under a relational semantics for real-world data processing. Also, we can see, that the GPGPU version which utilizes sparse matrices is significantly faster than the other implementations especially on big graphs. For example, for Geospacies it more than 7 times faster in both relational and single-path scenarios. Note, that for GPGPU versions we include the time required for data transferring and format conversions. We can conclude, that the cost of computing matrices with PathIndexes for single-path query semantics is not high. On average, it is about 2 times slower than the reachability matrix calculation. The additional running time of the path extraction is presented in figure 7 (boxplots are standard, outliers are omitted). As we can see, this time is small and linear in the length of the path. Finally, we conclude that the matrix-based algorithm paired with a suitable database and employing appropriate libraries for linear algebra is a promising way to make CFPQ with relational

and single-path query semantics applicable for real-world data analysis. We show that the SuiteSparse-based CPU implementation is performant enough to be comparable with GPGPU-based implementations on real-world data.

5 CONCLUSION AND FUTURE WORK

In this paper we propose a multiple-source modifications of Azimov's CFPQ algorithm and utilize it to provide full-stack support of CFPQ. For our solution, we implement a Cypher extension as a part of `libcypher-parser`, integrate the proposed algorithm into RedisGraph, and extend RedisGraph execution plan builder to support the extended Cypher queries. We demonstrate that our solution is applicable for real-world graph analyses.

In the future, it is necessary to provide formal translation of Cypher to linear algebra, or to determine a maximal subset of Cypher which can be translated to linear algebra. There is a number of works on the translation of a subset of SPARQL to linear algebra, such as [5, 10, 13]. Most of them are practical-oriented and do not provide full theoretical basis to translate querying language to linear algebra. Others discuss only partial cases and should be extended to cover real-world query languages. Deep investigation of this topic can help to determine the restrictions of linear algebra utilization for graph databases.

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Table 2: Index creation

Graph	G1						G2						Geo					
	MtxAll		Tns		MtxSingle		MtxAll		Tns		MtxSingle		MtxAll		Tns		MtxSingle	
	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem
pathways	0.04	91	0.02	123	0.01	671	0.01	49	0.01	122	0.01	671	—	—	—	—	—	—
go-hierarchy	22.12	38797	0.17	265	1.41	660	15.66	28447	0.24	252	0.84	671	—	—	—	—	—	—
enzyme	0.4	307	0.04	137	0.01	216	0.02	61	0.02	132	0.01	217	—	—	—	—	—	—
eclass_514en	25.02	14416	0.24	205	0.23	216	0.22	126	0.27	193	0.16	216	—	—	—	—	—	—
go	11.8	8290	1.58	282	1.45	215	1.13	990	1.27	243	0.93	217	—	—	—	—	—	—
geospecies	4.45	2691	0.08	218	0.06	2250	0.34	156	0.01	196	0.01	2251	32.06	44235	26.32	19537	15.54	22941
taxonomy	—	—	4.42	2018	2.73	1962	19.13	27232	3.56	1776	1.15	2250	—	—	—	—	—	—

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