# Module 2 Group Project
# Some Interesting Facts about the Earth

Section 2 Group 60
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*Introduction:*
Our project has three parts to it, we have centered our questions around the gravitation field of earth and how it is affected by different celestial bodies and different positions. Part A is about how the gravitation field changes depending on your position of the earth. We decided this was a reasonable question because gravity is a constant used in many parts of physics, so understanding how it changes it critical to one's physics knowledge. Part B is about using the earth as a frame of reference. Even though it is always moving, this question takes advantage of something called Coriolis Force in order to prove the question; this question is relevant because the motion of the earth affects many physical properties of the real world that have to be taken in to account when making more complex models. Part C is about how other celestial bodies affect the tides of earth and its magnitude and why the tides seem to follow a cyclical pattern every day. Our group found these questions the most exciting and relevant to the material we have learned in Module 2.

Figure 1: The Earth, our lovely home. From https://www.space.com/54-earth-history-composition-and-atmosphere.html

**Part A**

*Question:* It has been observed that the free fall acceleration, $g$, has different values at different locations on the Earth. Here is some data: (from $https://en.wikipedia.org/wiki/Gravitational\_acceleration$)

$g_{poles} = 9.832m/s^2$

$g_{latitude=45^\circ} = 9.806m/s^2$

$g_{equator} = 9.780m/s^2$

Discuss the reason of this and find $g$ as a function of latitude $\varphi$. Assume that the Earth is a perfect sphere with a constant radius. **Hint:** Consider the Earth's spinning.

Some constants that may be used:

$m_e$: the mass of the Earth.

$G$: gravitational constant.

$\omega_e$: the angular speed of the Earth's spinning.

$r_e$: the radius of the Earth.

*Approach:*

1. An object on the Earth rotates with respect to the Earth's axis as the Earth spins. As we all know, a rotating object requires a centripetal force to maintain its rotation. This force is provided by the Earth's gravitational attraction acting on the object. The overall force the object experiences is the weight $w = mg$. According to the diagram, it is the vector difference of the gravitational force and the centripetal force (see the diagram).

2. We can use this vector difference relation to calculate the magnitude of $w$ by using the cosine law. Then we just divide $w$ by the mass of the object $m$ to get the result we desire.

*Target Quantity:*

Free fall acceleration $g$ as a function of latitude $\varphi$.

*Quantitive Relationships that May be Useful:*

The weight of an object near the Earth's surface: $w = mg$

Gravitational force: $F_g = \frac{Gm_1m_2}{r^2}$

Centripetal force: $F_c = m\omega^2 r$

*Diagram:*

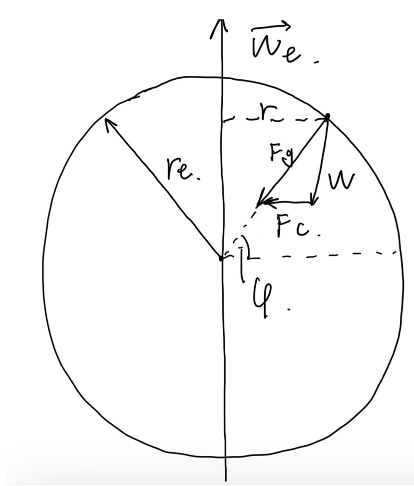


Figure 2: Diagram for this problem

*Solution:*

As the diagram shows, the vector difference of the gravitational force $F_g$ and the centripetal force $F_c$ is the weight $w$ that an object experiences. When the latitude is varying, the distance from a point on the Earth's surface to the Earth's axis would be different. This causes the centripetal force to be different, which would ultimately result in the varying of the weight $w$ at different latitudes.

