Lab12

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0.1 SKOLTECH, Experimental Data Processing

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In [1]:
```
import numpy as np
import scipy as sp
from matplotlib import pyplot as plt
from numpy.linalg import inv
import matplotlib as mplb
from matplotlib.font_manager import FontProperties
%matplotlib inline
from numpy.random import normal
from mpl_toolkits.mplot3d import Axes3D
mplb.rc('xtick', labelsize=5)
mplb.rc('ytick', labelsize=5)
```

In [78]:
```
def kalman(X_0, P_0, z, T, h, R, Q, dh):
    X = np.zeros((len(z),*(X_0.shape)))
    P = np.zeros((len(z),*(P_0.shape)))
    K = np.zeros((len(z), X_0.shape[0], z.shape[1]))
    XF= np.zeros_like(X)
    Xp= np.zeros_like(X)
    for i, _ in enumerate(z):
        #Prediction
        Xp[i] = X[i] = T.dot(X[i-1] if i > 0 else X_0)
        P[i] = T.dot((P[i-1] if i > 0 else P_0).dot(T.transpose())) + Q
        #Filtration
        #tmp1 = inv(H.dot(P[i].dot(H.transpose())) + R[i])
        #tmp2 = H.transpose().dot(tmp1)
        #K[i] = P[i].dot(tmp2)
        tmp1 = inv(dh(Xp[i]).dot(P[i].dot(dh(Xp[i]).transpose())) + R)
        tmp2 = dh(Xp[i]).transpose().dot(tmp1)
        K[i] = P[i].dot(tmp2)
        X[i] = Xp[i] + K[i].dot(z[i] - h(Xp[i]))

        P[i] = (np.identity(X_0.shape[0]) - K[i].dot(dh(Xp[i]))).dot(P[i])
    return X, K, P, Xp
```

```python
def generate_acc_trajectory(sigma_a_2, sigma_n_2, N, x_0, v_0, t, a_bias = 0):
    if sigma_a_2 == 0:
        a = np.zeros(N) + a_bias
    else:
        a = np.random.normal(0, sigma_a_2 ** 0.5, N) + a_bias
    v = np.ones(N) * v_0
    x = np.ones(N) * x_0
    for i, a_i in enumerate(a[:-1]):
        v[i+1] = v[i] + a_i*t
    dx = (v * t + a * t * t / 2)
    for i, dx_i in enumerate(dx[:-1]):
        x[i+1] = x[i] + dx_i
    #v2 = np.ones(N) * v_0 + a.dot(np.triu(np.ones((N, N)), 1)) * t
    #x2 = np.ones(N) * x_0 + (v2 * t + a * t * t / 2).dot(np.triu(np.ones((N, N)), 1)
    z = x + np.random.normal(0, sigma_n_2 ** 0.5, N)
    return x, z

def convert_to_polar(X):
    x = X[:,0,0]
    y = X[:,2,0]
    D = (x**2 + y**2) ** 0.5
    b = np.arctan(x / y)
    return b, D
```

