

XCPC Algorithm Template (II)

Jianly's Online version

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日录

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1 杂类

1.1 int128 输出流自定义

```
using i128 = __int128;
3
    std::ostream &operator<<(std::ostream &os, i128 n) {</pre>
4
        std::string s;
5
        while (n) {
 6
            s += '0' + n % 10;
 7
            n /= 10;
8
9
        std::reverse(s.begin(), s.end());
10
        return os << s;
11 }
```

1.2 常用库函数重载

```
using i64 = long long;
    using i128 = __int128;
 4
    i64 ceilDiv(i64 n, i64 m) {
 5
        if (n >= 0) {
 6
            return (n + m - 1) / m;
 7
        } else {
 8
            return n / m;
 9
10
11
12
    i64 floorDiv(i64 n, i64 m) {
13
        if (n >= 0) {
14
            return n / m;
15
        } else {
16
            return (n - m + 1) / m;
17
        }
18
    }
19
20 template<class T>
21
    void chmax(T &a, T b) {
22
        if (a < b) {
23
            a = b;
24
        }
25
26
27
    i128 gcd(i128 a, i128 b) {
28
        return b ? gcd(b, a % b) : a;
29
```

/END/

图与网络 2

强连通分量缩点 (SCC) 2.1

```
struct SCC {
 1
 2
         int n;
 3
         std::vector<std::vector<int>> adj;
 4
         std::vector<int> stk;
 5
         std::vector<int> dfn, low, bel;
 6
        int cur, cnt;
 7
 8
        SCC() {}
 9
         SCC(int n) {
10
             init(n);
11
12
13
         void init(int n) {
14
             this->n = n;
15
             adj.assign(n, {});
16
             dfn.assign(n, -1);
17
             low.resize(n);
18
             bel.assign(n, -1);
19
             stk.clear();
20
             cur = cnt = 0;
21
         }
22
23
         void addEdge(int u, int v) {
24
             adj[u].push_back(v);
25
26
27
         void dfs(int x) {
28
             dfn[x] = low[x] = cur++;
29
             stk.push_back(x);
30
31
             for (auto y : adj[x]) {
32
                 if (dfn[y] == -1) {
33
                     dfs(y);
34
                     low[x] = std::min(low[x], low[y]);
35
                 } else if (bel[y] == -1) {
36
                     low[x] = std::min(low[x], dfn[y]);
37
                 }
38
             }
39
40
             if (dfn[x] == low[x]) {
41
                 int y;
42
                 do {
43
                     y = stk.back();
44
                     bel[y] = cnt;
45
                     stk.pop_back();
46
                 } while (y != x);
47
                 cnt++;
48
             }
49
50
51
         std::vector<int> work() {
52
             for (int i = 0; i < n; i++) {
53
                 if (dfn[i] == -1) {
54
                     dfs(i);
55
                 }
56
             }
57
             return bel;
58
```

割边与割边缩点 (EBCC) 2.2

```
1
    std::set<std::pair<int, int>> E;
 2
 3
    struct EBCC {
 4
        int n;
 5
         std::vector<std::vector<int>> adj;
 6
         std::vector<int> stk;
 7
         std::vector<int> dfn, low, bel;
 8
        int cur, cnt;
 9
10
         EBCC() {}
11
         EBCC(int n) {
12
             init(n);
13
         }
14
15
         void init(int n) {
16
             this->n = n;
17
             adj.assign(n, {});
18
             dfn.assign(n, -1);
19
             low.resize(n);
20
             bel.assign(n, -1);
21
             stk.clear();
22
             cur = cnt = 0;
23
         }
24
25
         void addEdge(int u, int v) {
26
             adj[u].push_back(v);
27
             adj[v].push_back(u);
28
         }
29
30
         void dfs(int x, int p) {
31
             dfn[x] = low[x] = cur++;
32
             stk.push_back(x);
33
34
             for (auto y : adj[x]) {
35
                 if (y == p) {
36
                     continue;
37
                 }
38
                 if (dfn[y] == -1) {
39
                     E.emplace(x, y);
40
                     dfs(y, x);
41
                     low[x] = std::min(low[x], low[y]);
42
                 } else if (bel[y] == -1 && dfn[y] < dfn[x]) {
43
                     E.emplace(x, y);
44
                     low[x] = std::min(low[x], dfn[y]);
45
                 }
46
             }
47
48
             if (dfn[x] == low[x]) {
49
                 int y;
50
                 do {
51
                     y = stk.back();
52
                     bel[y] = cnt;
53
                     stk.pop_back();
54
                 } while (y != x);
55
                 cnt++;
56
             }
57
         }
58
```

```
59
         std::vector<int> work() {
60
             dfs(0, -1);
61
             return bel;
62
         }
63
64
         struct Graph {
65
             int n;
66
             std::vector<std::pair<int, int>> edges;
67
             std::vector<int> siz;
68
             std::vector<int> cnte;
69
         };
70
         Graph compress() {
71
             Graph g;
72
             g.n = cnt;
73
             g.siz.resize(cnt);
74
             g.cnte.resize(cnt);
75
             for (int i = 0; i < n; i++) {
76
                  g.siz[bel[i]]++;
77
                  for (auto j : adj[i]) {
78
                      if (bel[i] < bel[j]) {</pre>
79
                          g.edges.emplace_back(bel[i], bel[j]);
80
                      } else if (i < j) {</pre>
81
                          g.cnte[bel[i]]++;
82
                      }
83
                  }
84
             }
85
             return g;
86
         }
87
    };
```

二分图最大权匹配 (MaxAssignment 基于KM) 【久远】 2.3

```
1
    template<class T>
 2
    struct MaxAssignment {
 3
         public:
 4
             T solve(int nx, int ny, std::vector<std::vector<T>>> a) {
 5
                 assert(0 <= nx && nx <= ny);</pre>
 6
                 assert(int(a.size()) == nx);
 7
                 for (int i = 0; i < nx; ++i) {
 8
                     assert(int(a[i].size()) == ny);
 9
                     for (auto x : a[i])
10
                          assert(x >= 0);
11
                 }
12
13
                 auto update = [&](int x) {
14
                     for (int y = 0; y < ny; ++y) {
15
                          if (lx[x] + ly[y] - a[x][y] < slack[y]) {
16
                              slack[y] = lx[x] + ly[y] - a[x][y];
17
                              slackx[y] = x;
18
                          }
19
                     }
20
                 };
21
22
                 costs.resize(nx + 1);
23
                 costs[0] = 0;
24
                 lx.assign(nx, std::numeric_limits<T>::max());
25
                 ly.assign(ny, 0);
26
                 xy.assign(nx, -1);
27
                 yx.assign(ny, -1);
28
                 slackx.resize(ny);
29
                 for (int cur = 0; cur < nx; ++cur) {
30
                     std::queue<int> que;
```

```
31
                     visx.assign(nx, false);
32
                     visy.assign(ny, false);
33
                     slack.assign(ny, std::numeric_limits<T>::max());
34
                     p.assign(nx, -1);
35
36
                     for (int x = 0; x < nx; ++x) {
37
                         if (xy[x] == -1) {
38
                              que.push(x);
39
                              visx[x] = true;
40
                             update(x);
41
                         }
42
                     }
43
44
                     int ex, ey;
45
                     bool found = false;
46
                     while (!found) {
47
                         while (!que.empty() && !found) {
48
                              auto x = que.front();
49
                              que.pop();
50
                             for (int y = 0; y < ny; ++y) {
51
                                  if (a[x][y] == lx[x] + ly[y] && !visy[y]) {
52
                                      if (yx[y] == -1) {
53
                                          ex = x;
54
                                          ey = y;
55
                                          found = true;
56
                                          break;
57
58
                                      que.push(yx[y]);
59
                                      p[yx[y]] = x;
60
                                      visy[y] = visx[yx[y]] = true;
61
                                      update(yx[y]);
62
                                  }
63
                              }
64
                         }
65
                         if (found)
66
                             break;
67
68
                         T delta = std::numeric_limits<T>::max();
69
                         for (int y = 0; y < ny; ++y)
70
                             if (!visy[y])
71
                                  delta = std::min(delta, slack[y]);
72
                         for (int x = 0; x < nx; ++x)
73
                              if (visx[x])
74
                                  lx[x] -= delta;
75
                         for (int y = 0; y < ny; ++y) {
76
                              if (visy[y]) {
77
                                  ly[y] += delta;
78
                              } else {
79
                                  slack[y] -= delta;
80
                              }
81
82
                         for (int y = 0; y < ny; ++y) {
83
                              if (!visy[y] \&\& slack[y] == 0) {
84
                                  if (yx[y] == -1) {
85
                                      ex = slackx[y];
86
                                      ey = y;
87
                                      found = true;
88
                                      break;
89
                                  }
90
                                  que.push(yx[y]);
91
                                  p[yx[y]] = slackx[y];
92
                                  visy[y] = visx[yx[y]] = true;
93
                                  update(yx[y]);
94
```

```
95
 96
                      }
 97
 98
                      costs[cur + 1] = costs[cur];
99
                      for (int x = ex, y = ey, ty; x != -1; x = p[x], y = ty) {
100
                          costs[cur + 1] += a[x][y];
101
                          if (xy[x] != -1)
102
                              costs[cur + 1] -= a[x][xy[x]];
103
                          ty = xy[x];
104
                          xy[x] = y;
105
                          yx[y] = x;
106
                      }
107
                  }
108
                  return costs[nx];
109
              }
110
              std::vector<int> assignment() {
111
                  return xy;
112
113
              std::pair<std::vector<T>, std::vector<T>> labels() {
114
                  return std::make_pair(lx, ly);
115
116
              std::vector<T> weights() {
117
                  return costs;
118
              }
119
          private:
120
              std::vector<T> lx, ly, slack, costs;
121
              std::vector<int> xy, yx, p, slackx;
122
              std::vector<bool> visx, visy;
123
     };
```

2.4 一般图最大匹配 (Graph 带花树算法) 【久远】

```
1
    struct Graph {
 2
        int n;
 3
         std::vector<std::vector<int>> e;
 4
        Graph(int n) : n(n), e(n) {}
 5
        void addEdge(int u, int v) {
 6
             e[u].push_back(v);
 7
             e[v].push_back(u);
 8
 9
         std::vector<int> findMatching() {
10
             std::vector<int> match(n, -1), vis(n), link(n), f(n), dep(n);
11
12
             // disjoint set union
13
             auto find = [&](int u) {
14
                 while (f[u] != u)
15
                     u = f[u] = f[f[u]];
16
                 return u;
17
             };
18
19
             auto lca = [&](int u, int v) {
20
                 u = find(u);
21
                 v = find(v);
22
                 while (u != v) {
23
                     if (dep[u] < dep[v])</pre>
                         std::swap(u, v);
24
25
                     u = find(link[match[u]]);
26
                 }
27
                 return u;
28
             };
29
30
             std::queue<int> que;
```

```
31
             auto blossom = [\&](int u, int v, int p) {
32
                 while (find(u) != p) {
33
                     link[u] = v;
34
                     v = match[u];
                     if (vis[v] == 0) {
35
36
                         vis[v] = 1;
37
                         que.push(v);
38
39
                     f[u] = f[v] = p;
40
                     u = link[v];
41
                 }
42
            };
43
44
             // find an augmenting path starting from u and augment (if exist)
45
             auto augment = [&](int u) {
46
47
                 while (!que.empty())
48
                     que.pop();
49
50
                 std::iota(f.begin(), f.end(), 0);
51
52
                 // vis = 0 corresponds to inner vertices, vis = 1 corresponds to outer
    vertices
53
                 std::fill(vis.begin(), vis.end(), -1);
54
55
                 que.push(u);
56
                 vis[u] = 1;
57
                 dep[u] = 0;
58
59
                 while (!que.empty()){
60
                     int u = que.front();
61
                     que.pop();
62
                     for (auto v : e[u]) {
63
                         if (vis[v] == -1) {
64
65
                             vis[v] = 0;
66
                             link[v] = u;
67
                             dep[v] = dep[u] + 1;
68
69
                             // found an augmenting path
70
                             if (match[v] == -1) {
71
                                 for (int x = v, y = u, temp; y != -1; x = temp, y = x
    == -1 ? -1 : link[x]) {
72
                                      temp = match[y];
73
                                      match[x] = y;
74
                                      match[y] = x;
75
                                 }
76
                                 return;
77
                             }
78
79
                             vis[match[v]] = 1;
80
                             dep[match[v]] = dep[u] + 2;
81
                             que.push(match[v]);
82
83
                         } else if (vis[v] == 1 && find(v) != find(u)) {
84
                             // found a blossom
85
                             int p = lca(u, v);
86
                             blossom(u, v, p);
87
                             blossom(v, u, p);
88
                         }
89
                     }
90
                 }
91
92
             };
```

```
93
 94
              // find a maximal matching greedily (decrease constant)
 95
              auto greedy = [\&]() {
 96
 97
                  for (int u = 0; u < n; ++u) {
 98
                      if (match[u] != -1)
 99
                          continue;
100
                      for (auto v : e[u]) {
101
                          if (match[v] == -1) {
102
                               match[u] = v;
103
                               match[v] = u;
104
                              break;
105
                          }
106
                      }
107
                  }
108
              };
109
110
              greedy();
111
112
              for (int u = 0; u < n; ++u)
113
                  if (match[u] == -1)
114
                      augment(u);
115
116
              return match;
117
          }
118
     };
```

TwoSat (2-Sat) 2.5

```
1
    struct TwoSat {
 2
        int n;
 3
         std::vector<std::vector<int>> e;
 4
        std::vector<bool> ans;
 5
        TwoSat(int n) : n(n), e(2 * n), ans(n) {}
 6
         void addClause(int u, bool f, int v, bool g) {
 7
             e[2 * u + !f].push_back(2 * v + g);
 8
             e[2 * v + !g].push_back(2 * u + f);
 9
10
         bool satisfiable() {
11
             std::vector<int> id(2 * n, -1), dfn(2 * n, -1), low(2 * n, -1);
12
             std::vector<int> stk;
13
             int now = 0, cnt = 0;
14
             std::function<void(int)> tarjan = [&](int u) {
15
                 stk.push_back(u);
16
                 dfn[u] = low[u] = now++;
17
                 for (auto v : e[u]) {
18
                     if (dfn[v] == -1) {
19
                         tarjan(v);
20
                         low[u] = std::min(low[u], low[v]);
21
                     } else if (id[v] == -1) {
22
                         low[u] = std::min(low[u], dfn[v]);
23
                     }
24
25
                 if (dfn[u] == low[u]) {
26
                     int v;
27
                     do {
28
                         v = stk.back();
29
                         stk.pop_back();
30
                         id[v] = cnt;
31
                     } while (v != u);
32
                     ++cnt;
33
                 }
```

```
34
             };
35
             for (int i = 0; i < 2 * n; ++i) if (dfn[i] == -1) tarjan(i);
36
             for (int i = 0; i < n; ++i) {
37
                 if (id[2 * i] == id[2 * i + 1]) return false;
38
                 ans[i] = id[2 * i] > id[2 * i + 1];
39
             }
40
             return true;
41
42
         std::vector<bool> answer() { return ans; }
43
    };
```

最大流 2.6

最大流 (Flow 旧版其一, 整数应用) 2.6.1

```
template<class T>
 2
    struct Flow {
 3
         const int n;
 4
         struct Edge {
 5
             int to;
 6
             T cap;
             Edge(int to, T cap) : to(to), cap(cap) {}
 8
 9
         std::vector<Edge> e;
10
         std::vector<std::vector<int>> g;
11
         std::vector<int> cur, h;
12
         Flow(int n) : n(n), g(n) {}
13
14
         bool bfs(int s, int t) {
15
             h.assign(n, -1);
16
             std::queue<int> que;
17
             h[s] = 0;
18
             que.push(s);
19
             while (!que.empty()) {
20
                 const int u = que.front();
21
                 que.pop();
22
                 for (int i : g[u]) {
23
                      auto [v, c] = e[i];
24
                      if (c > 0 & h[v] == -1) {
25
                          h[v] = h[u] + 1;
26
                          if (v == t) {
27
                              return true;
28
                          }
29
                          que.push(v);
30
                     }
31
                 }
32
33
             return false;
34
35
36
         T dfs(int u, int t, T f) {
37
             if (u == t) {
38
                 return f;
39
             }
40
             auto r = f;
41
             for (int &i = cur[u]; i < int(g[u].size()); ++i) {</pre>
42
                 const int j = g[u][i];
43
                 auto [v, c] = e[j];
44
                 if (c > 0 & h[v] == h[u] + 1) {
45
                      auto a = dfs(v, t, std::min(r, c));
46
                     e[j].cap -= a;
47
                     e[j ^ 1].cap += a;
```

```
48
                     r -= a;
49
                     if (r == 0) {
50
                         return f;
51
                     }
52
                 }
53
             }
54
             return f - r;
55
56
         void addEdge(int u, int v, T c) {
57
             g[u].push_back(e.size());
58
             e.emplace_back(v, c);
59
             g[v].push_back(e.size());
60
             e.emplace_back(u, 0);
61
62
         T maxFlow(int s, int t) {
63
             T ans = 0;
64
             while (bfs(s, t)) {
65
                 cur.assign(n, 0);
66
                 ans += dfs(s, t, std::numeric_limits<T>::max());
67
68
             return ans;
69
         }
70
    };
```

