Multiple-Source Context-Free Path Querying in Terms of Linear Algebra

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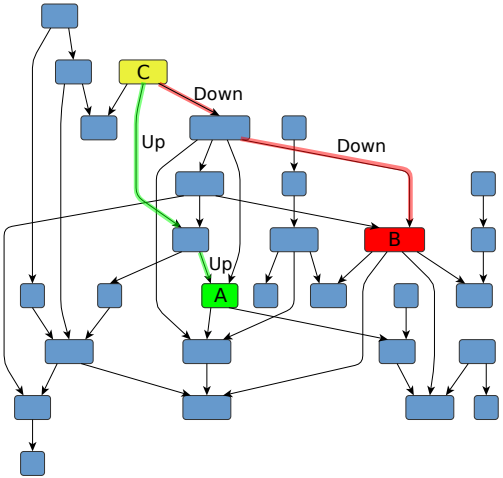
Formal Language Constrained Path Querying



Navigation through an edge-labeled graph

- **Path** specifies a **word** formed by labels of edges
- **Paths constraint** is a **language**: path is good if related word in the give language
- Constraints expressiveness is related to **formal languages classes**

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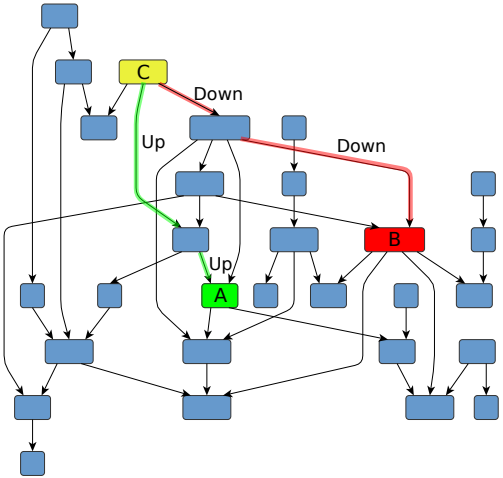

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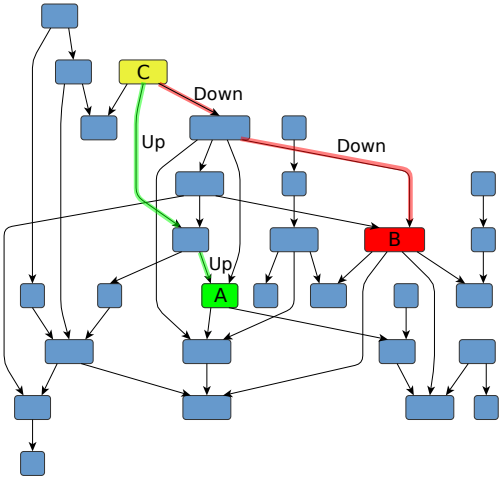
Regular path queries (RPQ)

- **Regular** languages are used as constraints
- Which nodes are reachable from **C** by arbitrary number of **Up** and **Down** edges?
- $\mathcal{L} = (\text{Up} \mid \text{Down})^*$

Context-Free Path Querying (CFPQ)



The constraint is a **context-free language**

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form $\mathbf{Up}^n\,\mathbf{Down}^n$?
- Find all paths of form $\mathbf{Up}^n\,\mathbf{Down}^n$ which start from the node A

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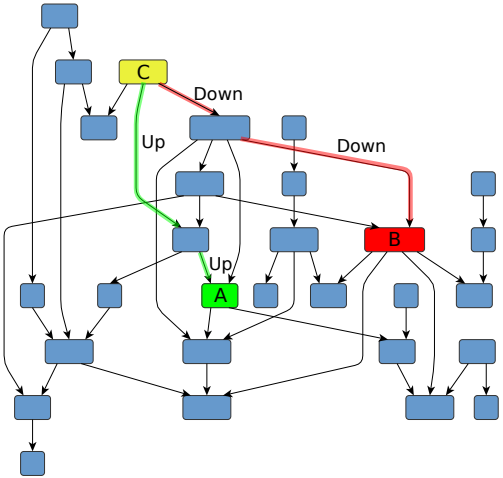

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Applications

- Static code analysis [T. Reps, et al, 1995]
- Graph segmentation [H. Miao, et al, 2019]
- Biological data analysis [P. Sevon, et al, 2008]
- ...

The Problem

There is no support of of CFPQ in real-world graph analysis systems (graph databases)

[1] Jochem Kuijpers, George Fletcher, Nikolay Yakovets, and Tobias Lindaaker. 2019. An Experimental Study of Context-Free Path Query Evaluation Methods.

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- There is no full-stack support of CFPQ
  - Grammars instead of full-featured queries
  - Custom graph storage instead of real-world graph database

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Proposed Solution

- **Multiple-Source CFPQ** allows one to reduce computations in real-world cases

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- **Multiple-Source CFPQ** allows one to reduce computations in real-world cases
- **Cypher** extended with **path patterns**[3] allows one to express context-free constraints

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Proposed Solution

- **Multiple-Source CFPQ** allows one to reduce computations in real-world cases
- **Cypher** extended with **path patterns**[3] allows one to express context-free constraints
- **RedisGraph** database
  - Provides graph storage with matrix-based representation
  - Contains linear algebra based query engine (SuitSparse:GraphBLAS[4] is used)
  - Allows one to use Cypher for querying (libcypher.parser[5] is used)

