Multiple-Source Context-Free Path Querying in Terms of Linear Algebra

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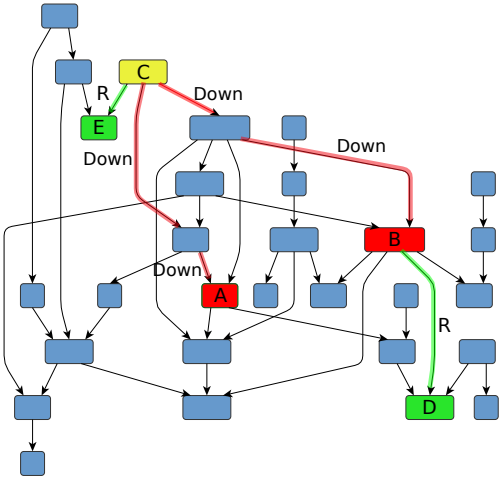
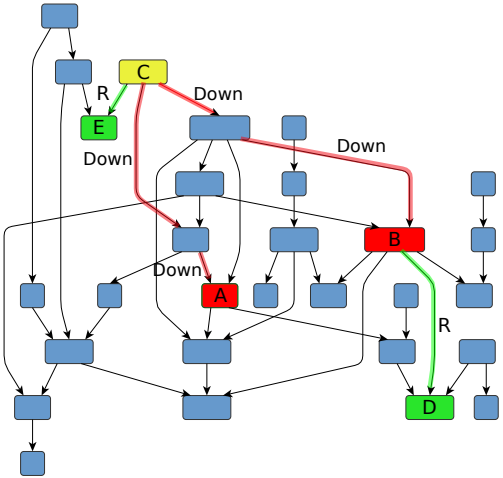
Formal Language Constrained Path Querying



Navigation through an edge-labeled graph

- **Path** specifies a **word** formed by labels of edges
- **Paths constraint** is a **language**: the word specified by the path should be in the given language
- Th expressiveness of constraints is related to **formal languages classes**

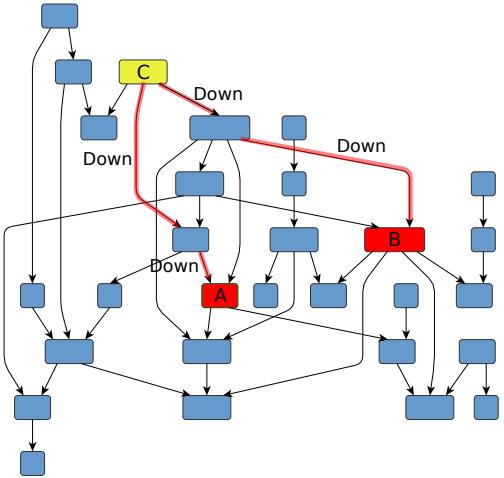
Regular Path Queries (RPQ)



Regular languages as constraints

- Which nodes are reachable from **C** by arbitrary number of **R** and **Down** edges?
- Regular language $\mathcal{L} = (R \mid Down)^*$

Context-Free Path Queries (CFPQ)



Context-free languages as constraints

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form $\overline{\textbf{Down}}^n\,\textbf{Down}^n$ between A and B?
- Context-free grammar:
  $sameLvl \rightarrow \overline{Down}\ sameLvl\ Down \mid \varepsilon$

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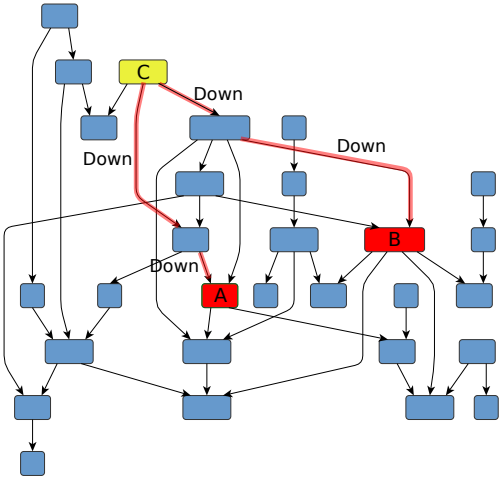

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Applications

- Static code analysis [T. Reps, et al, 1995]
- Graph segmentation [H. Miao, et al, 2019]
- Biological data analysis [P. Sevon, et al, 2008] ...

