

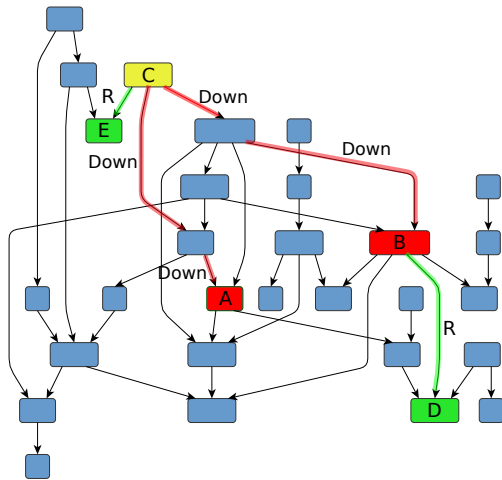
Multiple-Source Context-Free Path Querying in Terms of Linear Algebra

Arseniy Terekhov, Vlada Pogozhelskaya, Vadim Abzalov,
Timur Zinnatulin, **Semyon Grigorev**

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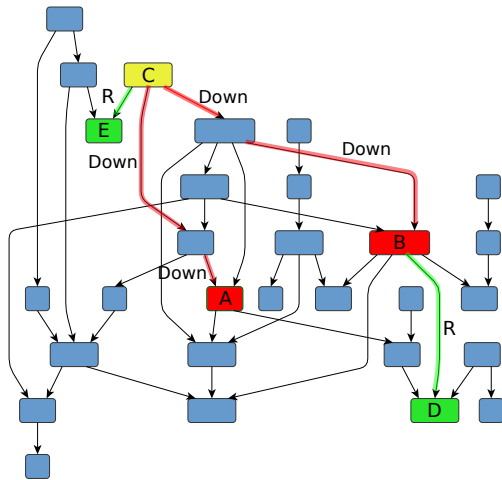
Formal Language Constrained Path Querying



Navigation through an edge-labeled graph

- **Path** specifies a **word** formed by the labels of the edges
- **Paths constraint** is a **language**: the word specified by the path should be in the given language
- The expressiveness of constraints is related to **formal languages classes**

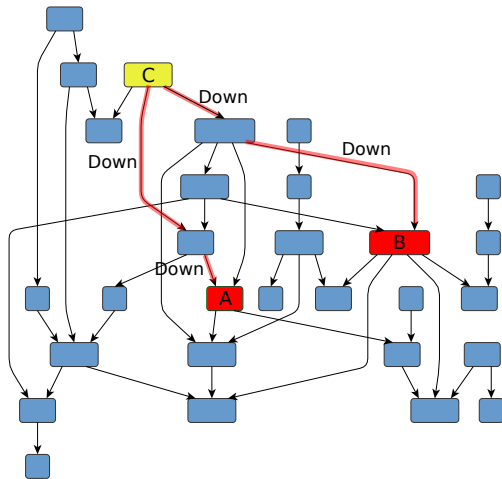
Regular Path Queries (RPQ)



Regular languages as constraints

- Which nodes are reachable from **C** by arbitrary number of **R** and **Down** edges?
- Regular language $\mathcal{L} = (R \mid \text{Down})^*$

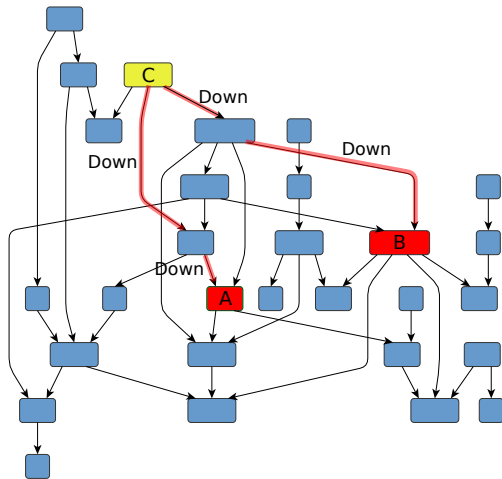
Context-Free Path Queries (CFPQ)



Context-free languages as constraints

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form $\overline{\text{Down}}^n \text{Down}^n$ between A and B?
- Context-free grammar:
 $\text{SameLvl} \rightarrow \overline{\text{Down}} \text{SameLvl} \text{Down} \mid \varepsilon$

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Applications

- Static code analysis [T. Reps, et al, 1995]
- Graph segmentation [H. Miao, et al, 2019]
- Biological data analysis [P. Sevon, et al, 2008] ...