The distance from an object on the Earth's surface to the Earth's axis is:

$$r = r_e \cos\varphi \tag{1}$$

The centripetal force required for the object to rotate with the Earth is:

$$F_c = m\omega_e^2 r \tag{2}$$

Substitute (1) into (2), we would get:

$$F_c = m\omega_e^2 r_e \cos\varphi \tag{3}$$

The gravitational force acting on the object is:

$$F_g = \frac{Gm_e m}{r_e^2} \tag{4}$$

Where $m$ is the mass of the object.
According to the diagram, we have the following geometrical relation (by using the cosine law):

$$w^2 = F_g^2 + F_c^2 - 2F_g F_c \cos\varphi$$

$$w = \sqrt{F_g^2 + F_c^2 - 2F_g F_c \cos\varphi} \tag{5}$$

Substitute (3) and (4) into (5), we would get:

$$w = \sqrt{(\frac{Gm_e m}{r_e^2})^2 + (m\omega_e^2 r_e \cos\varphi)^2 - 2\frac{Gm_e m}{r_e^2} m\omega_e^2 r_e \cos^2\varphi}$$

$$w = m\sqrt{(\frac{Gm_e}{r_e^2} - w_e^2 r_e \cos\varphi)^2 + \frac{2Gm_e w_e^2}{r_e}(\cos\varphi - \cos^2\varphi)}$$

$$g = \frac{w}{m} = \sqrt{(\frac{Gm_e}{r_e^2} - w_e^2 r_e \cos\varphi)^2 + \frac{2Gm_e w_e^2}{r_e}(\cos\varphi - \cos^2\varphi)} \tag{6}$$

*Evaluation:*

$$[m/s^2] = \sqrt{(\frac{[m^3kg^{-1}s^{-2}][kg]}{[m^2]} - [s^{-2}][m])^2 + \frac{[m^3kg^{-1}s^{-2}][kg][s^{-2}]}{[m]}} = \sqrt{[m]^2/[s]^4}$$

Therefore, the answer has proper dimensions.
No numerical value is given, so we cannot determine whether the answer is reasonable regarding its order of magnitude.
All original questions have been answered.

**Part B**

*Question:*

When tackling most physics problems, we can view the Earth's surface as an inertial frame of reference, in which Newton's Laws of Motion are valid. Strictly speaking, however, the Earth's surface is **not** an inertial frame of reference because of the Earth's spinning! Luckily, we can introduce something called the inertia force (or persudo force) to allow Newton's Laws to still work. A famous example of that is the Coriolis Force: $f_c = 2m\mathbf{v} \times \omega_{\mathbf{e}}$, where $m$ is the mass of an object on the Earth's surface, $\mathbf{v}$ is the velocity of the object and $\omega_{\mathbf{e}}$ is the angular velocity of the Earth's spinning. The cross here is vector product. What's the direction of the Coriolis Force acting on an object if the object is in the northern hemisphere? Use this result to explain why all typhoons in the northern hemisphere rotate counterclockwise. You do not need any calculation for the above two questions. Draw diagrams and explain.

*Approach:*

1. In order to determine the direction of the Coriolis Force on an object, we first need to determine the direction of this object's velocity and the direction of the Earth's rotational angular velocity, then use the right hand rule to determine the direction of their cross product. After that, we project that vector onto the Earth's surface, since the vertical component of the force would be cancelled by gravity and does not cause any effect. The projected vector would be along the direction of the Coriolis Force on this object.

2. In order to explain why all typhoons in the northern hemisphere rotate counterclockwise, we need to determine how flows of air are deflected by the Coriolis Force when they move towards the center of a typhoon.

*Target Quantity*:

The direction of Coriolis Force $f_c$ on an object that is in the northern hemisphere.

*Quantitive Relationships that May be Useful:*

The formula for Coriolis Force (given in the problem): $f_c = 2m\mathbf{v} \times \omega_{\mathbf{e}}$.

*Diagrams and Solutions:*

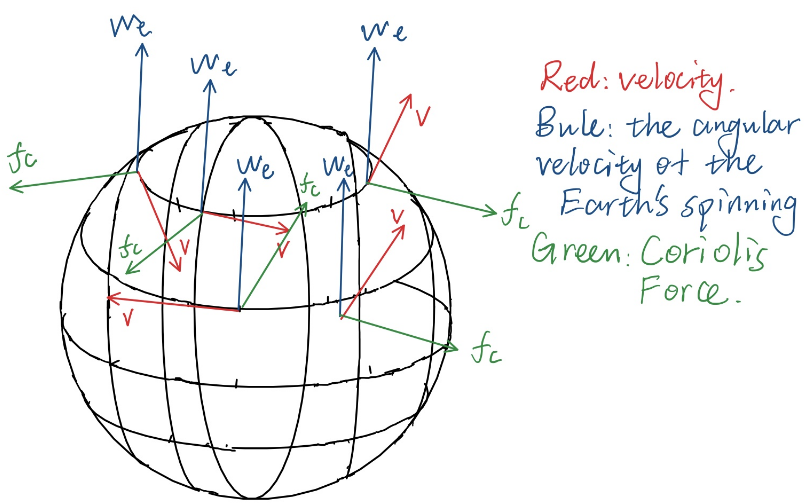


Figure 3: The direction of all vectors.

