Module 4 Group Project
Rocket Science!!

Section 2 Group 33
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i) In *University Physics with Modern Physics* section 8.6, we have derived the famous Tsiolkovsky rocket equation:

$$v - v_0 = v_{ex} \ln \frac{m_0}{m}$$

where v is the final speed of the rocket, v_0 is the initial speed, v_{ex} is the the relative speed of the ejected fuel, m_0 is the initial total mass of the rocket and m is the final mass. This equation only works when gravity is negligible. Modify this equation when gravity is not negligible. Suppose the gravitational acceleration is g (assume it is constant) and after a time period of τ the rocket runs out of fuel.

ii) The common value for v_{ex} is about $2.0 \sim 3.0 km/s$. Suppose we can make the ratio of the mass of the rocket shell to the mass of the fuel the same as the ratio of the mass of an egg's shell to the total mass of the egg white and the yolk (about 1/10). Use this ratio to calculate the change of the rocket's speed. Compare it to the first cosmic velocity ($7.9 km/s$). Can you see why we cannot use a single-stage rocket to enter the orbit?

iii) In order to enter the orbit, we have to use multistage rockets. A typical multistage rocket has three stages. Suppose our rocket has the following data for each stage:

	the mass of the shell	the mass of the fuel	the mass of this stage	accumulated mass
stage three	1	6	7	7
stage two	10	60	70	77
stage one	100	600	700	777

Use these data to calculate the change of speed of each stage and the final speed of the rocket. Does this speed enable us to enter the orbit? (You can assume the effect of gravity is negligible for this part.)

iv) Now our spacecraft has entered orbit, but it encounters a meteorite shower! Initially, the spacecraft moves in the opposite direction of the flow of meteorite particles at a speed of v. Then, it turns around and moves along the direction of the flow of meteorite particles at the same speed of v. At this time, the tractive force of its engine is 1/4 of that at the begining. Find the speed V of the meteorite particles in terms of v. Assume that our spacecraft is a perfect cylinder with flat surfaces (surface area S) on its two ends. Suppose the collision between the meteorite particles and the spacecraft is perfectly elastic.

v) An astronaut comes out of the spacecraft to fix the apparatus damaged by the meteorite shower. His total mass is M (including his space suit and other necessary equipment). Suddenly, by accident, the rope that connects him with the spacecraft breaks and he is at rest at a distance d from the spacecraft. He has some oxygen (total mass m_0) in his space suit. There is a nozzle on the suit that can eject the oxygen at a speed v_0. He has to eject the oxygen to go back to the spacecraft, but he also needs oxygen to breathe. His respiratory rate is R. If $M = 100kg$, $m_0 = 0.5kg$, $d = 45m$, $v_0 = 50m/s$ and $R = 2.5 \times 10^{-4} kg/s$, how much oxygen can he eject in order to come back to the spacecraft safely?

Approach:
i) During the time period of τ, the change of velocity caused by gravity is $g\tau$. This term should be subtracted from the original equation.

ii) Use the data provided and the Tsiolkovsky rocket equation to calculate the change of speed for the single-stage rocket. Then compare it to the first cosmic velocity. If it is less than that, the rocket will not be able to enter the orbit.

iii) Use the data provided in the data table to calculate the change of speed for each stage of the rocket. The final speed should be the sum of the change of speed for each stage. Then, substitute the data for v_{ex} and see whether the final speed can exceed the first cosmic velocity. If so, the rocket can enter the orbit.

iv) First we need to calculate the impulse exerted by the meteorite particles on the spacecraft. This equals the change of the meteorite particles' momentum. We can use the condition of perfectly elastic collision to find this change of momentum. Then by **the Impulse-Momentum theorem**, the net impulse should be zero in order to maintain a constant velocity for the spacecraft. So the impulse exterted by the engine should have the same magnitude as that exerted by the meteorite flow. Use this relation to solve for V.

v) Use the **the Conservation of Momentum**, we can calculate the speed v that the astronaut can reach when he ejects the oxygen of mass m. Then we can use this to calculate the time for him to return to the spacecraft at this speed. In order to return to the spacecraft safely, the time for him to return should be less or equal to the remaining time for respiration. Use this relation, we can calculate the amount of oxygen that can be ejected safely.