Problem Statement

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- J. Kuijpers, et al¹: existing algorithms are too slow to be used in practical applications (in the context of Neo4j)

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- A. Terekhov, et al²: linear algebra based CFPQ algorithm can be performant enough

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- J. Kuijpers, et al¹: existing algorithms are too slow to be used in practical applications (in the context of Neo4j)
- A. Terekhov, et al²: linear algebra based CFPQ algorithm can be performant enough
- There is no full-stack support of CFPQ
 - ▶ Grammars instead of full-featured queries
 - ▶ Custom graph storage instead of a mature graph database

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- **Multiple-Source CFPQ** to process only the relevant subgraph

Proposed Solution

- **Multiple-Source CFPQ** to process only the relevant subgraph
- **Cypher** extended with **path patterns**³ to express context-free constraints

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<https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc>

Proposed Solution

- **Multiple-Source CFPQ** to process only the relevant subgraph
- **Cypher** extended with **path patterns**³ to express context-free constraints
- **RedisGraph** database
 - ▶ Graph storage with matrix-based representation
 - ▶ Linear algebra based query engine (SuiteSparse:GraphBLAS⁴ is used)
 - ▶ Cypher for querying (libcypher-parser⁵ is used)

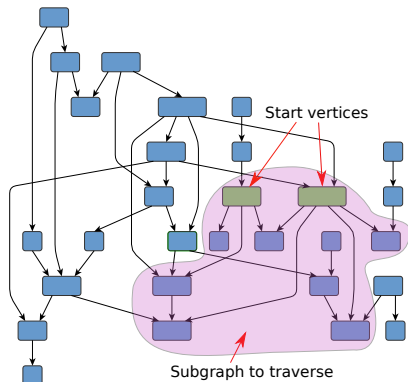
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Multiple-Source CFPQ

An improved version of Rustam Azimov CFPQ algorithm⁶

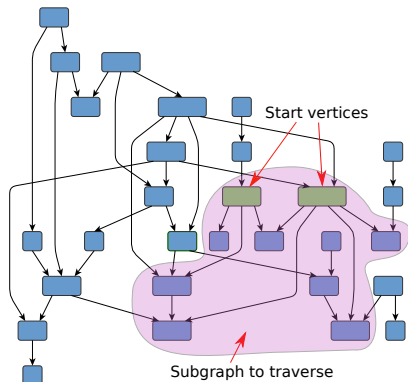


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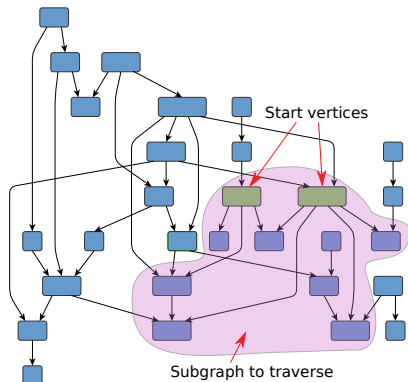
- The set of start vertices can be specified



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Multiple-Source CFPQ

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- The set of start vertices can be specified
- Only the relevant subgraph will be processed

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Multiple-Source CFPQ

```
1: function MULTISRCCFPQ( $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ ,  $G = (N, \Sigma, P, S)$ ,  $Src$ )
2:    $T \leftarrow \{T^A \mid A \in N, T^A[i, j] \leftarrow \text{false, for all } i, j\}$ 
3:    $TSrc \leftarrow \{TSrc^A \mid A \in N, TSrc^A[i, j] \leftarrow \text{false, for all } i, j\}$ 
4:   for all  $v \in Src$  do  $TSrc^S[v, v] \leftarrow \text{true}$ 
5:    $MSrc \leftarrow TSrc^S$ 
6:   for all  $A \rightarrow x \in P \mid x \in \Sigma_E$  do
7:     for all  $(v, to) \in E \mid x \in \lambda_E(v, to)$  do  $T^A[v, to] \leftarrow \text{true}$ 
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16:  return  $MSrc * T^S$ 
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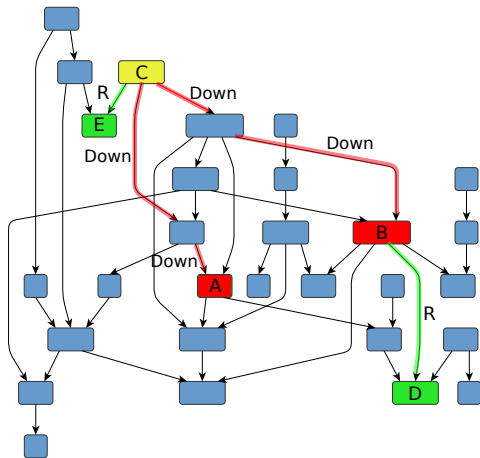
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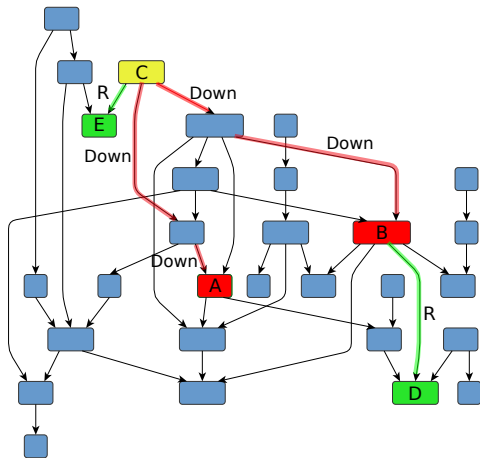
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Cypher Extension



