GRADES-NDA 2020

Context-Free Path Querying with Single-Path Semantics by Matrix Multiplication

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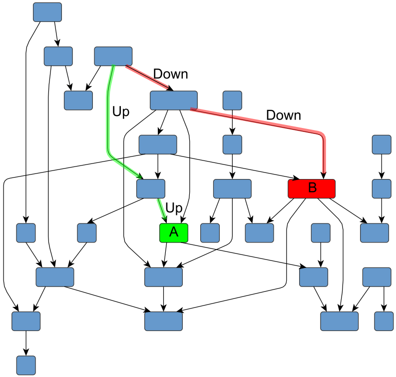
Context-Free Path Querying



Navigation through a graph

- Are nodes A and B on the same level of hierarchy?
- Is there a path of form **Up**^{n} **Down**^{n}?
- Find all paths of form **Up**^{n} **Down**^{n} which start from the node A

Context-Free Path Querying: Relational Query Semantics

- $\mathbb{G} = (\Sigma, N, P)$ — context-free grammar in normal form
  - $A \rightarrow BC$, where $A, B, C \in N$
  - $A \rightarrow x$, where $A \in N, x \in \Sigma \cup \{\varepsilon\}$
  - $L(\mathbb{G}, A) = \{\omega \mid A \Rightarrow^* \omega\}$

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  - $v \xrightarrow{l} u \in E$
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- $\omega(\pi) = \omega(v_0 \xrightarrow{l_0} v_1 \xrightarrow{l_1} \cdots \xrightarrow{l_{n-2}} v_{n-1} \xrightarrow{l_{n-1}} v_n) = l_0 l_1 \cdots l_{n-1}$

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- $R_A = \{(n, m) \mid \exists n\pi m$, such that $\omega(\pi) \in L(\mathbb{G}, A)\}$

Matrix-Based Algorithm: Relational Query Semantics

Algorithm Context-free path querying algorithm

function $\textsc{evalCFPQ}(D = (V, E, L), G = (\Sigma, N, P))$
 $n \leftarrow |V|$
 $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i}$ is a matrix $n \times n$, $T^{A_i}_{k,l} \leftarrow \texttt{false}\}$
 for all $(i, x, j) \in E$, $A_k \mid A_k \to x \in P$ **do** $T^{A_k}_{i,j} \leftarrow \texttt{true}$
 for all $A_k \mid A_k \to \varepsilon \in P$ **do**
  for all $i \in \{0, \ldots, n-1\}$ **do** $T^{A_k}_{i,i} \leftarrow \texttt{true}$
 while any matrix in T is changing **do**
  for $A_i \to A_j A_k \in P$ **do** $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \times T^{A_k})$
 return T

- $R_A = \{(n, m) \mid \exists n\pi m, \text{ such that } \omega(\pi) \in L(\mathbb{G}, A)\}$ — answers for the relational query semantics

Context-Free Path Querying: Single-Path Query Semantics

- $R_A = \{(n, m) \mid \exists n\pi m, \text{ such that } \omega(\pi) \in L(\mathbb{G}, A)\}$ — answers for the relational query semantics
- For all $A \in N$, for all $(n, m) \in R_A$ also return some such path $n\pi m$
  - usually the shortest path is returned
  - returned path can be used as a proof of existence

Research Questions

- Can we extend the matrix-based CFPQ algorithm to single-path query semantics?
- What the cost of such extension?
- Can we achieve high performance of CFPQ integrated with existing graph database?
- Does using GPGPU still improve performance over CPU versions?

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$$PI_1 \oplus PI_2 = \begin{cases} PI_1, & \text{if } PI_1.height \le PI_2.height \\ PI_2, & \text{otherwise} \end{cases}$$

Matrix-Based Algorithm: Single-Path Query Semantics

Algorithm CFPQ algorithm w.r.t. single-path query semantics

function $\textsc{evalCFPQ}(D = (V, E), G = (N, \Sigma, P))$
    $n \leftarrow |\mathrm{V}|$
    $T \leftarrow \{ T^{A_i} \mid A_i \in N,\ T^{A_i}$ is a matrix $n \times n,\ T^{A_i}_{k,l} \leftarrow \bot \}$
    for all $(i, x, j) \in E,\ A_k \mid A_k \to x \in P$ **do** $T^{A_k}_{i,j} \leftarrow (i, j, i, 1, 1)$
    for $A_k \mid A_k \to \varepsilon \in P$ **do** $T^{A_k}_{i,i} \leftarrow (i, i, i, 1, 0)$
    while any matrix in T is changing **do**
        for $A_i \to A_j A_k \in P$ **do** $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} \odot T^{A_k})$
    return T

Matrix-Based Algorithm: Technical Details

- We can remove *length* or *height* to reduce memory consumption
- The PathIndex operations can be represented as bitwise atomic operations
- We still can use existing high-performance libraries for matrix operations if they support the creation of custom operations

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  - The path which forms a string with minimal height of derivation tree
  - The shortest path
- Linear complexity in the length of the extracted path

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    - **RG_SPARSE**$_{rel}$ — relational query semantics, uses low-latency on-chip shared memory for the hash table of each row of the result matrix
    - **RG_SPARSE**$_{path}$ — single-path query semantics, operating over PathIndex semiring

Dataset[1]