Conceptual Components:
Rocket Propulsion
Impulse-Momentum theorem
Conservation of Momentum
Perfectly elastic collision

Target Quantities:
i) The Tsiolkovsky rocket equation modified by gravity.
ii) The change of the single-stage rocket's speed Δv.
ii) The change of the rocket's speed for each stage Δv_i. The final speed of the rocket v_f.
iv) The speed of the meteorite flow V.
v) The amount of the oxygen m that can be ejected.

Quantitive Relationships that May be Useful:
Tsiolkovsky rocket equation:

$$v - v_0 = v_{ex} \ln \frac{m_0}{m}$$

the Impulse-Momentum theorem:

$$\mathbf{J} = \int_{t_1}^{t_2} \Sigma \mathbf{F} dt = \mathbf{p}_2 - \mathbf{p}_1$$

Conservation of Momentum:

$$\Sigma \mathbf{p}_f = \Sigma \mathbf{p}_i$$

Change of velocity:

$$\Delta \mathbf{v} = \int_{t_1}^{t_2} \mathbf{a} dt$$

Solution:
i)
During the time period of τ, the change of speed caused by gravity is:

$$\Delta v_g = \int_0^{\tau} -g dt$$

$$\Delta v_g = -g\tau \tag{1}$$

Add this term to the Tsiolkovsky rocket equation, we get the equation modified by gravity:

$$v - v_0 = v_{ex} \ln \frac{m_0}{m} - g\tau \tag{2}$$

ii)
Let the mass of the shell be m_s and the mass of the fuel be m'. The initial mass $m_0 = m_s + m'$. According to the assumption of the question, $m' = 10m_s$. Here, we first consider the Tsiolkovsky rocket equation without the modification of gravity:

$$\Delta v = v - v_0 = v_{ex} \ln \frac{m_0}{m} = v_{ex} \ln \frac{m_s + m'}{m_s} = v_{ex} \ln \frac{10m_s + m_s}{m_s} = 2.4v_{ex} \tag{3}$$

According to the assumption of the question, the typical value for v_{ex} is about $2.0 \sim 3.0km/s$.
When $v_{ex} = 2km/s$,

$$\Delta v = 2.4v_{ex} = 4.8km/s \tag{4}$$

When $v_{ex} = 3km/s$,

$$\Delta v = 2.4v_{ex} = 7.2km/s \tag{5}$$

We can see that both of them are less than the first cosmic velocity $7.9km/s$. When the rocket equation is modified by gravity, these values will become even smaller (since the term $-g\tau$ is always negative). So, with a single-stage rocket, we cannot enter the orbit.

iii)
Since the effect of gravity has been assumed to be negligible, we use the original rocket equation for this part.

For the first stage, the initial mass $m_{01} = 777$ and the final mass $m_{s1} = 100 + 77 = 177$. According to the rocket equation, the speed change for this stage is:

$$\Delta v_1 = v_{ex} \ln \frac{m_{01}}{m_{s1}} = v_{ex} \ln \frac{777}{177} = 1.48v_{ex} \tag{6}$$

Then, we discard the shell of the first stage and fire the engine of the second stage.

For stage two, the initial mass $m_{02} = 77$ and the final mass $m_{s2} = 10 + 7 = 17$. According to the rocket equation, Δv_2 is:

$$\Delta v_2 = v_{ex} \ln \frac{m_{02}}{m_{s2}} = v_{ex} \ln \frac{77}{17} = 1.51v_{ex} \tag{7}$$

After that, we simply discard the shell of the second stage and fire the engine of the third stage.

For stage three, the initial mass is $m_{03} = 7$ and the final mass is $m_{s3} = 1$. Substitute these into the rocket equation and we can get:

$$\Delta v_3 = v_{ex} \ln \frac{m_{03}}{m_{s3}} = v_{ex} \ln \frac{7}{1} = 1.95v_{ex} \tag{8}$$

The final speed of the rocket equals the sum of the change of speed for each stage:

$$v_f = 1.48v_{ex} + 1.51v_{ex} + 1.95v_{ex} = 4.94v_{ex} \tag{9}$$

Even when we take the least value for v_{ex}: $v_{ex} = 2.0km/s$

$$v_f = 4.94 \times 2.0km/s = 9.9km/s$$

The final speed of the rocket exceeds the first cosmic velocity, which means it can enter the orbit.

iv)

