Вариант 1

№1

$A=\{1;2;3;4;5\}$; $X \setminus A=\{6;7\}$; $A \cap X=\{1;3;5\}$

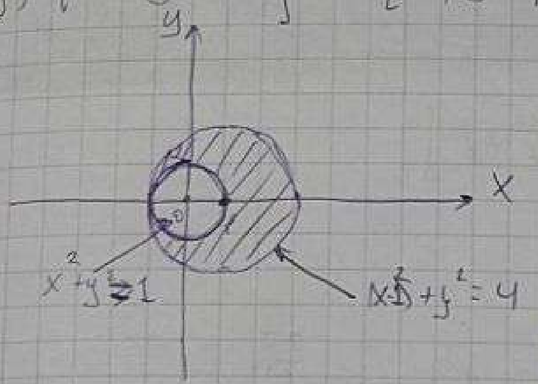
Н-ти: X

Реш-е: $A \cap X=\{1;3;5\} \Rightarrow \{1,3,5\} \in X$
$X \setminus A=\{6;7\} \Rightarrow \{6;7\} \in X$
$\Rightarrow X=\{1,3,5,6,7\}$

Ответ: $X=\{1;3;5;6;7\}$

№2

$\{(x,y) \mid x^2+y^2 \geq 1\} \cap \{(x,y) \mid (x-1)^2+y^2 \leq 4\}$



№3

$f: \mathbb{R} \to \mathbb{R}$; $f(x)=x^2-5x+6=(x-3)(x-2)$

Н-ти: $f^{-1}([2,+\infty))$

Реш-е. $f^{-1}=\frac{1}{f}=\frac{1}{(x-3)(x-2)}$

$f^{-1}([2,+\infty))=(0,+\infty)$, т.к. при $x \to 2$ $f^{-1}(x) \to +\infty$, при $x \to +\infty$ $f^{-1}(x) \to 0$

Ответ: $f^{-1}([2,+\infty))=(0;+\infty)$

№4

$x_n=\frac{n+1}{n}$

$\lim\limits_{n\to\infty} x_n=\lim\limits_{n\to\infty} \frac{n+1}{n}=\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)=1$

$\max x_n=2$ $\quad$ $\inf x_n=2$

$\min x_n$ – не сущ. $\quad$ $\sup x_n=1$

$E=(1;2]$

№5

$\forall \varepsilon > 0 \ \exists n_0 = n(\varepsilon) > 0 \ : \ \forall n \geq n_0 \ \ |a_n - a| < \varepsilon$

$a_n = \frac{5n+15}{6-n}, \ a = -5$

$\left| \frac{5n+15}{6-n} + 5 \right| < \varepsilon$

$\left| \frac{45}{6-n} \right| < \varepsilon$

$45 < \varepsilon |6-n|$

$|6-n| > \frac{45}{\varepsilon}$

$|n-6| > \frac{45}{\varepsilon}$

для $n \geq 6: \ n > \frac{45}{\varepsilon} + 6$

для $n \leq 6: \ n < -\frac{45}{\varepsilon} + 6$ $\Rightarrow n_0 = \left[\pm \frac{45}{\varepsilon} + 6 \right]$

Ответ: $n_0 = \left[\pm \frac{45}{\varepsilon} + 6 \right]$

№6

$$\lim_{n\to\infty} \frac{(3-n)^3}{(n+1)^2 - (n+1)^3} = \left| \frac{\infty}{\infty} \right| = \lim_{n\to\infty} \frac{27 - 27n + 9n^2 - n^3}{-2n^2 - n - n^3} = \lim_{n\to\infty} \frac{\frac{27}{n^3} - \frac{9}{n} + \frac{27}{n^2} - 1}{-\frac{2}{n} - \frac{1}{n^2} - 1} = \frac{-1}{-1} = 1$$

Ответ: 1

№7

$$\lim_{n\to\infty} \frac{n\sqrt[6]{n} + \sqrt[5]{32n^{10}+1}}{(n+\sqrt[4]{n})\sqrt[3]{n^3-1}} = \left| \frac{\infty}{\infty} \right| = \lim_{n\to\infty} \frac{n\sqrt[6]{n} + \sqrt[5]{32n^{10}+1}}{\sqrt[3]{n^6-n^3} + \sqrt[3]{n^{\frac{15}{4}}} - \sqrt[4]{n^3}} =$$

$$= \lim_{n\to\infty} \frac{\frac{1}{\sqrt[6]{n^5}} + \sqrt[5]{\frac{1}{n^{10}} + 32}}{\sqrt[3]{\frac{1}{n^{\frac{9}{4}}}} - \frac{1}{n^{\frac{21}{4}}} + \sqrt[3]{1 + \frac{1}{n^3}}} = \frac{\sqrt[5]{32}}{1} = 2$$

Ответ: 2.

№8

$$\lim_{n\to\infty} \left(\sqrt{(n+2)(n+1)} - \sqrt{(n-1)(n+3)} \right) = |\infty - \infty| = \lim_{n\to\infty} \left(\sqrt{n^2+n-2} - \sqrt{n^2+2n-3} \right) =$$

$$= \lim_{n\to\infty} \frac{1-n}{\sqrt{n^2+n-2} + \sqrt{n^2+2n-3}} = \lim_{n\to\infty} \frac{\frac{1}{n} - 1}{\sqrt{\frac{1}{n} - \frac{2}{n^2} + 1} + \sqrt{\frac{2}{n} - \frac{3}{n^2} + 1}} = \frac{-1}{2}$$

Ответ: $-\frac{1}{2}$.

№9

$$\lim_{n\to\infty}\frac{(2n+1)!+(2n+2)!}{(2n+3)!}=\lim_{n\to\infty}\frac{\cancel{(2n+1)!}(1+(2n+2))}{\cancel{(2n+1)!}(2n+2)(2n+3)}=\lim_{n\to\infty}\frac{\cancel{2n+3}}{(2n+2)\cancel{(2n+3)}}=\lim_{n\to\infty}\frac{1}{2n+2}=$$

$=0$

Ответ: 0

№10

$$\lim_{n\to\infty}\left(\frac{n^3+n+1}{n^3+2}\right)^{2n^2}=\lim_{n\to\infty}\left(1+\frac{n-1}{n^3+2}\right)^{2n^2}=\lim_{n\to\infty}\left(1+\frac{n-1}{n^3+2}\right)^{\frac{n^3+2}{n-1}\cdot\frac{2n^2(n-1)}{n^3+2}}=\left|\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e\right|=$$

$$=\lim_{n\to\infty}e^{\frac{2n^2(n-1)}{n^3+2}}=e^2,\text{ т.к.}$$

$$\lim_{n\to\infty}\frac{2n^3-2n^2}{n^3+2}=\lim_{n\to\infty}\frac{2-\frac{2}{n}}{1+\frac{2}{n^3}}=2$$

Ответ: e^2