1 Generate a true trajectory

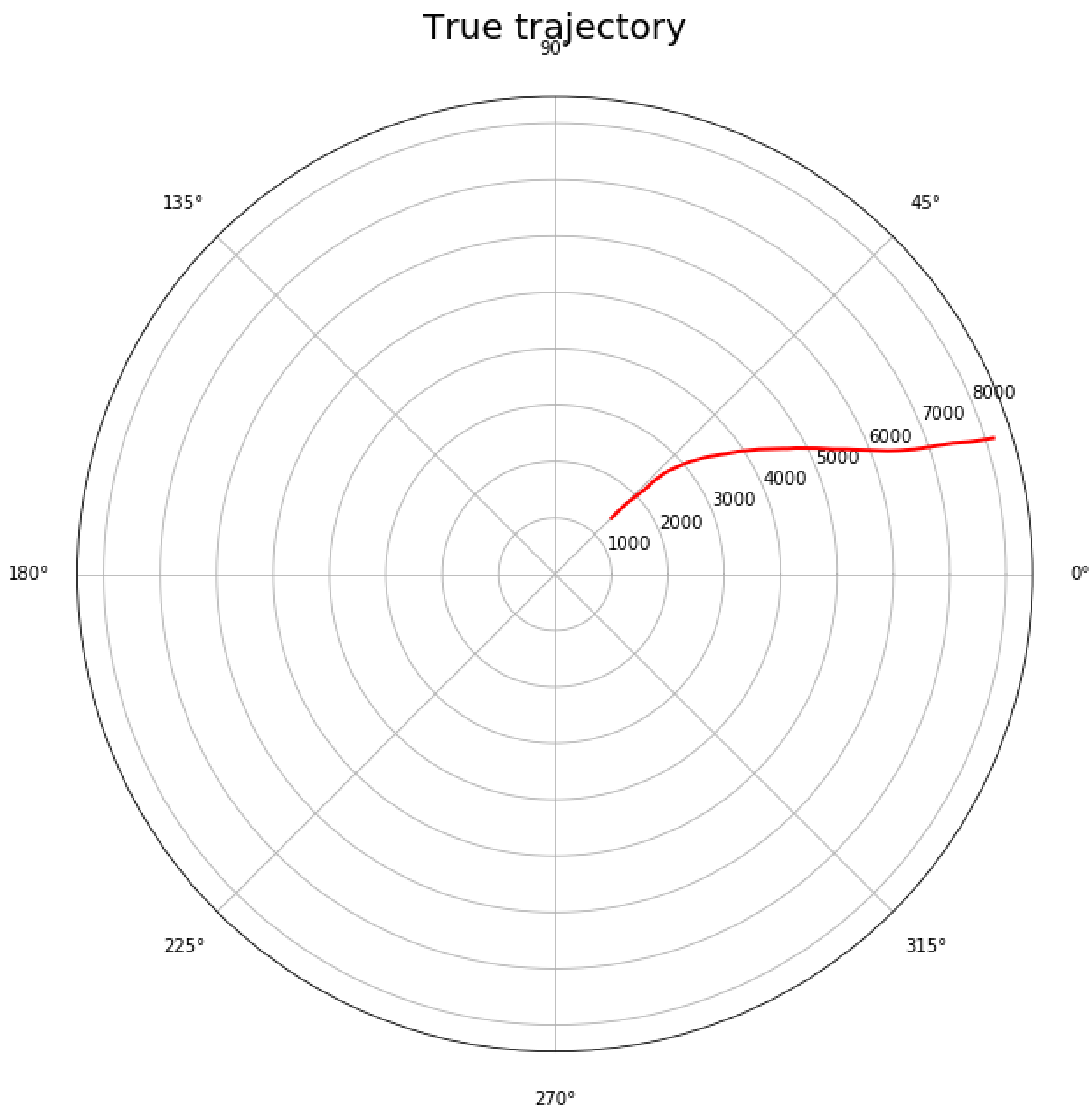
```python
In [5]: N = 500
        t = 1
        x_0 = 1000
        y_0 = 1000
        sigma_a_2 = 0.3 ** 2
        v_x = 10
        v_y = 10

        x, _ = generate_acc_trajectory(sigma_a_2, 1, N, x_0, v_x, t)
        y, _ = generate_acc_trajectory(sigma_a_2, 1, N, y_0, v_y, t)
```

2 Generate also true values of range D and azimut β

```python
In [6]: D = (x**2 + y**2) ** 0.5
        b = np.arctan(x / y)
```

```python
In [93]: fig = plt.figure(figsize=(10, 10))
         ax = fig.add_subplot(111, polar=True)
         ax.set_title('True trajectory', fontsize = 20)
         ax.plot(b, D, 'r', linewidth=2)
```

```
ax.xaxis.set_tick_params(labelsize=10)
ax.yaxis.set_tick_params(labelsize=10)
```