2.6.2 最大流 (Flow 旧版其二, 浮点数应用)

```
1
    template<class T>
 2
    struct Flow {
 3
         const int n;
 4
         struct Edge {
 5
             int to;
 6
             T cap;
 7
             Edge(int to, T cap) : to(to), cap(cap) {}
 8
         };
 9
         std::vector<Edge> e;
10
         std::vector<std::vector<int>> g;
11
         std::vector<int> cur, h;
12
         Flow(int n) : n(n), g(n) {}
13
14
         bool bfs(int s, int t) {
15
             h.assign(n, -1);
16
             std::queue<int> que;
17
             h[s] = 0;
18
             que.push(s);
19
             while (!que.empty()) {
20
                 const int u = que.front();
21
                 que.pop();
22
                 for (int i : g[u]) {
23
                     auto [v, c] = e[i];
24
                     if (c > 0 & h[v] == -1) {
25
                         h[v] = h[u] + 1;
26
                          if (v == t) {
27
                              return true;
28
29
                          que.push(v);
30
                     }
31
                 }
32
             }
33
             return false;
34
35
36
        T dfs(int u, int t, T f) {
```

```
if (u == t) {
38
                 return f;
39
             }
40
             auto r = f;
41
             double res = 0;
42
             for (int &i = cur[u]; i < int(g[u].size()); ++i) {</pre>
43
                 const int j = g[u][i];
44
                 auto [v, c] = e[j];
45
                 if (c > 0 \&\& h[v] == h[u] + 1) {
46
                     auto a = dfs(v, t, std::min(r, c));
47
                     res += a;
48
                     e[j].cap -= a;
49
                     e[j ^ 1].cap += a;
50
                     r -= a;
51
                     if (r == 0) {
52
                         return f;
53
54
                 }
55
56
             return res;
57
58
         void addEdge(int u, int v, T c) {
59
             g[u].push_back(e.size());
60
             e.emplace back(v, c);
61
             g[v].push_back(e.size());
62
             e.emplace_back(u, 0);
63
64
         T maxFlow(int s, int t) {
65
             T ans = 0;
66
             while (bfs(s, t)) {
67
                 cur.assign(n, 0);
68
                 ans += dfs(s, t, 1E100);
69
             }
70
             return ans;
71
         }
72
    };
```

最大流 (MaxFlow 新版) 2.6.3

```
constexpr int inf = 1E9;
    template<class T>
 3
    struct MaxFlow {
 4
         struct _Edge {
 5
             int to;
 6
             T cap;
 7
             _Edge(int to, T cap) : to(to), cap(cap) {}
 8
         };
 9
10
         int n;
11
         std::vector<_Edge> e;
12
         std::vector<std::vector<int>> g;
13
         std::vector<int> cur, h;
14
        MaxFlow() {}
15
16
        MaxFlow(int n) {
17
             init(n);
18
19
20
         void init(int n) {
21
             this->n = n;
22
             e.clear();
23
             g.assign(n, {});
```

```
24
             cur.resize(n);
25
             h.resize(n);
26
        }
27
28
         bool bfs(int s, int t) {
29
             h.assign(n, -1);
30
             std::queue<int> que;
31
             h[s] = 0;
32
             que.push(s);
33
             while (!que.empty()) {
34
                 const int u = que.front();
35
                 que.pop();
36
                 for (int i : g[u]) {
37
                     auto [v, c] = e[i];
38
                     if (c > 0 & h[v] == -1) {
39
                         h[v] = h[u] + 1;
40
                         if (v == t) {
41
                             return true;
42
43
                         que.push(v);
44
                     }
45
                 }
46
47
             return false;
48
        }
49
50
        T dfs(int u, int t, T f) {
51
             if (u == t) {
52
                 return f;
53
54
             auto r = f;
55
             for (int &i = cur[u]; i < int(g[u].size()); ++i) {</pre>
56
                 const int j = g[u][i];
57
                 auto [v, c] = e[j];
58
                 if (c > 0 \&\& h[v] == h[u] + 1) {
59
                     auto a = dfs(v, t, std::min(r, c));
60
                     e[j].cap -= a;
61
                     e[j ^ 1].cap += a;
62
                     r -= a;
63
                     if (r == 0) {
64
                         return f;
65
                     }
66
                 }
67
             }
68
             return f - r;
69
70
        void addEdge(int u, int v, T c) {
71
             g[u].push_back(e.size());
72
             e.emplace_back(v, c);
73
             g[v].push_back(e.size());
74
             e.emplace_back(u, 0);
75
76
        T flow(int s, int t) {
77
             T ans = 0;
78
             while (bfs(s, t)) {
79
                 cur.assign(n, 0);
80
                 ans += dfs(s, t, std::numeric_limits<T>::max());
81
82
             return ans;
83
        }
84
85
         std::vector<bool> minCut() {
86
             std::vector<bool> c(n);
87
             for (int i = 0; i < n; i++) {
```

```
88
                  c[i] = (h[i] != -1);
 89
 90
              return c;
 91
          }
 92
 93
          struct Edge {
 94
              int from;
 95
              int to;
 96
              T cap;
 97
              T flow;
 98
          };
 99
          std::vector<Edge> edges() {
100
              std::vector<Edge> a;
101
              for (int i = 0; i < e.size(); i += 2) {
102
                  Edge x;
103
                  x.from = e[i + 1].to;
104
                  x.to = e[i].to;
105
                  x.cap = e[i].cap + e[i + 1].cap;
106
                  x.flow = e[i + 1].cap;
107
                  a.push_back(x);
108
109
              return a;
110
          }
111
     };
```

费用流 2.7

费用流(MCFGraph 最小费用可行流) 2.7.1

```
1
    struct MCFGraph {
 2
         struct Edge {
 3
             int v, c, f;
 4
             Edge(int v, int c, int f) : v(v), c(c), f(f) {}
 5
         };
 6
         const int n;
 7
         std::vector<Edge> e;
 8
         std::vector<std::vector<int>> g;
 9
         std::vector<i64> h, dis;
10
         std::vector<int> pre;
11
         bool dijkstra(int s, int t) {
12
             dis.assign(n, std::numeric_limits<i64>::max());
13
             pre.assign(n, -1);
14
             std::priority_queue<std::pair<i64, int>, std::vector<std::pair<i64, int>>,
    std::greater<std::pair<i64, int>>> que;
15
             dis[s] = 0;
16
             que.emplace(0, s);
17
             while (!que.empty()) {
18
                 i64 d = que.top().first;
19
                 int u = que.top().second;
20
                 que.pop();
21
                 if (dis[u] < d) continue;</pre>
22
                 for (int i : g[u]) {
23
                     int v = e[i].v;
24
                     int c = e[i].c;
25
                     int f = e[i].f;
26
                     if (c > 0 \&\& dis[v] > d + h[u] - h[v] + f) {
27
                          dis[v] = d + h[u] - h[v] + f;
28
                          pre[v] = i;
29
                          que.emplace(dis[v], v);
30
                     }
31
                 }
32
             }
```

```
33
             return dis[t] != std::numeric_limits<i64>::max();
34
35
        MCFGraph(int n) : n(n), g(n) \{ \}
36
        void addEdge(int u, int v, int c, int f) {
37
            if (f < 0) {
38
                 g[u].push_back(e.size());
39
                 e.emplace_back(v, 0, f);
40
                 g[v].push_back(e.size());
41
                 e.emplace_back(u, c, -f);
42
             } else {
43
                 g[u].push_back(e.size());
44
                 e.emplace_back(v, c, f);
45
                 g[v].push_back(e.size());
46
                 e.emplace_back(u, 0, -f);
47
             }
48
         }
49
         std::pair<int, i64> flow(int s, int t) {
50
             int flow = 0;
51
             i64 cost = 0;
52
             h.assign(n, 0);
53
            while (dijkstra(s, t)) {
54
                 for (int i = 0; i < n; ++i) h[i] += dis[i];
55
                 int aug = std::numeric limits<int>::max();
56
                 for (int i = t; i != s; i = e[pre[i] ^ 1].v) aug = std::min(aug,
    e[pre[i]].c);
57
                 for (int i = t; i != s; i = e[pre[i] ^ 1].v) {
58
                     e[pre[i]].c -= aug;
59
                     e[pre[i] ^ 1].c += aug;
60
                 }
61
                 flow += aug;
62
                 cost += i64(aug) * h[t];
63
64
            return std::make_pair(flow, cost);
65
        }
66
    };
```

2.7.2 费用流 (MCFGraph 最小费用最大流)

代码同上,但是需要注释掉建边限制。以下为参考:

```
1
    void addEdge(int u, int v, int c, int f) { // 可行流
 2
        if (f < 0) {
 3
             g[u].push_back(e.size());
 4
             e.emplace_back(v, 0, f);
 5
             g[v].push_back(e.size());
 6
             e.emplace_back(u, c, -f);
 7
        } else {
 8
             g[u].push_back(e.size());
 9
             e.emplace_back(v, c, f);
10
             g[v].push_back(e.size());
11
             e.emplace_back(u, 0, -f);
12
        }
13
    }
```

```
void addEdge(int u, int v, int c, int f) { // 最大流

g[u].push_back(e.size());

e.emplace_back(v, c, f);

g[v].push_back(e.size());

e.emplace_back(u, 0, -f);

}
```

2.8 树链剖分 (HLD)

```
1
    struct HLD {
 2
         int n;
 3
         std::vector<int> siz, top, dep, parent, in, out, seq;
 4
         std::vector<std::vector<int>> adj;
 5
         int cur;
 6
 7
        HLD() {}
 8
        HLD(int n) {
 9
             init(n);
10
         }
11
         void init(int n) {
12
             this->n = n;
13
             siz.resize(n);
14
             top.resize(n);
15
             dep.resize(n);
16
             parent.resize(n);
17
             in.resize(n);
18
             out.resize(n);
19
             seq.resize(n);
20
             cur = 0;
21
             adj.assign(n, {});
22
23
         void addEdge(int u, int v) {
24
             adj[u].push_back(v);
25
             adj[v].push_back(u);
26
27
         void work(int root = 0) {
28
             top[root] = root;
29
             dep[root] = 0;
30
             parent[root] = -1;
31
             dfs1(root);
32
             dfs2(root);
33
34
         void dfs1(int u) {
35
             if (parent[u] != -1) {
36
                 adj[u].erase(std::find(adj[u].begin(), adj[u].end(), parent[u]));
37
38
39
             siz[u] = 1;
40
             for (auto &v : adj[u]) {
41
                 parent[v] = u;
42
                 dep[v] = dep[u] + 1;
43
                 dfs1(v);
44
                 siz[u] += siz[v];
45
                 if (siz[v] > siz[adj[u][0]]) {
46
                     std::swap(v, adj[u][0]);
47
                 }
48
             }
49
         }
50
         void dfs2(int u) {
51
             in[u] = cur++;
52
             seq[in[u]] = u;
53
             for (auto v : adj[u]) {
54
                 top[v] = v == adj[u][0] ? top[u] : v;
55
                 dfs2(v);
56
57
             out[u] = cur;
58
         }
59
         int lca(int u, int v) {
60
             while (top[u] != top[v]) {
61
                 if (dep[top[u]] > dep[top[v]]) {
```

```
62
                      u = parent[top[u]];
 63
                  } else {
 64
                      v = parent[top[v]];
 65
 66
              }
 67
              return dep[u] < dep[v] ? u : v;</pre>
 68
          }
 69
 70
          int dist(int u, int v) {
 71
              return dep[u] + dep[v] - 2 * dep[lca(u, v)];
 72
          }
 73
 74
          int jump(int u, int k) {
 75
              if (dep[u] < k) {</pre>
 76
                  return -1;
 77
 78
 79
              int d = dep[u] - k;
 80
 81
              while (dep[top[u]] > d) {
 82
                  u = parent[top[u]];
 83
 84
 85
              return seq[in[u] - dep[u] + d];
 86
          }
 87
 88
          bool isAncester(int u, int v) {
 89
              return in[u] <= in[v] && in[v] < out[u];</pre>
 90
          }
 91
 92
          int rootedParent(int u, int v) {
 93
              std::swap(u, v);
 94
              if (u == v) {
 95
                  return u;
 96
 97
              if (!isAncester(u, v)) {
 98
                  return parent[u];
99
100
              auto it = std::upper_bound(adj[u].begin(), adj[u].end(), v, [&](int x, int
     y) {
101
                  return in[x] < in[y];</pre>
102
              }) - 1;
103
              return *it;
104
          }
105
106
          int rootedSize(int u, int v) {
107
              if (u == v) {
108
                  return n;
109
110
              if (!isAncester(v, u)) {
111
                  return siz[v];
112
113
              return n - siz[rootedParent(u, v)];
114
          }
115
116
          int rootedLca(int a, int b, int c) {
117
              return lca(a, b) ^ lca(b, c) ^ lca(c, a);
118
          }
119
     };
```

/END/

数论、几何、多项式

快速幂 3.1

```
int power(int a, i64 b, int p) {
2
       int res = 1;
3
       for (; b; b /= 2, a = 1LL * a * a % p) {
4
           if (b % 2) {
5
               res = 1LL * res * a % p;
6
7
8
       return res;
9
  }
```

欧拉筛 3.2

```
1
    std::vector<int> minp, primes;
 2
 3
    void sieve(int n) {
 4
        minp.assign(n + 1, 0);
 5
        primes.clear();
 6
 7
        for (int i = 2; i <= n; i++) {
 8
             if (minp[i] == 0) {
 9
                 minp[i] = i;
10
                 primes.push_back(i);
11
            }
12
13
            for (auto p : primes) {
14
                 if (i * p > n) {
15
                     break;
16
17
                 minp[i * p] = p;
18
                 if (p == minp[i]) {
19
                     break;
20
21
            }
22
        }
23
   }
```

莫比乌斯函数筛 (莫比乌斯函数/反演) 3.3

```
1
    std::unordered map<int, Z> fMu;
 3
    constexpr int N = 1E7;
 4
    std::vector<int> minp, primes;
 5
    std::vector<Z> mu;
 6
 7
    void sieve(int n) {
 8
        minp.assign(n + 1, 0);
 9
        mu.resize(n);
10
        primes.clear();
11
12
        mu[1] = 1;
13
        for (int i = 2; i <= n; i++) {
14
             if (minp[i] == 0) {
15
                 mu[i] = -1;
16
                 minp[i] = i;
17
                 primes.push_back(i);
```

```
18
19
20
             for (auto p : primes) {
21
                 if (i * p > n) {
22
                     break;
23
24
                 minp[i * p] = p;
25
                 if (p == minp[i]) {
26
                     break;
27
                 }
28
                 mu[i * p] = -mu[i];
29
             }
30
31
32
         for (int i = 1; i \leftarrow n; i++) {
33
             mu[i] += mu[i - 1];
34
         }
35
    }
36
37
38
    Z sumMu(int n) {
39
        if (n <= N) {
40
             return mu[n];
41
42
         if (fMu.count(n)) {
43
             return fMu[n];
44
45
         if (n == 0) {
46
            return 0;
47
48
        Z ans = 1;
49
         for (int l = 2, r; l <= n; l = r + 1) {
50
             r = n / (n / 1);
51
             ans -= (r - 1 + 1) * sumMu(n / 1);
52
         }
53
        return ans;
54
    }
55
56
    int main() {
57
         std::ios::sync_with_stdio(false);
58
         std::cin.tie(nullptr);
59
60
         sieve(N);
61
62
        int L, R;
63
         std::cin >> L >> R;
64
        L -= 1;
65
66
         Z ans = 0;
67
         for (int l = 1, r; l \leftarrow R; l = r + 1) {
68
             r = R / (R / 1);
69
             if (1 <= L) {
70
                 r = std::min(r, L / (L / 1));
71
72
73
             ans += (power(Z(2), R / 1 - L / 1) - 1) * (sumMu(r) - sumMu(1 - 1));
74
         }
75
76
         std::cout << ans << "\n";</pre>
77
78
        return 0;
79
```

求解单个数的欧拉函数 3.4

```
1
    int phi(int n) {
 2
        int res = n;
 3
        for (int i = 2; i * i <= n; i++) {
 4
             if (n % i == 0) {
 5
                 while (n \% i == 0) {
 6
                     n /= i;
 7
 8
                 res = res / i * (i - 1);
 9
             }
10
11
        if (n > 1) {
12
            res = res / n * (n - 1);
13
        }
14
        return res;
15
   }
```

扩展欧几里得 (exGCD) 3.5

```
int exgcd(int a, int b, int &x, int &y) {
2
       if (!b) {
3
           x = 1, y = 0;
4
           return a;
5
       }
6
       int g = exgcd(b, a \% b, y, x);
7
       y -= a / b * x;
8
       return g;
9 }
```

组合数 (Comb, with. MInt & MLong) 3.6

```
struct Comb {
         int n;
 3
         std::vector<Z> _fac;
 4
         std::vector<Z> _invfac;
 5
         std::vector<Z> _inv;
 6
 7
         Comb() : n\{0\}, _fac\{1\}, _invfac\{1\}, _inv\{0\} {}
 8
         Comb(int n) : Comb() {
 9
             init(n);
10
         }
11
12
         void init(int m) {
13
             m = std::min(m, Z::getMod() - 1);
14
             if (m <= n) return;</pre>
15
             _fac.resize(m + 1);
16
             _invfac.resize(m + 1);
17
             _{inv.resize(m + 1);}
18
19
             for (int i = n + 1; i \leftarrow m; i++) {
20
                 _fac[i] = _fac[i - 1] * i;
21
22
             _invfac[m] = _fac[m].inv();
23
             for (int i = m; i > n; i--) {
24
                 _invfac[i - 1] = _invfac[i] * i;
25
                 _inv[i] = _invfac[i] * _fac[i - 1];
26
             }
27
             n = m;
28
         }
```

```
29
30
        Z fac(int m) {
31
             if (m > n) init(2 * m);
32
             return _fac[m];
33
34
         Z invfac(int m) {
35
            if (m > n) init(2 * m);
36
            return _invfac[m];
37
38
         Z inv(int m) {
39
            if (m > n) init(2 * m);
40
            return _inv[m];
41
42
        Z binom(int n, int m) {
43
             if (n < m | m < 0) return 0;
44
             return fac(n) * invfac(m) * invfac(n - m);
45
46 } comb;
```