RDF Name	#V	#E
univ-bench	179	413
pizza	671	2,604
wine	733	2,450
core	1,323	8,684
pathways	6,238	37,196
go-hierarchy	45,007	1,960,436
enzyme	48,815	219,390
eclass_514en	239,111	1,047,454
go	272,770	1,068,622
geospecies	450,609	4,622,922

[1] Queries is based on the context-free grammars for nested parentheses

Evaluation

- OS: Ubuntu 18.04
- CPU: Intel core i7 6700 3,4GHz
- RAM: DDR4 64 Gb
- GPGPU: NVIDIA GeForce 1070 (8Gb RAM)

Evaluation: CFPQ[2]

Name	Relational semantics index						Single path semantics index			
	RG_CPU_{rel}		RG_CUSP_{rel}		RG_SPARSE_{rel}		RG_CPU_{path}		RG_SPARSE_{path}	
	Time	Mem	Time	Mem	Time	Mem	Time	Mem	Time	Mem
univ-bench	0.002	0.3	0.010	0.1	0.005	0.1	0.013	0.3	0.007	0.1
pizza	0.030	1.8	0.021	4.0	0.006	0.1	0.075	5.5	0.009	0.1
wine	0.017	3.5	0.032	6.0	0.009	0.1	0.117	7.1	0.015	0.2
core	0.004	0.3	0.022	2.0	0.010	0.1	0.002	0.3	0.016	0.1
pathways	0.011	0.1	0.019	0.1	0.007	0.1	0.021	0.5	0.021	2.0
go-hierarchy	0.091	16.3	0.433	650.0	0.108	121.2	0.976	92.0	0.336	125.0
enzyme	0.018	5.9	0.021	0.1	0.018	4.0	0.029	8.1	0.043	6.0
eclass_514en	0.067	13.8	0.075	14.0	0.166	16.0	0.195	31.2	0.496	26.0
go	0.604	28.8	0.590	70.0	0.365	30.2	1.286	75.7	0.739	45.4
geospecies	7.146	16934.2	—	—	0.856	5274	15.134	35803.6	1.935	5282

[2] Time in seconds and memory is measured in megabytes

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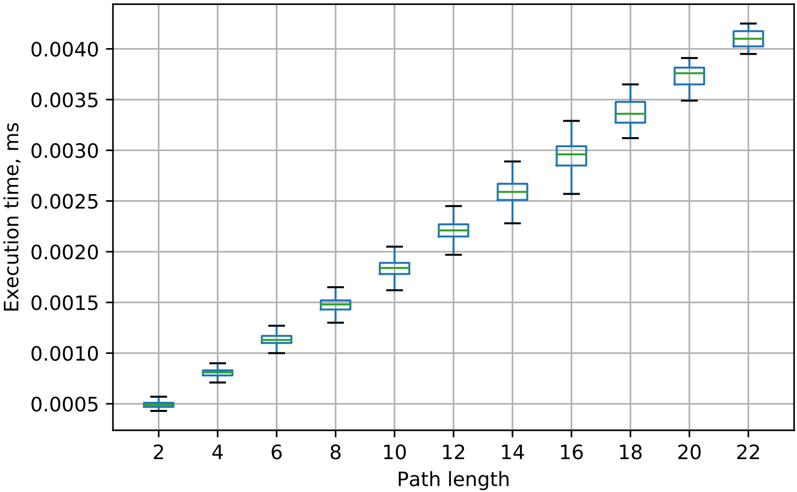
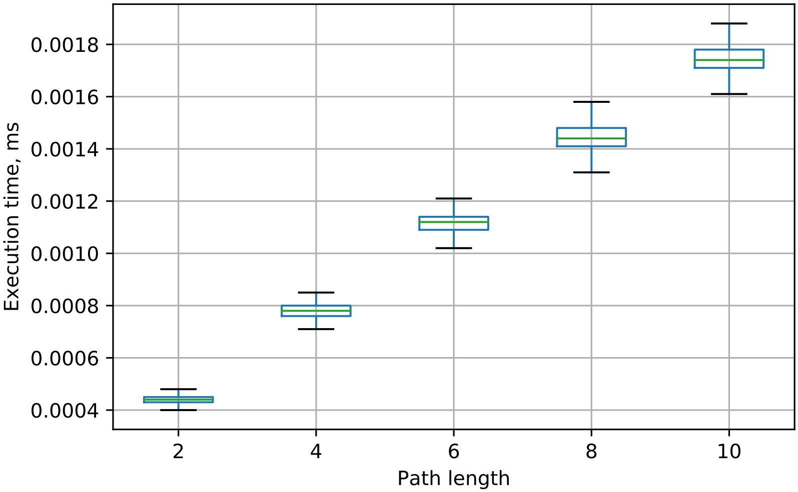
Evaluation: Path Extraction Time For *go*



Evaluation: Path Extraction Time For *geospecies*



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- Dataset is published: both graphs and queries
  - Link: `https://github.com/JetBrains-Research/CFPQ_Data`
- Implementations are available on GitHub
  - Link: `https://github.com/YaccConstructor/RedisGraph`

Future Research

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- Improve the dataset
  - Include real-world cases from the area of static code analysis
  - Find new applications that required CFPQ, such as graph segmentation

Acknowledgments

- Special thanks to
  - Gábor Szárnyas for turning our attention to SuiteSparse and GraphBLAS
  - George Fletcher for telling us about his research on CFPQ for Neo4j
  - Roi Lipman for great help with RedisGraph graph database

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Thanks!