#Савончик Егор 153505
#Лабораторная работа 3.2
#Обыкновенные дифференциальные уравнения высших порядков
#Вариант 10

> #**номер 1** : Решите уравнения и сравните с результатами, полученными в #Maple. Постройте в одной системе координат несколько интегральных кривых.

> $\#x=(y(x))^{(2)}\cdot\ln\left((y(x))^{(2)}\right)$

> $reshenie := dsolve\left(x-(y(x))^{(2)}\cdot\ln\left((y(x))^{(2)}\right), y(x)\right);$

$$reshenie := y(x) = \frac{1}{108}\,\frac{\left(18\,\mathrm{LambertW}(x)^2+15\,\mathrm{LambertW}(x)+4\right)x^3}{\mathrm{LambertW}(x)^3}+\_C1\,x+\_C2 \qquad (1)$$

> *#замена* $y^{(2)}=t$*, также из этого следует* $dy^2=t\cdot dx^2$

> $eq := x=t\cdot\ln(t);$

$$eq := x=t\ln(t) \qquad (2)$$

> $dx^2=diff(rhs(eq), t, t)$

$$dx^2=\frac{1}{t} \qquad (3)$$

> *#после подстановки* dy^2

$\frac{dy^2}{t}=\frac{1}{t}\cdot dt^2;$

#разделим обе части на dt^2 и *умножим на* t

$'diff(y, t, t)=1';$

$$\frac{dy^2}{t}=\frac{dt^2}{t}$$

$$\frac{\partial^2}{\partial t^2}\,y=1 \qquad (4)$$

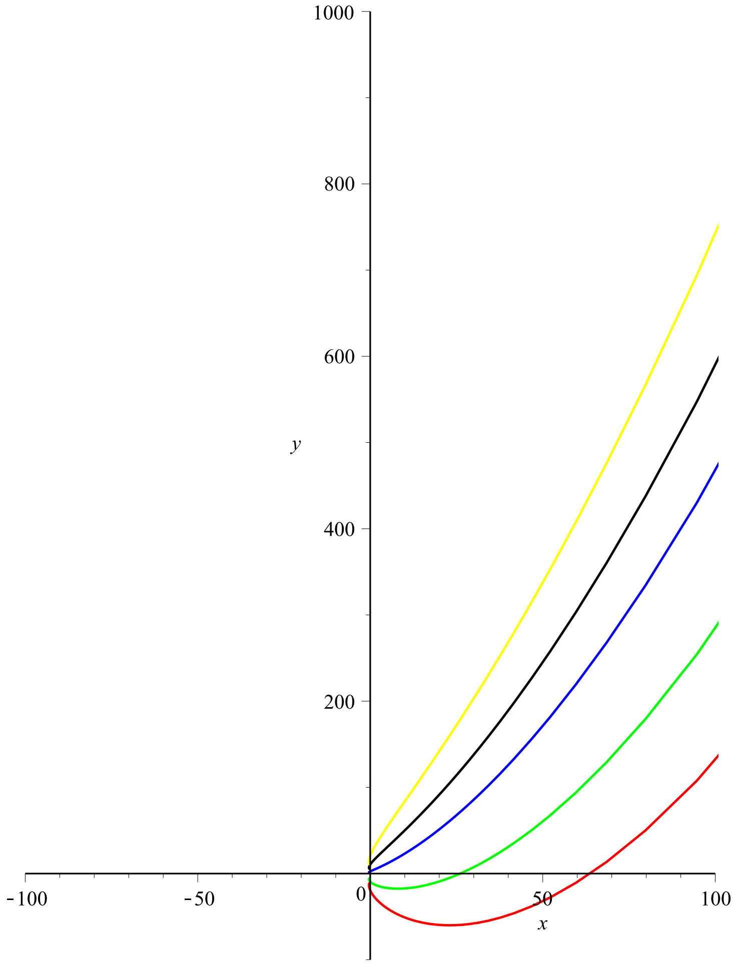
> $otvet := dsolve(diff(y(t), t, t)-1, y(t))$

$$otvet := y(t)=\frac{1}{2}\,t^2+\_C1\,t+\_C2 \qquad (5)$$

> *#построим график полученного решения*

$plot(\{[rhs(eq), subs(\_C1=1, \_C2=1, rhs(otvet)), t=-infinity..infinity],$
$[rhs(eq), subs(\_C1=5, \_C2=5, rhs(otvet)), t=-infinity..infinity], [rhs(eq),$
$subs(\_C1=10, \_C2=10, rhs(otvet)), t=-infinity..infinity], [rhs(eq),$
$subs(\_C1=-5, \_C2=-5, rhs(otvet)), t=-infinity..infinity], [rhs(eq),$