3.7 二项式 (Binomial 任意模数计算)

```
std::vector<std::pair<int, int>> factorize(int n) {
 2
         std::vector<std::pair<int, int>> factors;
 3
         for (int i = 2; static_cast<long long>(i) * i <= n; i++) {
 4
             if (n \% i == 0) {
 5
                 int t = 0;
 6
                 for (; n \% i == 0; n /= i)
 7
                     ++t;
 8
                 factors.emplace_back(i, t);
 9
             }
10
         }
11
        if (n > 1)
12
             factors.emplace_back(n, 1);
13
        return factors;
14
15
    constexpr int power(int base, i64 exp) {
16
        int res = 1;
17
         for (; exp > 0; base *= base, exp /= 2) {
18
             if (exp % 2 == 1) {
19
                 res *= base;
20
21
         }
22
        return res;
23
24
    constexpr int power(int base, i64 exp, int mod) {
25
        int res = 1 % mod;
26
        for (; exp > 0; base = 1LL * base * base % mod, exp /= 2) {
27
             if (exp % 2 == 1) {
28
                 res = 1LL * res * base % mod;
29
30
         }
31
        return res;
32
33
    int inverse(int a, int m) {
34
        int g = m, r = a, x = 0, y = 1;
35
        while (r != 0) {
36
             int q = g / r;
37
             g %= r;
38
             std::swap(g, r);
39
             x -= q * y;
40
             std::swap(x, y);
41
        }
```

```
42
         return x < 0? x + m : x;
 43
 44
     int solveModuloEquations(const std::vector<std::pair<int, int>> &e) {
 45
         int m = 1;
 46
         for (std::size_t i = 0; i < e.size(); i++) {
 47
              m *= e[i].first;
 48
 49
         int res = 0;
 50
          for (std::size_t i = 0; i < e.size(); i++) {
 51
              int p = e[i].first;
 52
              res = (res + 1LL * e[i].second * (m / p) * inverse(m / p, p)) % m;
 53
          }
 54
         return res;
 55
 56
     constexpr int N = 1E5;
 57
     class Binomial {
 58
          const int mod;
 59
     private:
 60
         const std::vector<std::pair<int, int>> factors;
 61
          std::vector<int> pk;
 62
         std::vector<std::vector<int>> prod;
 63
          static constexpr i64 exponent(i64 n, int p) {
 64
              i64 \text{ res} = 0;
 65
              for (n /= p; n > 0; n /= p) {
 66
                  res += n;
 67
              }
 68
              return res;
 69
 70
          int product(i64 n, std::size_t i) {
 71
              int res = 1;
 72
              int p = factors[i].first;
 73
              for (; n > 0; n \neq p) {
 74
                  res = 1LL * res * power(prod[i].back(), n / pk[i], pk[i]) % pk[i] *
     prod[i][n % pk[i]] % pk[i];
 75
 76
              return res;
 77
 78
     public:
 79
         Binomial(int mod) : mod(mod), factors(factorize(mod)) {
 80
              pk.resize(factors.size());
 81
              prod.resize(factors.size());
 82
              for (std::size_t i = 0; i < factors.size(); i++) {</pre>
 83
                  int p = factors[i].first;
 84
                  int k = factors[i].second;
 85
                  pk[i] = power(p, k);
 86
                  prod[i].resize(std::min(N + 1, pk[i]));
 87
                  prod[i][0] = 1;
 88
                  for (int j = 1; j < prod[i].size(); j++) {
 89
                      if (j % p == 0) {
 90
                          prod[i][j] = prod[i][j - 1];
 91
                      } else {
 92
                          prod[i][j] = 1LL * prod[i][j - 1] * j % pk[i];
 93
                      }
 94
                  }
 95
              }
 96
 97
          int operator()(i64 n, i64 m) {
 98
              if (n < m || m < 0) {
 99
                  return 0;
100
101
              std::vector<std::pair<int, int>> ans(factors.size());
102
              for (int i = 0; i < factors.size(); i++) {</pre>
103
                  int p = factors[i].first;
104
                  int k = factors[i].second;
```

```
105
                  int e = exponent(n, p) - exponent(m, p) - exponent(n - m, p);
106
                  if (e >= k) {
107
                      ans[i] = std::make_pair(pk[i], 0);
108
                  } else {
109
                      int pn = product(n, i);
110
                      int pm = product(m, i);
111
                      int pd = product(n - m, i);
112
                      int res = 1LL * pn * inverse(pm, pk[i]) % pk[i] * inverse(pd,
     pk[i]) % pk[i] * power(p, e) % pk[i];
113
                      ans[i] = std::make_pair(pk[i], res);
114
115
              }
116
             return solveModuloEquations(ans);
117
         }
118
     };
```

素数测试与因式分解 (Miller-Rabin & Pollard-Rho) 3.8

```
1
     i64 mul(i64 a, i64 b, i64 m) {
 2
         return static_cast<__int128>(a) * b % m;
 3
 4
     i64 power(i64 a, i64 b, i64 m) {
 5
         i64 \text{ res} = 1 \% \text{ m};
 6
         for (; b; b >>= 1, a = mul(a, a, m))
 7
             if (b & 1)
 8
                 res = mul(res, a, m);
 9
         return res;
10
11
     bool isprime(i64 n) {
12
         if (n < 2)
13
             return false;
14
         static constexpr int A[] = {2, 3, 5, 7, 11, 13, 17, 19, 23};
15
         int s = __builtin_ctzll(n - 1);
16
         i64 d = (n - 1) >> s;
17
         for (auto a : A) {
18
             if (a == n)
19
                 return true;
20
             i64 x = power(a, d, n);
21
             if (x == 1 | x == n - 1)
22
                 continue;
23
             bool ok = false;
24
             for (int i = 0; i < s - 1; ++i) {
25
                 x = mul(x, x, n);
26
                 if (x == n - 1) {
27
                      ok = true;
28
                      break;
29
                 }
30
             }
31
             if (!ok)
32
                 return false;
33
         }
34
         return true;
35
36
     std::vector<i64> factorize(i64 n) {
37
         std::vector<i64> p;
38
         std::function < void(i64) > f = [\&](i64 n) {
39
             if (n <= 10000) {
40
                 for (int i = 2; i * i <= n; ++i)
41
                      for (; n \% i == 0; n /= i)
42
                          p.push_back(i);
43
                 if (n > 1)
44
                      p.push_back(n);
                                              22
```

```
45
                 return;
46
             }
47
             if (isprime(n)) {
48
                 p.push_back(n);
49
                 return;
50
             }
51
             auto g = [\&](i64 x) {
52
                 return (mul(x, x, n) + 1) \% n;
53
             };
54
             i64 \times 0 = 2;
55
             while (true) {
56
                 i64 x = x0;
57
                 i64 y = x0;
58
                 i64 d = 1;
59
                 i64 power = 1, lam = 0;
60
                 i64 v = 1;
61
                 while (d == 1) {
62
                     y = g(y);
63
                     ++lam;
64
                     v = mul(v, std::abs(x - y), n);
65
                     if (lam % 127 == 0) {
66
                         d = std::gcd(v, n);
67
                          v = 1;
68
                     }
69
                     if (power == lam) {
70
                          x = y;
                          power *= 2;
71
72
                          lam = 0;
73
                          d = std::gcd(v, n);
74
                         v = 1;
75
                     }
76
                 }
77
                 if (d != n) {
78
                     f(d);
79
                     f(n / d);
80
                     return;
81
82
                 ++x0;
83
             }
84
         };
85
         f(n);
86
         std::sort(p.begin(), p.end());
87
         return p;
88
    }
```

3.9 平面几何

```
1
    template<class T>
 2
     struct Point {
 3
         Tx;
 4
         Ty;
 5
         Point(T x_{=} = 0, T y_{=} = 0) : x(x_{-}), y(y_{-}) {}
 6
 7
         template<class U>
 8
         operator Point<U>() {
 9
             return Point<U>(U(x), U(y));
10
11
         Point &operator+=(Point p) & {
12
             x += p.x;
13
             y += p.y;
14
             return *this;
15
         }
```

```
16
         Point &operator-=(Point p) & {
17
             x \rightarrow p.x;
18
             y -= p.y;
19
             return *this;
20
21
         Point &operator*=(T v) & {
22
             x *= v;
23
             y *= v;
24
             return *this;
25
         }
26
         Point operator-() const {
27
             return Point(-x, -y);
28
29
         friend Point operator+(Point a, Point b) {
30
             return a += b;
31
32
         friend Point operator-(Point a, Point b) {
33
             return a -= b;
34
35
         friend Point operator*(Point a, T b) {
36
            return a *= b;
37
38
         friend Point operator*(T a, Point b) {
39
             return b *= a;
40
         }
41
         friend bool operator==(Point a, Point b) {
42
             return a.x == b.x && a.y == b.y;
43
44
         friend std::istream &operator>>(std::istream &is, Point &p) {
45
            return is >> p.x >> p.y;
46
         }
47
         friend std::ostream &operator<<(std::ostream &os, Point p) {</pre>
48
             return os << "(" << p.x << ", " << p.y << ")";
49
         }
50
    };
51
52
    template<class T>
53
    T dot(Point<T> a, Point<T> b) {
54
         return a.x * b.x + a.y * b.y;
55
56
57
    template<class T>
58
    T cross(Point<T> a, Point<T> b) {
59
         return a.x * b.y - a.y * b.x;
60
61
62
    template<class T>
63
    T square(Point<T> p) {
64
         return dot(p, p);
65
    }
66
67
    template<class T>
68
    double length(Point<T> p) {
69
         return std::sqrt(double(square(p)));
70
    }
71
72
    long double length(Point<long double> p) {
73
        return std::sqrt(square(p));
74
75
76
    template<class T>
77
    struct Line {
78
         Point<T> a;
79
        Point<T> b;
```

```
Line(Point<T> a_ = Point<T>(), Point<T> b_ = Point<T>()) : a(a_), b(b_) {}
 81
     };
 82
 83
     template<class T>
     Point<T> rotate(Point<T> a) {
 85
         return Point(-a.y, a.x);
 86
 87
 88
     template<class T>
 89
     int sgn(Point<T> a) {
 90
          return a.y > 0 \mid \mid (a.y == 0 && a.x > 0) ? 1 : -1;
 91
 92
 93
     template<class T>
 94
     bool pointOnLineLeft(Point<T> p, Line<T> 1) {
 95
          return cross(1.b - 1.a, p - 1.a) > 0;
 96
 97
 98
     template<class T>
 99
     Point<T> lineIntersection(Line<T> l1, Line<T> l2) {
100
         return 11.a + (11.b - 11.a) * (cross(12.b - 12.a, 11.a - 12.a) / cross(12.b -
     12.a, l1.a - l1.b));
101
102
103
     template<class T>
104
     bool pointOnSegment(Point<T> p, Line<T> 1) {
105
          return cross(p - 1.a, 1.b - 1.a) == 0 \& std::min(1.a.x, 1.b.x) <= p.x \& p.x
     <= std::max(1.a.x, 1.b.x)
106
         && std::min(1.a.y, 1.b.y) \le p.y & p.y \le std::max(1.a.y, 1.b.y);
107
108
109
     template<class T>
110
     bool pointInPolygon(Point<T> a, std::vector<Point<T>> p) {
111
          int n = p.size();
112
         for (int i = 0; i < n; i++) {
113
              if (pointOnSegment(a, Line(p[i], p[(i + 1) \% n]))) {
114
                  return true;
115
              }
116
          }
117
118
         int t = 0;
119
          for (int i = 0; i < n; i++) {
120
              auto u = p[i];
121
              auto v = p[(i + 1) \% n];
122
              if (u.x < a.x && v.x >= a.x && pointOnLineLeft(a, Line(v, u))) {
123
                 t ^= 1;
124
125
              if (u.x >= a.x \& v.x < a.x \& pointOnLineLeft(a, Line(u, v))) {
126
                 t ^= 1;
127
              }
128
          }
129
130
         return t == 1;
131
     }
132
133
     // 0 : not intersect
134
     // 1 : strictly intersect
135
     // 2 : overlap
136
     // 3 : intersect at endpoint
137
     template<class T>
138
     std::tuple<int, Point<T>, Point<T>> segmentIntersection(Line<T> 11, Line<T> 12) {
139
          if (std::max(l1.a.x, l1.b.x) < std::min(l2.a.x, l2.b.x)) {
140
              return {0, Point<T>(), Point<T>()};
141
```

```
if (std::min(11.a.x, 11.b.x) > std::max(12.a.x, 12.b.x)) {
143
              return {0, Point<T>(), Point<T>()};
144
          }
145
          if (std::max(l1.a.y, l1.b.y) < std::min(l2.a.y, l2.b.y)) {
146
              return {0, Point<T>(), Point<T>()};
147
148
         if (std::min(l1.a.y, l1.b.y) > std::max(l2.a.y, l2.b.y)) {
149
              return {0, Point<T>(), Point<T>()};
150
151
          if (cross(11.b - 11.a, 12.b - 12.a) == 0) {
152
              if (cross(l1.b - l1.a, l2.a - l1.a) != 0) {
153
                  return {0, Point<T>(), Point<T>()};
154
              } else {
155
                  auto maxx1 = std::max(l1.a.x, l1.b.x);
156
                  auto minx1 = std::min(l1.a.x, l1.b.x);
157
                  auto maxy1 = std::max(l1.a.y, l1.b.y);
158
                  auto miny1 = std::min(l1.a.y, l1.b.y);
159
                  auto maxx2 = std::max(12.a.x, 12.b.x);
160
                  auto minx2 = std::min(12.a.x, 12.b.x);
161
                  auto maxy2 = std::max(12.a.y, 12.b.y);
162
                  auto miny2 = std::min(12.a.y, 12.b.y);
163
                  Point<T> p1(std::max(minx1, minx2), std::max(miny1, miny2));
164
                  Point<T> p2(std::min(maxx1, maxx2), std::min(maxy1, maxy2));
165
                  if (!pointOnSegment(p1, l1)) {
166
                      std::swap(p1.y, p2.y);
167
168
                  if (p1 == p2) {
169
                      return {3, p1, p2};
170
                  } else {
171
                      return {2, p1, p2};
172
173
              }
174
          }
175
         auto cp1 = cross(12.a - 11.a, 12.b - 11.a);
         auto cp2 = cross(12.a - 11.b, 12.b - 11.b);
176
177
          auto cp3 = cross(l1.a - l2.a, l1.b - l2.a);
178
          auto cp4 = cross(l1.a - l2.b, l1.b - l2.b);
179
180
         if ((cp1 > 0 && cp2 > 0) || (cp1 < 0 && cp2 < 0) || (cp3 > 0 && cp4 > 0) ||
      (cp3 < 0 && cp4 < 0)) {
181
             return {0, Point<T>(), Point<T>()};
182
         }
183
184
         Point p = lineIntersection(l1, l2);
185
          if (cp1 != 0 && cp2 != 0 && cp3 != 0 && cp4 != 0) {
186
              return {1, p, p};
187
          } else {
188
              return {3, p, p};
189
          }
190
     }
191
192
     template<class T>
193
     bool segmentInPolygon(Line<T> 1, std::vector<Point<T>> p) {
194
         int n = p.size();
195
          if (!pointInPolygon(l.a, p)) {
196
              return false;
197
198
         if (!pointInPolygon(l.b, p)) {
199
              return false;
200
          }
201
          for (int i = 0; i < n; i++) {
202
              auto u = p[i];
203
              auto v = p[(i + 1) \% n];
204
              auto w = p[(i + 2) \% n];
```

```
205
              auto [t, p1, p2] = segmentIntersection(l, Line(u, v));
206
207
              if (t == 1) {
208
                  return false;
209
              }
210
              if (t == 0) {
211
                  continue;
212
213
              if (t == 2) {
214
                  if (pointOnSegment(v, 1) && v != 1.a && v != 1.b) {
215
                      if (cross(v - u, w - v) > 0) {
216
                          return false;
217
218
                  }
219
              } else {
220
                  if (p1 != u && p1 != v) {
221
                      if (pointOnLineLeft(l.a, Line(v, u))
222
                          pointOnLineLeft(l.b, Line(v, u))) {
223
                          return false;
224
225
                  } else if (p1 == v) {
226
                      if (1.a == v) {
227
                          if (pointOnLineLeft(u, 1)) {
228
                              if (pointOnLineLeft(w, 1)
229
                                  && pointOnLineLeft(w, Line(u, v))) {
230
                                  return false;
231
                              }
232
                          } else {
233
                              if (pointOnLineLeft(w, 1)
234
                                  pointOnLineLeft(w, Line(u, v))) {
235
                                  return false;
236
                              }
237
                          }
238
                      } else if (1.b == v) {
239
                          if (pointOnLineLeft(u, Line(1.b, 1.a))) {
240
                              if (pointOnLineLeft(w, Line(1.b, 1.a))
241
                                  && pointOnLineLeft(w, Line(u, v))) {
242
                                  return false;
243
                              }
244
                          } else {
245
                              if (pointOnLineLeft(w, Line(1.b, 1.a))
246
                                  pointOnLineLeft(w, Line(u, v))) {
247
                                  return false;
248
                              }
249
                          }
250
                      } else {
251
                          if (pointOnLineLeft(u, 1)) {
252
                              if (pointOnLineLeft(w, Line(1.b, 1.a))
253
                                  pointOnLineLeft(w, Line(u, v))) {
254
                                  return false;
255
                              }
256
                          } else {
257
                              if (pointOnLineLeft(w, 1)
258
                                  pointOnLineLeft(w, Line(u, v))) {
259
                                  return false;
260
                              }
261
                          }
262
                      }
263
                  }
264
              }
265
          }
266
          return true;
267
268
```

```
269
     template<class T>
270
     std::vector<Point<T>> hp(std::vector<Line<T>> lines) {
271
         std::sort(lines.begin(), lines.end(), [&](auto l1, auto l2) {
272
              auto d1 = 11.b - 11.a;
273
              auto d2 = 12.b - 12.a;
274
275
              if (sgn(d1) != sgn(d2)) {
276
                  return sgn(d1) == 1;
277
278
279
              return cross(d1, d2) > 0;
280
         });
281
282
         std::deque<Line<T>> ls;
283
         std::deque<Point<T>> ps;
284
         for (auto 1 : lines) {
285
              if (ls.empty()) {
286
                  ls.push_back(1);
287
                  continue;
288
              }
289
290
              while (!ps.empty() && !pointOnLineLeft(ps.back(), 1)) {
291
                  ps.pop back();
292
                  ls.pop_back();
293
              }
294
295
              while (!ps.empty() && !pointOnLineLeft(ps[0], 1)) {
296
                  ps.pop front();
297
                  ls.pop_front();
298
              }
299
300
             if (cross(1.b - 1.a, ls.back().b - ls.back().a) == 0) {
301
                  if (dot(1.b - 1.a, ls.back().b - ls.back().a) > 0) {
302
303
                      if (!pointOnLineLeft(ls.back().a, 1)) {
304
                          assert(ls.size() == 1);
305
                          ls[0] = 1;
306
                      }
307
                      continue;
308
                  }
309
                  return {};
310
              }
311
312
              ps.push_back(lineIntersection(ls.back(), 1));
313
              ls.push_back(1);
314
         }
315
316
         while (!ps.empty() && !pointOnLineLeft(ps.back(), ls[0])) {
317
              ps.pop_back();
318
              ls.pop_back();
319
         }
320
         if (ls.size() <= 2) {
321
              return {};
322
         }
323
         ps.push_back(lineIntersection(ls[0], ls.back()));
324
325
         return std::vector(ps.begin(), ps.end());
326
```