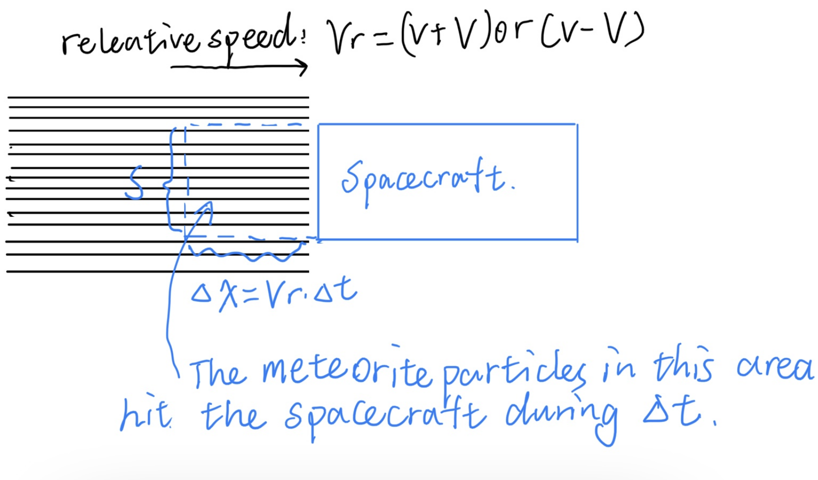

Figure 1: Diagram for this part

The spacecraft is chosen as the reference. Suppose the density of the meteorite flow is ρ. As shown on the diagram, during the time interval Δt, the meteorite particles in the labelled area hit the space craft. The total mass of these particles is:

$$m = \rho S v_r \Delta t \tag{10}$$

Where v_r is the speed of the meteorite particles relative to the spacecraft. When the spacecraft moves opposite to the meteorite flow, the relative speed $v_{r1} = v + V$. When the spacecraft moves along the meteorite flow, the relative speed $v_{r2} = v - V$. Substitute these into equation (10), we can get the mass of meteorite particles that hit the spacecraft in these two cases:

$$m_1 = \rho S(v + V)\Delta t \tag{11}$$

$$m_2 = \rho S(v - V)\Delta t \tag{12}$$

As we have chosen the spacecraft as the reference, and the collision is perfectly elastic, the speed at which the particles rebound equals their initial speed relative to the spacecraft. Therefore, the impulse they exert on the spacecraft (which is the momentum they transferred to the spacecraft) equals:

$$I = p_t = m\Delta v_m = m(2v_r) = 2mv_r \tag{13}$$

Substitute equation (11), (12), $v_{r1} = v + V$ and $v_{r2} = v - V$ into the above equation, we can get:

$$I_1 = p_{t1} = 2m_1 v_{r1} = 2\rho S(v + V)^2 \Delta t \tag{14}$$

$$I_2 = p_{t2} = 2m_2 v_{r2} = 2\rho S(v - V)^2 \Delta t \tag{15}$$

By the Impulse-Momentum theorem, since the spacecraft maintains a constant velocity, the net impulse that is transferred to the spacecraft equals zero. That is, the sum of the impulse exerted by the engine and that exerted by the meteorite particles is zero:

$$F_1\Delta t - I_1 = F_1\Delta t - 2\rho S(v + V)^2\Delta t = 0$$

$$F_1\Delta t = 2\rho S(v + V)^2\Delta t \tag{16}$$

$$F_2\Delta t - I_2 = F_2\Delta t - 2\rho S(v - V)^2\Delta t = 0$$

$$F_2\Delta t = 2\rho S(v - V)^2\Delta t \tag{17}$$

By the assumption of the question, we have the relation between the tractive forces:

$$\frac{F_1}{F_2} = \frac{4}{1}$$

Substitute equation (16) and (17) into this equation:

$$\frac{2\rho S(v+V)^2\Delta t}{2\rho S(v-V)^2\Delta t} = \frac{4}{1}$$

$$\frac{(v+V)^2}{(v-V)^2} = \frac{4}{1} \tag{18}$$

Solve this equation and we can get:

$$V = \frac{1}{3}v \text{ or } V = 3v$$

Since m_2 is positive, $(v-V) > 0$. Therefore, $V = \frac{1}{3}v$ is the only reasonable solution.

v)

