Multiple-Source Context-Free Path Querying in Terms of Linear Algebra

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ABSTRACT

Context-Free Path Querying (CFPQ) allows one to express path constraints in navigational graph queries as context-free grammars. Although there are many algorithms developed for CFPQ, no graph database provides full-stack support of CFPQ. The Azimov's CFPQ algorithm is applicable for real-world graph analysis, as shown by Arseniy Terekhov. In this work we provide a modification to Azimov's algorithm for multiple-source CFPQ which makes the algorithm more practical and eases the integration into RedisGraph graph database. We also implement a Cypher graph query language extension for context-free constraints. Thus we provide the first full-stack support of CFPQ for graph databases. Our evaluation shows that the provided solution is suitable for the real-world graph analysis.

1 INTRODUCTION

Language-constrained path querying [3] is a way to search for paths in edge-labeled graphs where constraints are formulated in terms of a formal language. The language restricts the set of accepted paths: the sentence formed by the labels of a path should be in the language. Regular languages are the most popular class of constraints used as navigational queries in graph databases. In some cases, regular languages are not expressive enough and context-free languages are used instead. Context-free path querying (CFPQ), can be used for RDF analysis [27], biological data analysis [22], static code analysis [20, 28], and in other areas.

CFPQ have been studied a lot since the problem was first stated by Mihalis Yannakakis in 1990 [26]. Jelle Hellings investigates various aspects of CFPQ in [8–10]. A number of CFPQ algorithms were proposed: (G)LL and (G)LR based algorithms by Ciro M. Medeiros et al. [15], Fred C. Santos et al. [21], Semyon Grigorev et al. [7], and Ekaterina Verbitskaia et al. [24]; CYK-based algorithm by Xiaowang Zhang et al. [27]; combinators-based approach to CFPQ by Ekaterina Verbitskaia et al. [25]. Nevertheless, the application of context-free constraints for real-world data analysis still faces many problems. The first problem is bad performance of the proposed algorithms on real-world data, as shown by Jochem Kuijpers et al. [14]. The second problem is that no graph database provides full-stack support of CFPQ, since most effort was made in developping algorithms and researching their theoretical properties. This fact hinders research of problems which can be reduced to CFPQ, thus it hinders the development of new solutions for them. For example, graph segmentation in data provenance analysis was recently reduced to CFPQ [17], but evaluation of the proposed approach was complicated because no graph database supported CFPQ.

Rustam Azimov proposed a matrix-based algorithm for CFPQ in [2]. This algorithm provides a solution performant enough for real-world data analysis, as shown by Nikita Mishim et al. in [18] and Arseniy Terekhov et al. in [23]. This algorithm computes reachability or provides a single path which satisfies constraints for *every* vertex pair in the graph. Namely it solves *all-pairs* context-free path querying problem. In many real-world scenarios it is redundant to handle all possible pairs, instead one can provide one or a relatively small set of start vertices.

While all-pairs context-free path querying is a problem well studied, there is no, best to our knowledge, solutions for the single-source and multiple-source CFPQ. In this work we propose a matrix-based *multiple-source* (and *single-source* as a partial case) CFPQ algorithm.

We also provide full-stack support of CFPQ for the RedisGraph[1] [4] graph database. We implement a Cypher query language extension[2] that makes it possible to use context-free constraints, and extend the RedisGraph to support this extension. As far as we know, it is the first full-stack implementation of CFPQ.

To sum up, we make the following contributions in this paper.

(1) We modify Azimov's matrix-based CFPQ algorithm and provide a multiple-source matrix-based CFPQ algorithm. As a partial case, it is possible to use our algorithm in a single-source scenario. Our modification is still based on linear algebra, hence it is simple to implement and allows one to use high-performance libraries and utilize modern parallel hardware for queries evaluation.

[1] RedisGraph graph database Web-page: https://redislabs.com/redis-enterprise/redis-graph/. Access date: 19.07.2020.

[2] Proposal which describes path patterns specification syntax for Cypher query language: https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc. The proposed syntax allows one to specify context-free constraints. Access date: 19.07.2020.

(2) We evaluate two versions of the proposed algorithm: with caching of results and without caching (naive). Caching is aimed to reduce repeated calculation of the same data. Our evaluation shows that the naive version is more performant and memory-efficient than the version with results caching in almost the all cases. We believe, it is a good choice for implementation in real-world graph database.

(3) We provide full-stack support of CFPQ by extending the RedisGraph graph database. To do it, we extended Cypher with syntax for context-free constraints, implemented the proposed algorithm in a RedisGraph backend, and supported the new syntax in the RedisGraph query execution engine. Finally, we evaluate the proposed solution and show that it is performant and memory-efficient enough to be applicable for real-world graph querying.

2 PRELIMINARIES

In this section we introduce common definitions in graph theory and formal language theory which are used in this paper. Also, we provide a brief description of Azimov's algorithm which is used as a base of our solution.

2.1 Basic Definitions of Graph Theory

In this paper we use a labeled directed graph as a data model and define it as follows.

Definition 2.1. *Labeled directed graph* is a tuple of six elements $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$, where

- V is a finite set of vertices. For simplicity, we assume that the vertices are natural numbers from 0 to $|V| - 1$.
- $E \subseteq V \times V$ is a set of edges.
- Σ_V and Σ_E are sets of labels of vertices and edges respectively, such that $\Sigma_V \cap \Sigma_E = \varnothing$.
- $\lambda_V : V \longrightarrow 2^{\Sigma_V}$ is a function that maps a vertex to a set of its labels, which can be empty.
- $\lambda_E : E \longrightarrow 2^{\Sigma_E} \setminus \{\varnothing\}$ is a function that maps an edge to a non-empty set of its labels, so each edge must have at least one label.

□

Labeled graph is the basis of the widely-used *property graph* data model [1] and allows one to use not only edge labels but also vertex labels in navigation queries.

An example of the labeled directed graph D_1 is presented in figure 1. Here the sets of labels $\Sigma_V = \{x, y\}$ and $\Sigma_E = \{a, b, c, d\}$. We omit vertex labels set if it is empty.

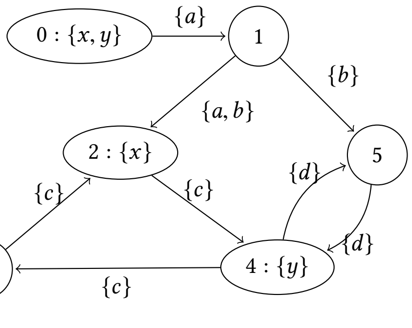

Figure 1: The input graph D_1

Definition 2.2. Path π in the graph $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ is a finite sequence of vertices and edges $(v_0, e_0, v_1, e_1, ..., e_{n-1}, v_n)$, where $\forall i, 0 \leq i \leq n : v_i \in V; \forall j, 1 \leq j \leq n : e_j = (v_j, v_{j+1}) \in E$.