This diagram clearly illustrates the relative directions of some $\mathbf{v}$, $\omega_{\mathbf{e}}$ and $f_c$ (determined by the right hand rule) when the object is in the northern hemisphere. From the diagram, it is clear that when $f_c$ is projected onto the Earth's surface, it is always to the right of $\mathbf{v}$. We can draw a conclusion that when an object is in the northern hemisphere, the Coriolis Force on it is always to the right of its velocity.

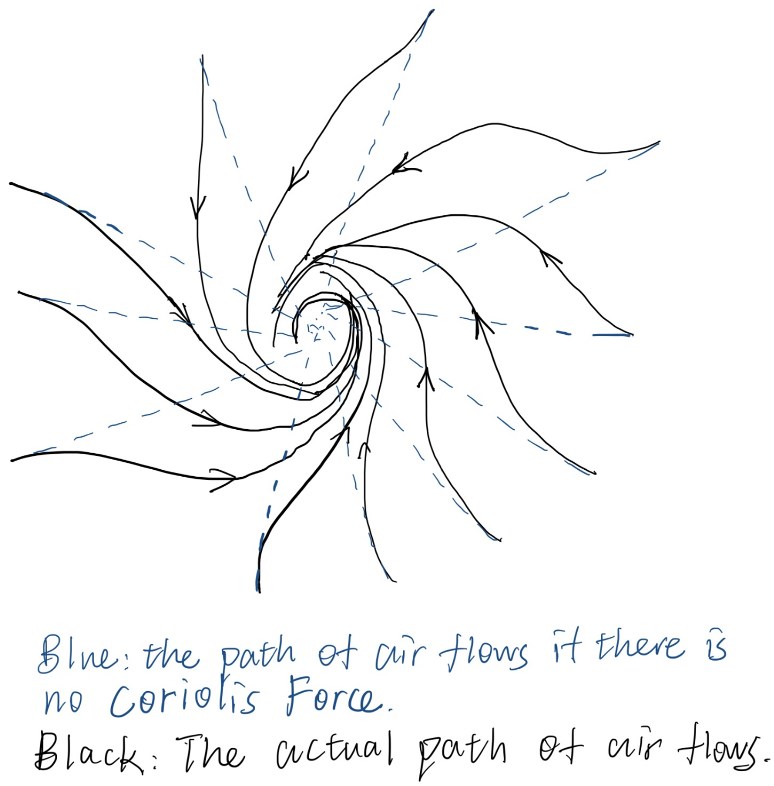


Figure 4: The air flows of a typhoon.

This diagram illustrates the air flows of a typhoon. As we can see in the diagram, the air flows are deflected to the right by the Coriolis Force when they move towards the center. This is the reason why all typhoons in the northern hemisphere rotate counterclockwise.

*Evaluation:*
We cannot evaluate the dimension and reasonableness of the answers since there is no mathematical derivation in this problem.
All questions have been answered.

**Part C**

*Question:*

Tides are the rise and fall of sea levels caused by the gravitational force exerted by the Moon and the Sun. Usually, the same location experiences two high tides every day. Explain this by considering the gravitational force exerted by the Moon or the Sun and the Earth's spinning. It is a surprising fact that although the gravitational force exerted by the Sun on the Earth is 200 times larger than that by the Moon, the high tides caused by the Sun have only about half the magnitude compared to those caused by the Moon. Verify this with calculations. Use the following data:

the mass of the Sun: $M_S = 2.0 \times 10^{30} kg$

the mass of the Moon: $M_M = 7.34 \times 10^{22} kg$

The distance from the Sun to the Earth: $r_S = 1.5 \times 10^{11} m$

The distance from the Moon to the Earth: $r_M = 3.82 \times 10^{8} m$

The radius of the Earth: $r_e = 6.371 \times 10^{6} m$

The following relation may be useful:

When $x \ll 1$:

$$\frac{1}{(1-x)^2} \approx 1 + 2x$$

*Approach:*

1. As the Earth rotates around the Sun (or the Moon), the centripetal force for this rotation would be equal to the gravitational force exerted by the Sun (or the Moon) on the Earth's center of mass. Since everything on the Earth always rotates with it, they experience the same centripetal acceleration. The centripetal force needed for an object to rotate with the Earth around the Sun (or the Moon) is provided by the Sun (Moon)'s gravitational force on it. According to the diagram, we can see the overall gravitational force exerted by the Sun (Moon) that an object experiences equals the vector difference of the gravitational force (by the Sun/Moon) and the centripetal force.

2. From 1 we can derive the overall gravitational force exerted by the Sun (Moon) that an object experiences. The magnitude of this force depends on the position of the object relative to the center of the Earth. But since the Earth is spinning, the same place on the surface of the Earth has different positions relative to the center of the Earth at different times of the day. After some derivations, we would see that when this place is at a position that is nearest or furthest from the Sun (Moon), the overall gravitational force it experiences would point radially outward. This causes a high tide. It is clear that the same place on the surface of the Earth would go through two positions that have a radially outward force every day. Therefore, this place experiences two high tides every day.

3. Use the formula for gravitational force to calculate the gravitational force exerted on the object by the Sun and by the Moon respectively. Then compare the magnitudes of these two forces. Since the magnitude of a high tide is propotional to the magnitude of the gravitational force exerted by the Sun/Moon, the ratio of these two forces would equal the ratio of the magnitudes of high tides.

*Target Quantity:*

The overall gravitational force exterted by the Sun/Moon that an object on the surface of the Earth experiences: $f_{ex}$; The ratio of the $f_{ex}$ caused by the Moon to the $f_{ex}$ caused by the Sun.

*Quantitive Relationships that May be Useful:*

Gravitational force: $\mathbf{F}_g = \frac{Gm_1m_2}{r^2}\hat{\mathbf{e}}_{12}$

Centripetal force: $\mathbf{F}_c = m\mathbf{a}_c$

*Diagram:*

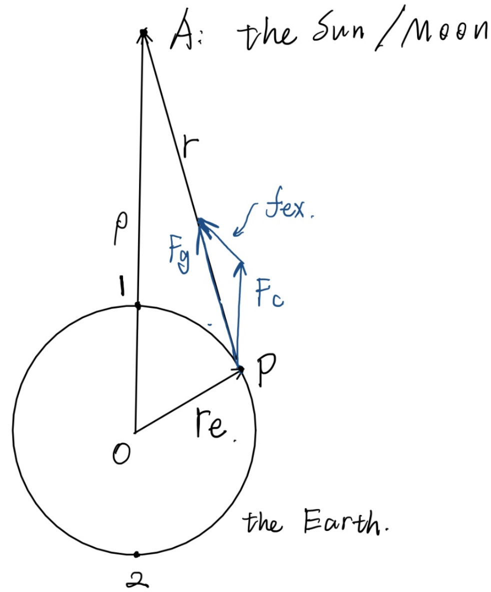


Figure 5: Diagram for this problem

*Solution:*
$A$ represents the Sun or the Moon. According to the diagram, the gravitational force on the Earth exerted by $A$ is:

$$\mathbf{F}_{ge} = \frac{GMm_e}{\rho^2}\hat{\mathbf{e}}_{OA} \tag{7}$$

Where $M$ is the mass of $A$, $m_e$ is the mass of the Earth, $\rho$ is the distance labelled on the diagram and $\hat{\mathbf{e}}_{OA}$ is the unit vector pointing from $O$ to $A$. This gravitational force provides the centripetal force for the Earth's rotation around $A$. Therefore, the centripetal acceleration $\mathbf{a}_c$ is:

$$\mathbf{a}_c = \frac{\mathbf{F}_c}{m_e} = \frac{\mathbf{F}_{ge}}{m_e} = \frac{GM}{\rho^2}\hat{\mathbf{e}}_{OA} \tag{8}$$