3.10 静态凸包

3.10.1 静态凸包 (with. Point)

```
1
    struct Point {
         i64 x;
 3
         i64 y;
 4
        Point(i64 x = 0, i64 y = 0) : x(x), y(y) {}
 5
    };
 6
    bool operator==(const Point &a, const Point &b) {
 8
        return a.x == b.x && a.y == b.y;
 9
10
11
    Point operator+(const Point &a, const Point &b) {
12
        return Point(a.x + b.x, a.y + b.y);
13
14
15
    Point operator-(const Point &a, const Point &b) {
16
        return Point(a.x - b.x, a.y - b.y);
17
18
19
    i64 dot(const Point &a, const Point &b) {
20
         return a.x * b.x + a.y * b.y;
21
    }
22
23
    i64 cross(const Point &a, const Point &b) {
24
        return a.x * b.y - a.y * b.x;
25
26
27
    void norm(std::vector<Point> &h) {
28
        int i = 0;
29
         for (int j = 0; j < int(h.size()); j++) {
30
             if (h[j].y < h[i].y \mid | (h[j].y == h[i].y && h[j].x < h[i].x)) {
31
                 i = j;
32
33
34
        std::rotate(h.begin(), h.begin() + i, h.end());
35
    }
36
37
    int sgn(const Point &a) {
38
        return a.y > 0 | (a.y == 0 && a.x > 0) ? 0 : 1;
39
    }
40
41
    std::vector<Point> getHull(std::vector<Point> p) {
42
         std::vector<Point> h, l;
43
         std::sort(p.begin(), p.end(), [&](auto a, auto b) {
44
             if (a.x != b.x) {
45
                 return a.x < b.x;
46
             } else {
47
                 return a.y < b.y;
48
49
        });
50
         p.erase(std::unique(p.begin(), p.end()), p.end());
51
        if (p.size() <= 1) {
52
             return p;
53
        }
54
55
        for (auto a : p) {
56
             while (h.size() > 1 \&\& cross(a - h.back(), a - h[h.size() - 2]) \leftarrow 0)  {
57
                 h.pop_back();
58
59
             while (1.size() > 1 \& cross(a - 1.back(), a - 1[1.size() - 2]) >= 0) {
```

29

```
60
                 1.pop_back();
61
62
             1.push_back(a);
63
             h.push_back(a);
64
         }
65
66
        1.pop_back();
67
         std::reverse(h.begin(), h.end());
68
        h.pop_back();
69
        1.insert(1.end(), h.begin(), h.end());
70
        return 1;
71
   }
```

静态凸包 (with. std::complex) 3.10.2

```
1
     using Point = std::complex<i64>;
 2
 3
     #define x real
 4
     #define y imag
 5
 6
     auto dot(const Point &a, const Point &b) {
 7
         return (std::conj(a) * b).x();
 8
 9
10
     auto cross(const Point &a, const Point &b) {
11
         return (std::conj(a) * b).y();
12
13
14
     auto rot(const Point &p) {
15
         return Point(-p.y(), p.x());
16
17
18
     auto complexHull(std::vector<Point> a) {
19
         std::sort(a.begin(), a.end(), [&](auto a, auto b) {
20
             if (a.x() != b.x()) {
21
                  return a.x() < b.x();
22
              } else {
23
                  return a.y() < b.y();
24
              }
25
         });
26
27
         std::vector<Point> 1, h;
28
29
         for (auto p : a) {
30
             \label{eq:while of the constraint} \mbox{while } (1.size() \mbox{ } \mbox{$>$ 1$ \&\& cross(l.back() - l[l.size() - 2], p - l.back()) <= 0)$}
     {
31
                  1.pop_back();
32
              }
33
34
             while (h.size() > 1 \& cross(h.back() - h[h.size() - 2], p - h.back()) >= 0)
     {
35
                  h.pop_back();
36
              }
37
38
              1.push_back(p);
39
             h.push_back(p);
40
41
42
         std::reverse(h.begin(), h.end());
43
44
         h.insert(h.end(), l.begin() + 1, l.end() - 1);
45
```

```
46
    return h;
47
    }
48
49
    int sgn(Point p) {
50
         if (p.y() > 0 | | (p.y() == 0 && p.x() < 0)) {
51
             return 0;
52
         } else {
53
             return 1;
54
55
    }
```

多项式相关 3.11

多项式相关 (Poly, with. Z) 3.11.1

```
1
    std::vector<int> rev;
     std::vector<Z> roots{0, 1};
 3
     void dft(std::vector<Z> &a) {
 4
         int n = a.size();
 5
 6
         if (int(rev.size()) != n) {
 7
             int k = __builtin_ctz(n) - 1;
 8
             rev.resize(n);
 9
             for (int i = 0; i < n; i++) {
10
                 rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
11
             }
12
         }
13
14
         for (int i = 0; i < n; i++) {
15
             if (rev[i] < i) {</pre>
16
                 std::swap(a[i], a[rev[i]]);
17
18
19
         if (int(roots.size()) < n) {</pre>
20
             int k = __builtin_ctz(roots.size());
21
             roots.resize(n);
22
             while ((1 << k) < n) {
23
                 Z = power(Z(3), (P - 1) >> (k + 1));
24
                 for (int i = 1 \iff (k - 1); i \iff (1 \iff k); i++) 
25
                      roots[2 * i] = roots[i];
26
                     roots[2 * i + 1] = roots[i] * e;
27
                 }
28
                 k++;
29
             }
30
         }
31
         for (int k = 1; k < n; k *= 2) {
32
             for (int i = 0; i < n; i += 2 * k) {
33
                 for (int j = 0; j < k; j++) {
34
                     Z u = a[i + j];
35
                     Z v = a[i + j + k] * roots[k + j];
36
                     a[i + j] = u + v;
37
                     a[i + j + k] = u - v;
38
                 }
39
             }
40
         }
41
42
     void idft(std::vector<Z> &a) {
43
         int n = a.size();
44
         std::reverse(a.begin() + 1, a.end());
45
         dft(a);
46
         Z inv = (1 - P) / n;
47
         for (int i = 0; i < n; i++) {
```

```
a[i] *= inv;
 49
         }
 50
 51
     struct Poly {
 52
          std::vector<Z> a;
 53
          Poly() {}
 54
          explicit Poly(int size, std::function<Z(int)> f = [](int) { return 0; }) :
     a(size) {
 55
              for (int i = 0; i < size; i++) {
 56
                  a[i] = f(i);
 57
 58
          }
 59
         Poly(const std::vector<Z> &a) : a(a) {}
 60
          Poly(const std::initializer_list<Z> &a) : a(a) {}
 61
          int size() const {
 62
              return a.size();
 63
 64
         void resize(int n) {
 65
             a.resize(n);
 66
 67
         Z operator[](int idx) const {
 68
              if (idx < size()) {</pre>
 69
                  return a[idx];
 70
              } else {
 71
                  return 0;
 72
 73
 74
          Z &operator[](int idx) {
 75
              return a[idx];
 76
 77
         Poly mulxk(int k) const {
 78
              auto b = a;
 79
              b.insert(b.begin(), k, 0);
 80
              return Poly(b);
 81
 82
         Poly modxk(int k) const {
 83
              k = std::min(k, size());
 84
              return Poly(std::vector<Z>(a.begin(), a.begin() + k));
 85
 86
          Poly divxk(int k) const {
 87
              if (size() <= k) {
 88
                  return Poly();
 89
 90
              return Poly(std::vector<Z>(a.begin() + k, a.end()));
 91
 92
          friend Poly operator+(const Poly &a, const Poly &b) {
 93
              std::vector<Z> res(std::max(a.size(), b.size()));
 94
              for (int i = 0; i < int(res.size()); i++) {
 95
                  res[i] = a[i] + b[i];
 96
              }
 97
              return Poly(res);
 98
99
          friend Poly operator-(const Poly &a, const Poly &b) {
100
              std::vector<Z> res(std::max(a.size(), b.size()));
101
              for (int i = 0; i < int(res.size()); i++) {</pre>
102
                  res[i] = a[i] - b[i];
103
104
              return Poly(res);
105
106
         friend Poly operator-(const Poly &a) {
107
              std::vector<Z> res(a.size());
108
              for (int i = 0; i < int(res.size()); i++) {
109
                  res[i] = -a[i];
110
```

```
111
             return Poly(res);
112
113
         friend Poly operator*(Poly a, Poly b) {
114
              if (a.size() == 0 | b.size() == 0) {
115
                  return Poly();
116
117
              if (a.size() < b.size()) {</pre>
118
                  std::swap(a, b);
119
120
              if (b.size() < 128) {
121
                  Poly c(a.size() + b.size() - 1);
122
                  for (int i = 0; i < a.size(); i++) {
123
                      for (int j = 0; j < b.size(); j++) {
124
                          c[i + j] += a[i] * b[j];
125
126
                  }
127
                  return c;
128
129
              int sz = 1, tot = a.size() + b.size() - 1;
130
              while (sz < tot) {
131
                  sz *= 2;
132
133
              a.a.resize(sz);
134
              b.a.resize(sz);
135
              dft(a.a);
136
              dft(b.a);
137
              for (int i = 0; i < sz; ++i) {
138
                  a.a[i] = a[i] * b[i];
139
              }
140
              idft(a.a);
141
              a.resize(tot);
142
              return a;
143
144
         friend Poly operator*(Z a, Poly b) {
145
              for (int i = 0; i < int(b.size()); i++) {
146
                  b[i] *= a;
147
148
              return b;
149
150
          friend Poly operator*(Poly a, Z b) {
151
              for (int i = 0; i < int(a.size()); i++) {
152
                  a[i] *= b;
153
              }
154
             return a;
155
156
          Poly &operator+=(Poly b) {
157
             return (*this) = (*this) + b;
158
159
          Poly &operator-=(Poly b) {
160
              return (*this) = (*this) - b;
161
162
          Poly &operator*=(Poly b) {
163
             return (*this) = (*this) * b;
164
165
         Poly &operator*=(Z b) {
166
              return (*this) = (*this) * b;
167
168
         Poly deriv() const {
169
              if (a.empty()) {
170
                  return Poly();
171
172
              std::vector<Z> res(size() - 1);
              for (int i = 0; i < size() - 1; ++i) {
173
174
                  res[i] = (i + 1) * a[i + 1];
```

```
175
176
              return Poly(res);
177
          }
178
          Poly integr() const {
179
              std::vector<Z> res(size() + 1);
180
              for (int i = 0; i < size(); ++i) {
181
                  res[i + 1] = a[i] / (i + 1);
182
183
              return Poly(res);
184
185
          Poly inv(int m) const {
186
              Poly x{a[0].inv()};
187
              int k = 1;
188
              while (k < m) {
189
                  k *= 2;
190
                  x = (x * (Poly{2} - modxk(k) * x)).modxk(k);
191
192
              return x.modxk(m);
193
194
          Poly log(int m) const {
195
              return (deriv() * inv(m)).integr().modxk(m);
196
197
          Poly exp(int m) const {
198
              Poly x\{1\};
199
              int k = 1;
200
              while (k < m) {
201
                  k *= 2;
202
                  x = (x * (Poly{1} - x.log(k) + modxk(k))).modxk(k);
203
              }
204
              return x.modxk(m);
205
206
          Poly pow(int k, int m) const {
207
              int i = 0;
208
              while (i < size() && a[i].val() == 0) {
209
                  i++;
210
211
              if (i == size() | 1LL * i * k >= m) {
212
                  return Poly(std::vector<Z>(m));
213
214
              Z v = a[i];
215
              auto f = divxk(i) * v.inv();
216
              return (f.\log(m - i * k) * k).exp(m - i * k).mulxk(i * k) * power(v, k);
217
218
          Poly sqrt(int m) const {
219
              Poly x\{1\};
220
              int k = 1;
221
              while (k < m) {
222
                  k *= 2;
223
                  x = (x + (modxk(k) * x.inv(k)).modxk(k)) * ((P + 1) / 2);
224
              }
225
              return x.modxk(m);
226
227
          Poly mulT(Poly b) const {
228
              if (b.size() == 0) {
229
                  return Poly();
230
231
              int n = b.size();
232
              std::reverse(b.a.begin(), b.a.end());
233
              return ((*this) * b).divxk(n - 1);
234
235
          std::vector<Z> eval(std::vector<Z> x) const {
236
              if (size() == 0) {
237
                  return std::vector<Z>(x.size(), 0);
238
```

```
239
              const int n = std::max(int(x.size()), size());
240
              std::vector<Poly> q(4 * n);
241
              std::vector<Z> ans(x.size());
242
              x.resize(n);
243
              std::function < void(int, int, int) > build = [\&](int p, int l, int r) {
244
                  if (r - 1 == 1) {
245
                      q[p] = Poly{1, -x[1]};
246
                  } else {
247
                      int m = (1 + r) / 2;
248
                      build(2 * p, 1, m);
249
                      build(2 * p + 1, m, r);
250
                      q[p] = q[2 * p] * q[2 * p + 1];
251
252
              };
253
              build(1, 0, n);
254
              std::function<void(int, int, int, const Poly \&)> work = [\&](int p, int 1,
     int r, const Poly &num) {
255
                  if (r - 1 == 1) {
256
                      if (l < int(ans.size())) {</pre>
257
                          ans[1] = num[0];
258
                      }
259
                  } else {
260
                      int m = (1 + r) / 2;
                      work(2 * p, 1, m, num.mulT(q[2 * p + 1]).modxk(m - 1));
261
262
                      work(2 * p + 1, m, r, num.mulT(q[2 * p]).modxk(r - m));
263
264
              };
265
              work(1, 0, n, mulT(q[1].inv(n)));
266
              return ans;
267
          }
268
     };
```