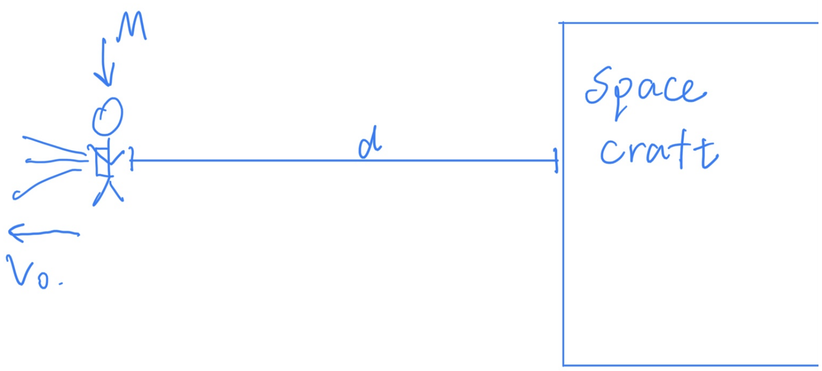

Figure 2: Diagram for this part

We choose the spacecraft as the reference. We view the astronaut and the equipment he takes as a system. Since there is no external force on this system, its momentum is conserved before and after the ejection of the oxygen. By the Conservation of Momentum, we have:

$$(M-m)v + m(v - v_0) = p_0 = 0 \tag{19}$$

where v is the astronaut's speed after ejecting the oxygen and m is the mass of the oxygen ejected. Rearrange this equation and we can get:

$$v = \frac{m}{M}v_0 \tag{20}$$

Therefore, the time needed for the astronaut to return to the space craft is:

$$t_1 = \frac{d}{v} = \frac{Md}{mv_0} \tag{21}$$

After the oxygen of mass m is ejected, the mass of the remaining oxygen is $m_0 - m$. The remaining time that the astronaut can breathe is

$$t_2 = \frac{m_0 - m}{R} \tag{22}$$

In order to return to the spacecraft safely, we must have $t_2 \geq t_1$. That is:

$$\frac{m_0 - m}{R} \geq \frac{Md}{mv_0}$$

$$m^2 - m_0 m + \frac{MRd}{v_0} \leq 0 \tag{23}$$

Solve this inequality, we get:

$$\frac{m_0}{2} - \frac{1}{2v_0}\sqrt{(m_0 v_0)^2 - 4v_0 MRd} \leq m \leq \frac{m_0}{2} + \frac{1}{2v_0}\sqrt{(m_0 v_0)^2 - 4v_0 MRd} \tag{24}$$

Substitute the data: $M = 100kg$, $m_0 = 0.5kg$, $d = 45m$, $v_0 = 50m/s$ and $R = 2.5 \times 10^{-4} kg/s$ into the above equation:

$$\frac{0.5kg}{2} - \frac{1}{2 \times 50m/s}\sqrt{(0.5kg \times 50m/s)^2 - 4 \times 50m/s \times 100kg \times 2.5 \times 10^{-4}kg/s \times 45m} \leq m$$

$$m \leq \frac{0.5kg}{2} + \frac{1}{2 \times 50m/s}\sqrt{(0.5kg \times 50m/s)^2 - 4 \times 50m/s \times 100kg \times 2.5 \times 10^{-4}kg/s \times 45m}$$

$$0.05kg \leq m \leq 0.45kg \tag{25}$$

Evaluation:

a) Dimension check:

i) $\frac{[m]}{[s]} = \frac{[m]}{[s]} \cdot 1 + \frac{[m]}{[s]^2}[s]$

ii) $\frac{[m]}{[s]} = \frac{[m]}{[s]} \cdot 1$

iii) The same as ii)

iv) $[N][s] = \frac{[kg]}{[m]^3}[m]^2(\frac{[m]}{[s]})^2[s]$

$\frac{[m]}{[s]} = \frac{[m]}{[s]}$

v) $[kg] + \frac{1}{[m]/[s]}\sqrt{([kg]\frac{[m]}{[s]})^2 + \frac{[m]}{[s]}[kg]\frac{[kg]}{[s]}[m]} = [kg]$

All answers have proper dimensions.

b) Orders of magnitude

i) No numerical value.

ii) The answer is reasonable regarding its order of magnitude.

iii) The answer is reasonable regarding its order of magnitude.

iv) No numerical value.

v) The answer is reasonable regarding its order of magnitude.

c) All questions have been answered.