We denote the set of all paths in the graph D as $\pi(D)$. □

Definition 2.3. An *adjacency matrix* M of the graph D is a matrix of size $|V| \times |V|$, such that

$$M[i,j] = \begin{cases} \lambda_E((i,j)) & , (i,j) \in E \\ \varnothing & , otherwise \end{cases}$$

□

Adjacency matrix M of the graph D_1 (fig. 1) is the following:

$$M = \begin{pmatrix} \varnothing & \{a\} & \varnothing & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \{a,b\} & \varnothing & \varnothing & \{b\} \\ \varnothing & \varnothing & \varnothing & \varnothing & \{c\} & \varnothing \\ \varnothing & \varnothing & \{c\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{c\} & \varnothing & \{d\} \\ \varnothing & \varnothing & \varnothing & \varnothing & \{d\} & \varnothing \end{pmatrix}.$$

Definition 2.4. Let M be an adjacency matrix of the graph D. Then the *adjacency matrix of label* $l \in \Sigma_E$ of graph D is a matrix $\mathcal{E}^l$ of size $|V| \times |V|$, such that

$$\mathcal{E}^l[i,j] = \begin{cases} 1 & , l \in M[i,j] \\ 0 & , otherwise \end{cases}$$

□

Definition 2.5. A *boolean decomposition of adjacency matrix* M of the graph D is a set of Boolean matrices $\mathcal{E} = \{\mathcal{E}^l \mid l \in \Sigma_E\}$, where $\mathcal{E}^l$ is the adjacency matrix of label l. □

For example, the boolean decomposition of the adjacency matrix M of the graph D_1 is the set of Boolean matrices $\mathcal{E}^a$, $\mathcal{E}^b$, $\mathcal{E}^c$ and $\mathcal{E}^d$:

$$\mathcal{E}^a = \begin{pmatrix} . & . & . & . & . & . \\ 1 & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix}, \mathcal{E}^b = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & 1 & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & 1 & . & . & . & . \end{pmatrix},$$

$$\mathcal{E}^c = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & 1 & . & . \\ . & . & . & . & 1 & . \\ . & . & 1 & . & . & . \\ . & . & . & . & . & . \end{pmatrix}, \mathcal{E}^d = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & 1 \\ . & . & . & . & 1 & . \end{pmatrix}.$$

Definition 2.6. A *vertex label matrix* H of the graph D is a matrix of size $|V| \times |V|$, such that

$$H[i,j] = \begin{cases} \lambda_V(i) & , i = j \\ \varnothing & , otherwize \end{cases}$$

□

The vertex label matrix H of the example graph D_1 is

$$H = \begin{pmatrix} \{x,y\} & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \{x\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \varnothing & \{y\} & \varnothing \\ \varnothing & \varnothing & \varnothing & \varnothing & \varnothing & \varnothing \end{pmatrix}.$$

Definition 2.7. Let H be a vertex label matrix of graph D. Then the *vertices matrix of label* l is a matrix $\mathcal{V}^l$ of size $|V| \times |V|$, such that

$$\mathcal{V}^l[i,j] = \begin{cases} 1 & , l \in H[i,j] \\ 0 & , otherwise \end{cases}$$

Definition 2.8. A *boolean decomposition of vertex label matrix* H of the graph D is the set of Boolean matrices $\mathcal{V} = \{V^l \mid l \in \Sigma\}$, where $\mathcal{V}^l$ is a vertices matrix of label l.

Vertex label matrix H of the graph D_1 can be decomposed into a set of the following Boolean matrices:

$$\mathcal{V}^x = \begin{pmatrix} 1 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix}, \mathcal{V}^y = \begin{pmatrix} 1 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & . \end{pmatrix}.$$

2.2 Basic Definitions of Formal Languages

We use context-free grammars as paths constraints, thus in this subsection we define context-free languages and grammars.

Definition 2.9. A *context-free grammar* is a tuple $G = (N, \Sigma, P, S)$, where

- N is a finite set of nonterminals
- Σ is a finite set of terminals, $N \cap \Sigma = \varnothing$
- P is a finite set of productions of the form $A \to \alpha$, where $A \in N,\ \alpha \in (N \cup \Sigma)^*$
- S is the start nonterminal

□

Definition 2.10. A *context-free language* is a language generated by a context-free grammar G:

$$L(G) = \{w \in \Sigma^* \mid S \overset{*}{\underset{G}{\Longrightarrow}} w\}$$

Where $S \overset{*}{\underset{G}{\Longrightarrow}} w$ denotes that a string w can be generated from a starting non-terminal S using some sequence of production rules from P. □

Definition 2.11. A context-free grammar $G = (N, \Sigma, P, S)$ is in *Chomsky normal form* if every production in P has one of the following forms:

- $A \to BC$, where $A \in N,\ B,\ C \in N \setminus \{S\}$
- $A \to a$, where $A \in N,\ a \in \Sigma$
- $S \to \varepsilon$, where ε is an empty string.

□

Definition 2.12. A context-free grammar $G = (N, \Sigma, P, S)$ is in *weak Chomsky normal form* if every production in P has one of the following forms:

- $A \to BC$, where $A, B, C \in N$
- $A \to a$, where $A \in N, a \in \Sigma$
- $A \to \varepsilon,\ A \in N$

□

In other words, weak Chomsky normal form differs from Chomsky normal form in the following:

- ε can be derived from any non-terminal;
- S can occur in the right-hand side of productions.

The matrix-based CFPQ algorithms process grammars only in weak Chomsky normal form, but every context-free grammar can be transformed into the equivalent grammar in this form.

Consider the example of the context-free grammar $G_1 = (N, \Sigma, P, S)$, where $N = \{S\},\ \Sigma = \{c, d, y\}$, and P contains two rules:

$$\begin{aligned} S &\to c\ S\ d \\ S &\to c\ y\ d \end{aligned} \tag{1}$$

This grammar generates the context-free language:

$$L(G_1) = \{c^n y d^n, n \in \mathbb{N}\}.$$

The following grammar G_1^{wcnf} is a result of the transformation of G_1 to weak Chomsky normal form:

$$\begin{array}{ll} S \to C\ E & C \to c \\ S \to C\ S_1 & Y \to y \\ E \to Y\ D & D \to d \\ S_1 \to S\ D & \end{array}$$

2.3 Context-Free Path Querying

Definition 2.13. Let $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ be a labeled graph, $G = (N, \Sigma_V \cup \Sigma_E, P, S)$ be a context free grammar. Then a *context free relation* with grammar G on the labeled graph D is the relation $R_{G,D} \subseteq V \times V$:

$$R_{G,D} = \{(v_1, v_n) \in V \times V \mid \exists \pi = (v_1, e_1, v_2, \ldots, e_n, v_n) \in \pi(D) : \\ l(\pi) \cap L(G) \neq \varnothing\},$$

where $l(\pi) \subset (\Sigma_V \cup \Sigma_E)^*$ is the set of labels along the path π:

$$l(\pi) = \lambda_V(v_1)^* \cdot \lambda_E(e_1) \cdot \lambda_V(v_2)^* \cdot \lambda_E(e_2) \cdot \ldots \cdot \lambda_E(e_n) \cdot \lambda_V(v_n)^*$$

□

For example, π is a path from vertex 2 to vertex 5 in the labeled graph presented in figure 1:

$$\pi = 2 : \{x\} \xrightarrow{\{c\}} 4 : \{y\} \xrightarrow{\{d\}} 5.$$

Labels along π form the set of sequences $l(\pi) = \{x^m c y^n d \mid n \geq 0, m \geq 0\}$. Only one of these sequences satisfies context-free constraints of the grammar G_1: cyd. The derivation of the sequence is the following:

$$S \Rightarrow CE \Rightarrow cE \Rightarrow cYD \Rightarrow cyD \Rightarrow cyd$$

Hence $l(\pi) \cap L(G_1) \neq \varnothing$ and the pair $(3, 6) \in R_{G_1,D}$.

Take a closer look at the definition of path labels, namely that it allows for zero or more repetitions of a label of each vertex. This makes it possible to omit vertex labels or, if there are many vertex labels, to use them in an arbitrary order. It also permits to write a query which uses one vertex label multiple times. This definition may appear strange in some cases, but it depends on the semantics of the graph query language. To formalize the semantics is future work, so we will stick to this definition in this paper.

Finally, we can define context-free path querying problem.

Definition 2.14. *Context-free path querying problem* is the problem of finding context-free relation $R_{G,D}$ for a given directed labeled graph D and a context-free grammar G. □

In other words, the result of context-free path query evaluation is a set of vertex pairs such that there is a path between them and this path forms a word from the given language.