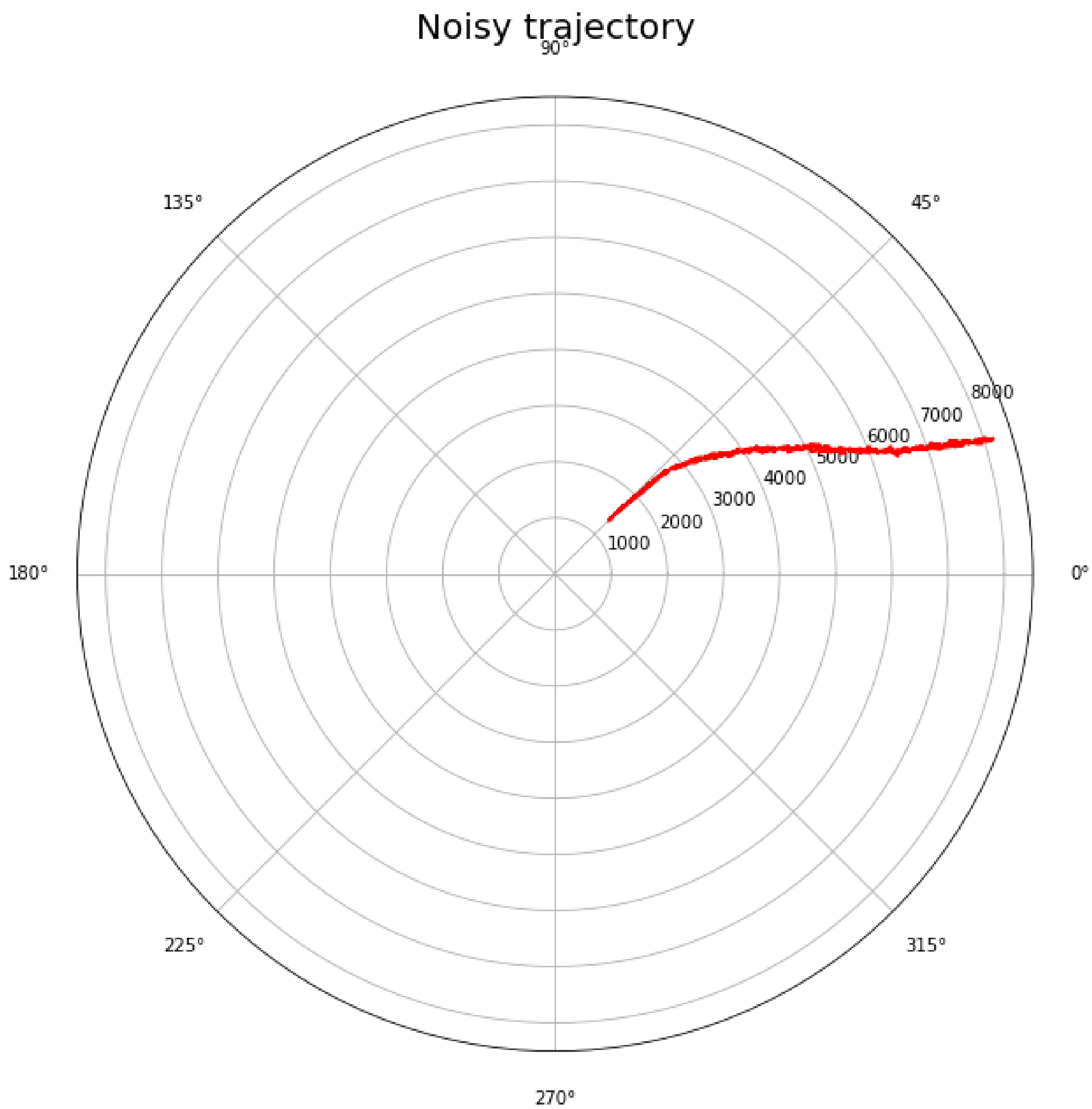


3 Generate measurements D^m and β^m

In [60]:
```
sigma_D = 50
sigma_b = 0.004
D_n = np.random.normal(0, sigma_D, N)
b_n = np.random.normal(0, sigma_b, N)
D_m = D + D_n
b_m = b + b_n
z = np.zeros((N, 2, 1))
```

```
        z[:, 0, 0] = D_m
        z[:, 1, 0] = b_m
```

```
In [92]: fig = plt.figure(figsize=(10, 10))
         ax = fig.add_subplot(111, polar=True)
         ax.set_title('Noisy trajectory', fontsize = 20)
         ax.plot(b_m, D_m, 'r', linewidth=2)
         ax.xaxis.set_tick_params(labelsize=10)
         ax.yaxis.set_tick_params(labelsize=10)
```



4 Initial conditions for Kalman filter algorithm

```
In [12]: X_0 = np.array([[D_m[0] * np.sin(b_m[0])], [0], [D_m[0] * np.cos(b_m[0])], [0]])
         P_0 = np.eye(4) * 10 ** 10
```

5 Create the transition matrix

```
In [28]: T = np.zeros((4, 4))
         T[0:2, 0:2] = np.array([[1, t], [0, 1]])
         T[2:4, 2:4] = np.array([[1, t], [0, 1]])
```

6 Calculate state noise covariance matrix Q

```
In [27]: G = np.zeros((4, 2))
         G[0:2, 0:1] = np.array([[t ** 2 / 2], [t]])
         G[2:4, 1:2] = np.array([[t ** 2 / 2], [t]])
         Q = G.dot(G.transpose() * sigma_a_2)
```

7 Create the measurement noise covariance matrix R

```
In [74]: R = np.array([[sigma_D ** 2, 0], [0, sigma_b ** 2]])
```