Problem Statement

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- J. Kuijpers, et al[1]: existing algorithms are too slow to be practical (in the context of Neo4j)

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- J. Kuijpers, et al[1]: existing algorithms are too slow to be practical (in the context of Neo4j)
- A. Terekhov, et al[2]: linear algebra based CFPQ algorithm can be performant enough
- There is no full-stack support of CFPQ
  - Grammars instead of full-featured queries
  - Custom graph storage instead of a mature graph database

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- **Cypher** extended with **path patterns**[3] to express context-free constraints

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https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc

Proposed Solution

- **Multiple-Source CFPQ** to process only the relevant subset of a graph
- **Cypher** extended with **path patterns**[3] to express context-free constraints
- **RedisGraph** database
  - Graph storage with matrix-based representation
  - Linear algebra based query engine (SuiteSparse:GraphBLAS[4] is used)
  - Cypher for querying (libcypher-parser[5] is used)

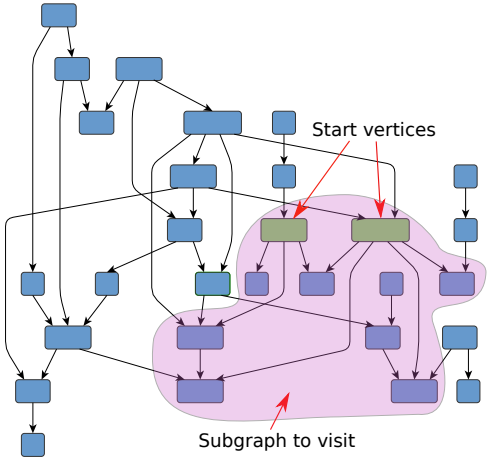
[3] Tobias Lindaaker, Path Patterns for Cypher, 2017, `https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc`

[4] Timothy A. Davis. 2019. Algorithm 1000: SuiteSparse:GraphBLAS: Graph Algorithms in the Language of Sparse Linear Algebra

[5] Chris Leishman, `https://github.com/cleishm/libcypher-parser`

Multiple-Source CFPQ

An improved version of Rustam Azimov CFPQ algorithm[6]



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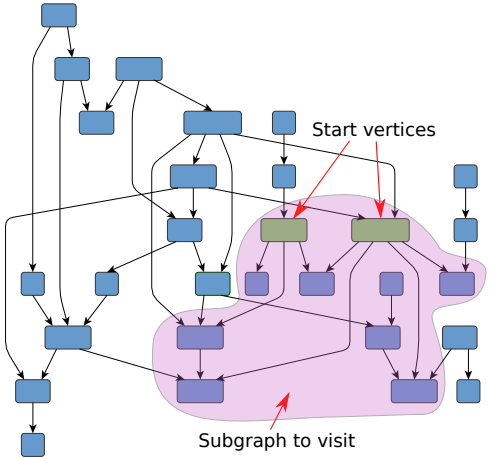
- The set of start vertices can be specified



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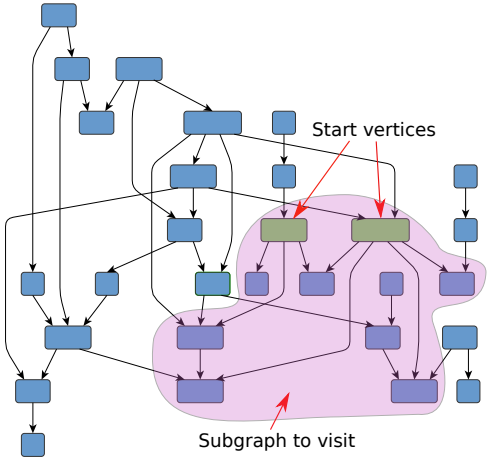
Multiple-Source CFPQ

An improved version of Rustam Azimov CFPQ algorithm[6]

- The set of start vertices can be specified
- Only the relevant subgraph will be processed



