# Lab13

October 19, 2017

### 0.1 SKOLTECH, Experimental Data Processing

### 0.2 Evgenii Safronov, Mikhail Kurenkov, Taras Melnik

```
In [20]: import numpy as np
         import scipy as sp
         from matplotlib import pyplot as plt
         from numpy.linalg import inv
         import matplotlib as mplb
         from matplotlib.font_manager import FontProperties
         %matplotlib inline
         from numpy.random import normal
         from mpl_toolkits.mplot3d import Axes3D
         mplb.rc('xtick', labelsize=5)
         mplb.rc('ytick', labelsize=5)
         import math
```

```
In [53]: def kalman(X_0, P_0, z, T, h, R, Q, dh, start_step = 0):
             N = len(z)
             X = np.zeros((len(z),*(X_0.shape)))
             P = np.zeros((len(z),*(P_0.shape)))
             Xp= np.zeros_like(X)
             for i in range(N):
                 #Prediction
                 Xp[i] = X[i] = T.dot(X[i-1] if i > 0 else X_0)
                 P[i] = T.dot((P[i-1] if i > 0 else P_0).dot(T.transpose())) + Q
                 #Filtration
                 if (i >= start_step):
                     tmp1 = inv(dh[i](Xp[i]).dot(P[i].dot(dh[i](Xp[i]).transpose())) + R[i])
                     tmp2 = dh[i](Xp[i]).transpose().dot(tmp1)
                     K = P[i].dot(tmp2)
                     X[i] = Xp[i] + K.dot(z[i] - h[i](Xp[i]))
                     P[i] = (np.identity(X_0.shape[0]) - K.dot(dh[i](Xp[i]))).dot(P[i])
             return X, Xp

         def generate_acc_trajectory(sigma_a_2, sigma_n_2, N, x_0, v_0, t, a_bias = 0):
             if sigma_a_2 == 0:
                 a = np.zeros(N) + a_bias
```

```
        else:
            a = np.random.normal(0, sigma_a_2 ** 0.5, N) + a_bias
        v = np.ones(N) * v_0
        x = np.ones(N) * x_0
        for i, a_i in enumerate(a[:-1]):
            v[i+1] = v[i] + a_i*t
        dx = (v * t + a * t * t / 2)
        for i, dx_i in enumerate(dx[:-1]):
            x[i+1] = x[i] + dx_i
        #v2 = np.ones(N) * v_0 + a.dot(np.triu(np.ones((N, N)), 1)) * t
        #x2 = np.ones(N) * x_0 + (v2 * t + a * t * t / 2).dot(np.triu(np.ones((N, N)), 1)
        z = x + np.random.normal(0, sigma_n_2 ** 0.5, N)
        return x, z

    def convert_to_polar(X):
        x = X[:,0,0]
        y = X[:,2,0]
        D = (x**2 + y**2) ** 0.5
        b = np.arctan(x / y)
        return b, D
```