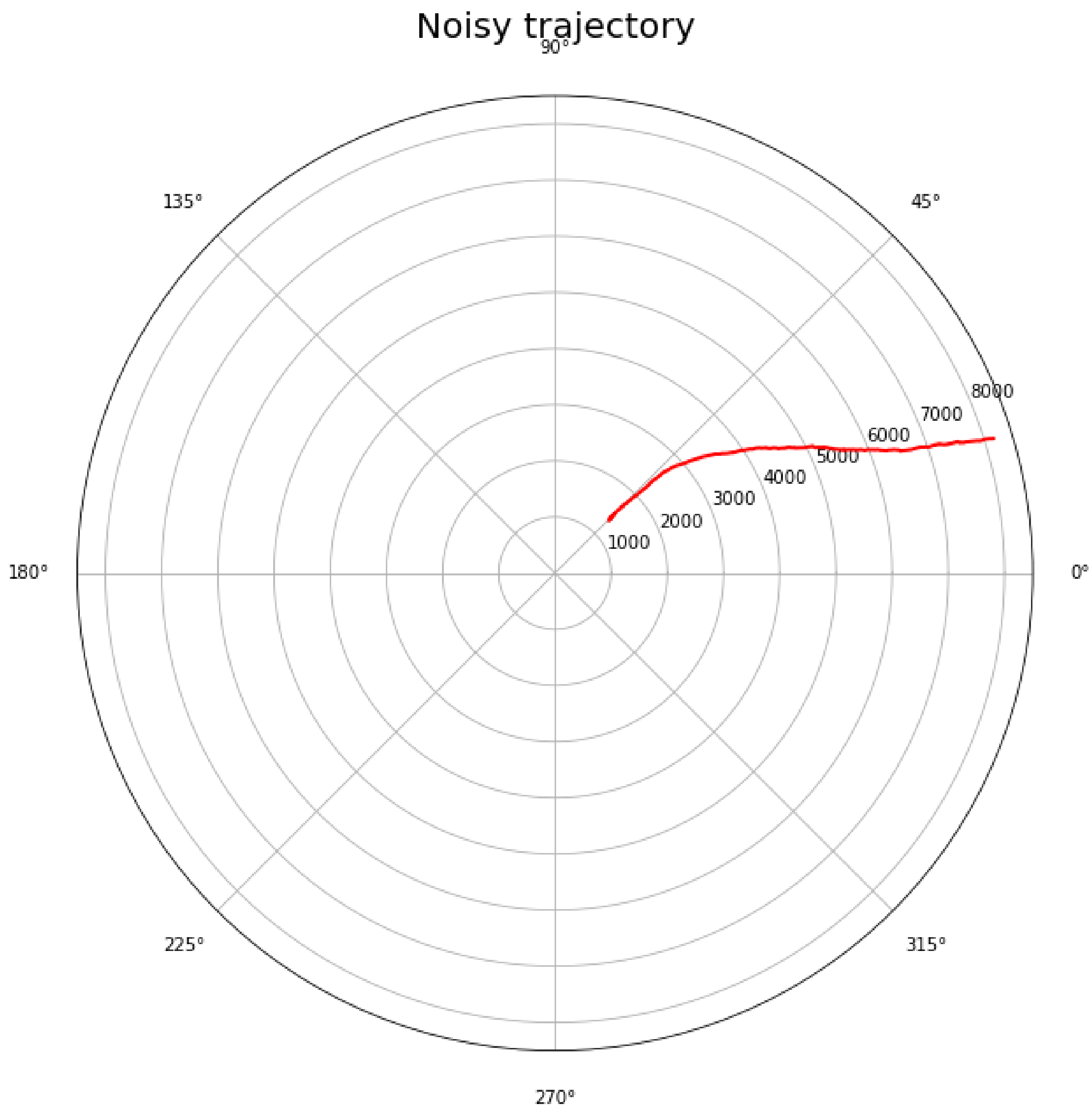
8 linearize measurement equation

```
In [75]: def h(X):
             x = X[0]
             y = X[2]
             H = np.zeros((2, 1))
             H[0] = (x ** 2 + y ** 2) ** 0.5
             H[1] = np.arctan(x / y)
             return H

         def dh(X):
             x = X[0, 0]
             y = X[2, 0]
             dH = np.zeros((2, 4))
             dH[0, 0] = x / (x ** 2 + y ** 2) ** 0.5
             dH[0, 2] = y / (x ** 2 + y ** 2) ** 0.5
             dH[1, 0] = y / (x ** 2 + y ** 2)
             dH[1, 2] = - x / (x ** 2 + y ** 2)
             return dH
```

9 Develop Kalman filter algorithm

```
In [90]: X, K, _, Xp = kalman(X_0, P_0, z, T, h, R, Q, dh)
```

```
In [91]: b_f, D_f = convert_to_polar(Xp)
         fig = plt.figure(figsize=(10, 10))
         ax = fig.add_subplot(111, polar=True)
         ax.set_title('Noisy trajectory', fontsize = 20)
         ax.plot(b_f, D_f, 'r', linewidth=2)
         ax.xaxis.set_tick_params(labelsize=10)
         ax.yaxis.set_tick_params(labelsize=10)
```