The context-free relation R_{G_1,D_1} for the graph D_1 and the context-free free grammar G_1 is the following:

$$R_{G_1,D_1} = \{(2, 4), (2, 5), (3, 4), (3, 5), (4, 4), (4, 5)\}.$$

Note that any relation $R_{G,D}$ can be represented as a Boolean matrix:

$$T[i, j] = 1 \iff (i, j) \in R_{G,D}.$$

In our example, R_{G_1,D_1} can be represented as follows:

$$T = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & 1 \\ . & . & . & . & 1 & 1 \\ . & . & . & . & 1 & 1 \\ . & . & . & . & . & . \end{pmatrix}.$$

Definition 2.15. Suppose *Src* is a given set of start vertices, then *multiple-source context-free path querying problem* for the given *Src*, directed labeled graph D and context-free grammar G is to find a context-free relation

$$R^{Src}_{G,D} \subseteq Src \times V \subseteq R_{G,D}.$$

Thus we restrict start vertices of the paths of interest to be a vertices from the given set □

As a special case, a *single-source CFPQ* is when *Src* is a singleton set. If we set $Src = \{2\}$ in the previous example, then the result is:

$$R^{\{2\}}_{G_1,D_1} = \{(2,4),(2,5)\}.$$

We can represent the $R^{\{2\}}_{G_1,D_1}$ as a Boolean matrix:

$$T = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & 1 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix}.$$

2.4 Matrix-Based Algorithm

Our algorithm is based on the Azimov's CFPQ algorithm [2] which is based on matrix operations. This algorithm allows one to use high-performance linear algebra libraries and utilize modern parallel hardware for CFPQ.

Let $G = (N, \Sigma, P, S)$ be the input grammar, the input labeled graph $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ and language L over alphabet Σ. The matrix-based algorithm for CFPQ can be expressed in terms of operations over Boolean matrices as presented in Algorithm 1. Using Boolean matrices simplifies the implementation of the algorithm.

Algorithm 1 Context-free path querying algorithm

```
function evalCFPQ($D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$,
        $G = (N, \Sigma, P, S)$)
    $n \leftarrow |V|$
    $T \leftarrow \{T^{A_i} \mid A_i \in N, T^{A_i}[i,j] \leftarrow false$, for all $i,j\}$
    for all $(i,j) \in E, A_k \mid \lambda_E(i,j) = x, A_k \to x \in P$ do $T^{A_k}_{i,j} \leftarrow$ true
    for all $A_k \mid A_k \to \varepsilon \in P$ do
        for all $i \in \{0,\dots,n-1\}$ do $T^{A_k}[i,i] \leftarrow true$
    while any matrix in $T$ is changing do
        for $A_i \to A_j A_k \in P$ do $T^{A_i} \leftarrow T^{A_i} + (T^{A_j} * T^{A_k})$
    return $T$
```

Note, that the provided algorithm computes not only the context-free relation $R_{G,D}$ but a set of context-free relations $R_{A,D} \subseteq V \times V$ for every $A \in N$. Thus it provides information about paths which form words derivable from any nonterminal in the given grammar. Also, this algorithm handles only the edge labels.

As was shown by Nikita Mishin et al. [18] and Arseniy Terekhov et al. [23], this algorithm can be implemented using various high-performance programming techniques (including GPGPU utilization), and it is applicable for real-world graph analysis. But this algorithm solves *all-pairs* version of CFPQ: it finds all pairs of vertices in the given graph, such that there exist a path between them which forms a word in the given language. Thus it is impractical in cases when we are only interested in paths which start from the specific set of vertices, especially if this set is relatively small. Moreover, Azimov's algorithm operates over an adjacency matrix of the whole input graph, and as a result it requires a huge amount of memory, which may be a problem for a real-world graph database.

3 MATRIX-BASED MULTIPLE-SOURCE CFPQ ALGORITHM

In this section we introduce two versions of the multiple-source matrix-based CFPQ algorithm. This algorithm is a modification of Azimov's matrix-based algorithm for CFPQ and its core idea is to cut off those vertices which are not in the selected set of start vertices.

In order to simplify Azimov's algorithm modification and the final algorithm description, we assume the input graph to have only edge labels. Note, that we always can convert the original graph into such form. To do it, we should add loops into vertices in the following way: for the vertex i we add an edge $i \xrightarrow{x} i$ iff $\lambda_V(i) = x$ and $x \neq \varnothing$. This way we can switch to an edge-labeled graph with the same number of vertices while preserving the defined semantics of CFPQ.

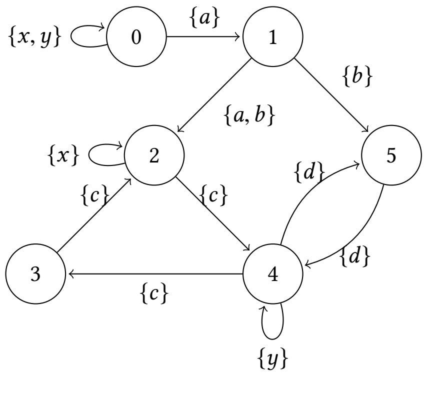

Figure 2: The example of D_1': the modified input graph D_1

The adjacency matrix M of the graph D_1' is

$$M = \begin{pmatrix} \{x,y\} & \{a\} & \varnothing & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \{a,b\} & \varnothing & \varnothing & \{b\} \\ \varnothing & \varnothing & \{x\} & \varnothing & \{c\} & \varnothing \\ \varnothing & \varnothing & \{c\} & \varnothing & \varnothing & \varnothing \\ \varnothing & \varnothing & \varnothing & \{c\} & \{y\} & \{d\} \\ \varnothing & \varnothing & \varnothing & \varnothing & \{d\} & \varnothing \end{pmatrix}.$$

Note that this transformation is impractical for real-world graphs, thus we use it only for algorithm description.

The first version of multiple-source algorithm is the Azimov's algorithm equipped with vertices filtering. Let $G = (N, \Sigma, P, S)$ be the input context-free grammar, $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$ be the input graph and *Src* be the input set of start vertices. The result of the algorithm is a Boolean matrix which represents relation $R^{Src}_{S,D}$.

Algorithm 2 Multiple-source context-free path querying algorithm

```
1:  function MultiSrcCFPQNaive(
            $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$,
            $G = (N, \Sigma, P, S)$,
            $Src$)
2:      $T \leftarrow \{T^A \mid A \in N, T^A[i, j] \leftarrow false$, for all $i, j\}$
3:      $TSrc \leftarrow \{TSrc^A \mid A \in N, TSrc^A[i, j] \leftarrow false$, for all $i, j\}$
4:      for all $v \in Src$ do                          ▷ Input matrix initialization
5:          $TSrc^S[v, v] \leftarrow true$
6:      $MSrc \leftarrow TSrc^S$
7:      for all $A \rightarrow x \in P$ do                ▷ Simple rules initialization
8:          for all $(v, to) \in E, \lambda_E(v, to) = x$ do
9:              $T^A[v, to] \leftarrow true$
10:     while $T$ or $TSrc$ is changing do                   ▷ Algorithm's body
11:         for all $A \rightarrow BC \in P$ do
12:             $M \leftarrow TSrc^A * T^B$
13:             $T^A \leftarrow T^A + M * T^C$
14:             $TSrc^B \leftarrow TSrc^B + TSrc^A$
15:             $TSrc^C \leftarrow TSrc^C +$ getDst($M$)
16:     return $MSrc * T^S$
17: function getDst($M$)
18:     $A[i, j] \leftarrow false$
19:     for all $(v, to) \in V^2 \mid M[v, to] = true$ do
20:         $A[to, to] \leftarrow true$
21:     return A
```

In order to solve the single-source and multiple-source CFPQ problem, we modified the Azimov's algorithm. Each time, when a grammar rule is applied (see line **8** of Algorithm 1: Boolean matrix multiplication $T^A = T^A + T^B \cdot T^C$ for each $A \rightarrow BC \in P$), only vertices of interest should be stored. To do it, we added one more matrix multiplication: $T^A = T^A + (TSrc^A \cdot T^B) \cdot T^C$, where $TSrc^A$—matrix of start vertices for the current iteration (lines **11-13** of the Algorithm 2). In the end of every iteration of for loop, it is necessary to update the set of source vertices paths from which we need to calculate. To do it, the we call the function getDst (see lines **17-21**), in line **15**. Thus, the modified algorithm supports the frontier of the vertices of interest and updates it on each iteration. As a result, it only computes the paths which starts from the small set of selected vertices.