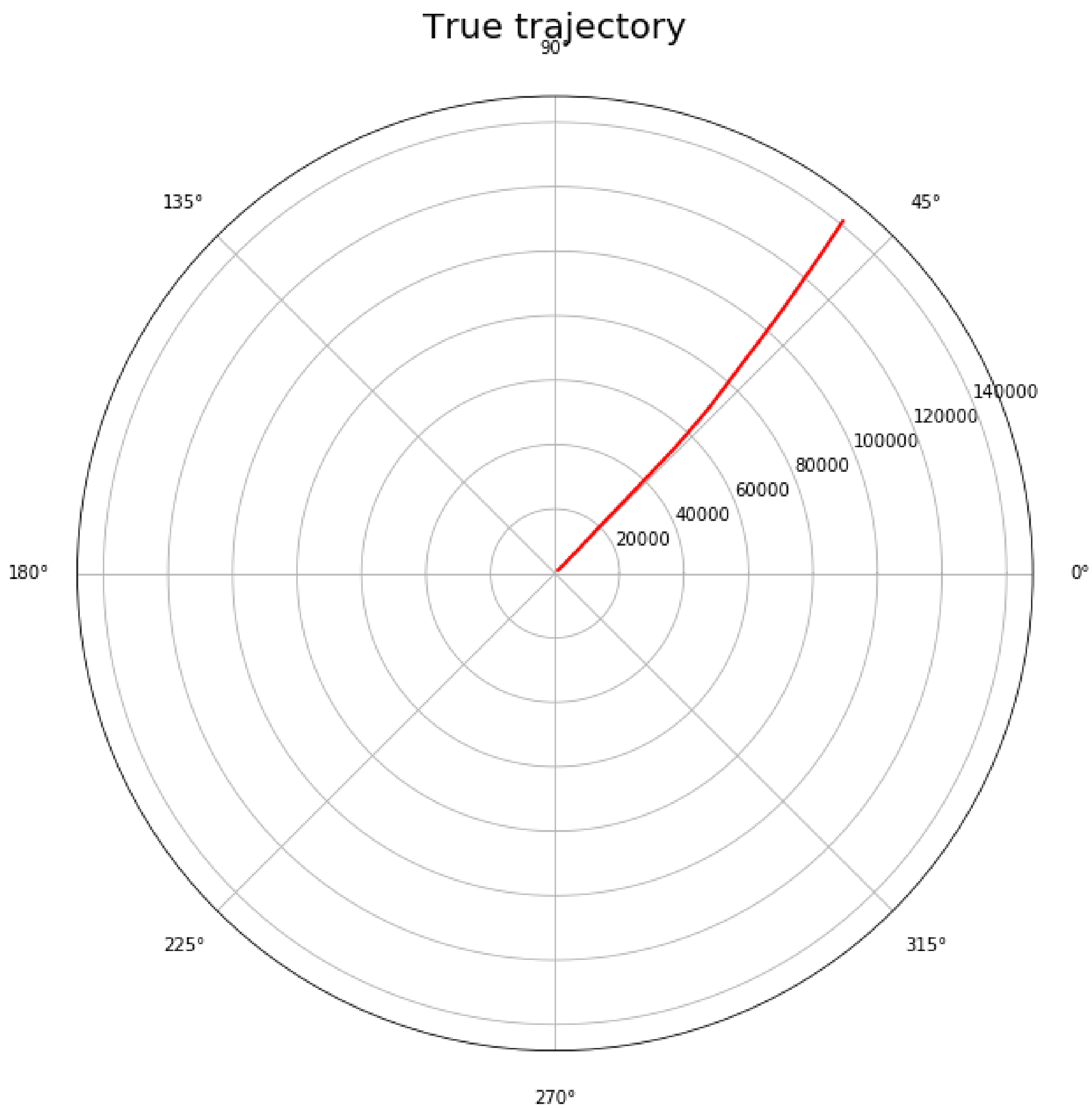
# 1 Generate a true trajectory

In [6]:
```
N = 500
t = 2
x_0 = 1000
y_0 = 1000
sigma_a_2 = 0.3 ** 2
v_x = 100
v_y = 100

x, _ = generate_acc_trajectory(sigma_a_2, 1, N, x_0, v_x, t)
y, _ = generate_acc_trajectory(sigma_a_2, 1, N, y_0, v_y, t)
```

# 2 Generate also true values of range $D$ and azimut $\beta$

In [7]:
```
D = (x**2 + y**2) ** 0.5
b = np.arctan(x / y)
```

In [8]:
```
fig = plt.figure(figsize=(10, 10))
ax = fig.add_subplot(111, polar=True)
ax.set_title('True trajectory', fontsize = 20)
ax.plot(b, D, 'r', linewidth=2)
ax.xaxis.set_tick_params(labelsize=10)
ax.yaxis.set_tick_params(labelsize=10)
```




## 3 Generate measurements $D^m$ and $\beta^m$

```
In [32]: sigma_D = 50
         sigma_b = 0.004
         z = [np.zeros(1) for x in range(N)]
         for i in range(0, N, 2):
             D_n = np.random.normal(0, sigma_D, 1)
             b_n = np.random.normal(0, sigma_b, 1)
             z[i] = np.array([D_n + D[i],b_n + b[i]])
```

## 4 Generate more accurate measurements of azimuth

In [33]:
```
sigma_b_add = 0.001
for i in range(1, N, 2):
    b_n = np.random.normal(0, sigma_b_add, 1)
    z[i] = np.array([b_n + b[i]])
```

## 5 Initial conditions for Kalman filter algorithm

In [37]:
```
x_1 = z[0][0, 0] * math.sin(z[0][1, 0])
x_3 = z[2][0, 0] * math.sin(z[2][1, 0])
y_1 = z[0][0, 0] * math.cos(z[0][1, 0])
y_3 = z[2][0, 0] * math.cos(z[2][1, 0])
X_0 = np.array([[x_3],[(x_3 - x_1) / 2 / t], [y_3],[(y_3 - y_1) / 2 / t]])
P_0 = np.eye(4) * 10 ** 4
```

## 6 Develop Kalman filter algorithm to estimate state vector

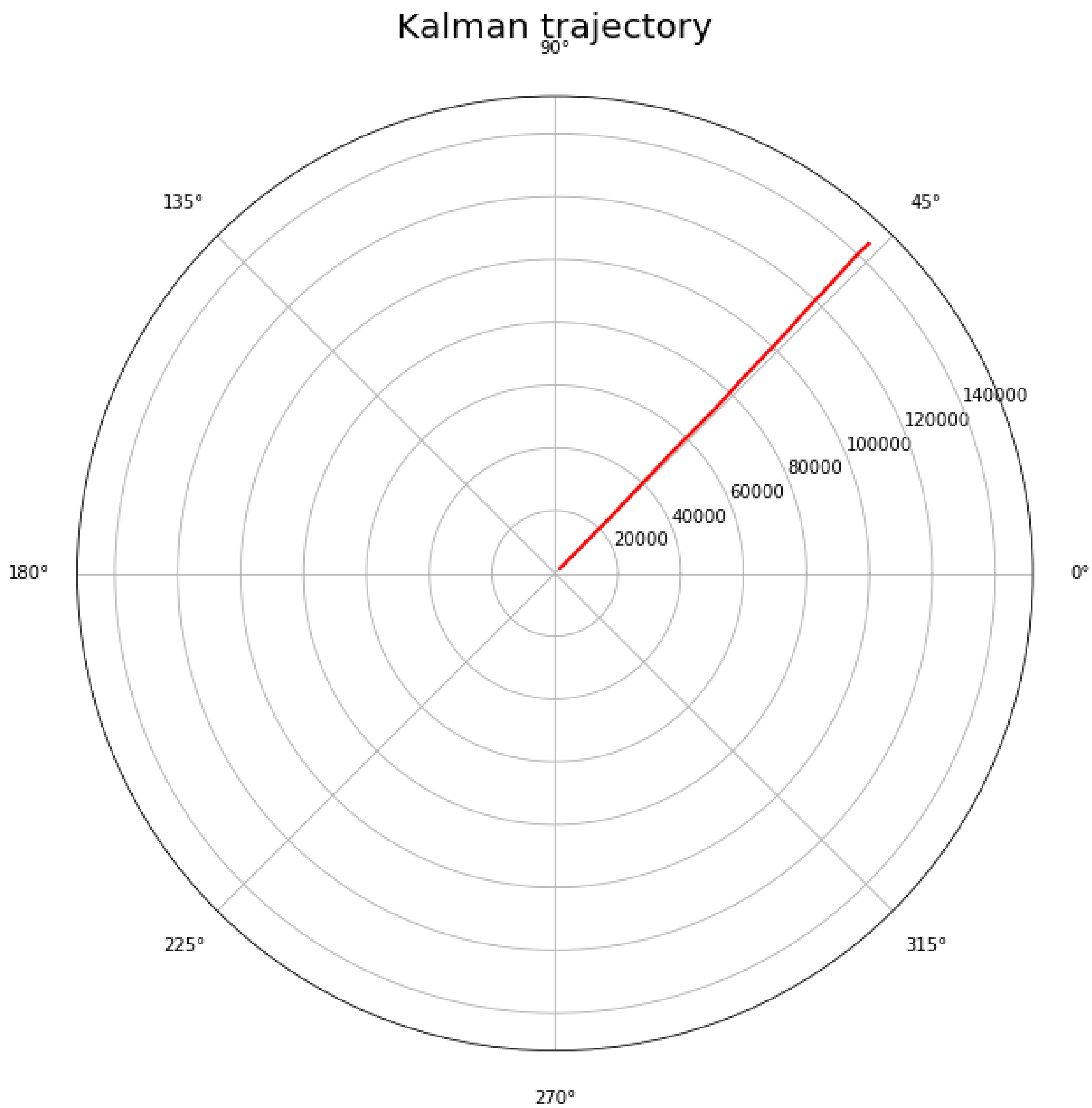
In [40]:
```
def h_1(X):
    x = X[0]
    y = X[2]
    H = np.zeros((2, 1))
    H[0] = (x ** 2 + y ** 2) ** 0.5
    H[1] = np.arctan(x / y)
    return H

def dh_1(X):
    x = X[0, 0]
    y = X[2, 0]
    dH = np.zeros((2, 4))
    dH[0, 0] = x / (x ** 2 + y ** 2) ** 0.5
    dH[0, 2] = y / (x ** 2 + y ** 2) ** 0.5
    dH[1, 0] = y / (x ** 2 + y ** 2)
    dH[1, 2] = - x / (x ** 2 + y ** 2)
    return dH

def h_2(X):
    x = X[0]
    y = X[2]
    H = np.zeros((1, 1))
    H[0] = np.arctan(x / y)
    return H

def dh_2(X):
    x = X[0, 0]
    y = X[2, 0]
    dH = np.zeros((1, 4))
```

```
        dH[0, 0] = y / (x ** 2 + y ** 2)
        dH[0, 2] = - x / (x ** 2 + y ** 2)
        return dH

    R_1 = np.array([[sigma_D ** 2, 0], [0, sigma_b ** 2]])
    R_2 = np.array([[sigma_b_add ** 2]])
```

```
In [45]: T = np.zeros((4, 4))
         T[0:2, 0:2] = np.array([[1, t], [0, 1]])
         T[2:4, 2:4] = np.array([[1, t], [0, 1]])

         G = np.zeros((4, 2))
         G[0:2, 0:1] = np.array([[t ** 2 / 2], [t]])
         G[2:4, 1:2] = np.array([[t ** 2 / 2], [t]])
         Q = G.dot(G.transpose() * sigma_a_2)


         dh = list(range(N))
         h = list(range(N))
         R = list(range(N))
         for i in range(0, N, 2):
             h[i] = h_1
             dh[i] = dh_1
             R[i] = R_1

         for i in range(1, N, 2):
             h[i] = h_2
             dh[i] = dh_2
             R[i] = R_2
```

```
In [54]: X, Xp = kalman(X_0, P_0, z, T, h, R, Q, dh, 3)
```

```
In [74]: b_f, D_f = convert_to_polar(Xp)
         fig = plt.figure(figsize=(10, 10))
         ax = fig.add_subplot(111, polar=True)
         ax.set_title('Kalman trajectory', fontsize = 20)
         ax.plot(b_f, D_f, 'r', linewidth=2)
         ax.xaxis.set_tick_params(labelsize=10)
         ax.yaxis.set_tick_params(labelsize=10)
```