$subs(\_C1 = -10, \_C2 = -10, rhs(otvet)), t = -infinity..infinity], \}, x = -100..100, y = -100..1000, color = [red, green, blue, black, yellow])$



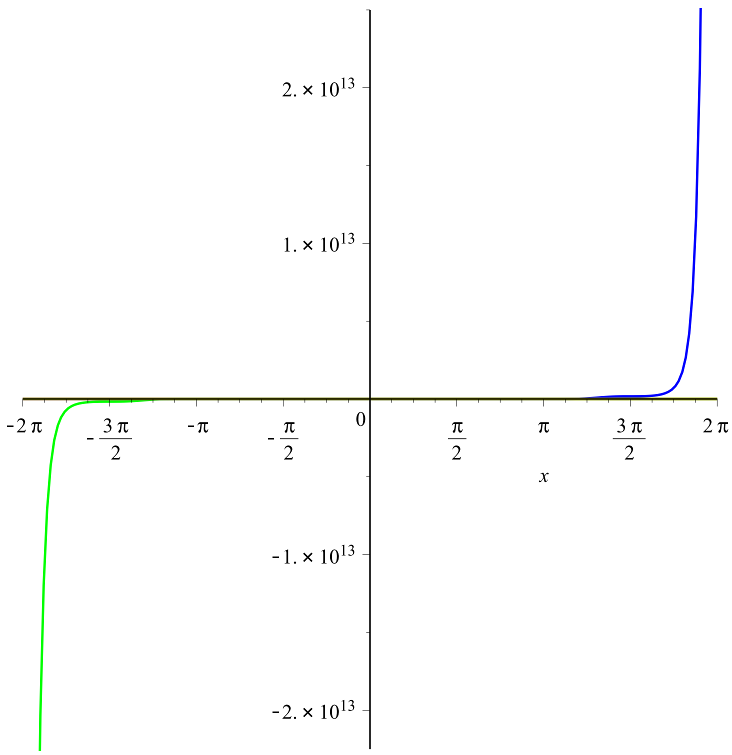
> #номер 1.2

> $\#\cos(x)\cdot\left(y(x)\cdot(y(x))^{(2)}-\left((y(x))^{(1)}\right)^2\right)+2\cdot y(x)\cdot(y(x))^{(1)}\cdot\sin(x)=0$

> $reshenie := dsolve\left(\cos(x)\cdot\left(y(x)\cdot(y(x))^{(2)}-\left((y(x))^{(1)}\right)^2\right)+2\cdot y(x)\cdot(y(x))^{(1)}\cdot\sin(x), y(x)\right);$

$$reshenie := y(x) = \mathrm{e}^{\frac{1}{2}\_C1\,x}\,\mathrm{e}^{\frac{1}{4}\_C1\sin(2x)}\,\_C2 \quad (6)$$

> $plot(\{subs(\_C1=1, \_C2=1, rhs(reshenie)), subs(\_C1=5, \_C2=5, rhs(reshenie)), subs(\_C1=10, \_C2=10, rhs(reshenie)), subs(\_C1=-5, \_C2=-5, rhs(reshenie)), subs(\_C1=-10, \_C2=-10, rhs(reshenie))\}, color=[red, green, blue, black, yellow]);$