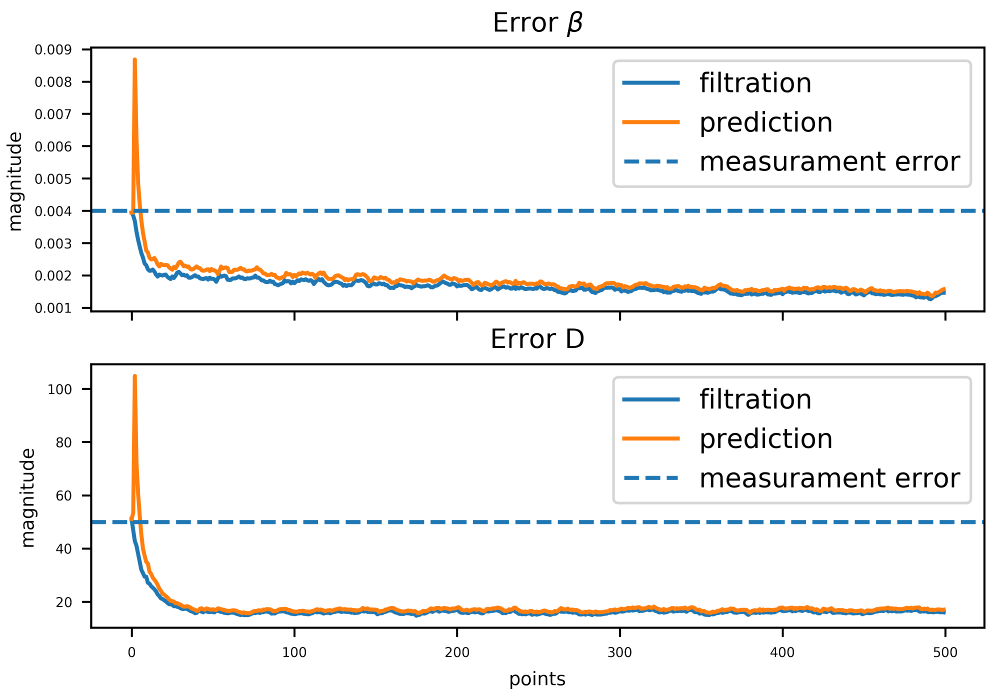
10 Run Kalman filter algorithm over $M = 500$ runs

```
In [99]: M = 500
         N = 500
```

```
t = 1
x_0 = 1000
y_0 = 1000
sigma_a_2 = 0.3 ** 2
v_x = 10
v_y = 10
sigma_D = 50
sigma_b = 0.004

error_b_f = np.zeros((N, M))
error_D_f = np.zeros((N, M))
error_b_p = np.zeros((N, M))
error_D_p = np.zeros((N, M))

for i in range(M):
    x, _ = generate_acc_trajectory(sigma_a_2, 1, N, x_0, v_x, t)
    y, _ = generate_acc_trajectory(sigma_a_2, 1, N, y_0, v_y, t)

    D = (x**2 + y**2) ** 0.5
    b = np.arctan(x / y)
    D_n = np.random.normal(0, sigma_D, N)
    b_n = np.random.normal(0, sigma_b, N)
    D_m = D + D_n
    b_m = b + b_n
    z = np.zeros((N, 2, 1))
    z[:, 0, 0] = D_m
    z[:, 1, 0] = b_m

    X_0 = np.array([[D_m[0] * np.sin(b_m[0])], [0], [D_m[0] * np.cos(b_m[0])], [0]])
    P_0 = np.eye(4) * 10 ** 10

    X, K, _, Xp = kalman(X_0, P_0, z, T, h, R, Q, dh)

    b_f, D_f = convert_to_polar(X)
    b_p, D_p = convert_to_polar(Xp)

    error_b_f[:,i] = (b - b_f) ** 2
    error_D_f[:,i] = (D - D_f) ** 2
    error_b_p[:,i] = (b - b_p) ** 2
    error_D_p[:,i] = (D - D_p) ** 2
```

In [109]:
```
fig, ax = plt.subplots(2,1, figsize=(6,4), dpi = 600, sharex = True)
ax[0].set_title(r'Error $\beta$', fontsize = 10)
ax[0].plot( (np.sum(error_b_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[0].plot( (np.sum(error_b_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[0].set_ylabel('magnitude', fontsize = 7)
ax[0].axhline(sigma_b, label = "measurament error", linestyle='--')
ax[0].legend(loc='upper right')
```

```
ax[1].set_title('Error D', fontsize = 10)
ax[1].plot( (np.sum(error_D_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[1].plot( (np.sum(error_D_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[1].set_xlabel('points', fontsize = 7)
ax[1].set_ylabel('magnitude', fontsize = 7)
ax[1].axhline(sigma_D, label = "measurament error", linestyle='--')
ax[1].legend(loc='upper right');
```



10.1 One can see from this plot that measurement error is times higher than true estimaiton error of filtration and prediction

11 Conclusion

11.1 Today we have learned extended Kalman filter

In [ ]: