Lab9

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0.1 SKOLTECH, Experimental Data Processing

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In [2]:
```
import numpy as np
import scipy as sp
from matplotlib import pyplot as plt
from numpy.linalg import inv
import matplotlib as mplb
from matplotlib.font_manager import FontProperties
%matplotlib inline
from numpy.random import normal
from mpl_toolkits.mplot3d import Axes3D
mplb.rc('xtick', labelsize=5)
mplb.rc('ytick', labelsize=5)
```

In [101]:
```
def kalman(X_0, P_0, z, T, H, R, Q):
    X = np.zeros((len(z),*(X_0.shape)))
    P = np.zeros((len(z),*(P_0.shape)))
    K = np.zeros((len(z),*(H.transpose().shape)))
    XF= np.zeros_like(X)
    Xp= np.zeros_like(X)
    T6 = np.linalg.matrix_power(T,6)
    for i, _ in enumerate(z):
        #Prediction
        Xp[i] = X[i] = T.dot(X[i-1] if i > 0 else X_0)
        P[i] = T.dot((P[i-1] if i > 0 else P_0).dot(T.transpose())) + Q
        #Filtration
        tmp1 = inv(H.dot(P[i].dot(H.transpose())) + R[i])
        tmp2 = H.transpose().dot(tmp1)
        K[i] = P[i].dot(tmp2)
        X[i] = X[i] + K[i].dot(z[i] - H.dot(X[i]))
        P[i] = (np.identity(X_0.shape[0]) - K[i].dot(H)).dot(P[i])
    return X, K, P, Xp
```

In [77]:
```
def convert_to_polar(X):
    x = X[:,0,0]
    y = X[:,2,0]
    D = (x**2 + y**2) ** 0.5
```

```
        b = np.arctan(x / y)
        return b, D
```

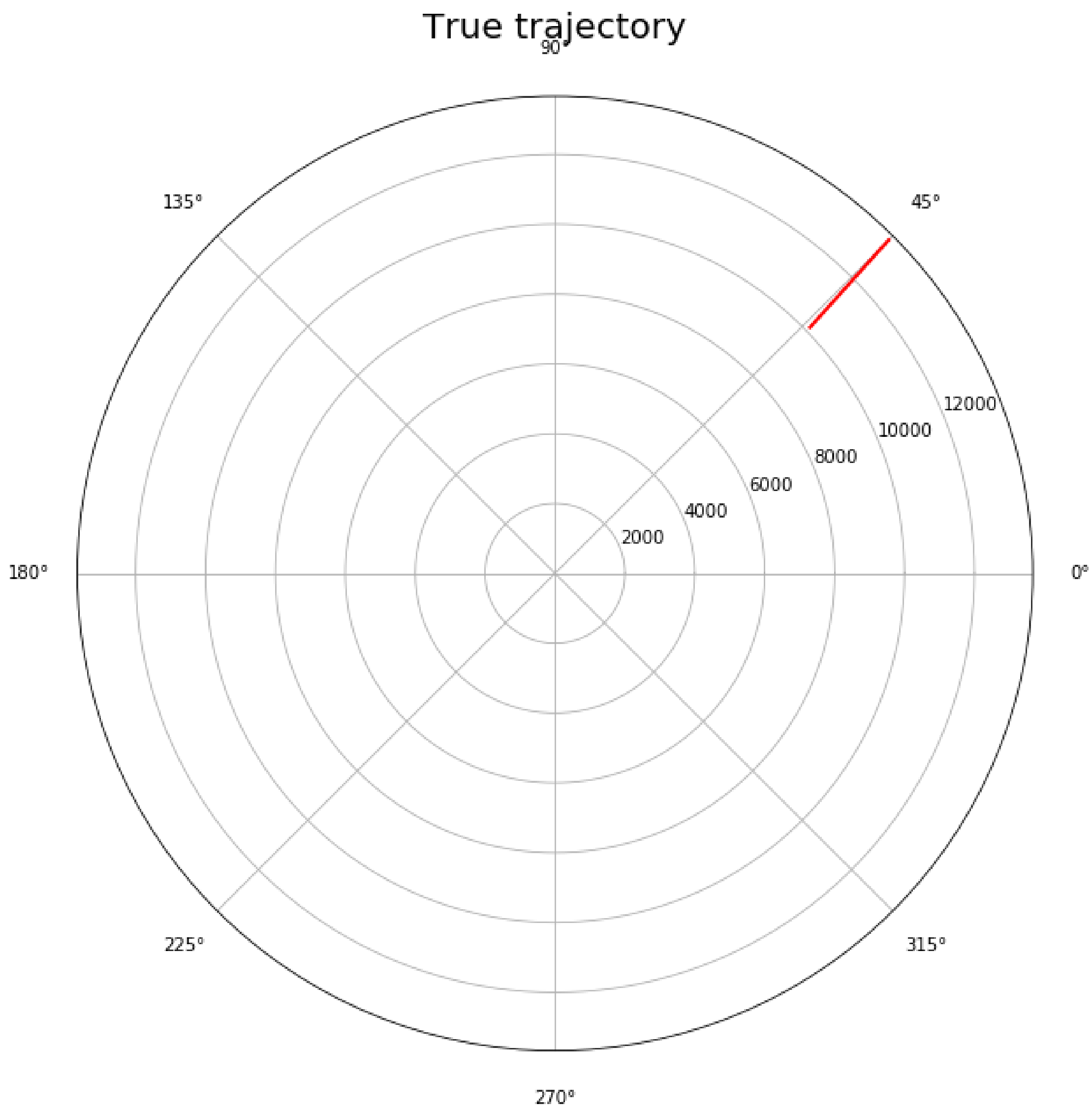
1 Generate a true trajectory

```
In [5]: N = 26
        t = 2
        vx_0 = -50
        vy_0 = -45
        x_0 = 13500 / 2 ** 0.5
        y_0 = 13500 / 2 ** 0.5
        vx = np.ones((N)) * vx_0
        vy = np.ones((N)) * vy_0
        x = np.ones((N)) * x_0 + (vx * t).dot(np.triu(np.ones((N, N)), 1))
        y = np.ones((N)) * x_0 + (vy * t).dot(np.triu(np.ones((N, N)), 1))
```

2 Generate also true values of range D and azimut β

```
In [6]: D = (x**2 + y**2) ** 0.5
        b = np.arctan(x / y)
```

```
In [25]: fig = plt.figure(figsize=(10, 10))
         ax = fig.add_subplot(111, polar=True)
         ax.set_title('True trajectory', fontsize = 20)
         ax.plot(b, D, 'r', linewidth=2)
         ax.xaxis.set_tick_params(labelsize=10)
         ax.yaxis.set_tick_params(labelsize=10)
```

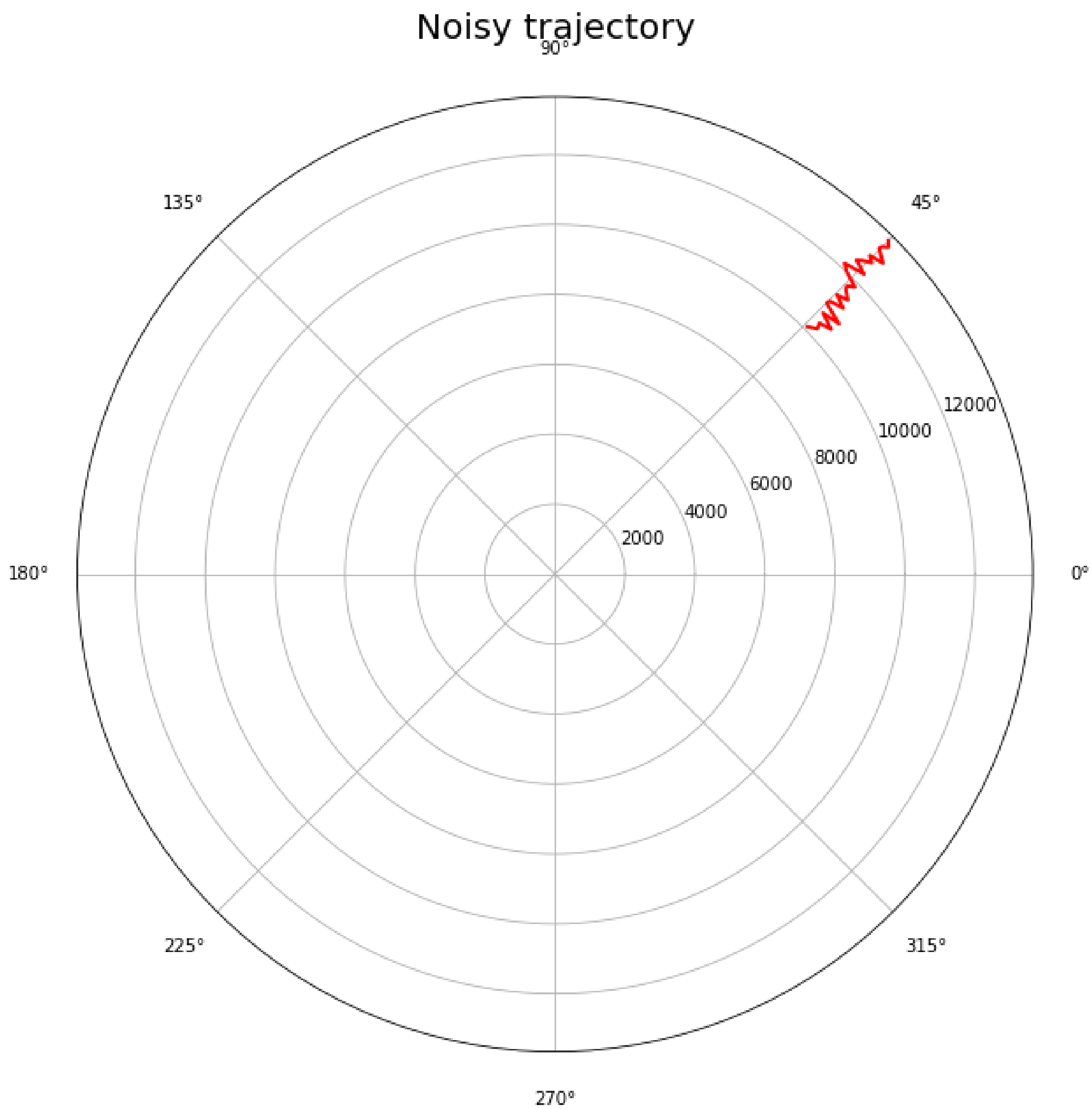


3 Generate measurements D^m and β^m

In [26]:
```
sigma_D = 20
sigma_b = 0.02
D_n = np.random.normal(0, sigma_D, N)
b_n = np.random.normal(0, sigma_b, N)
D_m = D + D_n
b_m = b + b_n
```

In [29]:
```
fig = plt.figure(figsize=(10, 10))
ax = fig.add_subplot(111, polar=True)
ax.set_title('Noisy trajectory', fontsize = 20)
```

```
ax.plot(b_m, D_m, 'r', linewidth=2)
ax.xaxis.set_tick_params(labelsize=10)
ax.yaxis.set_tick_params(labelsize=10)
```



4 Transform polar coordinates

```
In [82]: x_m = D_m * np.sin(b_m)
         y_m = D_m * np.cos(b_m)
```

5 Create the measurement vector

```
In [83]: z = np.array([[x_m, y_m]]).transpose()
```

6 Initial conditions for Kalman filter algorithm

In [84]:
```
X_0 = np.array([[40000], [-20], [40000], [-20]])
P_0 = np.eye(4) * 10 ** 10
```

7 Create the transition matrix and observation matrix H

In [85]:
```
T = np.zeros((4, 4))
T[0:2, 0:2] = np.array([[1, t], [0, 1]])
T[2:4, 2:4] = np.array([[1, t], [0, 1]])
H = np.array([[1, 0, 0, 0], [0, 0, 1, 0]])
```

8 Create the measurement error covariance matrix R

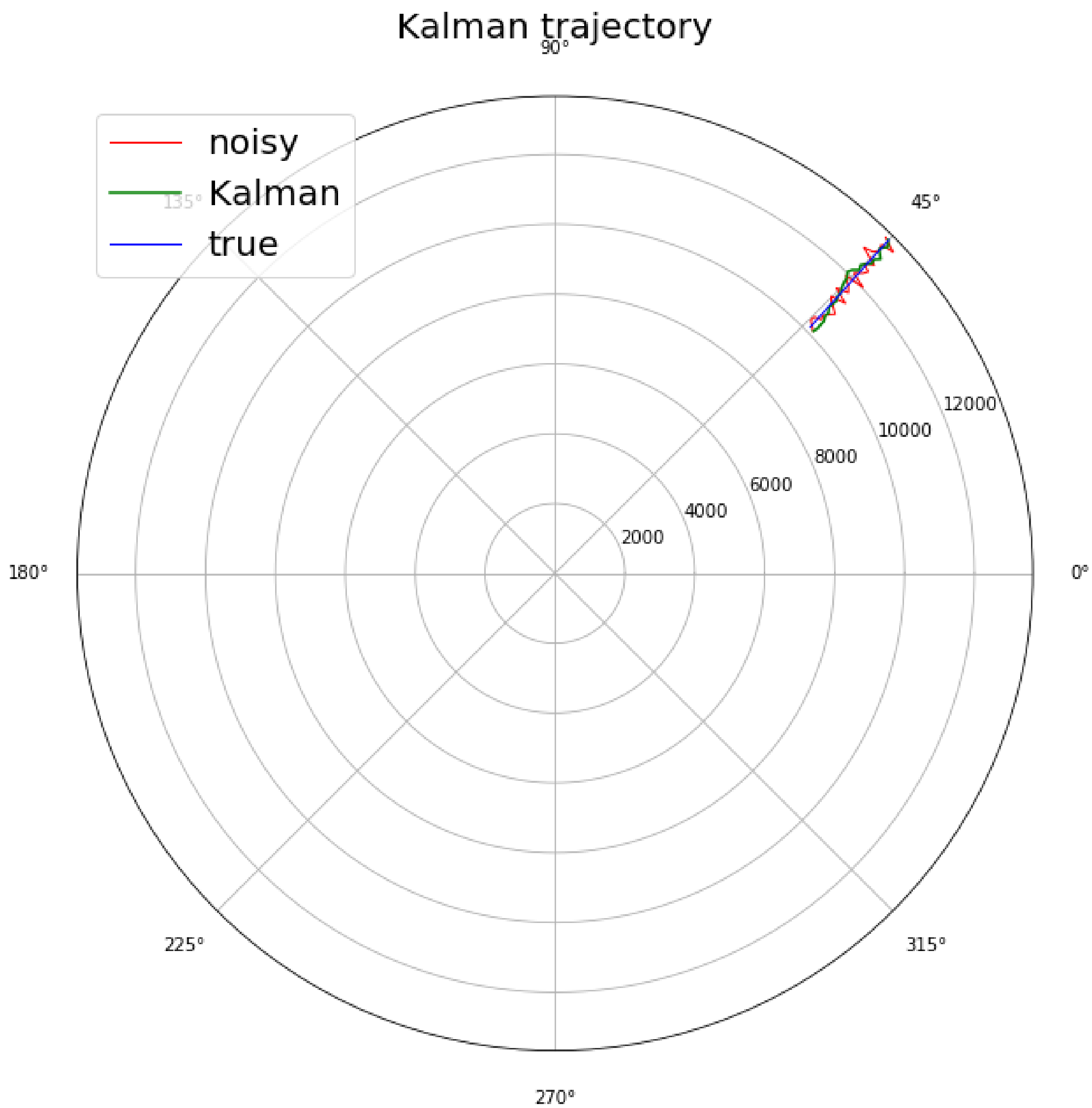
In [103]:
```
R = np.zeros((N, 2, 2))
R[:, 0, 0] = np.sin(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.cos(b_m) ** 2 * sigma_b
R[:, 0, 1] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
R[:, 1, 0] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
R[:, 1, 1] = np.cos(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.sin(b_m) ** 2 * sigma_b
```

9 Develop Kalman filter algorithm

In [159]:
```
X, K, _, Xp = kalman(X_0, P_0, z, T, H, R, Q=0)
```

In [160]:
```
fig = plt.figure(figsize=(10, 10))
ax = fig.add_subplot(111, polar=True)
ax.set_title('Kalman trajectory', fontsize = 20)
ax.plot(b_m, D_m, 'r', linewidth=1, label="noisy")
ax.plot(*convert_to_polar(X), 'g', linewidth=1.5, label="Kalman")
ax.plot(b, D, 'b', linewidth=1, label="true")
ax.xaxis.set_tick_params(labelsize=10)
ax.yaxis.set_tick_params(labelsize=10)
ax.legend(loc = 'upper left', fontsize=20)
```

Out[160]:
```
<matplotlib.legend.Legend at 0x240197f4ba8>
```



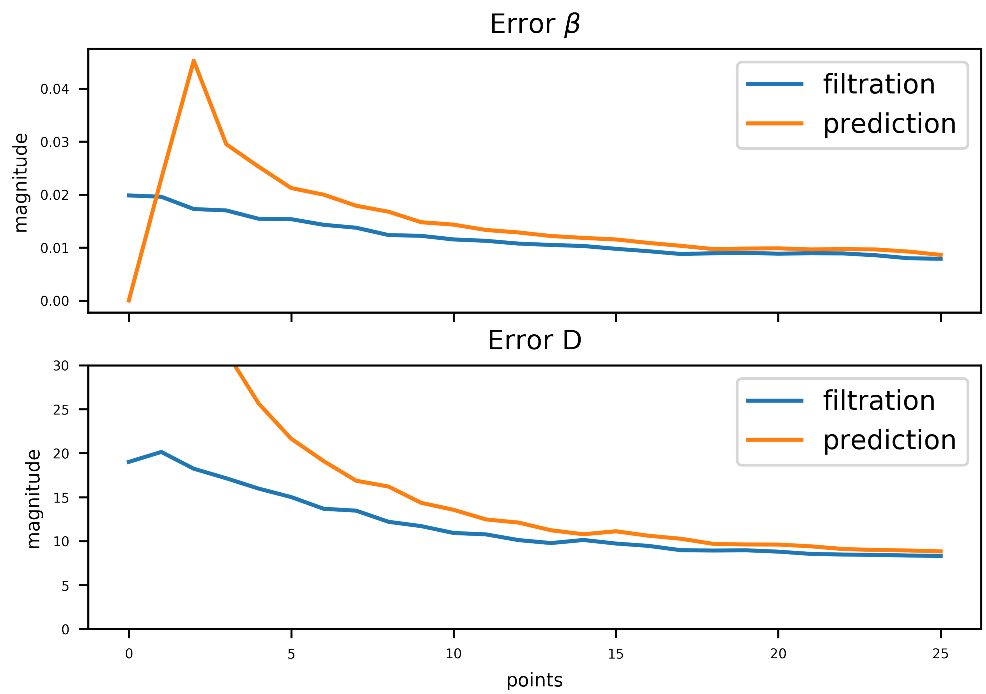
10 Run Kalman filter algorithm over M = 500

In [174]:
```
M = 500
N = 26
t = 2
vx_0 = -50
vy_0 = -45
x_0 = 13500 / 2 ** 0.5
y_0 = 13500 / 2 ** 0.5
sigma_D = 20
sigma_b = 0.02
```

```
error_b_f = np.zeros((N, M))
error_D_f = np.zeros((N, M))
error_b_p = np.zeros((N, M))
error_D_p = np.zeros((N, M))

for i in range(M):
    vx = np.ones((N)) * vx_0
    vy = np.ones((N)) * vy_0
    x = np.ones((N)) * x_0 + (vx * t).dot(np.triu(np.ones((N, N)), 1))
    y = np.ones((N)) * x_0 + (vy * t).dot(np.triu(np.ones((N, N)), 1))

    D = (x**2 + y**2) ** 0.5
    b = np.arctan(x / y)
    D_n = np.random.normal(0, sigma_D, N)
    b_n = np.random.normal(0, sigma_b, N)
    D_m = D + D_n
    b_m = b + b_n

    x_m = D_m * np.sin(b_m)
    y_m = D_m * np.cos(b_m)

    z = np.array([[x_m, y_m]]).transpose()

    R = np.zeros((N, 2, 2))
    R[:, 0, 0] = np.sin(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.cos(b_m) ** 2 * sigm
    R[:, 0, 1] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
    R[:, 1, 0] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
    R[:, 1, 1] = np.cos(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.sin(b_m) ** 2 * sigm

    X, _, _, Xp = kalman(X_0, P_0, z, T, H, R, Q=0)

    b_f, D_f = convert_to_polar(X)
    b_p, D_p = convert_to_polar(Xp)

    error_b_f[:,i] = (b - b_f) ** 2
    error_D_f[:,i] = (D - D_f) ** 2
    error_b_p[:,i] = (b - b_p) ** 2
    error_D_p[:,i] = (D - D_p) ** 2
```

In [175]:
```
fig, ax = plt.subplots(2,1, figsize=(6,4), dpi = 600, sharex = True)
ax[0].set_title(r'Error $\beta$', fontsize = 10)
ax[0].plot( (np.sum(error_b_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[0].plot( (np.sum(error_b_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[0].legend(loc='upper right')
ax[0].set_ylabel('magnitude', fontsize = 7)

ax[1].set_title('Error D', fontsize = 10)
ax[1].plot( (np.sum(error_D_f,axis=1)/(M-1))**0.5, label = 'filtration')
```

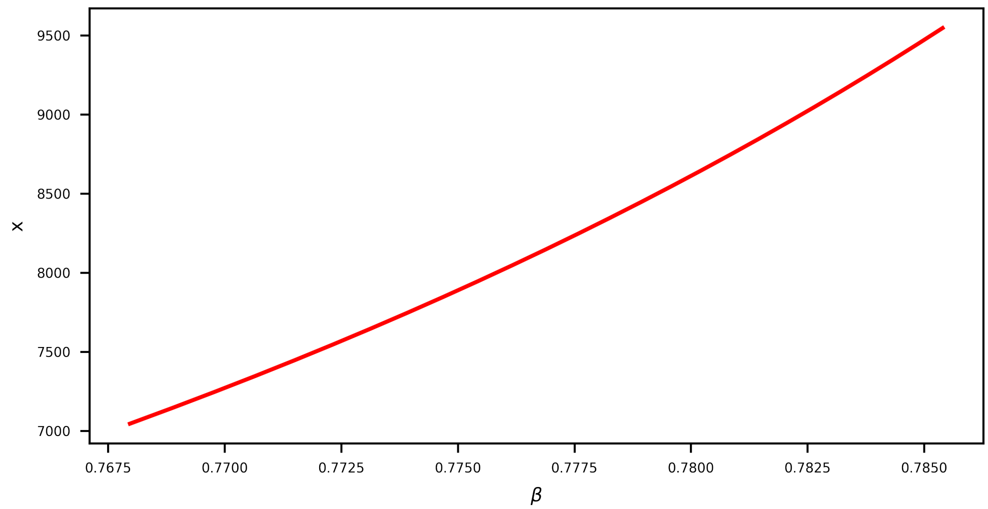
```
ax[1].plot( (np.sum(error_D_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[1].legend(loc='upper right')
ax[1].set_xlabel('points', fontsize = 7)
ax[1].set_ylabel('magnitude', fontsize = 7)
ax[1].set_ylim(0, 30);
```



10.0.1 The error of D decreases, but the error of β remains same

11 Analyze dependence of coordinate on azimuth

In [154]:
```
fig, ax = plt.subplots(1,1, figsize=(6,3), dpi = 600)
ax.plot(b, x, 'r')
ax.set_ylabel('x', fontsize = 7)
ax.set_xlabel(r'$\beta$', fontsize = 7);
```



11.0.1 The dependence in this region is exactly linear

12 Calculate condition number of covariance matrix

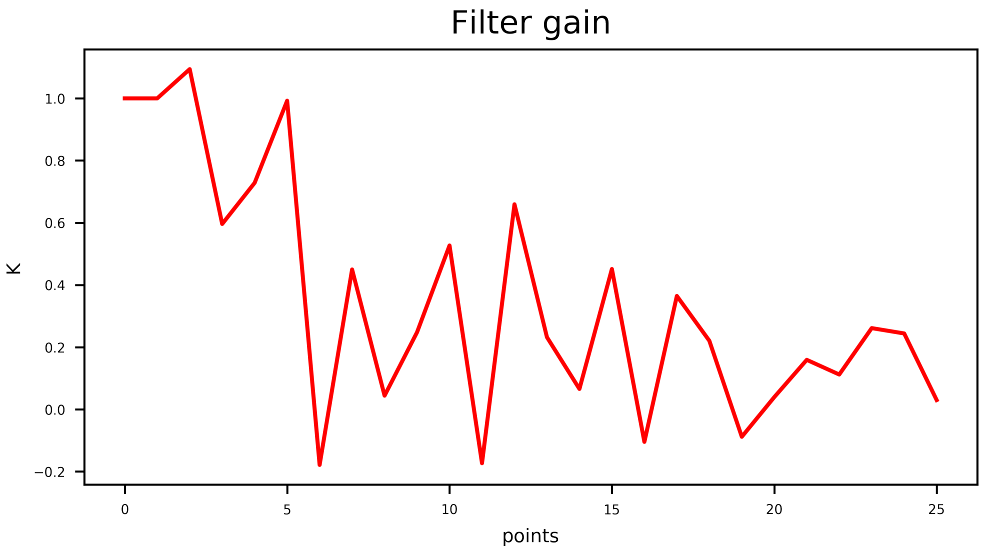
```
In [156]: cond_number = D_m ** 2 * sigma_b ** 2 / sigma_D ** 2
          print(cond_number)
```

```
[ 182.09785286  178.36107179  175.78829697  171.45512117  167.47121624
  165.25321911  161.27610204  156.87779471  155.06462771  151.54397988
  146.92958174  144.76474709  142.09394633  138.2738676   134.97935683
  131.6038484   128.81102606  125.48400279  123.35960915  119.36559495
  116.88271523  113.22491665  110.86830678  109.01074934  106.17093135
  102.72925903]
```

12.0.1 Condition numbers are small compared with 1000, but large compared with 1

13 Analyze filter gain

```
In [163]: fig, ax = plt.subplots(1,1, figsize=(6,3), dpi = 600)
          ax.set_title("Filter gain")
          ax.plot(K[:,0,0], 'r')
          ax.set_ylabel('K', fontsize = 7)
          ax.set_xlabel(r'points', fontsize = 7);
```



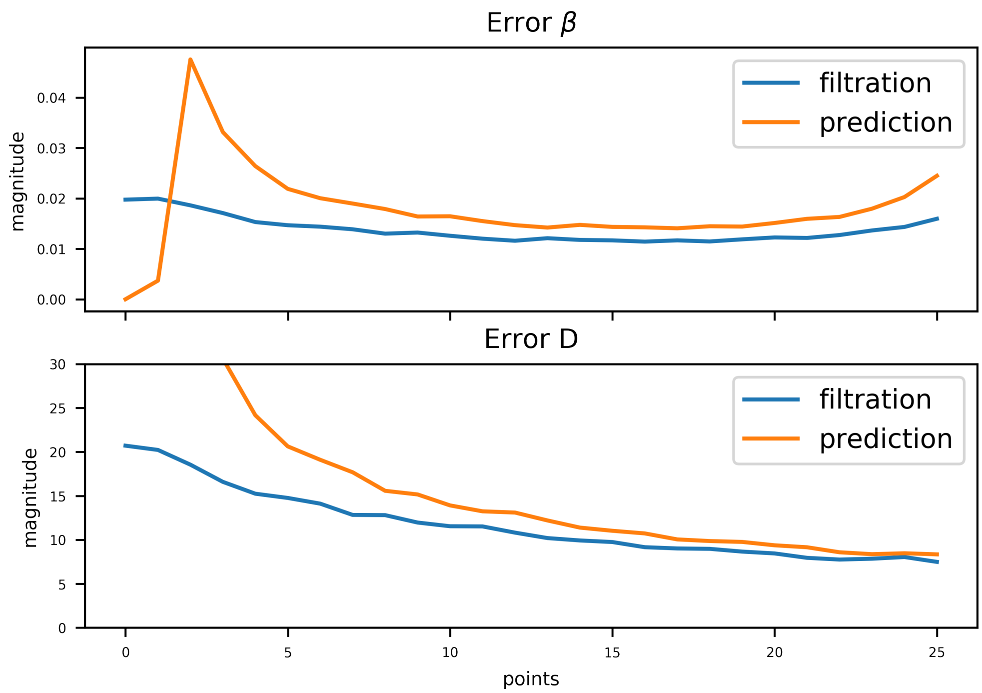
14 Run filter again M = 500 when an object starts it motion at a quite close distance from an observer

15 Calculate true errors of estimation

In [176]:
```
M = 500
N = 26
t = 2
vx_0 = -50
vy_0 = -45
x_0 = 3500 / 2 ** 0.5
y_0 = 3500 / 2 ** 0.5
sigma_D = 20
sigma_b = 0.02

error_b_f = np.zeros((N, M))
error_D_f = np.zeros((N, M))
error_b_p = np.zeros((N, M))
error_D_p = np.zeros((N, M))

for i in range(M):
    vx = np.ones((N)) * vx_0
    vy = np.ones((N)) * vy_0
    x = np.ones((N)) * x_0 + (vx * t).dot(np.triu(np.ones((N, N)), 1))
    y = np.ones((N)) * x_0 + (vy * t).dot(np.triu(np.ones((N, N)), 1))

    D = (x**2 + y**2) ** 0.5
```

```
        b = np.arctan(x / y)
        D_n = np.random.normal(0, sigma_D, N)
        b_n = np.random.normal(0, sigma_b, N)
        D_m = D + D_n
        b_m = b + b_n

        x_m = D_m * np.sin(b_m)
        y_m = D_m * np.cos(b_m)

        z = np.array([[x_m, y_m]]).transpose()

        R = np.zeros((N, 2, 2))
        R[:, 0, 0] = np.sin(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.cos(b_m) ** 2 * sigm
        R[:, 0, 1] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
        R[:, 1, 0] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
        R[:, 1, 1] = np.cos(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.sin(b_m) ** 2 * sigm

        X, _, _, Xp = kalman(X_0, P_0, z, T, H, R, Q=0)

        b_f, D_f = convert_to_polar(X)
        b_p, D_p = convert_to_polar(Xp)

        error_b_f[:,i] = (b - b_f) ** 2
        error_D_f[:,i] = (D - D_f) ** 2
        error_b_p[:,i] = (b - b_p) ** 2
        error_D_p[:,i] = (D - D_p) ** 2
```

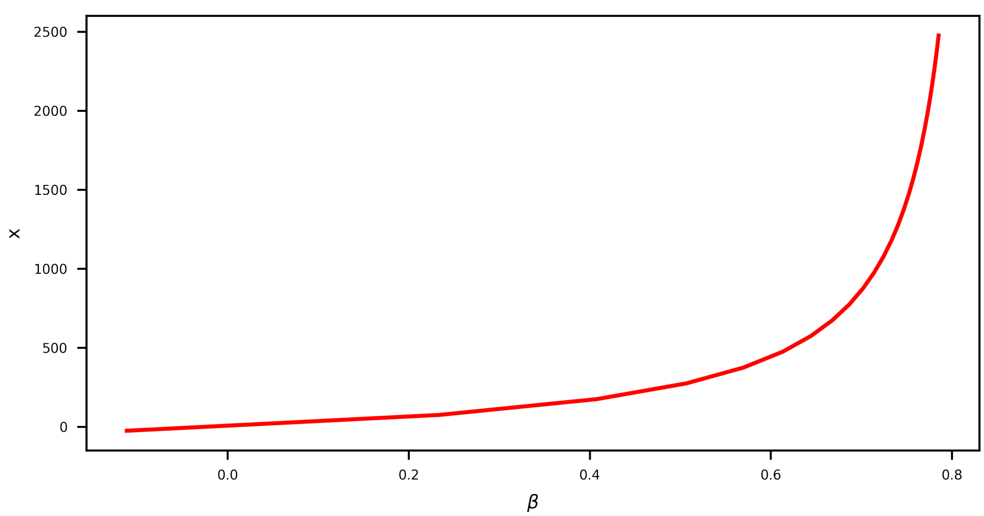
In [177]:
```
fig, ax = plt.subplots(2,1, figsize=(6,4), dpi = 600, sharex = True)
ax[0].set_title(r'Error $\beta$', fontsize = 10)
ax[0].plot( (np.sum(error_b_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[0].plot( (np.sum(error_b_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[0].legend(loc='upper right')
ax[0].set_ylabel('magnitude', fontsize = 7)

ax[1].set_title('Error D', fontsize = 10)
ax[1].plot( (np.sum(error_D_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[1].plot( (np.sum(error_D_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[1].legend(loc='upper right')
ax[1].set_xlabel('points', fontsize = 7)
ax[1].set_ylabel('magnitude', fontsize = 7)
ax[1].set_ylim(0, 30);
```



15.0.1 The error of D decreases, but the error after some point start to increase

16 Analyze dependence of coordinate x on azimuth β

In [178]:
```
fig, ax = plt.subplots(1,1, figsize=(6,3), dpi = 600)
ax.plot(b, x, 'r')
ax.set_ylabel('x', fontsize = 7)
ax.set_xlabel(r'$\beta$', fontsize = 7);
```



16.0.1 Dependence x from β is greatly non-linear

17 Calculate condition number of covariance matrix R

In [179]:
```
cond_number = D_m ** 2 * sigma_b ** 2 / sigma_D ** 2
print(cond_number)
```

```
[ 12.09054718  11.30747679  10.27856001   9.71461231   8.81775643
   7.97727593   7.06766931   6.5395941    5.9118608    5.31786983
   4.6373925    4.20657709   3.53585233   3.02156067   2.57135579
   2.33324084   1.81159972   1.42288073   1.21696928   0.94240178
   0.66452668   0.50443361   0.33988275   0.18528757   0.0972112
   0.0522159 ]
```

17.0.1 Condition numbers are less than 1 starting from some point

18 Make conclusions how linearization errors affect tracking accuracy and how important for tracking accuracy is starting position of a moving object (close or far from an observer)

18.0.1 When condition number is less than 1 and dependence x from β is non-linear error is big

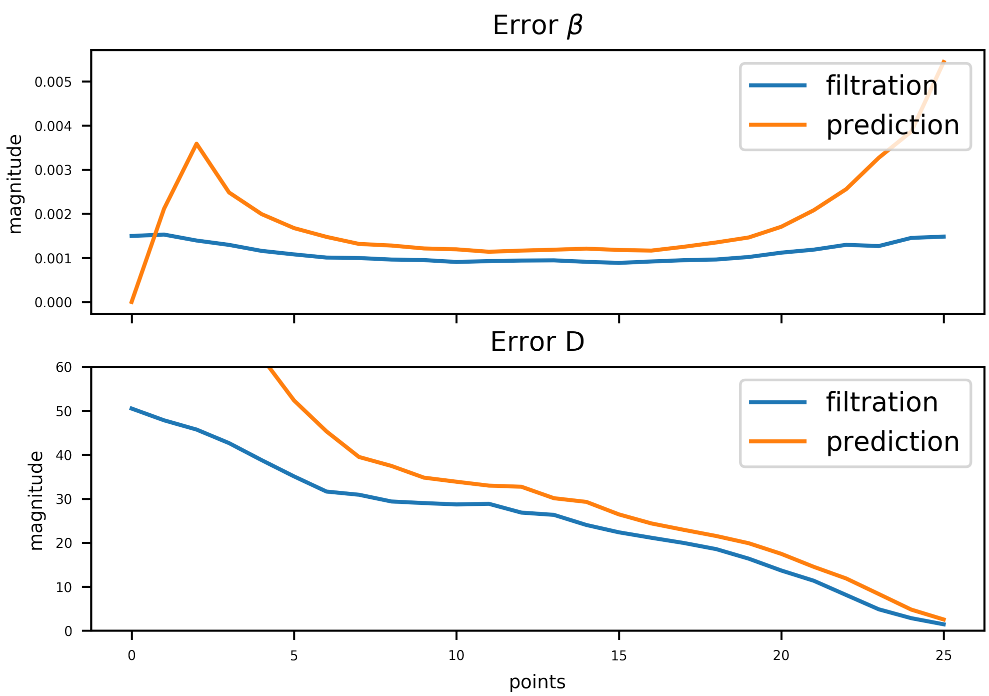
19 Run filter again over M = 500 runs

20 Repeat items 14, 15, 16, 17

In [182]:
```
M = 500
N = 26
t = 2
vx_0 = -50
vy_0 = -45
x_0 = 3500 / 2 ** 0.5
y_0 = 3500 / 2 ** 0.5
sigma_D = 50
sigma_b = 0.0015

error_b_f = np.zeros((N, M))
error_D_f = np.zeros((N, M))
error_b_p = np.zeros((N, M))
error_D_p = np.zeros((N, M))

for i in range(M):
```

```
    vx = np.ones((N)) * vx_0
    vy = np.ones((N)) * vy_0
    x = np.ones((N)) * x_0 + (vx * t).dot(np.triu(np.ones((N, N)), 1))
    y = np.ones((N)) * x_0 + (vy * t).dot(np.triu(np.ones((N, N)), 1))

    D = (x**2 + y**2) ** 0.5
    b = np.arctan(x / y)
    D_n = np.random.normal(0, sigma_D, N)
    b_n = np.random.normal(0, sigma_b, N)
    D_m = D + D_n
    b_m = b + b_n

    x_m = D_m * np.sin(b_m)
    y_m = D_m * np.cos(b_m)

    z = np.array([[x_m, y_m]]).transpose()

    R = np.zeros((N, 2, 2))
    R[:, 0, 0] = np.sin(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.cos(b_m) ** 2 * sigm
    R[:, 0, 1] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
    R[:, 1, 0] = np.sin(b_m) * np.cos(b_m) * (sigma_D ** 2 - D_m ** 2 * sigma_b ** 2)
    R[:, 1, 1] = np.cos(b_m) ** 2 * sigma_D ** 2 + D_m ** 2 * np.sin(b_m) ** 2 * sigm

    X, _, _, Xp = kalman(X_0, P_0, z, T, H, R, Q=0)

    b_f, D_f = convert_to_polar(X)
    b_p, D_p = convert_to_polar(Xp)

    error_b_f[:,i] = (b - b_f) ** 2
    error_D_f[:,i] = (D - D_f) ** 2
    error_b_p[:,i] = (b - b_p) ** 2
    error_D_p[:,i] = (D - D_p) ** 2
```

In [186]:
```
fig, ax = plt.subplots(2,1, figsize=(6,4), dpi = 600, sharex = True)
ax[0].set_title(r'Error $\beta$', fontsize = 10)
ax[0].plot( (np.sum(error_b_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[0].plot( (np.sum(error_b_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[0].legend(loc='upper right')
ax[0].set_ylabel('magnitude', fontsize = 7)

ax[1].set_title('Error D', fontsize = 10)
ax[1].plot( (np.sum(error_D_f,axis=1)/(M-1))**0.5, label = 'filtration')
ax[1].plot( (np.sum(error_D_p,axis=1)/(M-1))**0.5,  label = 'prediction')
ax[1].legend(loc='upper right')
ax[1].set_xlabel('points', fontsize = 7)
ax[1].set_ylabel('magnitude', fontsize = 7)
ax[1].set_ylim(0, 60);
```




```
In [187]: cond_number = D_m ** 2 * sigma_b ** 2 / sigma_D ** 2
          print(cond_number)
```

```
[  1.11202345e-02   1.00048914e-02   9.44945211e-03   8.40273088e-03
   7.78206575e-03   7.10849398e-03   6.71301163e-03   5.87895517e-03
   5.30022802e-03   4.58680140e-03   4.13663248e-03   3.77594061e-03
   3.05811398e-03   2.94256427e-03   2.43151238e-03   2.12362347e-03
   1.70786962e-03   1.34739879e-03   1.23562653e-03   8.75602419e-04
   7.45010662e-04   4.33451530e-04   4.00091751e-04   2.15290034e-04
   6.79161594e-05   5.80196668e-05]
```

20.0.1 Condition numbers are greatly less than 1 and they decrease over time

21 Conclusion

21.0.1 Thus when coordinates are closer to zero, conditions become more diffucult and errors become greater