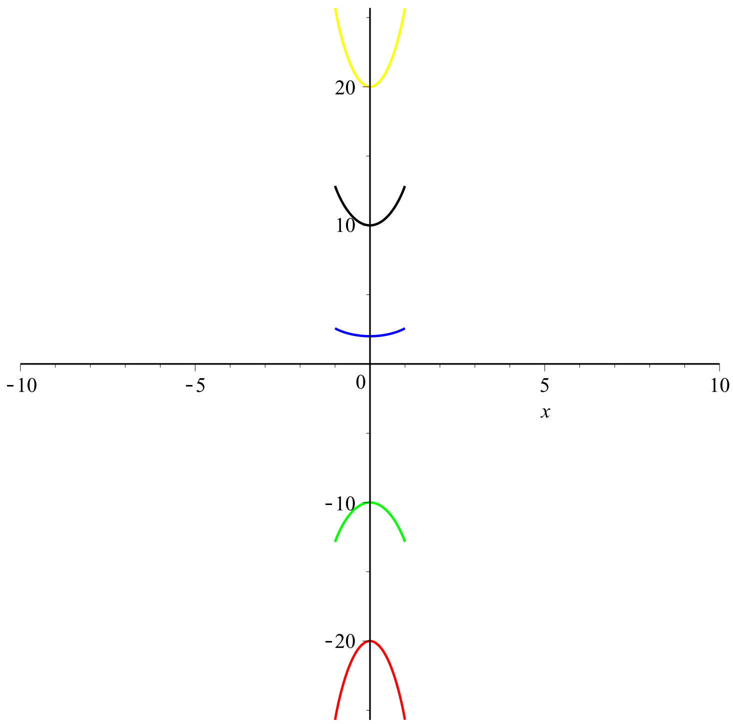
> **#номер 1.3**

> $\#(y(x))^{(2)} \cdot \mathrm{sqrt}(1-x^2) \cdot \arcsin(x) = (y(x))^{(1)}$

> $reshenie := dsolve\left((y(x))^{(2)} \cdot \mathrm{sqrt}(1-x^2) \cdot \arcsin(x) - (y(x))^{(1)}, y(x)\right);$

$$reshenie := y(x) = \_C1 + \left(x \arcsin(x) + \sqrt{-x^2+1}\right) \_C2 \tag{7}$$

> $plot(\{subs(\_C1=1, \_C2=1, rhs(reshenie)), subs(\_C1=5, \_C2=5, rhs(reshenie)), subs(\_C1=10, \_C2=10, rhs(reshenie)), subs(\_C1=-5, \_C2=-5, rhs(reshenie)), subs(\_C1=-10, \_C2=-10, rhs(reshenie))\}, color = [red, green, blue, black, yellow]);$



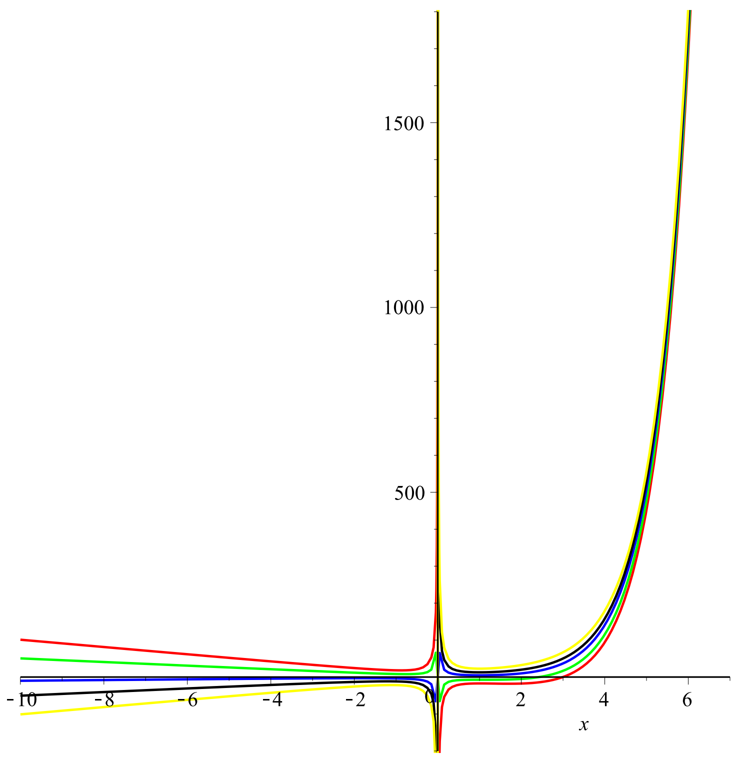
> **#номер 1.4**

> $\#(y(x))^{(2)} + \dfrac{(y(x))^{(1)}}{x} - \dfrac{y(x)}{x^2} = \exp(1)^x \cdot (1+x)$

> $reshenie := dsolve\left((y(x))^{(2)} + \frac{(y(x))^{(1)}}{x} - \frac{y(x)}{x^2} - \exp(1)^x \cdot (1+x), y(x) \right);$

$$reshenie := y(x) = \_C1\,x + \frac{\_C2}{x} + \frac{e^x\left(x^2 - 2x + 2\right)}{x} \qquad \textbf{(8)}$$

> $plot(\{subs(\_C1 = 1, \_C2 = 1, rhs(reshenie)), subs(\_C1 = 5, \_C2 = 5, rhs(reshenie)), subs(\_C1 = 10, \_C2 = 10, rhs(reshenie)), subs(\_C1 = -5, \_C2 = -5, rhs(reshenie)), subs(\_C1 = -10, \_C2 = -10, rhs(reshenie))\}, color = [red, green, blue, black, yellow]);$



> **#номер 2** : Найдите общее решение уравнения и сравните с результатом, #полученным в системе Maple.

> $\#(y(x))^{(3)}\cdot\coth(2\cdot x)=2\cdot(y(x))^{(2)}$

> $reshenie := dsolve\left((y(x))^{(3)}\cdot\coth(2\cdot x)-2\cdot(y(x))^{(2)}, y(x)\right);$

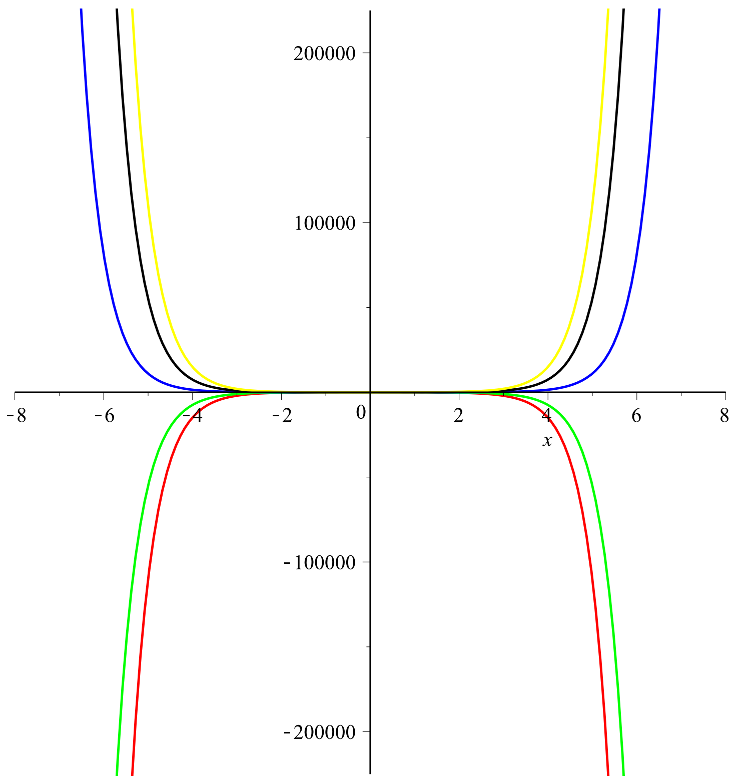
$$reshenie := y(x) = \int \int \frac{\_C1 \cosh(2\,x)}{\sqrt{\frac{\sinh(2\,x)+\cosh(2\,x)}{\sinh(2\,x)}}\sqrt{\frac{-\sinh(2\,x)+\cosh(2\,x)}{\sinh(2\,x)}}\sinh(2\,x)}\,\mathrm{d}x\,\mathrm{d}x + \_C2\,x + \_C3 \quad (9)$$

> *#конструкция под интегралом сводится к cosh(2x), если его интегрировать два раза то он будет иметь вид cosh(2x), константу уберет _C1*
#итоговый вид:

> $reshenie := y(x) = \_C1\cdot\cosh(2\cdot x)+\_C2\cdot x+\_C3;$

$$reshenie := y(x) = \_C1\cosh(2\,x)+\_C2\,x+\_C3 \quad (10)$$

> $plot(\{subs(\_C1=1, \_C2=1, \_C3=1, rhs(reshenie)), subs(\_C1=5, \_C2=5, \_C3=5, rhs(reshenie)), subs(\_C1=10, \_C2=10, \_C3=10, rhs(reshenie)), subs(\_C1=-5, \_C2=-5, \_C3=-5, rhs(reshenie)), subs(\_C1=-10, \_C2=-10, \_C3=-10, rhs(reshenie))\}, color=[red, green, blue, black, yellow]);;$



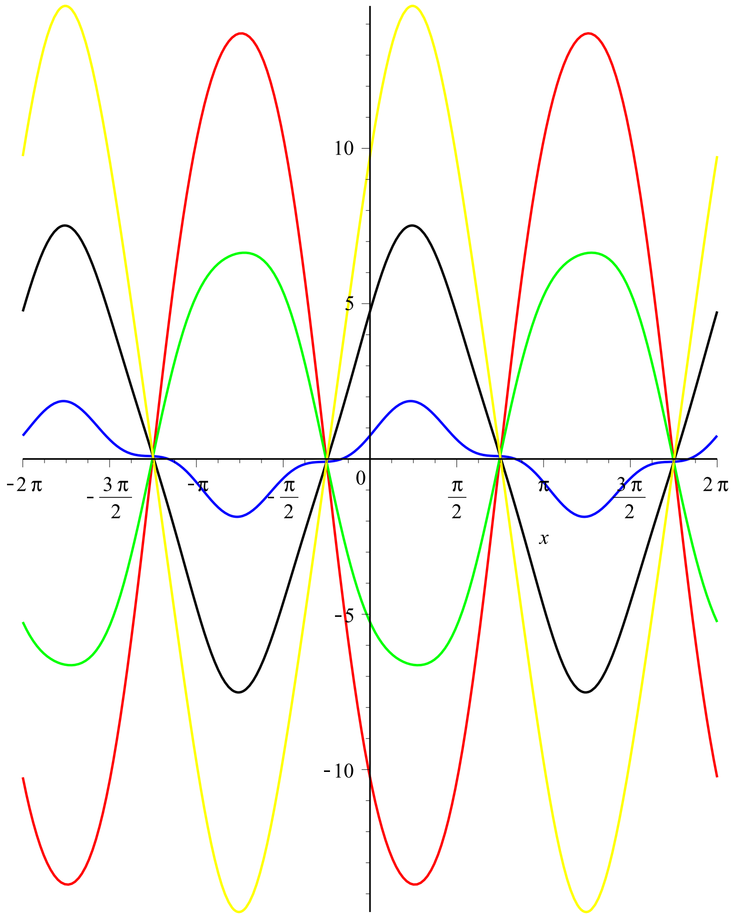
> **#номер 3** : Найдите общее решение дифференциального уравнения

> $\#(y(x))^{(2)} + y(x) = 2\cdot\cos(3\cdot x) - 3\cdot\sin(3\cdot x)$

> $reshenie := dsolve\Big((y(x))^{(2)} + y(x) - 2\cdot\cos(3\cdot x) + 3\cdot\sin(3\cdot x), y(x)\Big);$

$$reshenie := y(x) = \sin(x)\ \_C2 + \cos(x)\ \_C1 - \frac{1}{4}\cos(3\,x) + \frac{3}{8}\sin(3\,x) \qquad \textbf{(11)}$$

> $plot(\{subs(\_C1 = 1, \_C2 = 1, rhs(reshenie)), subs(\_C1 = 5, \_C2 = 5, rhs(reshenie)), subs(\_C1 = 10, \_C2 = 10, rhs(reshenie)), subs(\_C1 = -5, \_C2 = -5, rhs(reshenie)), subs(\_C1 = -10, \_C2 = -10, rhs(reshenie))\}, color = [red, green, blue, black, yellow]);$



10
5
-5
-10
-2 π
- 3π/2
-π
- π/2
0
π/2
π
3π/2
2 π
x