多项式相关 (Poly, with. MInt & MLong) 3.12

```
1
    std::vector<int> rev;
    template<int P>
    std::vector<MInt<P>> roots{0, 1};
 5
    template<int P>
 6
    constexpr MInt<P> findPrimitiveRoot() {
 7
        MInt\langle P \rangle i = 2;
 8
         int k = __builtin_ctz(P - 1);
 9
         while (true) {
10
             if (power(i, (P - 1) / 2) != 1) {
11
                 break;
12
             }
13
             i += 1;
14
15
         return power(i, (P - 1) >> k);
16
17
18
    template<int P>
19
    constexpr MInt<P> primitiveRoot = findPrimitiveRoot<P>();
20
21
    template<>
22
    constexpr MInt<998244353> primitiveRoot<998244353> {31};
23
24
    template<int P>
25
    constexpr void dft(std::vector<MInt<P>> &a) {
26
         int n = a.size();
27
28
         if (int(rev.size()) != n) {
```

```
29
             int k = __builtin_ctz(n) - 1;
30
             rev.resize(n);
31
             for (int i = 0; i < n; i++) {
32
                 rev[i] = rev[i >> 1] >> 1 | (i & 1) << k;
33
             }
34
        }
35
36
        for (int i = 0; i < n; i++) {
37
             if (rev[i] < i) {
38
                 std::swap(a[i], a[rev[i]]);
39
40
         }
41
         if (roots<P>.size() < n) {</pre>
42
             int k = __builtin_ctz(roots<P>.size());
43
             roots<P>.resize(n);
44
             while ((1 << k) < n) {
45
                 auto e = power(primitiveRoot<P>, 1 << (__builtin_ctz(P - 1) - k - 1));</pre>
46
                 for (int i = 1 \iff (k - 1); i \iff (1 \iff k); i++) 
47
                     roots<P>[2 * i] = roots<P>[i];
48
                     roots<P>[2 * i + 1] = roots<P>[i] * e;
49
                 }
50
                 k++;
51
             }
52
53
        for (int k = 1; k < n; k *= 2) {
54
             for (int i = 0; i < n; i += 2 * k) {
55
                 for (int j = 0; j < k; j++) {
56
                     MInt<P> u = a[i + j];
57
                     MInt<P> v = a[i + j + k] * roots<P>[k + j];
58
                     a[i + j] = u + v;
59
                     a[i + j + k] = u - v;
60
                 }
61
             }
62
        }
63
    }
64
65
    template<int P>
66
    constexpr void idft(std::vector<MInt<P>> &a) {
67
         int n = a.size();
68
         std::reverse(a.begin() + 1, a.end());
69
        dft(a);
70
        MInt<P> inv = (1 - P) / n;
71
        for (int i = 0; i < n; i++) {
72
             a[i] *= inv;
73
         }
74
75
76
    template<int P = 998244353>
77
    struct Poly : public std::vector<MInt<P>>> {
78
        using Value = MInt<P>;
79
80
        Poly() : std::vector<Value>() {}
81
        explicit constexpr Poly(int n) : std::vector<Value>(n) {}
82
83
         explicit constexpr Poly(const std::vector<Value> &a) : std::vector<Value>(a) {}
84
        constexpr Poly(const std::initializer_list<Value> &a) : std::vector<Value>(a)
    {}
85
86
         template<class InputIt, class = std::_RequireInputIter<InputIt>>
87
         explicit constexpr Poly(InputIt first, InputIt last) : std::vector<Value>
    (first, last) {}
88
89
         template<class F>
90
         explicit constexpr Poly(int n, F f) : std::vector<Value>(n) {
```

```
91
              for (int i = 0; i < n; i++) {
 92
                  (*this)[i] = f(i);
 93
              }
 94
         }
 95
 96
          constexpr Poly shift(int k) const {
 97
              if (k >= 0) {
 98
                  auto b = *this;
 99
                  b.insert(b.begin(), k, 0);
100
                  return b;
101
              } else if (this->size() <= -k) {</pre>
102
                  return Poly();
103
              } else {
104
                  return Poly(this->begin() + (-k), this->end());
105
106
          }
107
          constexpr Poly trunc(int k) const {
108
              Poly f = *this;
109
              f.resize(k);
110
              return f;
111
          }
112
          constexpr friend Poly operator+(const Poly &a, const Poly &b) {
113
              Poly res(std::max(a.size(), b.size()));
              for (int i = 0; i < a.size(); i++) {
114
115
                  res[i] += a[i];
116
117
              for (int i = 0; i < b.size(); i++) {
118
                  res[i] += b[i];
119
120
             return res;
121
          }
122
          constexpr friend Poly operator-(const Poly &a, const Poly &b) {
123
              Poly res(std::max(a.size(), b.size()));
124
              for (int i = 0; i < a.size(); i++) {
125
                  res[i] += a[i];
126
127
              for (int i = 0; i < b.size(); i++) {
128
                  res[i] -= b[i];
129
130
              return res;
131
          }
132
          constexpr friend Poly operator-(const Poly &a) {
133
              std::vector<Value> res(a.size());
134
              for (int i = 0; i < int(res.size()); i++) {
135
                  res[i] = -a[i];
136
137
              return Poly(res);
138
          }
139
          constexpr friend Poly operator*(Poly a, Poly b) {
140
              if (a.size() == 0 | b.size() == 0) {
141
                  return Poly();
142
143
              if (a.size() < b.size()) {
144
                  std::swap(a, b);
145
146
              int n = 1, tot = a.size() + b.size() - 1;
147
              while (n < tot) {
148
                  n *= 2;
149
150
              if (((P - 1) & (n - 1)) != 0 || b.size() < 128) {
151
                  Poly c(a.size() + b.size() - 1);
152
                  for (int i = 0; i < a.size(); i++) {
153
                      for (int j = 0; j < b.size(); j++) {
154
                          c[i + j] += a[i] * b[j];
```

```
155
156
                  }
157
                  return c;
158
159
              a.resize(n);
160
              b.resize(n);
161
              dft(a);
162
              dft(b);
163
              for (int i = 0; i < n; ++i) {
164
                  a[i] *= b[i];
165
166
              idft(a);
167
              a.resize(tot);
168
              return a;
169
170
         constexpr friend Poly operator*(Value a, Poly b) {
171
              for (int i = 0; i < int(b.size()); i++) {
172
                  b[i] *= a;
173
174
              return b;
175
          }
176
          constexpr friend Poly operator*(Poly a, Value b) {
177
              for (int i = 0; i < int(a.size()); i++) {
178
                  a[i] *= b;
179
              }
180
              return a;
181
182
          constexpr friend Poly operator/(Poly a, Value b) {
183
              for (int i = 0; i < int(a.size()); i++) {
184
                  a[i] /= b;
185
              }
186
              return a;
187
          }
188
          constexpr Poly &operator+=(Poly b) {
189
              return (*this) = (*this) + b;
190
191
          constexpr Poly &operator-=(Poly b) {
192
              return (*this) = (*this) - b;
193
194
          constexpr Poly &operator*=(Poly b) {
195
              return (*this) = (*this) * b;
196
          }
197
         constexpr Poly &operator*=(Value b) {
198
              return (*this) = (*this) * b;
199
          }
200
          constexpr Poly &operator/=(Value b) {
201
              return (*this) = (*this) / b;
202
          }
203
          constexpr Poly deriv() const {
204
              if (this->empty()) {
205
                  return Poly();
206
207
              Poly res(this->size() - 1);
208
              for (int i = 0; i < this->size() - 1; ++i) {
209
                  res[i] = (i + 1) * (*this)[i + 1];
210
211
              return res;
212
          }
213
          constexpr Poly integr() const {
214
              Poly res(this->size() + 1);
215
              for (int i = 0; i < this->size(); ++i) {
216
                  res[i + 1] = (*this)[i] / (i + 1);
217
              }
218
              return res;
```

```
219
         }
220
          constexpr Poly inv(int m) const {
221
              Poly x{(*this)[0].inv()};
222
              int k = 1;
223
              while (k < m) {
224
                  k *= 2;
225
                  x = (x * (Poly{2} - trunc(k) * x)).trunc(k);
226
227
              return x.trunc(m);
228
          }
229
          constexpr Poly log(int m) const {
230
              return (deriv() * inv(m)).integr().trunc(m);
231
232
          constexpr Poly exp(int m) const {
233
              Poly x\{1\};
234
              int k = 1;
235
              while (k < m) {
236
                  k *= 2;
237
                  x = (x * (Poly{1} - x.log(k) + trunc(k))).trunc(k);
238
239
             return x.trunc(m);
240
          }
241
          constexpr Poly pow(int k, int m) const {
242
              int i = 0;
243
              while (i < this->size() \&\& (*this)[i] == 0) {
244
245
246
              if (i == this->size() | 1LL * i * k >= m) {
247
                  return Poly(m);
248
249
             Value v = (*this)[i];
250
              auto f = shift(-i) * v.inv();
251
              return (f.log(m - i * k) * k).exp(m - i * k).shift(i * k) * power(v, k);
252
          }
253
          constexpr Poly sqrt(int m) const {
254
              Poly x\{1\};
255
              int k = 1;
256
              while (k < m) {
257
                  k *= 2;
258
                  x = (x + (trunc(k) * x.inv(k)).trunc(k)) * CInv<2, P>;
259
260
              return x.trunc(m);
261
          }
262
          constexpr Poly mulT(Poly b) const {
263
              if (b.size() == 0) {
264
                  return Poly();
265
266
              int n = b.size();
267
              std::reverse(b.begin(), b.end());
268
              return ((*this) * b).shift(-(n - 1));
269
          }
270
          constexpr std::vector<Value> eval(std::vector<Value> x) const {
271
              if (this->size() == 0) {
272
                  return std::vector<Value>(x.size(), 0);
273
274
              const int n = std::max(x.size(), this->size());
275
              std::vector<Poly> q(4 * n);
276
              std::vector<Value> ans(x.size());
277
              x.resize(n);
278
              std::function < void(int, int, int) > build = [\&](int p, int l, int r) {
279
                  if (r - 1 == 1) {
280
                      q[p] = Poly{1, -x[1]};
281
                  } else {
282
                      int m = (1 + r) / 2;
```

```
283
                      build(2 * p, 1, m);
284
                      build(2 * p + 1, m, r);
285
                      q[p] = q[2 * p] * q[2 * p + 1];
286
                  }
287
              };
288
              build(1, 0, n);
289
              std::function<void(int, int, int, const Poly &)> work = [&](int p, int l,
     int r, const Poly &num) {
290
                  if (r - 1 == 1) {
291
                      if (l < int(ans.size())) {</pre>
292
                          ans[1] = num[0];
293
                      }
294
                  } else {
295
                      int m = (1 + r) / 2;
296
                      work(2 * p, 1, m, num.mulT(q[2 * p + 1]).resize(m - 1));
297
                      work(2 * p + 1, m, r, num.mulT(q[2 * p]).resize(r - m));
298
                  }
299
              };
300
              work(1, 0, n, mulT(q[1].inv(n)));
301
              return ans;
302
          }
303
     };
304
305
     template<int P = 998244353>
306
     Poly<P> berlekampMassey(const Poly<P> &s) {
307
         Poly<P> c;
308
         Poly<P> oldC;
309
          int f = -1;
310
          for (int i = 0; i < s.size(); i++) {
311
              auto delta = s[i];
312
              for (int j = 1; j <= c.size(); j++) {
313
                  delta -= c[j - 1] * s[i - j];
314
315
              if (delta == 0) {
316
                  continue;
317
318
              if (f == -1) {
319
                  c.resize(i + 1);
320
                  f = i;
321
              } else {
322
                  auto d = oldC;
323
                  d *= -1;
                  d.insert(d.begin(), 1);
324
325
                  MInt<P> df1 = 0;
326
                  for (int j = 1; j <= d.size(); j++) {
327
                      df1 += d[j - 1] * s[f + 1 - j];
328
329
                  assert(df1 != 0);
330
                  auto coef = delta / df1;
331
                  d *= coef;
332
                  Poly<P> zeros(i - f - 1);
333
                  zeros.insert(zeros.end(), d.begin(), d.end());
334
                  d = zeros;
335
                  auto temp = c;
336
                  c += d;
337
                  if (i - temp.size() > f - oldC.size()) {
338
                      oldC = temp;
339
                      f = i;
340
                  }
341
              }
342
          }
343
          c *= -1;
         c.insert(c.begin(), 1);
344
345
         return c;
```

```
346
347
348
349
     template<int P = 998244353>
350
     MInt<P> linearRecurrence(Poly<P> p, Poly<P> q, i64 n) {
351
          int m = q.size() - 1;
352
          while (n > 0) {
353
              auto newq = q;
354
              for (int i = 1; i \leftarrow m; i += 2) {
355
                  newq[i] *= -1;
356
357
              auto newp = p * newq;
358
              newq = q * newq;
359
              for (int i = 0; i < m; i++) {
360
                  p[i] = newp[i * 2 + n % 2];
361
362
              for (int i = 0; i <= m; i++) {
363
                  q[i] = newq[i * 2];
364
365
              n /= 2;
366
367
          return p[0] / q[0];
368
369
370
      struct Comb {
371
          int n;
372
          std::vector<Z> _fac;
373
          std::vector<Z> _invfac;
374
          std::vector<Z> _inv;
375
376
          Comb() : n{0}, _fac{1}, _invfac{1}, _inv{0} {}
377
          Comb(int n) : Comb() {
378
              init(n);
379
          }
380
381
          void init(int m) {
382
              m = std::min(m, Z::getMod() - 1);
383
              if (m <= n) return;</pre>
384
              _fac.resize(m + 1);
385
              _invfac.resize(m + 1);
386
              _{inv.resize(m + 1);}
387
388
              for (int i = n + 1; i \leftarrow m; i++) {
389
                  _fac[i] = _fac[i - 1] * i;
390
391
              _invfac[m] = _fac[m].inv();
392
              for (int i = m; i > n; i--) {
393
                  _invfac[i - 1] = _invfac[i] * i;
394
                  _inv[i] = _invfac[i] * _fac[i - 1];
395
              }
396
              n = m;
397
          }
398
399
          Z fac(int m) {
400
              if (m > n) init(2 * m);
401
              return _fac[m];
402
          Z invfac(int m) {
403
404
              if (m > n) init(2 * m);
405
              return _invfac[m];
406
407
          Z inv(int m) {
408
              if (m > n) init(2 * m);
409
              return _inv[m];
```

```
410
411
         Z binom(int n, int m) {
412
             if (n < m | | m < 0) return 0;
413
             return fac(n) * invfac(m) * invfac(n - m);
414
         }
415
     } comb;
416
417
     Poly<P> get(int n, int m) {
418
         if (m == 0) {
419
             return Poly(n + 1);
420
          }
421
         if (m % 2 == 1) {
422
             auto f = get(n, m - 1);
423
             Z p = 1;
424
             for (int i = 0; i <= n; i++) {
425
                 f[n - i] += comb.binom(n, i) * p;
426
                 p *= m;
427
428
             return f;
429
         }
430
         auto f = get(n, m / 2);
431
         auto fm = f;
432
         for (int i = 0; i <= n; i++) {
433
             fm[i] *= comb.fac(i);
434
          }
435
         Poly pw(n + 1);
436
         pw[0] = 1;
437
         for (int i = 1; i <= n; i++) {
438
             pw[i] = pw[i - 1] * (m / 2);
439
          }
440
         for (int i = 0; i <= n; i++) {
441
             pw[i] *= comb.invfac(i);
442
          }
443
         fm = fm.mulT(pw);
444
         for (int i = 0; i \leftarrow n; i++) {
445
             fm[i] *= comb.invfac(i);
446
447
         return f + fm;
448
```

/END/

数据结构

树状数组 (Fenwick 新版) 4.1

```
1
    template <typename T>
 2
    struct Fenwick {
 3
        int n;
 4
        std::vector<T> a;
 5
 6
         Fenwick(int n_{=} = 0) {
 7
             init(n_);
 8
         }
 9
10
        void init(int n_) {
11
             n = n_{j}
12
             a.assign(n, T{});
13
         }
14
15
         void add(int x, const T &v) {
16
            for (int i = x + 1; i \le n; i += i \& -i) {
17
                 a[i - 1] = a[i - 1] + v;
18
19
        }
20
21
        T sum(int x) {
22
             T ans{};
23
             for (int i = x; i > 0; i -= i \& -i) {
24
                 ans = ans + a[i - 1];
25
26
             return ans;
27
        }
28
29
        T rangeSum(int 1, int r) {
30
             return sum(r) - sum(l);
31
32
33
        int select(const T &k) {
34
             int x = 0;
35
             T cur{};
36
             for (int i = 1 \ll std::_lg(n); i; i \neq 2) {
37
                 if (x + i \le n \&\& cur + a[x + i - 1] \le k) {
38
                     x += i;
39
                     cur = cur + a[x - 1];
40
                 }
41
             }
42
             return x;
43
44 };
```

并查集 (DSU) 4.2

```
1
   struct DSU {
2
       std::vector<int> f, siz;
3
4
       DSU() {}
5
       DSU(int n) {
6
            init(n);
7
        }
8
9
       void init(int n) {
```

```
10
             f.resize(n);
11
             std::iota(f.begin(), f.end(), 0);
12
             siz.assign(n, 1);
13
        }
14
15
        int find(int x) {
16
             while (x != f[x]) {
17
                 x = f[x] = f[f[x]];
18
19
            return x;
20
        }
21
22
        bool same(int x, int y) {
23
            return find(x) == find(y);
24
25
26
        bool merge(int x, int y) {
27
            x = find(x);
28
             y = find(y);
29
            if (x == y) {
30
                 return false;
31
32
            siz[x] += siz[y];
33
            f[y] = x;
34
            return true;
35
        }
36
37
        int size(int x) {
38
             return siz[find(x)];
39
40
    };
```

线段树 4.3

线段树 (SegmentTree 基础区间加乘) 4.3.1

```
1
    struct SegmentTree {
 2
         int n;
 3
         std::vector<int> tag, sum;
 4
         SegmentTree(int n_{-}) : n(n_{-}), tag(4 * n, 1), sum(4 * n) {}
 5
 6
         void pull(int p) {
 7
             sum[p] = (sum[2 * p] + sum[2 * p + 1]) % P;
 8
         }
 9
10
         void mul(int p, int v) {
11
             tag[p] = 1LL * tag[p] * v % P;
12
             sum[p] = 1LL * sum[p] * v % P;
13
         }
14
15
         void push(int p) {
16
             mul(2 * p, tag[p]);
17
             mul(2 * p + 1, tag[p]);
18
             tag[p] = 1;
19
         }
20
21
         int query(int p, int 1, int r, int x, int y) {
22
             if (1 >= y || r <= x) {
23
                 return 0;
24
             }
25
             if (1 >= x && r <= y) {
26
                 return sum[p];
```

```
28
             int m = (1 + r) / 2;
29
             push(p);
30
             return (query(2 * p, 1, m, x, y) + query(2 * p + 1, m, r, x, y)) % P;
31
        }
32
33
        int query(int x, int y) {
34
             return query(1, 0, n, x, y);
35
36
37
        void rangeMul(int p, int l, int r, int x, int y, int v) {
38
             if (1 >= y || r <= x) {
39
                 return;
40
             }
             if (1 >= x \& r <= y) {
41
42
                 return mul(p, v);
43
44
             int m = (1 + r) / 2;
45
             push(p);
46
             rangeMul(2 * p, 1, m, x, y, v);
47
             rangeMul(2 * p + 1, m, r, x, y, v);
48
             pull(p);
49
        }
50
51
        void rangeMul(int x, int y, int v) {
52
             rangeMul(1, 0, n, x, y, v);
53
         }
54
55
        void add(int p, int l, int r, int x, int v) {
56
             if (r - 1 == 1) {
57
                 sum[p] = (sum[p] + v) \% P;
58
                 return;
59
             }
             int m = (1 + r) / 2;
60
61
             push(p);
62
             if (x < m) {
63
                 add(2 * p, 1, m, x, v);
64
             } else {
65
                 add(2 * p + 1, m, r, x, v);
66
67
             pull(p);
68
        }
69
70
        void add(int x, int v) {
71
             add(1, 0, n, x, v);
72
73
    };
```