In case when one has a sequence of similar queries to the single graph, it may be useful to cache results of the query evaluation and share them between queries. This may help to avoid recalculation of the already computed results. To introduce sharing of results between queries, we modify the previous version of the algorithm. The modified version stores all the vertices the paths from which have already been calculated in cache *index*, which is used to filter such vertices in line **11** of Algorithm 3. Thus, the modified algorithm calculates paths from the particular vertex only once. Note, that CreateIndex function should be called first, and the created index can be shared between multiple calls of MultiSrcCFPQCache after that.

3.1 Example

Consider the first few steps of the proposed algorithm (without caching) on the graph D_1', grammar G_1^{wcnf}, and the set of start vertices $Src = \{2\}$. At the first step (lines **4–5**) $TSrc^S[2, 2]$ sets to *true*, all other cells have value *false*. Then the set of matrices T is initialized using rules $C \rightarrow c, D \rightarrow d, Y \rightarrow y$ as follows:

$$T^C = \mathcal{E}^c = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & 1 & . & . & . \\ . & . & . & 1 & . & . \\ . & . & . & . & . & . \end{pmatrix}, T^D = \mathcal{E}^d = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & 1 \\ . & . & . & . & 1 & . \end{pmatrix},$$

$$T^Y = \mathcal{V}^y = \begin{pmatrix} 1 & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & . \end{pmatrix}$$

The following computations take place at the first iteration of the while loop in line **9** for the grammar rule $S \rightarrow CE$. First, the matrix M is computed as follows

$$M = TSrc^S * T^C = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & 1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix} * \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & 1 & . & . & . \\ . & . & . & 1 & . & . \\ . & . & . & . & . & . \end{pmatrix} = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix}$$

Since matrix T^E is empty, T^S stays the same in line **12**, as well as $TSrc^C$ in line **13**. But the matrix $TSrc^E$ is updated (line **14**):

$$TSrc^E = TSrc^E + \text{getDist}(M) =$$

$$= \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \end{pmatrix} + \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & . \end{pmatrix} = \begin{pmatrix} . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & 1 & . \\ . & . & . & . & . & . \end{pmatrix}$$

This means that we are interested in paths that start from the vertex 4 and satisfy the constraints specified by nonterminal E.

Algorithm 3 Multiple-source context-free path querying algorithm with caching

```
1:  function CreateIndex(
            $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$,
            $G = (N, \Sigma, P, S)$)
2:      $T \leftarrow \{T^A \mid A \in N, T^A[i, j] \leftarrow false$, for all $i, j\}$
3:      $TSrc \leftarrow \{TSrc^A \mid A \in N, TSrc^A[i, j] \leftarrow false$, for all $i, j\}$
4:      for all $A \rightarrow x \in P$ do                ▷ Simple rules initialization
5:          for all $(v, to) \in E, \lambda_E(v, to) = x$ do
6:              $T^A[v, to] \leftarrow true$
7:      return $(T, TSrc)$
8:
9:  function MultiSrcCFPQCache(
            $D = (V, E, \Sigma_V, \Sigma_E, \lambda_V, \lambda_E)$,
            $G = (N, \Sigma, P, S)$,
            $Src$,
            $Index = (T, TSrc)$)
10:     $TNewSrc \leftarrow \{TNewSrc^A \mid A \in N, TNewSrc^A \leftarrow$
    false, for all $i, j\}\}$
11:     for all $v \in Src \mid TSrc[v, v] = false$ do
12:         $TNewSrc^S[v, v] \leftarrow true$
13:     $MSrc \leftarrow TNewSrc^S$
14:     while T or TNewSrc is changing do
15:         for all $A \rightarrow BC \in P$ do
16:             $M \leftarrow TNewSrc^A * T^B$
17:             $T^A \leftarrow T^A + M * T^C$
18:             $TNewSrc^B \leftarrow TNewSrc^B + TNewSrc^A \setminus TSrc^B$
19:             $TNewSrc^C \leftarrow TNewSrc^C +$ getDst(M) $\setminus TSrc^C$
20:     return $MSrc * T^S$ ▷ We want to return only relevant data, not
    all cached results
```

The second rule is $S \longrightarrow CS_1$ and its processing is similar to previous one. After processing,

$$TSrc^{S_1} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

The third rule is $E \longrightarrow YD$. Here M is computed as follows:

$$M = TSrc^E * T^Y = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Then the algorithm updates T^E as follows:

$$T^E = T^E + M * T^D =$$
$$= \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} + \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Thus we know that there exists a path from vertex 4 to vertex 5 such that it forms a word derivable from E.

The last rule is $S_1 \longrightarrow SD$. During processing this rule only $TSrc^S$ is updated, since T^S is empty:

$$TSrc^S = TSrc^S + TSrc^{S_1} =$$
$$= \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

This is the end of the first iteration.

At the second iteration, matrices M and then T^S are computed for rule $S \longrightarrow CE$ as follows:

$$M = TSrc^S * T^C = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

$$T^S = M * T^E = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} * \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix} = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Thus, we found the first path that satisfies our query. After all iterations finished, we get the final result:

$$T^S = \begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

Thus only vertices 4 and 5 are reachable from the vertex 2 by a path which forms a word derivable from the start nonterminal S.

Table 1: Graphs for CFPQ evaluation

Graph	#V	#E	#subClassOf	#type	#broaderTransitive
core	1323	3636	178	1412	0
pathways	6238	18 598	3117	3118	0
gohierarchy	45 007	980 218	490 109	0	0
enzyme	48 815	109 695	8163	14 989	0
eclass_514en	239 111	523 727	90 962	72 517	0
geospecies	450 609	2 311 461	0	89 062	20 867
go	272 770	534 311	90 512	58 483	0

3.2 Implementation Notes

All of the versions presented have been implemented[3] using GraphBLAS framework that allows one to represent graphs as matrices and to work with them in terms of linear algebra. For convenience, the code is written in Python using pygraphblas[4], which is a Python wrapper for GraphBLAS API and is based on SuiteSparse:GraphBLAS[5] [5] — the complete implementation of the GraphBLAS standard. This library is specialized to work with sparse matrices which appear in real graphs most often.

3.3 Algorithm Evaluation

We evaluate both described versions of the multiple-source algorithm on real-world graphs. For evaluation, we use a PC with Ubuntu 20.04 installed. It has Intel core i7-4790 CPU, 3.60GHz, and DDR3 32Gb RAM. As far as we evaluate only the execution time of the algorithm, we store each graph fully in RAM as its adjacency matrix in sparse format. Note, that graph loading time is not included in the result time of evaluation.