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Multiple-Source CFPQ

1: **function** MultiSrcCFPQ($D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$, $G = (N, \Sigma, P, S)$, Src)
2: $T \leftarrow \{T^A \mid A \in N, T^A[i,j] \leftarrow \mathit{false}, \text{for all } i,j\}$
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4: **for all** $v \in Src$ **do** $TSrc^S[v,v] \leftarrow \mathit{true}$
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10: **while** T *or* $TSrc$ *is changing* **do**
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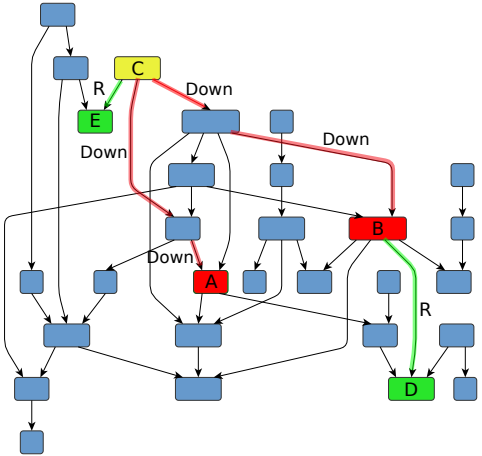
Cypher Extension




```
MATCH (u)-/[:R | :Down]*/->(v)
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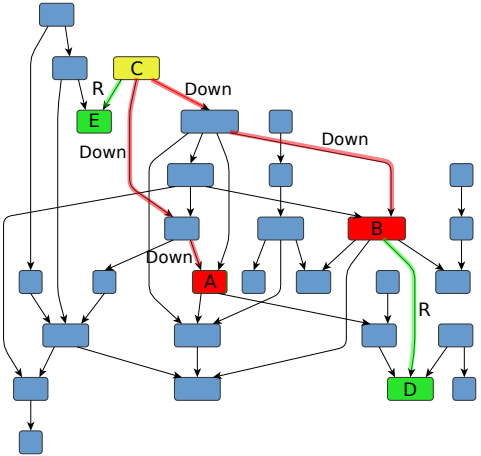


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Named path pattern

$SameLvl \rightarrow \overline{Down}\ SameLvl\ Down \mid \varepsilon$

```
PATH PATTERN SameLvl =
  ()-/[<:Down ~SameLvl :Down] | ()/->()
MATCH (u)-/ ~SameLvl /->(v)
RETURN u.name, v.name
```

Implementation Details

- Linear algebra based multiple-source CFPQ is implemented as a part of RedisGraph query engine
- Cypher parser is extended to support path patterns
- Path patterns are partially supported[7] in RedisGreaph query execution workflow

[7] Full support is a nontrivial challenge: formal description of the extension is required

Evaluation Setup

- Ubuntu 18.04, Intel Core i7-6700 CPU, 3.4GHz, DDR4 64Gb RAM
- Graphs are stored in RedisGraph with our extensions
- Queries are generated with template for given size of start set
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core	1323	4342	g_1
pathways	6238	18 598	g_1
gohierarchy	45 007	980 218	g_1
enzyme	48 815	109 695	g_1
eclass_514en	239 111	523 727	g_1
geospecies	450 609	2 311 461	*geo*
go	272 770	534 311	g_1

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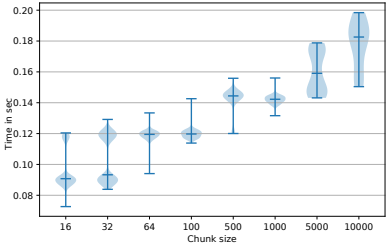
```
PATH PATTERN S =
  ()-/ [<:SubClassOf [~S | ()] :SubClassOf] | [<:Type [~S | ()] :Type] /->()
MATCH (src)-/ ~S /->()
WHERE {id_from} <= src.id and src.id <= {id_to}
RETURN count(*)
```

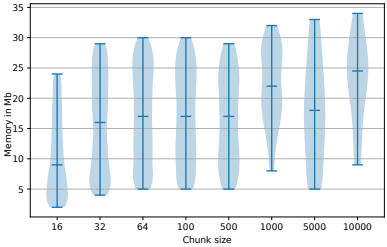
Evaluation Results

Time

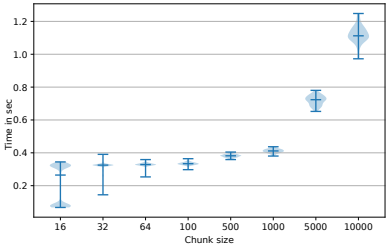
Memory

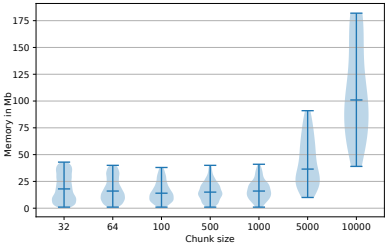
eclass_514en
Query: g1





geospecies
Query: geo





Conclusion

- Full-stack support for CFPQ in real-world applications which use RedisGraph database with Cypher query language
  - No more context-free grammars
  - No more custom graph formats and storages
- Reasonable performance of context-free path queries
  - Multiple-source scenario
  - Space-time ratio can be tuned
- Context-free path queries can be used in applications with well-established tools

Future Research

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  - Including path patterns
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  - To construct paths, not only reachability facts
  - The algorithm should be modified to get multiple-source version

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- Detailed evaluation
  - More graphs and queries, including RPQs
  - Scalability of the solution
  - Comparison with other graph query engines

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Contact Information

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Thanks!

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