线段树 (SegmentTree+Info 查找前驱后继) 4.3.2

```
1
    template<class Info>
 2
    struct SegmentTree {
 3
        int n;
 4
         std::vector<Info> info;
 5
        SegmentTree() : n(0) {}
 6
        SegmentTree(int n_, Info v_ = Info()) {
 7
             init(n_, v_);
 8
        }
 9
        template<class T>
10
         SegmentTree(std::vector<T> init_) {
11
             init(init_);
12
         }
```

```
void init(int n_, Info v_ = Info()) {
14
             init(std::vector(n_, v_));
15
         }
16
         template<class T>
17
         void init(std::vector<T> init_) {
18
             n = init_.size();
19
             info.assign(4 << std::__lg(n), Info());</pre>
             std::function<void(int, int, int)> build = [&](int p, int l, int r) {
20
21
                 if (r - l == 1) {
22
                     info[p] = init_[1];
23
                     return;
24
                 }
25
                 int m = (1 + r) / 2;
26
                 build(2 * p, l, m);
                 build(2 * p + 1, m, r);
27
28
                 pull(p);
29
30
             build(1, 0, n);
31
32
        void pull(int p) {
33
             info[p] = info[2 * p] + info[2 * p + 1];
34
35
         void modify(int p, int l, int r, int x, const Info &v) {
36
             if (r - 1 == 1) {
37
                 info[p] = v;
38
                 return;
39
40
             int m = (1 + r) / 2;
41
             if (x < m) {
42
                 modify(2 * p, 1, m, x, v);
43
             } else {
44
                 modify(2 * p + 1, m, r, x, v);
45
             }
46
             pull(p);
47
48
         void modify(int p, const Info &v) {
49
             modify(1, 0, n, p, v);
50
51
         Info rangeQuery(int p, int l, int r, int x, int y) {
52
             if (1 >= y || r <= x) {
53
                 return Info();
54
55
             if (1 >= x && r <= y) {
56
                 return info[p];
57
58
             int m = (1 + r) / 2;
59
             return rangeQuery(2 * p, 1, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
60
61
         Info rangeQuery(int 1, int r) {
62
             return rangeQuery(1, 0, n, 1, r);
63
         }
64
         template<class F>
65
         int findFirst(int p, int l, int r, int x, int y, F pred) {
66
             if (1 \ge y \mid | r \le x \mid | !pred(info[p])) {
67
                 return -1;
68
69
             if (r - 1 == 1) {
70
                 return 1;
71
             }
72
             int m = (1 + r) / 2;
73
             int res = findFirst(2 * p, 1, m, x, y, pred);
74
             if (res == -1) {
75
                 res = findFirst(2 * p + 1, m, r, x, y, pred);
76
```

```
return res;
 78
          }
 79
         template<class F>
 80
          int findFirst(int 1, int r, F pred) {
 81
              return findFirst(1, 0, n, l, r, pred);
 82
          }
 83
          template<class F>
 84
          int findLast(int p, int l, int r, int x, int y, F pred) {
 85
              if (1 \ge y \mid | r \le x \mid | !pred(info[p]))  {
 86
                  return -1;
 87
 88
              if (r - 1 == 1) {
 89
                  return 1;
 90
              }
 91
              int m = (1 + r) / 2;
 92
              int res = findLast(2 * p + 1, m, r, x, y, pred);
 93
              if (res == -1) {
 94
                  res = findLast(2 * p, 1, m, x, y, pred);
 95
 96
              return res;
 97
          }
 98
         template<class F>
99
          int findLast(int 1, int r, F pred) {
100
              return findLast(1, 0, n, 1, r, pred);
101
          }
102
     };
103
     struct Info {
104
          int cnt = 0;
105
          i64 sum = 0;
106
          i64 ans = 0;
107
     };
108
     Info operator+(Info a, Info b) {
109
          Info c;
110
          c.cnt = a.cnt + b.cnt;
111
          c.sum = a.sum + b.sum;
112
          c.ans = a.ans + b.ans + a.cnt * b.sum - a.sum * b.cnt;
113
         return c;
114
```

4.3.3 线段树 (SegmentTree+Info+Merge 区间合并)

```
1
    template<class Info>
 2
    struct SegmentTree {
 3
         int n;
 4
         std::vector<Info> info;
 5
         SegmentTree() : n(0) {}
 6
         SegmentTree(int n_, Info v_ = Info()) {
 7
             init(n_, v_);
 8
         }
 9
         template<class T>
10
         SegmentTree(std::vector<T> init_) {
11
             init(init );
12
13
         void init(int n_, Info v_ = Info()) {
14
             init(std::vector(n_, v_));
15
         }
16
        template<class T>
17
         void init(std::vector<T> init_) {
18
             n = init_.size();
19
             info.assign(4 << std::__lg(n), Info());</pre>
20
             std::function < void(int, int, int) > build = [&](int p, int l, int r) {
21
                 if (r - 1 == 1) {
                                            47
```

```
22
                     info[p] = init_[1];
23
                     return;
24
                 }
25
                 int m = (1 + r) / 2;
                 build(2 * p, 1, m);
26
27
                 build(2 * p + 1, m, r);
28
                 pull(p);
29
             };
30
             build(1, 0, n);
31
         }
32
         void pull(int p) {
33
             info[p] = info[2 * p] + info[2 * p + 1];
34
35
         void modify(int p, int l, int r, int x, const Info &v) {
36
             if (r - 1 == 1) {
37
                 info[p] = v;
38
                 return;
39
40
             int m = (1 + r) / 2;
41
            if (x < m) {
42
                 modify(2 * p, 1, m, x, v);
43
             } else {
44
                 modify(2 * p + 1, m, r, x, v);
45
             }
46
            pull(p);
47
48
         void modify(int p, const Info &v) {
49
             modify(1, 0, n, p, v);
50
51
         Info rangeQuery(int p, int l, int r, int x, int y) {
52
             if (1 >= y || r <= x) {
53
                 return Info();
54
             }
55
             if (1 >= x && r <= y) {
56
                 return info[p];
57
58
             int m = (1 + r) / 2;
59
             return rangeQuery(2 * p, 1, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
60
61
         Info rangeQuery(int 1, int r) {
62
             return rangeQuery(1, 0, n, l, r);
63
         }
64
        template<class F>
65
         int findFirst(int p, int l, int r, int x, int y, F pred) {
66
             if (1 \ge y \mid | r \le x \mid | !pred(info[p])) {
67
                 return -1;
68
69
             if (r - 1 == 1) {
70
                 return 1;
71
             }
72
             int m = (1 + r) / 2;
73
             int res = findFirst(2 * p, 1, m, x, y, pred);
74
             if (res == -1) {
75
                 res = findFirst(2 * p + 1, m, r, x, y, pred);
76
77
             return res;
78
         }
79
         template<class F>
80
         int findFirst(int 1, int r, F pred) {
81
             return findFirst(1, 0, n, 1, r, pred);
82
         }
83
         template<class F>
84
         int findLast(int p, int 1, int r, int x, int y, F pred) {
85
             if (1 >= y || r <= x || !pred(info[p])) {
```

```
86
                  return -1;
 87
              }
 88
              if (r - 1 == 1) {
 89
                  return 1;
 90
              }
 91
              int m = (1 + r) / 2;
 92
              int res = findLast(2 * p + 1, m, r, x, y, pred);
 93
              if (res == -1) {
 94
                  res = findLast(2 * p, 1, m, x, y, pred);
 95
              }
 96
              return res;
 97
          }
 98
          template<class F>
 99
          int findLast(int 1, int r, F pred) {
100
              return findLast(1, 0, n, l, r, pred);
101
          }
102
     };
103
104
     struct Info {
105
          int x = 0;
106
          int cnt = 0;
107
108
109
     Info operator+(Info a, Info b) {
110
          if (a.x == b.x) {
111
              return {a.x, a.cnt + b.cnt};
112
          } else if (a.cnt > b.cnt) {
113
              return {a.x, a.cnt - b.cnt};
114
          } else {
115
              return {b.x, b.cnt - a.cnt};
116
          }
117
```

懒标记线段树 4.4

懒标记线段树 (LazySegmentTree 基础区间修改) 4.4.1

```
1
    template<class Info, class Tag>
 2
    struct LazySegmentTree {
 3
         const int n;
 4
         std::vector<Info> info;
 5
         std::vector<Tag> tag;
 6
        LazySegmentTree(int n) : n(n), info(4 << std::__lg(n)), tag(4 << std::__lg(n))
    {}
 7
        LazySegmentTree(std::vector<Info> init) : LazySegmentTree(init.size()) {
 8
             std::function<void(int, int, int)> build = [&](int p, int 1, int r) {
 9
                 if (r - 1 == 1) {
10
                     info[p] = init[l];
11
                     return;
12
                 }
13
                 int m = (1 + r) / 2;
14
                 build(2 * p, 1, m);
15
                 build(2 * p + 1, m, r);
16
                 pull(p);
17
             };
18
             build(1, 0, n);
19
20
         void pull(int p) {
21
             info[p] = info[2 * p] + info[2 * p + 1];
22
         }
23
        void apply(int p, const Tag &v) {
24
             info[p].apply(v);
```

```
tag[p].apply(v);
26
        }
27
        void push(int p) {
28
             apply(2 * p, tag[p]);
29
             apply(2 * p + 1, tag[p]);
30
             tag[p] = Tag();
31
32
        void modify(int p, int l, int r, int x, const Info &v) {
33
             if (r - l == 1) {
34
                 info[p] = v;
35
                 return;
36
             }
37
             int m = (1 + r) / 2;
38
             push(p);
39
             if (x < m) {
40
                 modify(2 * p, 1, m, x, v);
41
             } else {
42
                 modify(2 * p + 1, m, r, x, v);
43
44
             pull(p);
45
         }
46
        void modify(int p, const Info &v) {
47
             modify(1, 0, n, p, v);
48
49
        Info rangeQuery(int p, int l, int r, int x, int y) {
50
             if (1 >= y || r <= x) {
51
                 return Info();
52
53
             if (1 >= x && r <= y) {
54
                 return info[p];
55
             }
56
             int m = (1 + r) / 2;
57
             push(p);
             return rangeQuery(2 * p, 1, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
58
59
60
        Info rangeQuery(int 1, int r) {
61
             return rangeQuery(1, 0, n, 1, r);
62
        void rangeApply(int p, int l, int r, int x, int y, const Tag &v) {
63
64
             if (1 >= y || r <= x) {
65
                 return;
66
67
             if (1 >= x && r <= y) {
68
                 apply(p, v);
69
                 return;
70
71
             int m = (1 + r) / 2;
72
             push(p);
73
             rangeApply(2 * p, 1, m, x, y, v);
74
             rangeApply(2 * p + 1, m, r, x, y, v);
75
             pull(p);
76
         }
77
        void rangeApply(int 1, int r, const Tag &v) {
78
             return rangeApply(1, 0, n, l, r, v);
79
80
        void half(int p, int l, int r) {
81
             if (info[p].act == 0) {
82
                 return;
83
84
             if ((info[p].min + 1) / 2 == (info[p].max + 1) / 2) {
85
                 apply(p, \{-(\inf o[p].min + 1) / 2\});
86
                 return;
87
             }
88
             int m = (1 + r) / 2;
```

```
89
              push(p);
 90
              half(2 * p, 1, m);
 91
              half(2 * p + 1, m, r);
 92
              pull(p);
 93
          }
 94
          void half() {
 95
              half(1, 0, n);
 96
 97
     };
 98
 99
     constexpr i64 inf = 1E18;
100
101
     struct Tag {
102
         i64 \text{ add} = 0;
103
104
         void apply(Tag t) {
105
              add += t.add;
106
107
     };
108
109
     struct Info {
110
         i64 min = inf;
111
         i64 max = -inf;
112
         i64 sum = 0;
113
         i64 act = 0;
114
115
         void apply(Tag t) {
116
              min += t.add;
117
              max += t.add;
118
              sum += act * t.add;
119
          }
120
     };
121
122
     Info operator+(Info a, Info b) {
123
         Info c;
124
          c.min = std::min(a.min, b.min);
125
          c.max = std::max(a.max, b.max);
126
         c.sum = a.sum + b.sum;
127
         c.act = a.act + b.act;
128
         return c;
129
```

懒标记线段树 (LazySegmentTree 查找前驱后继) 4.4.2

```
1
    template<class Info, class Tag>
 2
    struct LazySegmentTree {
 3
        int n;
 4
        std::vector<Info> info;
 5
        std::vector<Tag> tag;
 6
        LazySegmentTree() : n(0) {}
 7
        LazySegmentTree(int n_, Info v_ = Info()) {
 8
             init(n_, v_);
 9
10
        template<class T>
11
        LazySegmentTree(std::vector<T> init_) {
12
             init(init_);
13
14
         void init(int n_, Info v_ = Info()) {
15
             init(std::vector(n_, v_));
16
17
         template<class T>
18
        void init(std::vector<T> init_) {
```

```
19
             n = init_.size();
20
             info.assign(4 << std::__lg(n), Info());</pre>
21
             tag.assign(4 << std::__lg(n), Tag());</pre>
22
             std::function < void(int, int, int) > build = [&](int p, int l, int r) {
23
                 if (r - 1 == 1) {
24
                     info[p] = init_[1];
25
                     return;
26
                 }
27
                 int m = (1 + r) / 2;
28
                 build(2 * p, 1, m);
29
                 build(2 * p + 1, m, r);
30
                 pull(p);
31
             };
32
             build(1, 0, n);
33
         }
34
        void pull(int p) {
35
             info[p] = info[2 * p] + info[2 * p + 1];
36
37
         void apply(int p, const Tag &v) {
38
             info[p].apply(v);
39
             tag[p].apply(v);
40
41
         void push(int p) {
42
             apply(2 * p, tag[p]);
43
             apply(2 * p + 1, tag[p]);
44
             tag[p] = Tag();
45
46
         void modify(int p, int l, int r, int x, const Info &v) {
47
             if (r - 1 == 1) {
48
                 info[p] = v;
49
                 return;
50
             }
51
             int m = (1 + r) / 2;
52
             push(p);
53
             if (x < m) {
54
                 modify(2 * p, 1, m, x, v);
55
             } else {
56
                 modify(2 * p + 1, m, r, x, v);
57
58
             pull(p);
59
         }
60
         void modify(int p, const Info &v) {
61
             modify(1, 0, n, p, v);
62
63
         Info rangeQuery(int p, int l, int r, int x, int y) {
64
             if (1 >= y || r <= x) {
65
                 return Info();
66
67
             if (1 >= x \&\& r <= y) {
68
                 return info[p];
69
             }
70
             int m = (1 + r) / 2;
71
             push(p);
72
             return rangeQuery(2 * p, 1, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
73
74
         Info rangeQuery(int 1, int r) {
75
             return rangeQuery(1, 0, n, 1, r);
76
77
         void rangeApply(int p, int l, int r, int x, int y, const Tag \&v) {
78
             if (1 >= y || r <= x) {
79
                 return;
80
81
             if (1 >= x && r <= y) {
82
                 apply(p, v);
```

```
83
                  return;
 84
              }
 85
              int m = (1 + r) / 2;
 86
              push(p);
              rangeApply(2 * p, 1, m, x, y, v);
 87
 88
              rangeApply(2 * p + 1, m, r, x, y, v);
 89
              pull(p);
 90
 91
          void rangeApply(int 1, int r, const Tag &v) {
 92
              return rangeApply(1, 0, n, l, r, v);
 93
          }
 94
          template<class F>
 95
          int findFirst(int p, int l, int r, int x, int y, F pred) {
 96
              if (1 >= y || r <= x || !pred(info[p])) {
 97
                  return -1;
 98
 99
              if (r - 1 == 1) {
100
                  return 1;
101
              }
102
              int m = (1 + r) / 2;
103
              push(p);
104
              int res = findFirst(2 * p, 1, m, x, y, pred);
105
              if (res == -1) {
                  res = findFirst(2 * p + 1, m, r, x, y, pred);
106
107
              }
108
              return res;
109
          }
110
          template<class F>
111
          int findFirst(int 1, int r, F pred) {
112
              return findFirst(1, 0, n, l, r, pred);
113
          }
114
          template<class F>
115
          int findLast(int p, int l, int r, int x, int y, F pred) {
116
              if (1 >= y || r <= x || !pred(info[p])) {
117
                  return -1;
118
119
              if (r - 1 == 1) {
120
                  return 1;
121
122
              int m = (1 + r) / 2;
123
              push(p);
124
              int res = findLast(2 * p + 1, m, r, x, y, pred);
125
              if (res == -1) {
126
                  res = findLast(2 * p, 1, m, x, y, pred);
127
              }
128
              return res;
129
130
          template<class F>
131
          int findLast(int 1, int r, F pred) {
132
              return findLast(1, 0, n, 1, r, pred);
133
          }
134
     };
135
136
      struct Tag {
137
         i64 \ a = 0, \ b = 0;
138
          void apply(Tag t) {
139
              a = std::min(a, b + t.a);
140
              b += t.b;
141
          }
142
     };
143
144
     int k;
145
146
     struct Info {
```

```
147
          i64 x = 0;
148
          void apply(Tag t) {
149
              x += t.a;
150
              if (x < 0) {
151
                  x = (x \% k + k) \% k;
152
153
              x += t.b - t.a;
154
          }
155
      };
156
     Info operator+(Info a, Info b) {
157
          return {a.x + b.x};
158
```