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Multiple-Source CFPQ

1: **function** MultiSrcCFPQ($D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$, $G = (N, \Sigma, P, S)$, Src)
2: $T \leftarrow \{T^A \mid A \in N, T^A[i,j] \leftarrow \mathit{false}, \text{for all } i,j\}$
3: $TSrc \leftarrow \{TSrc^A \mid A \in N, TSrc^A[i,j] \leftarrow \mathit{false}, \text{for all } i,j\}$
4: **for all** $v \in Src$ **do** $TSrc^S[v,v] \leftarrow \mathit{true}$
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11: **for all** $A \to BC \in P$ **do**
12: $M \leftarrow TSrc^A * T^B$
13: $T^A \leftarrow T^A + M * T^C$
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16: **return** $MSrc * T^S$

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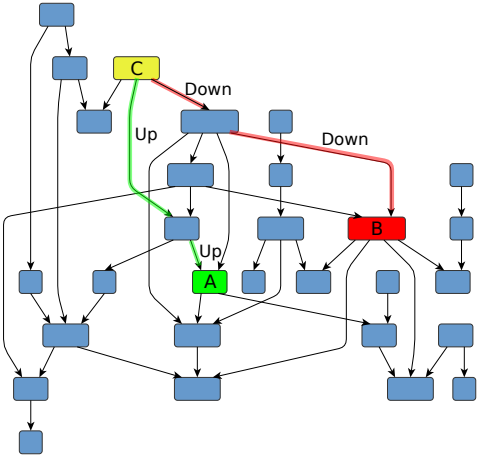
Cypher Extension

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MATCH (a)-[(Down | Up)*]->(b)
RETURN a.name, b.name
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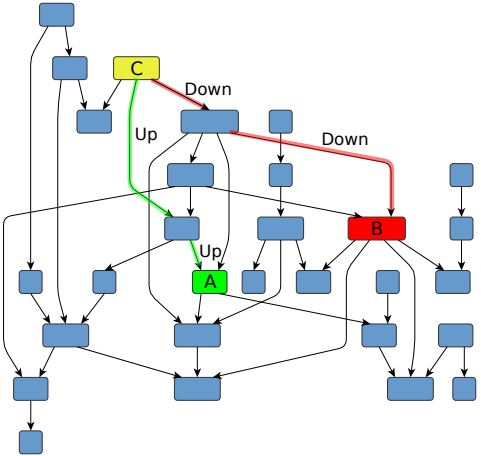


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Named path pattern

$\text{SameLvl} \rightarrow \overline{\text{Down}}\ \text{SameLvl}\ \text{Down} \mid \varepsilon$

```
PATH PATTERN SameLvl =
  ()-/ <:Down [~SameLvl | ()] :Down> /->()
MATCH (a)-/ ~SameLvl /->(b)
RETURN a.name, b.name
```

Implementation Details

- Linear algebra based multiple-source CFPQ is implemented as part of RedisGraph query engine
- Cypher parser is extended to support path patterns
- Path patterns are supported[6] in RedisGreaph query execution workflow

[6] Partially. Full support is a nontrivial challenge: formal description of the extension is required

Evaluation Setup

- Ubuntu 18.04, Intel Core i7-6700 CPU, 3.4GHz, DDR4 64Gb RAM
- Graphs stored in RedisGraph with our extensions
- Queries are generated with template for given size of start set
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core	1323	4342	g_1
pathways	6238	18 598	g_1
gohierarchy	45 007	980 218	g_1
enzyme	48 815	109 695	g_1
eclass_514en	239 111	523 727	g_1
geospecies	450 609	2 311 461	*geo*
go	272 770	534 311	g_1

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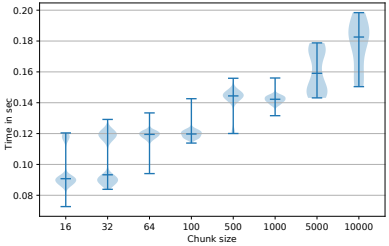
```
PATH PATTERN S =
  ()-/ [<:SubClassOf [~S | ()] :SubClassOf] | [<:Type [~S | ()] :Type] /->()
MATCH (src)-/ ~S /->()
WHERE {id_from} <= src.id and src.id <= {id_to}
RETURN count(*)
```

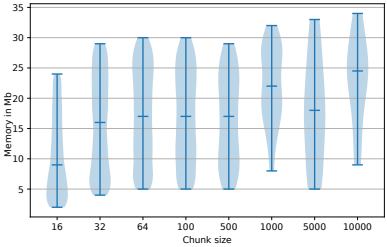
Evaluation Results

Time

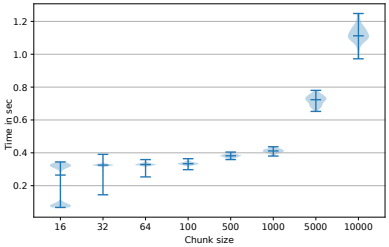
Memory

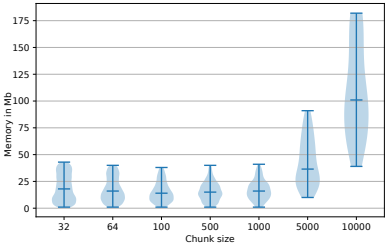
eclass_514en
Query: g1





geospecies
Query: geo





Summary

- Full-stack support for CFPQ in real-world graph query language (Cypher) on the top of real-world graph database (RedisGraph)
  - No more ugly context-free grammars
  - No more custom graph formats and storages
- Reasonable performance of context-free path queries
  - Multiple-source scenario
  - Space-time ratio can be tuned
- Context-free path queries can be used in applications with well-established tools

Future Research

- Mechanization of Cypher semantics in Coq
  - Including path patterns
  - Correctness of translation to linear algebra

[7] Egor Orachev, Ilya Epelbaum, R. Azimov and S. Grigorev. "Context-Free Path Querying by Kronecker Product." ADBIS (2020).

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- Detailed evaluation
  - More graphs and queries, including RPQs
  - Scalability of the solution
  - Comparison with other graph query engines

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Contact Information

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