```
MATCH (u)-/[:R | :Down]*-/-(v)  
RETURN u.name, v.name
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Named path pattern

$\text{SameLvl} \rightarrow \overline{\text{Down}} \text{ SameLvl} \text{ Down} \mid \epsilon$

```
PATH PATTERN SameLvl =
  ()-/[:Down ~SameLvl :Down] | ()/->()
MATCH (u)-/ ~SameLvl /-(v)
RETURN u.name, v.name
```

Implementation Details

- Linear algebra based multiple-source Context-Free Path Querying is implemented as a part of RedisGraph query engine
- Cypher parser is extended to support path patterns
- Path patterns are partially supported⁷ in RedisGreaph query execution workflow

⁷Full support is a nontrivial challenge: formal description of the extension is required

Evaluation Setup

- Ubuntu 18.04, Intel Core i7-6700 CPU, 3.4GHz, DDR4 64Gb RAM
- Graphs are stored in RedisGraph augmented with our extensions
- Queries are generated with template for the given size of the start set
- The union of all start sets is denoted V

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Graph	#V	#E	Q
core	1323	4342	g_1
pathways	6238	18 598	g_1
gohierarchy	45 007	980 218	g_1
enzyme	48 815	109 695	g_1
eclass_514en	239 111	523 727	g_1
geospecies	450 609	2 311 461	geo
go	272 770	534 311	g_1

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PATH PATTERN $S =$

$() - / [< : \text{SubClassOf} [\sim S \mid ()] : \text{SubClassOf}] \mid [< : \text{Type} [\sim S \mid ()] : \text{Type}] / - > ()$

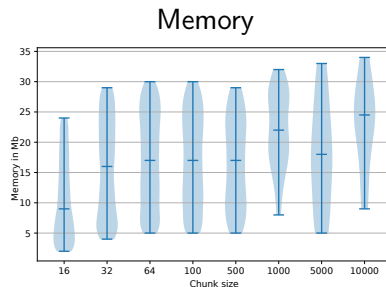
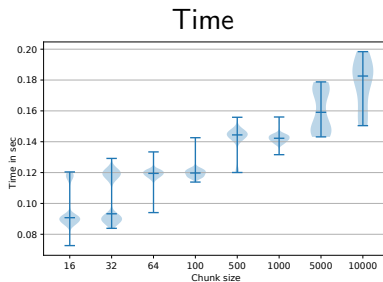
MATCH (src) - / $\sim S$ / - > ()

WHERE {id_from} <= src.id and src.id <= {id_to}

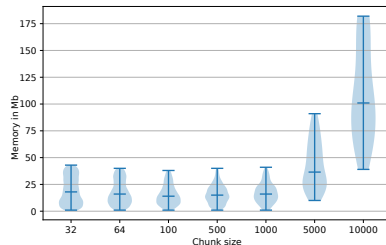
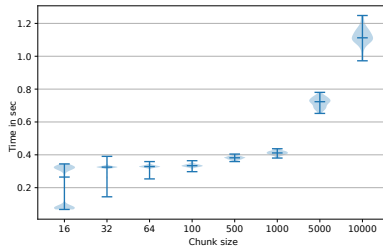
RETURN count(*)

Evaluation Results

eclass_514en
Query: g1



geospecies
Query: geo



- Full-stack support for CFPQ in real-world applications which use RedisGraph database with Cypher query language
 - ▶ No more context-free grammars
 - ▶ No more custom graph formats and storages
- Reasonable performance of context-free path queries
 - ▶ Multiple-source scenario
 - ▶ Space-time ratio can be tuned
- Context-free path queries can be used in applications with well-established tools

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 - ▶ The algorithm should be modified to get multiple-source version

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 - ▶ The algorithm should be modified to get multiple-source version
- Detailed evaluation
 - ▶ Include more graphs and queries, including RPQs
 - ▶ Evaluate the scalability of the solution
 - ▶ Compare with other graph query engines

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Thanks!