懒标记线段树 (LazySegmentTree 二分修改) 4.4.3

```
1
    constexpr int inf = 1E9 + 1;
 2
    template<class Info, class Tag>
 3
    struct LazySegmentTree {
 4
         const int n;
 5
         std::vector<Info> info;
 6
         std::vector<Tag> tag;
 7
        LazySegmentTree(int n) : n(n), info(4 << std::__lg(n)), tag(4 << std::__lg(n))
 8
        LazySegmentTree(std::vector<Info> init) : LazySegmentTree(init.size()) {
 9
             std::function < void(int, int, int) > build = [&](int p, int l, int r) {
10
                 if (r - 1 == 1) {
11
                     info[p] = init[l];
12
                     return;
13
                 }
14
                 int m = (1 + r) / 2;
15
                 build(2 * p, 1, m);
16
                 build(2 * p + 1, m, r);
17
                 pull(p);
18
             };
19
             build(1, 0, n);
20
21
        void pull(int p) {
22
             info[p] = info[2 * p] + info[2 * p + 1];
23
        }
24
        void apply(int p, const Tag &v) {
25
             info[p].apply(v);
26
             tag[p].apply(v);
27
         }
28
        void push(int p) {
29
             apply(2 * p, tag[p]);
30
             apply(2 * p + 1, tag[p]);
31
             tag[p] = Tag();
32
33
        void modify(int p, int l, int r, int x, const Info &v) {
34
             if (r - 1 == 1) {
35
                 info[p] = v;
36
                 return;
37
38
             int m = (1 + r) / 2;
39
             push(p);
40
             if (x < m) {
41
                 modify(2 * p, 1, m, x, v);
42
             } else {
43
                 modify(2 * p + 1, m, r, x, v);
44
45
             pull(p);
46
         }
```

```
void modify(int p, const Info &v) {
 48
              modify(1, 0, n, p, v);
 49
          }
 50
          Info rangeQuery(int p, int l, int r, int x, int y) {
 51
              if (1 >= y || r <= x) {
 52
                  return Info();
 53
 54
              if (1 >= x && r <= y) {
 55
                  return info[p];
 56
              }
 57
              int m = (1 + r) / 2;
 58
              push(p);
 59
              return rangeQuery(2 * p, 1, m, x, y) + rangeQuery(2 * p + 1, m, r, x, y);
 60
 61
         Info rangeQuery(int 1, int r) {
 62
              return rangeQuery(1, 0, n, 1, r);
 63
 64
         void rangeApply(int p, int l, int r, int x, int y, const Tag &v) {
 65
              if (1 >= y || r <= x) {
 66
                  return;
 67
 68
              if (1 >= x && r <= y) {
 69
                  apply(p, v);
 70
                  return;
 71
              }
 72
              int m = (1 + r) / 2;
 73
              push(p);
 74
              rangeApply(2 * p, 1, m, x, y, v);
 75
              rangeApply(2 * p + 1, m, r, x, y, v);
 76
              pull(p);
 77
          }
 78
         void rangeApply(int 1, int r, const Tag &v) {
 79
              return rangeApply(1, 0, n, l, r, v);
 80
 81
         void maintainL(int p, int l, int r, int pre) {
 82
              if (info[p].difl > 0 && info[p].maxlowl < pre) {</pre>
 83
                  return;
 84
 85
              if (r - 1 == 1) {
 86
                  info[p].max = info[p].maxlowl;
 87
                  info[p].maxl = info[p].maxr = 1;
 88
                  info[p].maxlowl = info[p].maxlowr = -inf;
 89
                  return;
 90
              }
 91
              int m = (1 + r) / 2;
 92
              push(p);
 93
              maintainL(2 * p, 1, m, pre);
94
              pre = std::max(pre, info[2 * p].max);
 95
              maintainL(2 * p + 1, m, r, pre);
 96
              pull(p);
 97
          }
 98
         void maintainL() {
99
              maintainL(1, 0, n, -1);
100
101
         void maintainR(int p, int 1, int r, int suf) {
102
              if (info[p].difr > 0 && info[p].maxlowr < suf) {</pre>
103
                  return;
104
105
              if (r - 1 == 1) {
106
                  info[p].max = info[p].maxlowl;
107
                  info[p].maxl = info[p].maxr = 1;
108
                  info[p].maxlowl = info[p].maxlowr = -inf;
109
                  return;
110
```

```
111
              int m = (1 + r) / 2;
112
              push(p);
113
              maintainR(2 * p + 1, m, r, suf);
114
              suf = std::max(suf, info[2 * p + 1].max);
115
              maintainR(2 * p, 1, m, suf);
116
              pull(p);
117
118
          void maintainR() {
119
              maintainR(1, 0, n, -1);
120
          }
121
      };
122
123
      struct Tag {
124
          int add = 0;
125
126
          void apply(Tag t) & {
127
              add += t.add;
128
129
     };
130
131
      struct Info {
132
          int max = -1;
133
          int maxl = -1;
134
          int maxr = -1;
135
          int difl = inf;
136
          int difr = inf;
137
          int maxlowl = -inf;
138
          int maxlowr = -inf;
139
140
          void apply(Tag t) & {
141
              if (max != -1) {
142
                  max += t.add;
143
144
              difl += t.add;
145
              difr += t.add;
146
          }
147
      };
148
149
      Info operator+(Info a, Info b) {
150
          Info c;
151
          if (a.max > b.max) {
152
              c.max = a.max;
153
              c.maxl = a.maxl;
154
              c.maxr = a.maxr;
155
          } else if (a.max < b.max) {</pre>
156
              c.max = b.max;
157
              c.maxl = b.maxl;
158
              c.maxr = b.maxr;
          } else {
159
160
              c.max = a.max;
161
              c.maxl = a.maxl;
162
              c.maxr = b.maxr;
163
          }
164
165
          c.difl = std::min(a.difl, b.difl);
166
          c.difr = std::min(a.difr, b.difr);
167
          if (a.max != -1) {
168
              c.difl = std::min(c.difl, a.max - b.maxlowl);
169
          }
170
          if (b.max != -1) {
171
              c.difr = std::min(c.difr, b.max - a.maxlowr);
172
          }
173
          if (a.max == -1) {
174
```

```
175
              c.maxlowl = std::max(a.maxlowl, b.maxlowl);
176
         } else {
177
             c.maxlowl = a.maxlowl;
178
         if (b.max == -1) {
179
180
             c.maxlowr = std::max(a.maxlowr, b.maxlowr);
181
          } else {
182
             c.maxlowr = b.maxlowr;
183
184
         return c;
185
```

4.5 取模类

4.5.1 **取模类 (MLong & MInt)**

```
1 constexpr int P = 998244353;
    using i64 = long long;
    // assume -P <= x < 2P
 4
    int norm(int x) {
 5
        if (x < 0) {
 6
            x += P;
 7
 8
        if (x >= P) {
9
            x -= P;
10
        }
11
        return x;
12
13
    template<class T>
14
    T power(T a, i64 b) {
15
        T res = 1;
16
        for (; b; b /= 2, a *= a) {
17
            if (b % 2) {
18
                 res *= a;
19
             }
20
21
        return res;
22
23
    struct Z {
24
        int x;
25
        Z(int x = 0) : x(norm(x)) \{\}
26
        Z(i64 x) : x(norm(x % P)) {}
27
         int val() const {
28
            return x;
29
         }
30
        Z operator-() const {
31
             return Z(norm(P - x));
32
33
        Z inv() const {
34
             assert(x != 0);
35
             return power(*this, P - 2);
36
37
         Z &operator*=(const Z &rhs) {
38
            x = i64(x) * rhs.x % P;
39
             return *this;
40
41
         Z &operator+=(const Z &rhs) {
42
             x = norm(x + rhs.x);
43
            return *this;
44
45
         Z &operator-=(const Z &rhs) {
46
             x = norm(x - rhs.x);
```

```
47
            return *this;
48
49
        Z &operator/=(const Z &rhs) {
50
            return *this *= rhs.inv();
51
52
        friend Z operator*(const Z &lhs, const Z &rhs) {
53
            Z res = lhs;
54
            res *= rhs;
55
             return res;
56
        }
57
        friend Z operator+(const Z &lhs, const Z &rhs) {
58
            Z res = lhs;
59
             res += rhs;
60
            return res;
61
         }
62
        friend Z operator-(const Z &lhs, const Z &rhs) {
63
            Z res = lhs;
64
            res -= rhs;
65
            return res;
66
67
        friend Z operator/(const Z &lhs, const Z &rhs) {
68
            Z res = lhs;
69
             res /= rhs;
70
             return res;
71
72
        friend std::istream &operator>>(std::istream &is, Z &a) {
73
            i64 v;
74
            is >> v;
75
             a = Z(v);
76
            return is;
77
78
         friend std::ostream &operator<<(std::ostream &os, const Z &a) {
79
            return os << a.val();
80
81
    };
```

4.5.2 取模类 (MLong & MInt 新版)

根据输入内容动态修改 MOD 的方法: Z::setMod(p); 。

```
1
    template<class T>
 2
    constexpr T power(T a, i64 b) {
 3
        T res = 1;
 4
        for (; b; b /= 2, a *= a) {
 5
            if (b % 2) {
 6
                 res *= a;
 7
             }
 8
        }
 9
        return res;
10
    }
11
12
    constexpr i64 mul(i64 a, i64 b, i64 p) {
        i64 res = a * b - i64(1.L * a * b / p) * p;
13
14
        res %= p;
15
        if (res < 0) {
16
            res += p;
17
        }
        return res;
18
19
20
    template<i64 P>
21
    struct MLong {
22
        i64 x;
```

```
23
         constexpr MLong() : x{} {}
24
         constexpr MLong(i64 x) : x{norm(x % getMod())} {}
25
26
        static i64 Mod;
27
         constexpr static i64 getMod() {
28
             if (P > 0) {
29
                 return P;
30
             } else {
31
                 return Mod;
32
             }
33
         }
34
         constexpr static void setMod(i64 Mod_) {
35
            Mod = Mod_{;}
36
         }
37
        constexpr i64 norm(i64 x) const {
38
             if (x < 0) {
39
                 x += getMod();
40
41
             if (x \ge getMod()) {
42
                x -= getMod();
43
             }
44
             return x;
45
         }
46
        constexpr i64 val() const {
47
            return x;
48
         }
49
         explicit constexpr operator i64() const {
50
             return x;
51
         }
52
         constexpr MLong operator-() const {
53
            MLong res;
54
             res.x = norm(getMod() - x);
55
             return res;
56
         }
57
        constexpr MLong inv() const {
58
             assert(x != 0);
59
             return power(*this, getMod() - 2);
60
61
        constexpr MLong &operator*=(MLong rhs) & {
62
             x = mul(x, rhs.x, getMod());
63
             return *this;
64
65
        constexpr MLong &operator+=(MLong rhs) & {
66
             x = norm(x + rhs.x);
67
             return *this;
68
         }
69
         constexpr MLong &operator-=(MLong rhs) & {
70
             x = norm(x - rhs.x);
71
            return *this;
72
         }
73
        constexpr MLong &operator/=(MLong rhs) & {
74
             return *this *= rhs.inv();
75
         }
76
        friend constexpr MLong operator*(MLong lhs, MLong rhs) {
77
             MLong res = 1hs;
78
             res *= rhs;
79
             return res;
80
         }
81
        friend constexpr MLong operator+(MLong lhs, MLong rhs) {
82
            MLong res = 1hs;
83
             res += rhs;
84
             return res;
85
         }
86
        friend constexpr MLong operator-(MLong lhs, MLong rhs) {
```

```
MLong res = 1hs;
 88
              res -= rhs;
 89
              return res;
 90
 91
         friend constexpr MLong operator/(MLong lhs, MLong rhs) {
 92
              MLong res = 1hs;
 93
              res /= rhs;
 94
              return res;
 95
 96
          friend constexpr std::istream &operator>>(std::istream &is, MLong &a) {
 97
              i64 v;
 98
              is >> v;
 99
              a = MLong(v);
100
              return is;
101
102
         friend constexpr std::ostream &operator<<(std::ostream &os, const MLong &a) {
103
              return os << a.val();
104
105
         friend constexpr bool operator==(MLong lhs, MLong rhs) {
106
              return lhs.val() == rhs.val();
107
108
         friend constexpr bool operator!=(MLong lhs, MLong rhs) {
109
              return lhs.val() != rhs.val();
110
         }
111
     };
112
113
     template<>
114
     i64 MLong<0LL>::Mod = i64(1E18) + 9;
115
116
     template<int P>
117
     struct MInt {
118
         int x;
119
          constexpr MInt() : x{} {}
120
         constexpr MInt(i64 x) : x{norm(x % getMod())} {}
121
122
         static int Mod;
123
          constexpr static int getMod() {
124
              if (P > 0) {
125
                  return P;
126
              } else {
127
                  return Mod;
128
129
          }
130
          constexpr static void setMod(int Mod_) {
131
              Mod = Mod_{;}
132
133
          constexpr int norm(int x) const {
134
              if (x < 0) {
135
                  x += getMod();
136
              }
137
              if (x >= getMod()) {
138
                 x -= getMod();
139
              }
140
              return x;
141
142
         constexpr int val() const {
143
              return x;
144
145
          explicit constexpr operator int() const {
146
              return x;
147
         }
148
          constexpr MInt operator-() const {
149
              MInt res;
150
              res.x = norm(getMod() - x);
```

```
151
             return res;
152
         }
153
         constexpr MInt inv() const {
154
              assert(x != 0);
155
              return power(*this, getMod() - 2);
156
157
          constexpr MInt &operator*=(MInt rhs) & {
158
              x = 1LL * x * rhs.x % getMod();
159
              return *this;
160
          }
161
          constexpr MInt &operator+=(MInt rhs) & {
162
              x = norm(x + rhs.x);
163
             return *this;
164
          }
165
          constexpr MInt &operator-=(MInt rhs) & {
166
             x = norm(x - rhs.x);
167
             return *this;
168
          }
169
          constexpr MInt &operator/=(MInt rhs) & {
170
             return *this *= rhs.inv();
171
172
         friend constexpr MInt operator*(MInt lhs, MInt rhs) {
173
             MInt res = lhs;
174
              res *= rhs;
175
             return res;
176
177
          friend constexpr MInt operator+(MInt lhs, MInt rhs) {
178
              MInt res = lhs;
179
              res += rhs;
180
             return res;
181
          }
182
          friend constexpr MInt operator-(MInt lhs, MInt rhs) {
183
             MInt res = lhs;
184
             res -= rhs;
185
             return res;
186
187
         friend constexpr MInt operator/(MInt lhs, MInt rhs) {
188
             MInt res = lhs;
189
              res /= rhs;
190
             return res;
191
192
          friend constexpr std::istream &operator>>(std::istream &is, MInt &a) {
193
             i64 v;
194
             is >> v;
195
              a = MInt(v);
196
              return is;
197
198
          friend constexpr std::ostream &operator<<(std::ostream &os, const MInt &a) {
199
             return os << a.val();
200
          }
201
          friend constexpr bool operator == (MInt lhs, MInt rhs) {
202
              return lhs.val() == rhs.val();
203
204
         friend constexpr bool operator!=(MInt lhs, MInt rhs) {
205
             return lhs.val() != rhs.val();
206
207
     };
208
209
     template<>
210
     int MInt<0>::Mod = 998244353;
211
212
     template<int V, int P>
213
     constexpr MInt<P> CInv = MInt<P>(V).inv();
214
```

```
215 | constexpr int P = 1000000007;
216 | using Z = MInt<P>;
```

4.6 状压RMQ (RMQ)

```
1
    template<class T,
 2
        class Cmp = std::less<T>>
 3
    struct RMQ {
 4
         const Cmp cmp = Cmp();
 5
         static constexpr unsigned B = 64;
 6
         using u64 = unsigned long long;
 7
         int n;
 8
         std::vector<std::vector<T>> a;
 9
         std::vector<T> pre, suf, ini;
10
         std::vector<u64> stk;
11
         RMQ() {}
12
         RMQ(const std::vector<T> &v) {
13
             init(v);
14
15
         void init(const std::vector<T> &v) {
16
             n = v.size();
17
             pre = suf = ini = v;
18
             stk.resize(n);
19
             if (!n) {
20
                 return;
21
             }
22
             const int M = (n - 1) / B + 1;
23
             const int lg = std::__lg(M);
24
             a.assign(lg + 1, std::vector<T>(M));
25
             for (int i = 0; i < M; i++) {
26
                 a[0][i] = v[i * B];
27
                 for (int j = 1; j < B && i * B + j < n; j++) {
28
                     a[0][i] = std::min(a[0][i], v[i * B + j], cmp);
29
30
             }
31
             for (int i = 1; i < n; i++) {
32
                 if (i % B) {
33
                     pre[i] = std::min(pre[i], pre[i - 1], cmp);
34
                 }
35
36
             for (int i = n - 2; i >= 0; i--) {
37
                 if (i % B != B - 1) {
38
                     suf[i] = std::min(suf[i], suf[i + 1], cmp);
39
                 }
40
             }
41
             for (int j = 0; j < lg; j++) {
42
                 for (int i = 0; i + (2 << j) <= M; i++) {
43
                     a[j + 1][i] = std::min(a[j][i], a[j][i + (1 << j)], cmp);
44
45
46
             for (int i = 0; i < M; i++) {
47
                 const int l = i * B;
48
                 const int r = std::min(1U * n, l + B);
49
                 u64 s = 0;
                 for (int j = 1; j < r; j++) {
50
51
                     while (s \&\& cmp(v[j], v[std::__lg(s) + 1])) {
52
                         s ^= 1ULL << std::__lg(s);
53
54
                     s = 1ULL \leftrightarrow (j - 1);
55
                     stk[j] = s;
56
                 }
57
             }
```

```
58
59
         T operator()(int 1, int r) {
60
             if (1 / B != (r - 1) / B) {
61
                 T ans = std::min(suf[1], pre[r - 1], cmp);
62
                 1 = 1 / B + 1;
63
                 r = r / B;
64
                 if (1 < r) {
65
                     int k = std::_lg(r - 1);
66
                     ans = std::min({ans, a[k][1], a[k][r - (1 << k)]}, cmp);
67
                 }
68
                 return ans;
69
             } else {
70
                 int x = B * (1 / B);
71
                 return ini[__builtin_ctzll(stk[r - 1] \Rightarrow (1 - x)) + 1];
72
             }
73
        }
74
    };
```