For evaluation we use graphs and queries from CFPQ_Data dataset[6]. Detailed information, such as the number of vertices and edges, and the number of edges with the specific label, on graphs used in evaluation is provided in table 1. We use classical same-generation queries g_1 (eq. 2) and g_2 (eq. 3) which are used in other works for CFPQ evaluation. We also use *geo* (eq. 4) query provided by Jochem Kuijpers et. al [14] for *geospecies* RDF. Note that in queries we use $\overline{x}$ notation to denote the inverse of the x relation and the respective edge.

$$\begin{aligned} S \to & \overline{subClassOf}\ S\ subClasOf \mid \overline{type}\ S\ type \\ & \mid \overline{subClassOf}\ subClassOf \mid \overline{type}\ type \end{aligned} \tag{2}$$

$$S \to \overline{subClassOf}\ S\ subClasOf \mid subClassOf \tag{3}$$

$$\begin{aligned} S \to & broaderTransitive\ S\ \overline{broaderTransitive} \\ & \mid broaderTransitive\ \overline{broaderTransitive} \end{aligned} \tag{4}$$

Our main goal is to compare the behavior of the two proposed versions of the algorithm. To do it, we measure query execution time of both versions for different sizes of star vertex set. Namely, for each graph we split all vertices into disjoint subsets of fixed size. After that, we evaluate queries using each subset as a set of start vertices. For algorithm with caching we initialize cache

[3] GitHub repository with implemented algorithms: https://github.com/JetBrains-Research/CFPQ_PyAlgo, last accessed 28.08.2020

[4] GitHub repository of PyGraphBLAS library: https://github.com/michelp/pygraphblas

[5] GitHub repository of SuiteSparse:GraphBLAS library: https://github.com/DrTimothyAldenDavis/SuiteSparse

[6] CFPQ_Data is a dataset for CFPQ evaluation which contains both synthetic and real-world data and queries https://github.com/JetBrains-Research/CFPQ_Data, last accessed 28.08.2020.

once for each chunk size and accumulate results for all chunks of the specific size.

We evaluate all three queries for each graph. The results of the evaluation are presented in figures 3–9. Here **Naive** is a naive version of multiple-source CFPQ, and **Cache** is a version with caching. We use standard violin plot with median to show the distribution of results; time is measured in seconds. For several input graphs we provide additional figures for smaller chunks in order to analyze these cases carefully: figures 10–12.

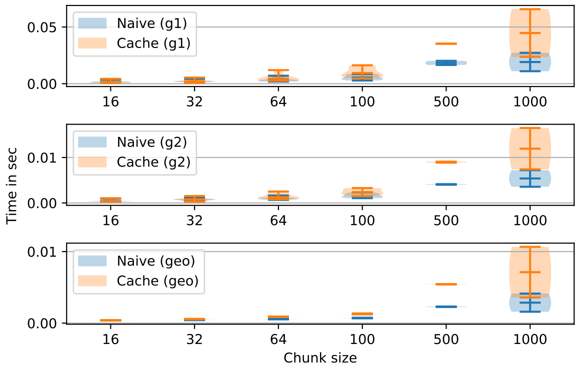

Figure 3: Performance of *core* graph querying

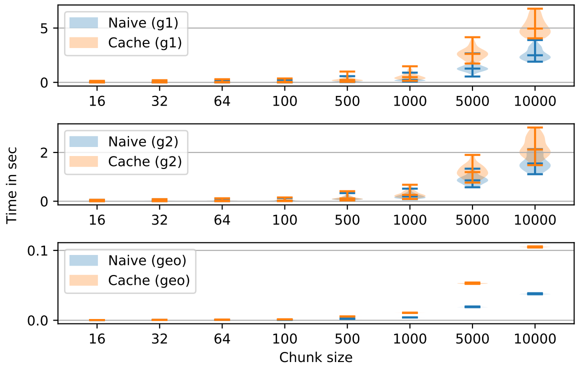

Figure 4: Performance of *go* graph querying

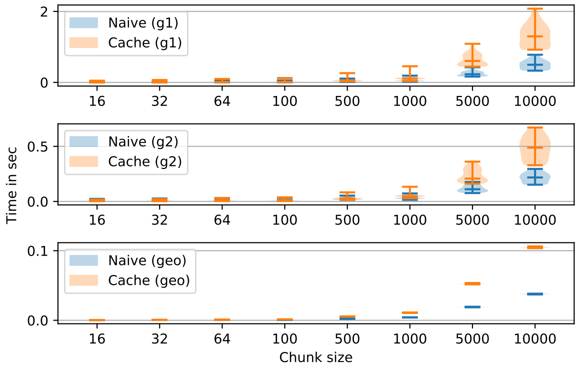

Figure 5: Performance of *eclass_514en* graph querying

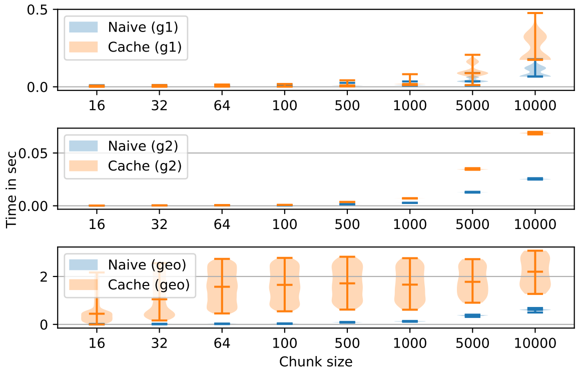

Figure 6: Performance of *geospecies* graph querying

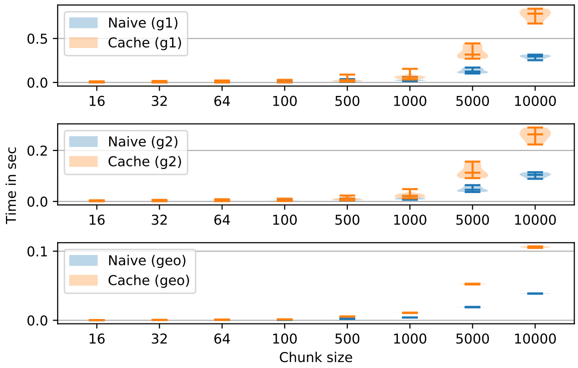

Figure 7: Performance of *enzyme* graph querying

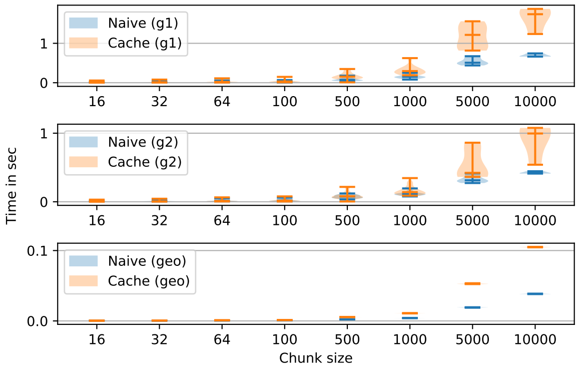

Figure 8: Performance of *gohierarchy* graph querying

First of all, we can see that even for the cases when the input graph does not contain edges used in a query, the chunk processing time grows with the size of the chunk. For example, consider the results for *geo* query for all graphs except *geospecies* and *enzyme*. Thus, the preliminary check of the existence of the edges of interest may improve performance.

We can also see, that the chunk processing time significantly depends on the graph structure. For example, *go* graph querying requires more than 5 seconds (fig. 4), while *geospecies* graph querying requires less than 0.5 seconds (fig. 6) for the chunks of size 10 000 and the query g_1.