Consider an object at $P$ with mass $m$. Since it rotates with the Earth around $A$, the centripetal force needed to maintain its rotation is:

$$\mathbf{F}_c = m\mathbf{a}_c = \frac{GMm}{\rho^2}\hat{\mathbf{e}}_{OA} \tag{9}$$

The gravitational force exerted by $A$ on the object is:

$$\mathbf{F}_g = \frac{GMm}{r^2}\hat{\mathbf{e}}_{PA} \tag{10}$$

Where $r$ is the distance labelled on the diagram and $\hat{\mathbf{e}}_{PA}$ is the unit vector pointing from $P$ to $A$. According to the diagram, we have the following relation between the forces:

$$\mathbf{f}_{ex} = \mathbf{F}_g - \mathbf{F}_c = \frac{GMm}{r^2}\hat{\mathbf{e}}_{PA} - \frac{GMm}{\rho^2}\hat{\mathbf{e}}_{OA} \tag{11}$$

It is clear that at position 1 or position 2 labelled on the diagram, $\mathbf{f}_{ex}$ points radially outward. High tides occur at these two positions. Since the Earth rotates around its axis once every day, $P$ would pass position 1 and position 2 respectively once every 24 hours. This is the reason why high tides occur twice every day.

Then consider the $\mathbf{f}_{ex}$ at position 1. We call it $\mathbf{f}_1$. It is clear that at position 1, we have:

$$\hat{\mathbf{e}}_{PA} = \hat{\mathbf{e}}_{OA}$$
$$r = \rho - r_e$$

By substituting the above equations into equation (11), we would derive:

$$\mathbf{f}_1 = \frac{GMm}{r^2}\hat{\mathbf{e}}_{PA} - \frac{GMm}{\rho^2}\hat{\mathbf{e}}_{OA} = (\frac{GMm}{(\rho - r_e)^2} - \frac{GMm}{\rho^2})\hat{\mathbf{e}}_{OA}$$
$$\mathbf{f}_1 = \frac{GMm}{\rho^2}(\frac{1}{(1 - \frac{r_e}{\rho})^2} - 1)\hat{\mathbf{e}}_{OA}$$

$\rho$ is the distance from the Earth to the Sun or to the Moon. $r_e$ is very small compared to these distances (see their orders of magnitude listed in the question). Therefore, $\frac{r_e}{\rho} \ll 1$, we can use the relation: when $x \ll 1$, $\frac{1}{(1-x)^2} \approx 1 + 2x$ on this equation:

$$\mathbf{f}_1 \approx \frac{GMm}{\rho^2}(1 + \frac{2r_e}{\rho} - 1)\hat{\mathbf{e}}_{OA} = \frac{2GMmr_e}{\rho^3}\hat{\mathbf{e}}_{OA} \tag{12}$$

Let $\mathbf{f}_{1S}$ be the $\mathbf{f}_1$ exerted by the Sun and $\mathbf{f}_{1M}$ be the $\mathbf{f}_1$ exerted by the Moon. By substituting the data for the Sun and the Moon, we would have:

$$\mathbf{f}_{1S} \approx \frac{2GM_S m r_e}{r_S^3}\hat{\mathbf{e}}_{OA} \tag{13}$$

$$\mathbf{f}_{1M} \approx \frac{2GM_M m r_e}{r_M^3}\hat{\mathbf{e}}_{OA} \tag{14}$$

$$\frac{f_{1M}}{f_{1S}} = \frac{\frac{M_M}{r_M^3}}{\frac{M_S}{r_S^3}} = \frac{\frac{7.34\times10^{22}kg}{(3.82\times10^8 m)^3}}{\frac{2.0\times10^{30}}{(1.5\times10^{11}m)^3}} = 2.2 \tag{15}$$

Since the magnitude of a high tide is propotional to the magnitude of the gravitational force exerted by the Sun/Moon, this ratio would equal the ratio of the magnitudes of high tides. This proves that the high tides caused by the Sun have only about half the magnitude compared to those caused by the Moon.

*Evaluation:*

$$[N] = \frac{[m^3kg^{-1}s^{-2}][kg]^2}{[m]^2}$$
$$[N] = \frac{[m^3kg^{-1}s^{-2}][kg]^2[m]}{[m]^3}$$

This verifies that the answers have proper dimensions.
The answer is reasonable regarding its order of magnitude.
All questions have been answered.