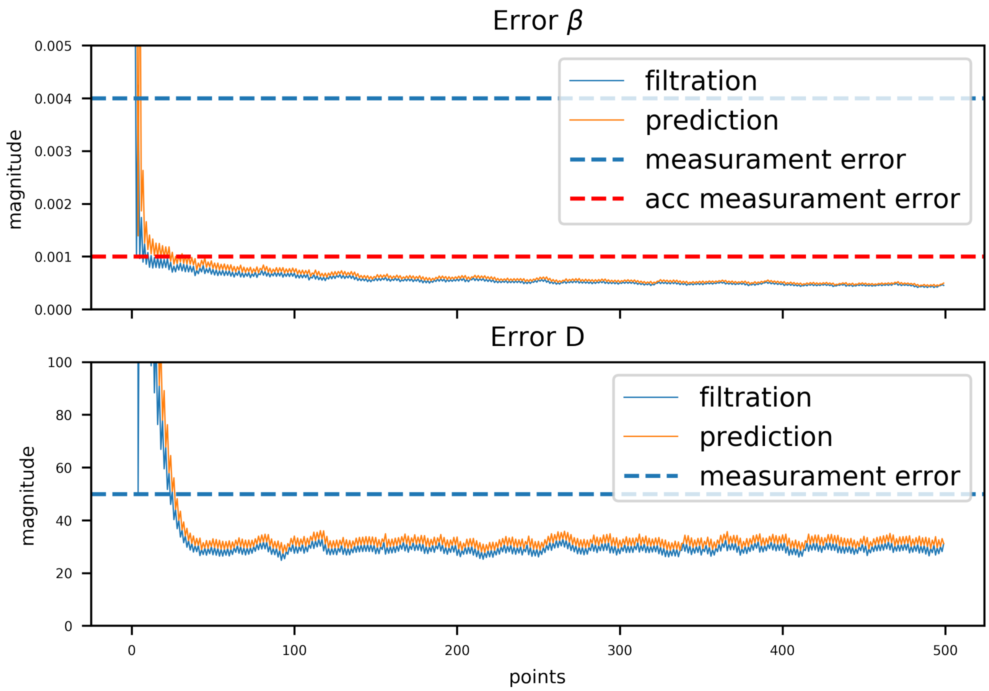



## 7 Run Kalman filter algorithm over $M = 500$ runs

```
In [67]: M = 500
         error_b_f = np.zeros((N, M))
         error_D_f = np.zeros((N, M))
         error_b_p = np.zeros((N, M))
         error_D_p = np.zeros((N, M))

         for i in range(M):
             x, _ = generate_acc_trajectory(sigma_a_2, 1, N, x_0, v_x, t)
             y, _ = generate_acc_trajectory(sigma_a_2, 1, N, y_0, v_y, t)
```

```
    D = (x**2 + y**2) ** 0.5
    b = np.arctan(x / y)
    z = [np.zeros(1) for x in range(N)]
    for j in range(0, N, 2):
        D_n = np.random.normal(0, sigma_D, 1)
        b_n = np.random.normal(0, sigma_b, 1)
        z[j] = np.array([D_n + D[j],b_n + b[j]])
    for j in range(1, N, 2):
        b_n = np.random.normal(0, sigma_b_add, 1)
        z[j] = np.array([b_n + b[j]])

    x_1 = z[0][0, 0] * math.sin(z[0][1, 0])
    x_3 = z[2][0, 0] * math.sin(z[2][1, 0])
    y_1 = z[0][0, 0] * math.cos(z[0][1, 0])
    y_3 = z[2][0, 0] * math.cos(z[2][1, 0])
    X_0 = np.array([[x_3],[(x_3 - x_1) / 2 / t], [y_3],[(y_3 - y_1) / 2 / t]])

    X, Xp = kalman(X_0, P_0, z, T, h, R, Q, dh, 3)

    b_f, D_f = convert_to_polar(X)
    b_p, D_p = convert_to_polar(Xp)

    error_b_f[:,i] = (b - b_f) ** 2
    error_D_f[:,i] = (D - D_f) ** 2
    error_b_p[:,i] = (b - b_p) ** 2
    error_D_p[:,i] = (D - D_p) ** 2
```

```
In [73]: fig, ax = plt.subplots(2,1, figsize=(6,4), dpi = 600, sharex = True)
         ax[0].set_title(r'Error $\beta$', fontsize = 10)
         ax[0].plot( (np.sum(error_b_f,axis=1)/(M-1))**0.5, label = 'filtration', linewidth=0.
         ax[0].plot( (np.sum(error_b_p,axis=1)/(M-1))**0.5,  label = 'prediction', linewidth=0
         ax[0].set_ylabel('magnitude', fontsize = 7)
         ax[0].axhline(sigma_b, label = "measurament error", linestyle='--')
         ax[0].axhline(sigma_b_add, label = "acc measurament error", color = 'r', linestyle='-
         ax[0].set_ylim((0, 0.005))
         ax[0].legend(loc='upper right')

         ax[1].set_title('Error D', fontsize = 10)
         ax[1].plot( (np.sum(error_D_f,axis=1)/(M-1))**0.5, label = 'filtration', linewidth=0.
         ax[1].plot( (np.sum(error_D_p,axis=1)/(M-1))**0.5,  label = 'prediction', linewidth=0
         ax[1].set_xlabel('points', fontsize = 7)
         ax[1].set_ylabel('magnitude', fontsize = 7)
         ax[1].axhline(sigma_D, label = "measurament error", linestyle='--')
         ax[1].set_ylim((0, 100))
         ax[1].legend(loc='upper right');
```




### 7.1 One can see from this plot that measurement error (even more accurate measurement error) is times higher than true estimaiton error of filtration and prediction. Also there is an oscillation of Error D and Error b which corespond to different measurements. It can be due-to different accuracy of measurements on odd and even steps.

## 8 Conclusion

### 8.1 Today we have learned extended Kalman filter with different sources of measurements

```
In [ ]:
```