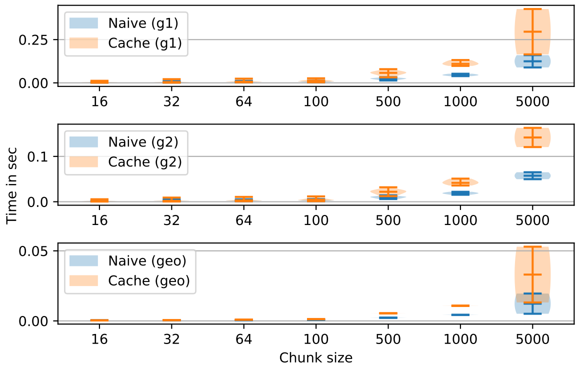

Figure 9: Performance of *pathways* graph querying

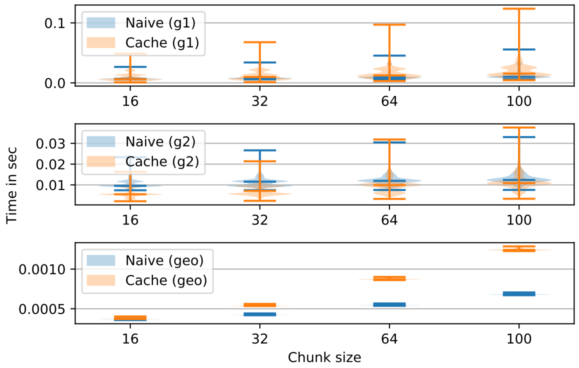

Figure 10: Performance of *eclass_514en* graph querying with small chunks

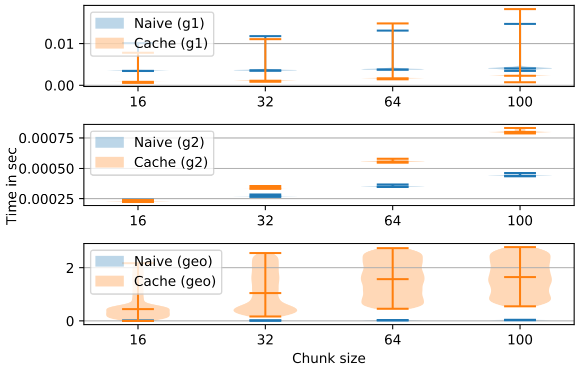

Figure 11: Performance of *geospecies* graph querying with small chunks

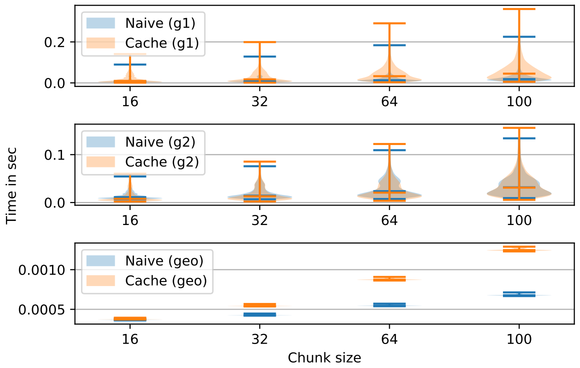

Figure 12: Performance of *go* graph querying with small chunks

The comparison of two versions of the algorithm shows that the algorithm without caching is significantly faster in almost all cases, even when the graph does not contain edges of interest. Analysis of results for small chunks (fig. 10–12) shows that it is not always true. For example, median time for algorithm with caching is slightly better than for the naive version for *eclass_514en* graph and query g_2 (fig. 10). On the other hand, algorithm with caching is drastically slower than the naive version for *geospecies* graph and *geo* query (fig.11). At the same time, for *go* graph and g_2 query, median time of both versions is comparable, while time for the worst queries is better for the naive version. Moreover, caching requires additional memory in comparison with the naive version of the algorithm. Thus we conclude, that the query results caching introduces significant overhead and does not lead to significant performance improvements. We can also conclude that small chunk processing using the naive version is fast enough: the worst time in our experiments is about 0.2 seconds (fig. 12, query G_1).

As a result, we conclude that caching is not useful for multiple-source CFPQ for the evaluated cases even if one wants to process several chunks sequentially, or even process full graph chunk-by-chunk. Thus, we believe that the naive version of the algorithm is better for implementation in a real-world graph database.

4 CFPQ FULL-STACK SUPPORT

In order to provide full-stack support of CFPQ it is necessatry to choose an appropriate graph database. It was shown by Arseniy Terekhov et al. [23] that matrix-based algorithm can be naturally integrated into RedisGraph graph database because the algorithm and the database both operate over a matrix representation of graphs. Moreover, RedisGraph supports Cypher as a query language and there is a proposal which describes Cypher extension for context-free constraints. Thus we chose RedisGraph as a base for our solution.

4.1 Cypher Extension

The first thing to do is to extend the Cypher parser to support the context-free constraints. Tobias Lindaaker proposed an extension for context free constraints to the Cypher syntax[7], which is not implemented in the Cypher parsers yet.

This extension introduces path patterns, which are a powerful alternative to the original Cypher relationship patterns. Path patterns allow one to express regular constrains over the basic patterns such as relationship and node patterns. Like relationship patterns, they can be specified in the MATCH clause.

The main feature which allows one to specify context-free constraints is *named path patterns*: a path pattern can be assigned a name which can be used in other patterns or within the same pattern. Named patterns can be defined in the PATH PATTERN clause. Using this feature, the structure of queries is pretty similar

[7] Formal syntax specification: https://github.com/thobe/openCypher/blob/rpq/cip/1.accepted/CIP2017-02-06-Path-Patterns.adoc#11-syntax. Access date: 19.07.2020.

to a context-free grammar in the Extended Backus-Naur Form (EBNF) [11].

Listing 4 Query based on the example grammar G_1 (eq. 1) written in Cypher with path patterns

```
PATH PATTERN S = ()-/ [:c ~S :d] | [:c (:y) :d] /->()
MATCH (v:x)-[:a | :c]->()-/ :b ~S /->(to)
RETURN v, to
```

An example of a query which uses named path patters is presented in listing 4. This query is based on the context-free grammar G_1 (eq. 1). Namely, the path pattern named S specifies exactly the same constraint as the grammar G_1. The MATCH clause consists of the relation pattern `[:a | :c]` and the path pattern `/:b ~S/`, and this path pattern references the named pattern S. The constraint specifies that a path of interest starts in a vertex labelled x, goes through an edge labelled either a or c, then the rest of the path is constrainted by a path pattern which starts with an edge b and follows with a path matched with S. The RETURN clause specifies what the result of the query is supposed to be. For the example graph D_1, this query returns the set of vertex pairs $\{(0, 4), (0, 5)\}$.

RedisGraph database supports a subset of the Cypher language and uses libcypher-parser[8] library to parse queries. We extend this library with the new syntax described in the proposal. Note that we implement[9] the complete syntax extension, not only the part necessary for simple CFPQ.

4.2 RedisGraph Extension

As a proof-of-concept, we implemented the multiple-source algorithm in the RedisGraph backend. We partially supported the proposed syntax extension in RedisGraph query execution engine so that one can specify the labels of edges and vertices and use named path patterns.

Processing the input as a whole may require a lot of memory. RedisGraph implements lazy evaluation: it creates execution strategy in terms of elementary operations each of which processes the input sequentially in *chunks*. This reduces memory consumption so that it does not depend on the input size which is crucial when dealing with big real-world graphs. However processing chunks comes with a time overhead. By changing the size of a chunk, a developer may adjust the ratio between the time and memory consumption so that it fits their needs.

We use subsets of the start vertices as chunks since it is most natural in the multiple source algorithm. We study how the size of a chunk affects the performance in the evaluation.

4.3 Evaluation

In order to demonstrate the applicability of the implemented extension for RedisGraph we evaluate it on the subset of cases provided in the section 3.3.

For RedisGraph evaluation, we used a PC with Ubuntu 18.04 installed. It has Intel Core i7-6700 CPU, 3.4GHz, and DDR4 64Gb RAM. RedisGraph with our extensions is installed from the GitHub repository[10].

4.3.1 Data Preparation. We use the same graphs which are presented in table 1 to evaluate RedisGraph-based solution.

Graphs are loaded into the RedisGraph database so that each vertex has a unique property id in the range $[0 \ldots |V| - 1]$, where $|V|$ is a number of vertices in the graph to load. This allows us to generate queries for start vertex set with specific size using templates. The template for the g_1 query is provided in listing 5. Here `{id_from}` and `{id_to}` are placeholders for the lower and the upper bounds for id. The example of the concrete query for the start set of size 16 is presented in listing 6.

Listing 5 Cypher query pattern for g_1

```
PATH PATTERN S =
        ()-/ [<:SubClassOf [~S | ()] :SubClassOf]
        | [<:Type [~S | ()] :Type] /->()
MATCH (src)-/ ~S /->()
WHERE {id_from} <= src.id and src.id <= {id_to}
RETURN count(*)
```

Listing 6 Query g_1 in Cypher using template from listing 5

```
PATH PATTERN S =
        ()-/ [<:SubClassOf [~S | ()] :SubClassOf]
        | [<:Type [~S | ()] :Type] /->()
MATCH (src)-/ ~S /->()
WHERE 15 <= src.id and src.id <= 31
RETURN count(*)
```

We implemented a query generator for the queries g_1 and *geo* to create concrete queries for all the start sets which are used in the previous experiment.

4.3.2 Evaluation Results. We use *geo* query for *geospecies* graph as one of the hardest queries, and g_1 query for other graphs. We measure time and memory consumption for each start set. The results are presented in figures 13–19.

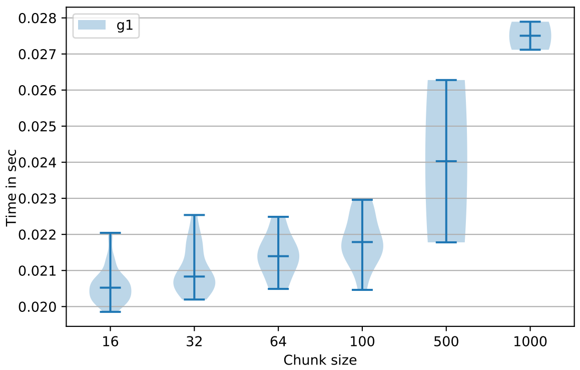

Figure 13: RedisGraph performance on *core* graph

We can see, that the execution time is comparable with the results of the previous experiment presented in section 3.3. The execution time for all sets, except the set of size 10 000 for *geospecies* graph (fig. 17) is less then 1 second. Moreover, for sets of size 16, processing median time is less then 0.1 second, except for *geospecies* graph.

[8] The libcypher-parser is an open-source parser library for Cypher query language. GitHub repository of the project: https://github.com/cleishm/libcypher-parser. Access date: 19.07.2020.

[9] The modified libcypher-parser library with support of syntax for path patterns: https://github.com/YaccConstructor/libcypher-parser. Access date: 19.07.2020.

[10] Sources of RedisGraph database with full-stack CFPQ support:https://github.com/YaccConstructor/RedisGraph/tree/path_patterns_dev. Access data: 19.07.2020.

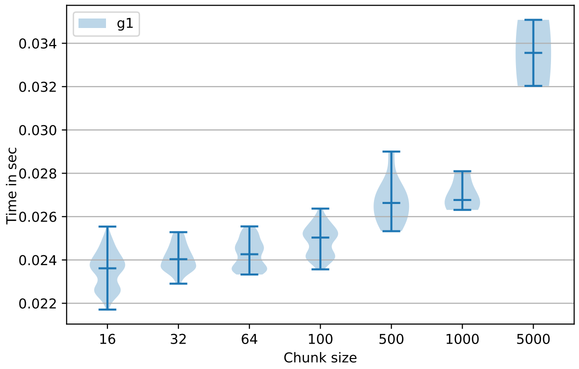

Figure 14: RedisGraph performance on *pathways* graph

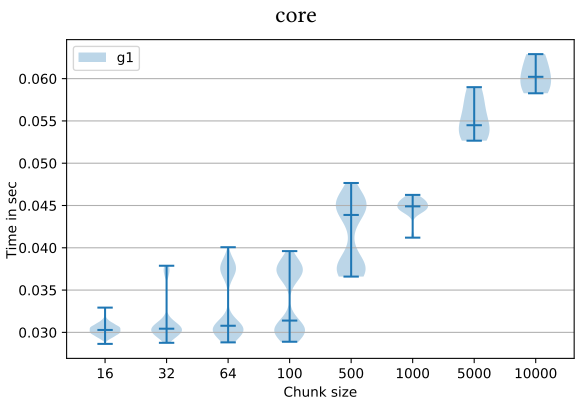

Figure 15: RedisGraph performance on *enzyme* graph

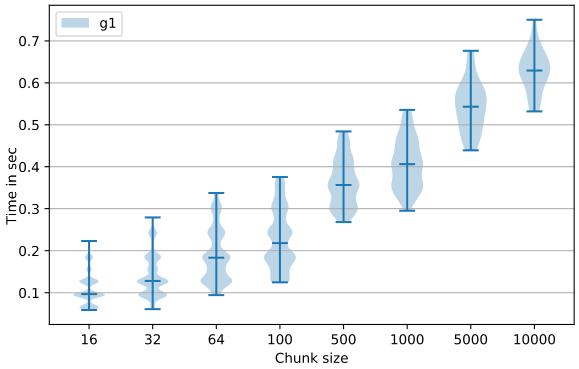

Figure 16: RedisGraph performance on *go* graph

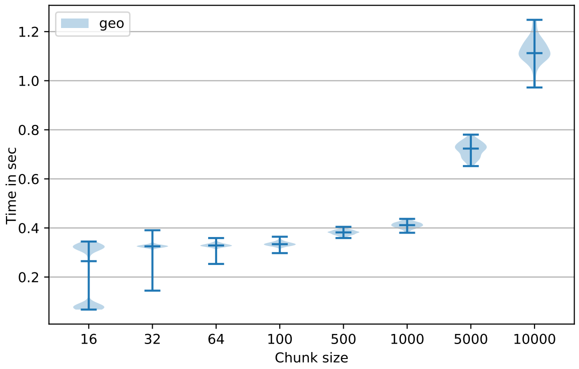

Figure 17: RedisGraph performance on *geospecies* graph

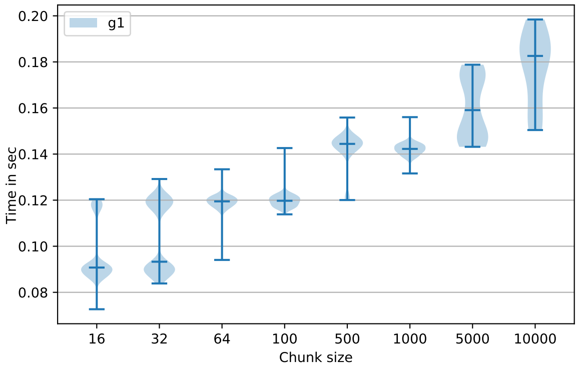

Figure 18: RedisGraph performance on *eclass_514en* graph

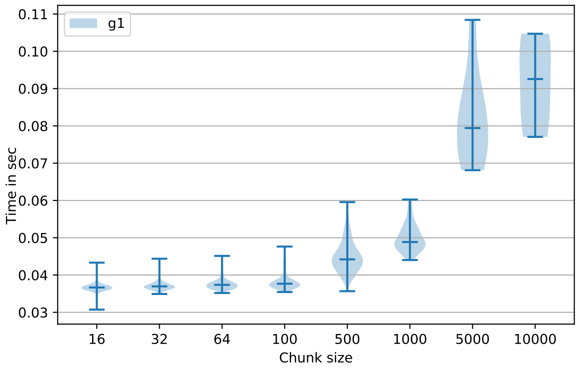

Figure 19: RedisGraph performance on *gohierarchy* graph

Memory consumption for the big graphs *eclass_514en* and *geospecies* is presented in figures 20 and 21 respectively. The amount of memory used depends on the graph and the query, but RedisGraph uses less that 50Mb of RAM to process one set for relatively small sets (≤ 1000). Note that RedisGraph includes memory management system, thus we measured all allocated memory, not only the memory really used for the query evaluation. As a result, we can conclude that the multiple-source CFPQ is significantly more memory efficient than creation of the complete reachability index and its filtering: processing the set of size 10 000 on *geospecies* graph requires less than 200Mb, while full index creation requires 16Gb [23].

Listing 7 Query g_1 in Cypher for all-pairs scenario evaluation

```
PATH PATTERN S =
          ()-/ [<:SubClassOf [~S | ()] :SubClassOf]
          | [<:Type [~S | ()] :Type] /->()
MATCH ()-/ ~S /->()
RETURN count(*)
```

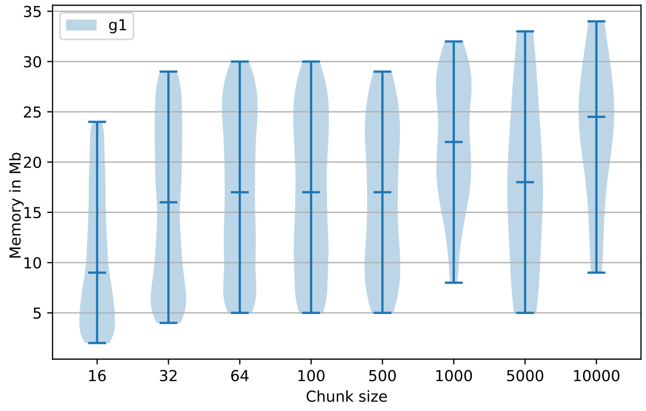

Figure 20: RedisGraph memory consumption on *eclass_514en* graph

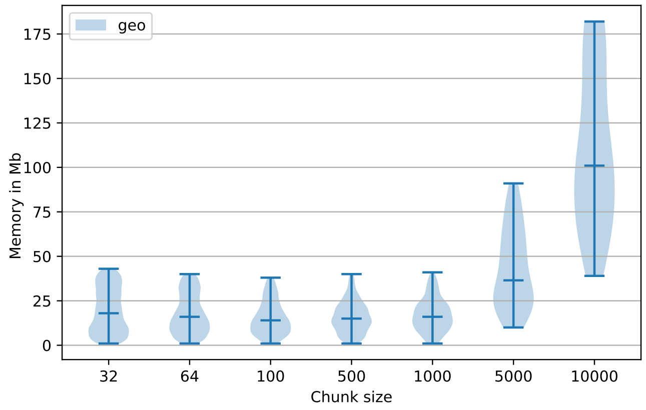

Figure 21: RedisGraph memory consumption on *geospecies* graph

We also evaluate how the chunk size affects the performance on the all-pairs reachability problem. We fix the size of a chunk to be 1000 for graphs of different sizes and measure time and memory required to process queries. Namely, we evaluate the query, presented in listing 7. It is similar to the query from the previous scenario, but it does not constraint vertices ids (it does not have the WHERE clause). We measure total processing time (in seconds) and total required memory (in Mb). Also, we compare our solution with the results of Arseniy Terekhov et al. from [23] in which the Azimov's algorithm was naively integrated with RedisGraph without support of lazy query evaluation and query language. Similar hardware and the same input graphs and queries were used. Results are provided in table 2.

Although chunk-by-chunk processing is slower, it still requires reasonable time. Moreover, if the chunk size is comparable with the graph size (*core* and *pathways* graphs), then the execution time is comparable with the monolithic processing. Thus one can decrease execution time by increasing the chunk size. On the other hand, even with relatively small chunks (*eclass_514*, *go* and *geospecies* graphs), when for chunk-by-chunk processing is 100 times slower, our results are still reasonable for some cases. For example, it requires more than 70 times less time for *geospecies* graph processing than the solution of Jochem Kuijpers et al. [14] which is based on Neo4j and requires more than 6000 seconds. Moreover, while the solution from [23] requires huge amount of memory (more than 16Gb for *geospecies* graph and *geo* query), our solution requires only 196Mb. We argue, that our solution is more suitable for general-purpose graph databases. First of all, the core scenario when the set of start vertices is relatively small can be handled efficiently. Second, all-pairs reachability, which is not a massive case, can be solved in reasonable time with low memory consumption. One can easily tune our solution to get the optimal time and memory consumption for their specific case.

Table 2: Full graph processing time by RedisGraph with chunks of size 1000, time is measured in seconds, memory in Mb (Chunks — the proposed solution, Mono — results from [23])

Graph	#V	#E	Query	Chunks		Mono
				Time	Mem	
core	1323	4342	g_1	0.003	2	0.004
pathways	6238	18 598	g_1	0.031	6	0.011
gohierarchy	45 007	980 218	g_1	0.847	62	0.091
enzyme	48 815	109 695	g_1	0.698	13	0.018
eclass_514en	239 111	523 727	g_1	18.825	35	0.067
geospecies	450 609	2 311 461	*geo*	80.979	196	7.146
go	272 770	534 311	g_1	72.034	40	0.604

Finally, we conclude that the provided solution is a promising way to implement CFPQ for real-world graph databases.

5 CONCLUSION AND FUTURE WORK

In this paper we propose two version of multiple-source modifications of Azimov's CFPQ algorithm. The evaluation of the proposed modifications on the real-world examples shows that caching of the queries results is not useful in the evaluated scenarios and the naive implementation is the best choice for the integration with a real-world graph database. We also provide the full-stack support of CFPQ. For our solution, we implement a Cypher extension as a part of `libcypher-parser`, integrate the proposed algorithm into RedisGraph, and extend RedisGraph execution plan builder to support the extended Cypher queries. We demonstrate that our solution is applicable for real-world graph analysis.

In the future, it is necessary to provide formal translation of Cypher to linear algebra, or to determine a maximal subset of Cypher which can be translated to linear algebra. There is a number of works on a subset of SPARQL to linear algebra translation, such as [6, 12, 13, 16]. Most of them are practical-oriented and do not provide full theoretical basis to translate querying language to linear algebra. Others discuss only partial cases and should be extended to cover real-world query languages. Deep investigation of this topic can help to determine the restrictions of linear algebra utilization for graph databases. Moreover, it can also improve the existing solutions.

The evaluation of the regular queries is possible in practice by using the CFPQ algorithm, since the regular queries are a partial case of the context-free queries. But it seems that the proposed solution is not optimal. It is important to provide an optimal unified algorithm for both RPQ and CFPQ to create a tool applicable to a real-world tasks. One of possible way to solve this problem is to use the tensor-based algorithm [19].

Another important task is to compare non-linear-algebra-based approaches to the multiple-source CFPQ with the proposed solution. In [14] Jochem Kuijpers et al. show that all-pairs CFPQ algorithms implemented in Neo4j demonstrate unreasonable performance on real-world data. At the same time, Arseniy Terekhov et.al. shows that matrix-based all-pairs CFPQ algorithm implemented in the appropriate linear algebra based graph database (RedisGraph) demonstrates good performance. But in the case of multiple-source scenario, when a number of start vertices is relatively small, non-linear-algebra-based solutions can be better, because such solutions naturally handle only subgraph required to answer the query. Thus detailed investigation and comparison of other approaches to evaluate multiple-source CFPQ is required in the future.

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