4.6.1 Splay

```
1
     struct Node {
 2
         Node *1 = nullptr;
 3
         Node *r = nullptr;
 4
         int cnt = 0;
 5
         i64 sum = 0;
 6
     };
 7
 8
     Node *add(Node *t, int 1, int r, int p, int v) {
 9
         Node *x = new Node;
10
         if (t) {
11
             *x = *t;
12
13
         x->cnt += 1;
14
         x \rightarrow sum += v;
15
         if (r - 1 == 1) {
16
             return x;
17
         }
18
         int m = (1 + r) / 2;
19
         if (p < m) {
20
             x->1 = add(x->1, 1, m, p, v);
21
         } else {
22
             x\rightarrow r = add(x\rightarrow r, m, r, p, v);
23
         }
24
         return x;
25
     }
26
27
     int find(Node *tl, Node *tr, int l, int r, int x) {
28
         if (r \le x) {
29
             return -1;
30
         }
31
         if (1 >= x) {
32
             int cnt = (tr ? tr->cnt : 0) - (tl ? tl->cnt : 0);
33
             if (cnt == 0) {
34
                  return -1;
35
36
             if (r - 1 == 1) {
37
                  return 1;
38
             }
39
         }
40
         int m = (1 + r) / 2;
41
         int res = find(tl ? tl->l : tl, tr ? tr->l : tr, l, m, x);
42
         if (res == -1) {
```

```
res = find(tl ? tl->r : tl, tr ? tr->r : tr, m, r, x);
 44
           }
 45
           return res;
 46
 47
 48
      std::pair<int, i64> get(Node *t, int 1, int r, int x, int y) {
 49
           if (1 >= y || r <= x || !t) {
 50
                return {0, 0LL};
 51
 52
           if (1 >= x && r <= y) {
 53
                return {t->cnt, t->sum};
 54
           }
 55
           int m = (1 + r) / 2;
 56
           auto [cl, sl] = get(t->1, 1, m, x, y);
 57
           auto [cr, sr] = get(t->r, m, r, x, y);
 58
           return {cl + cr, sl + sr};
 59
 60
 61
      struct Tree {
 62
           int add = 0;
 63
           int val = 0;
 64
           int id = 0;
 65
           Tree *ch[2] = {};
 66
           Tree *p = nullptr;
 67
      };
 68
 69
      int pos(Tree *t) {
 70
           return t \rightarrow p \rightarrow ch[1] == t;
 71
 72
 73
      void add(Tree *t, int v) {
 74
           t->val += v;
 75
           t->add += v;
 76
      }
 77
 78
      void push(Tree *t) {
 79
           if (t->ch[0]) {
 80
                add(t->ch[0], t->add);
 81
 82
           if (t->ch[1]) {
 83
               add(t->ch[1], t->add);
 84
 85
           t->add = 0;
 86
 87
 88
      void rotate(Tree *t) {
 89
           Tree *q = t->p;
 90
           int x = !pos(t);
 91
           q\rightarrow ch[!x] = t\rightarrow ch[x];
 92
           if (t\rightarrow ch[x]) t\rightarrow ch[x]\rightarrow p = q;
 93
           t\rightarrow p = q\rightarrow p;
 94
           if (q\rightarrow p) q\rightarrow p\rightarrow ch[pos(q)] = t;
 95
           t\rightarrow ch[x] = q;
 96
           q \rightarrow p = t;
 97
      }
 98
99
      void splay(Tree *t) {
100
           std::vector<Tree *> s;
101
           for (Tree *i = t; i \rightarrow p; i = i \rightarrow p) s.push_back(i \rightarrow p);
102
           while (!s.empty()) {
103
                push(s.back());
104
                s.pop_back();
105
           }
106
           push(t);
```

```
107
          while (t->p) {
108
               if (t->p->p) {
109
                   if (pos(t) == pos(t->p)) rotate(t->p);
110
                   else rotate(t);
111
112
               rotate(t);
113
          }
114
115
116
      void insert(Tree *&t, Tree *x, Tree *p = nullptr) {
117
          if (!t) {
118
               t = x;
119
               x \rightarrow p = p;
120
               return;
121
          }
122
123
          push(t);
124
          if (x->val < t->val) {
125
               insert(t\rightarrow ch[0], x, t);
126
          } else {
127
               insert(t->ch[1], x, t);
128
          }
129
130
131
      void dfs(Tree *t) {
132
          if (!t) {
133
               return;
134
135
          push(t);
136
          dfs(t->ch[0]);
137
          std::cerr << t->val << " ";
138
          dfs(t->ch[1]);
139
      }
140
141
      std::pair<Tree *, Tree *> split(Tree *t, int x) {
142
           if (!t) {
143
               return {t, t};
144
           }
145
          Tree *v = nullptr;
146
          Tree *j = t;
147
          for (Tree *i = t; i; ) {
148
               push(i);
149
               j = i;
150
               if (i\rightarrow val >= x) {
151
                   v = i;
152
                   i = i \rightarrow ch[0];
153
               } else {
154
                   i = i \rightarrow ch[1];
155
               }
156
          }
157
158
          splay(j);
159
          if (!v) {
160
               return {j, nullptr};
161
          }
162
163
          splay(v);
164
165
          Tree *u = v \rightarrow ch[0];
166
          if (u) {
167
               v\rightarrow ch[0] = u\rightarrow p = nullptr;
168
169
          // std::cerr << "split " << x << "\n";
170
          // dfs(u);
```

```
171
          // std::cerr << "\n";
172
          // dfs(v);
173
          // std::cerr << "\n";
174
          return {u, v};
175
176
177
      Tree *merge(Tree *1, Tree *r) {
178
           if (!1) {
179
               return r;
180
           }
181
          if (!r) {
182
               return 1;
183
184
          Tree *i = 1;
185
          while (i->ch[1]) {
186
               i = i \rightarrow ch[1];
187
188
          splay(i);
189
          i\rightarrow ch[1] = r;
190
          r \rightarrow p = i;
191
          return i;
192
```

```
struct Node {
 2
         Node *ch[2], *p;
 3
         bool rev;
 4
         int siz = 1;
 5
         Node() : ch{nullptr, nullptr}, p(nullptr), rev(false) {}
 6
     };
 7
     void reverse(Node *t) {
 8
         if (t) {
 9
              std::swap(t->ch[0], t->ch[1]);
10
             t->rev ^= 1;
11
         }
12
13
     void push(Node *t) {
14
         if (t->rev) {
15
             reverse(t->ch[0]);
16
             reverse(t->ch[1]);
17
              t->rev = false;
18
         }
19
20
     void pull(Node *t) {
21
         t->siz = (t->ch[0] ? t->ch[0]->siz : 0) + 1 + (t->ch[1] ? t->ch[1]->siz : 0);
22
23
     bool isroot(Node *t) {
24
         return t->p == nullptr || (t->p->ch[0] != t && t->p->ch[1] != t);
25
26
     int pos(Node *t) {
27
         return t \rightarrow p \rightarrow ch[1] == t;
28
29
     void pushAll(Node *t) {
30
         if (!isroot(t)) {
31
              pushAll(t->p);
32
33
         push(t);
34
35
     void rotate(Node *t) {
36
         Node *q = t \rightarrow p;
37
         int x = !pos(t);
38
         q\rightarrow ch[!x] = t\rightarrow ch[x];
39
         if (t->ch[x]) {
40
             t\rightarrow ch[x]\rightarrow p = q;
```

```
41
42
         t\rightarrow p = q\rightarrow p;
43
          if (!isroot(q)) {
44
              q \rightarrow p \rightarrow ch[pos(q)] = t;
45
46
         t\rightarrow ch[x] = q;
47
          q \rightarrow p = t;
48
          pull(q);
49
50
     void splay(Node *t) {
51
          pushAll(t);
52
          while (!isroot(t)) {
53
              if (!isroot(t->p)) {
54
                   if (pos(t) == pos(t->p)) {
55
                        rotate(t->p);
56
                   } else {
57
                       rotate(t);
58
59
              }
60
              rotate(t);
61
          }
62
         pull(t);
63
64
     void access(Node *t) {
65
          for (Node *i = t, *q = nullptr; i; q = i, i = i \rightarrow p) {
66
              splay(i);
67
              i\rightarrow ch[1] = q;
68
              pull(i);
69
          }
70
         splay(t);
71
72
     void makeroot(Node *t) {
73
         access(t);
74
         reverse(t);
75
76
     void link(Node *x, Node *y) {
77
         makeroot(x);
78
         x \rightarrow p = y;
79
80
     void split(Node *x, Node *y) {
81
         makeroot(x);
82
         access(y);
83
84
     void cut(Node *x, Node *y) {
85
          split(x, y);
86
          x->p = y->ch[0] = nullptr;
87
          pull(y);
88
89
     int dist(Node *x, Node *y) {
90
          split(x, y);
91
         return y->siz - 1;
92
    }
```

```
1
    struct Matrix : std::array<std::array<i64, 4>, 4> {
2
        Matrix(i64 v = 0) {
 3
            for (int i = 0; i < 4; i++) {
 4
                 for (int j = 0; j < 4; j++) {
 5
                     (*this)[i][j] = (i == j ? v : inf);
 6
                 }
 7
             }
 8
        }
9
    };
10
```

```
Matrix operator*(const Matrix &a, const Matrix &b) {
12
         Matrix c(inf);
13
         for (int i = 0; i < 3; i++) {
14
              for (int j = 0; j < 3; j++) {
15
                  for (int k = 0; k < 4; k++) {
16
                       c[i][k] = std::min(c[i][k], a[i][j] + b[j][k]);
17
18
19
             c[i][3] = std::min(c[i][3], a[i][3]);
20
21
         c[3][3] = 0;
22
         return c;
23
24
25
     struct Node {
26
         Node *ch[2], *p;
27
         i64 \text{ sumg} = 0;
28
         i64 \text{ sumh} = 0;
29
         i64 \text{ sumb} = 0;
30
         i64 g = 0;
31
         i64 h = 0;
32
         i64 b = 0;
33
         Matrix mat;
34
         Matrix prd;
35
         std::array<i64, 4> ans{};
36
         Node() : ch{nullptr, nullptr}, p(nullptr) {}
37
38
         void update() {
39
              mat = Matrix(inf);
40
              mat[0][0] = b + h - g + sumg;
41
             mat[1][1] = mat[1][2] = mat[1][3] = h + sumh;
42
              mat[2][0] = mat[2][1] = mat[2][2] = mat[2][3] = b + h + sumb;
43
              mat[3][3] = 0;
44
         }
45
     };
46
     void push(Node *t) {
47
48
49
     void pull(Node *t) {
50
         t->prd = (t->ch[0] ? t->ch[0]->prd : Matrix()) * t->mat * (t->ch[1] ? t->ch[1]-
     >prd : Matrix());
51
52
     bool isroot(Node *t) {
53
         return t->p == nullptr || (t->p->ch[0] != t && t->p->ch[1] != t);
54
55
     int pos(Node *t) {
56
         return t-p-ch[1] == t;
57
58
     void pushAll(Node *t) {
59
         if (!isroot(t)) {
60
              pushAll(t->p);
61
         }
62
         push(t);
63
64
     void rotate(Node *t) {
65
         Node *q = t \rightarrow p;
66
         int x = !pos(t);
67
         q\rightarrow ch[!x] = t\rightarrow ch[x];
68
         if (t->ch[x]) {
69
             t\rightarrow ch[x]\rightarrow p=q;
70
71
         t->p = q->p;
72
         if (!isroot(q)) {
73
             q \rightarrow p \rightarrow ch[pos(q)] = t;
```

```
74
 75
          t\rightarrow ch[x] = q;
 76
          q \rightarrow p = t;
 77
          pull(q);
 78
 79
      void splay(Node *t) {
 80
          pushAll(t);
 81
          while (!isroot(t)) {
 82
              if (!isroot(t->p)) {
 83
                   if (pos(t) == pos(t->p)) {
 84
                       rotate(t->p);
 85
                   } else {
 86
                       rotate(t);
 87
                   }
 88
              }
 89
              rotate(t);
 90
 91
          pull(t);
 92
 93
 94
      std::array<i64, 4> get(Node *t) {
 95
          std::array<i64, 4> ans;
 96
          ans.fill(inf);
 97
          ans[3] = 0;
 98
          for (int i = 0; i < 3; i++) {
 99
              for (int j = 0; j < 4; j++) {
100
                   ans[i] = std::min(ans[i], t->prd[i][j]);
101
102
          }
103
          return ans;
104
105
106
      void access(Node *t) {
107
          std::array<i64, 4> old{};
108
          for (Node *i = t, *q = nullptr; i; q = i, i = i \rightarrow p) {
109
              splay(i);
110
              if (i->ch[1]) {
111
                   auto res = get(i->ch[1]);
112
                   i->sumg += res[0];
113
                   i->sumh += std::min({res[1], res[2], res[3]});
114
                   i->sumb += std::min({res[0], res[1], res[2], res[3]});
115
116
              i\rightarrow ch[1] = q;
117
              i->sumg -= old[0];
118
              i->sumh -= std::min({old[1], old[2], old[3]});
119
              i->sumb -= std::min({old[0], old[1], old[2], old[3]});
120
              old = get(i);
121
              i->update();
122
              pull(i);
123
          }
124
          splay(t);
125
```

其他平衡树 4.7

```
1
   struct Node {
2
       Node *1 = nullptr;
3
       Node *r = nullptr;
4
       int sum = 0;
5
       int sumodd = 0;
6
7
       Node(Node *t) {
```

```
if (t) {
 9
                 *this = *t;
10
             }
11
        }
12
    };
13
14
    Node *add(Node *t, int 1, int r, int x, int v) {
15
        t = new Node(t);
16
         t\rightarrow sum += v;
17
         t->sumodd += (x \% 2) * v;
18
         if (r - 1 == 1) {
19
             return t;
20
         }
21
         int m = (1 + r) / 2;
22
         if (x < m) {
23
            t->1 = add(t->1, 1, m, x, v);
24
         } else {
25
             t->r = add(t->r, m, r, x, v);
26
         }
27
        return t;
28
    }
29
30
    int query1(Node *t1, Node *t2, int 1, int r, int k) {
31
         if (r - 1 == 1) {
32
             return 1;
33
34
         int m = (1 + r) / 2;
35
         int odd = (t1 & t1 - r ? t1 - r - sumodd : 0) - (t2 & t2 - r ? t2 - r - sumodd : 0);
36
         int cnt = (t1 && t1->r ? t1->r->sum : 0) - (t2 && t2->r ? t2->r->sum : 0);
37
         if (odd > 0 || cnt > k) {
38
             return query1(t1 ? t1->r : t1, t2 ? t2->r : t2, m, r, k);
39
         } else {
40
             return query1(t1 ? t1->l : t1, t2 ? t2->l : t2, l, m, k - cnt);
41
         }
42
    }
43
44
     std::array<int, 3> query2(Node *t1, Node *t2, int 1, int r, int k) {
45
         if (r - 1 == 1) {
46
             int cnt = (t1 ? t1->sumodd : 0) - (t2 ? t2->sumodd : 0);
47
             return {1, cnt, k};
48
         }
49
         int m = (1 + r) / 2;
50
         int cnt = (t1 & t1 - r ? t1 - r - sumodd : 0) - (t2 & t2 - r ? t2 - r - sumodd : 0);
51
         if (cnt > k) {
52
             return query2(t1 ? t1->r : t1, t2 ? t2->r : t2, m, r, k);
53
         } else {
54
             return query2(t1 ? t1->l : t1, t2 ? t2->l : t2, l, m, k - cnt);
55
56
    }
```

```
1
    struct Node {
2
        Node *1 = nullptr;
3
        Node *r = nullptr;
4
        int cnt = 0;
 5
    };
 6
 7
    Node *add(Node *t, int 1, int r, int x) {
8
        if (t) {
9
            t = new Node(*t);
10
         } else {
11
             t = new Node;
12
13
        t->cnt += 1;
```

```
if (r - 1 == 1) {
15
            return t;
16
17
        int m = (1 + r) / 2;
18
        if (x < m) {
19
            t->1 = add(t->1, 1, m, x);
20
         } else {
21
            t->r = add(t->r, m, r, x);
22
23
        return t;
24
    }
25
26
    int query(Node *t1, Node *t2, int l, int r, int x) {
27
        int cnt = (t2 ? t2->cnt : 0) - (t1 ? t1->cnt : 0);
28
        if (cnt == 0 | | 1 >= x) {
29
            return -1;
30
31
        if (r - 1 == 1) {
32
            return 1;
33
        }
34
        int m = (1 + r) / 2;
35
        int res = query(t1 ? t1->r : t1, t2 ? t2->r : t2, m, r, x);
36
        if (res == -1) {
37
            res = query(t1 ? t1->l : t1, t2 ? t2->l : t2, l, m, x);
38
        }
39
        return res;
40
```

```
1
      struct Info {
 2
           int imp = 0;
 3
           int id = 0;
 4
      };
 5
 6
      Info operator+(Info a, Info b) {
 7
           return {std::max(a.imp, b.imp), 0};
 8
 9
10
      struct Node {
11
           int w = rng();
12
           Info info;
13
           Info sum;
14
           int siz = 1;
15
           Node *1 = nullptr;
16
           Node *r = nullptr;
17
      };
18
19
      void pull(Node *t) {
20
           t->sum = t->info;
21
           t\rightarrow siz = 1;
22
           if (t\rightarrow 1) {
23
                t\rightarrow sum = t\rightarrow l\rightarrow sum + t\rightarrow sum;
24
                t\rightarrow siz += t\rightarrow l\rightarrow siz;
25
26
           if (t->r) {
27
                t\rightarrow sum = t\rightarrow sum + t\rightarrow r\rightarrow sum;
28
                t\rightarrow siz += t\rightarrow r\rightarrow siz;
29
           }
30
      }
31
32
      std::pair<Node *, Node *> splitAt(Node *t, int p) {
33
           if (!t) {
34
                 return {t, t};
35
```

```
if (p \leftarrow (t->l ? t->l->siz : 0)) {
37
             auto [l, r] = splitAt(t->l, p);
38
             t\rightarrow 1 = r;
39
             pull(t);
40
             return {1, t};
41
         } else {
42
             auto [l, r] = splitAt(t->r, p - 1 - (t->l ? t->l->siz : 0));
43
             t->r = 1;
44
             pull(t);
45
             return {t, r};
46
         }
47
48
49
     void insertAt(Node *&t, int p, Node *x) {
50
         if (!t) {
51
             t = x;
52
             return;
53
54
         if (x->w < t->w) {
55
             auto [1, r] = splitAt(t, p);
56
             t = x;
57
             t->1 = 1;
58
             t->r = r;
59
             pull(t);
60
             return;
61
62
         if (p \leftarrow (t->l ? t->l->siz : 0)) {
63
             insertAt(t->1, p, x);
64
         } else {
65
             insertAt(t->r, p - 1 - (t->l ? t->l->siz : 0), x);
66
         }
67
         pull(t);
68
    }
69
70
    Node *merge(Node *a, Node *b) {
71
         if (!a) {
72
             return b;
73
         }
74
         if (!b) {
75
             return a;
76
         }
77
78
         if (a->w < b->w) {
79
             a \rightarrow r = merge(a \rightarrow r, b);
80
             pull(a);
81
             return a;
82
         } else {
83
             b->1 = merge(a, b->1);
84
             pull(b);
85
             return b;
86
         }
87
88
89
    int query(Node *t, int v) {
90
         if (!t) {
91
             return 0;
92
93
         if (t->sum.imp < v) {
94
             return t->siz;
95
         }
96
         int res = query(t->r, v);
97
         if (res != (t->r ? t->r->siz : 0)) {
98
             return res;
99
```

```
100
         if (t->info.imp > v) {
101
              return res;
102
103
          return res + 1 + query(t->1, v);
104
105
106
     void dfs(Node *t) {
107
          if (!t) {
108
              return;
109
          }
110
          dfs(t->1);
111
          std::cout << t->info.id << " ";</pre>
112
          dfs(t->r);
113 }
```

```
struct Node {
 2
          Node *1 = nullptr;
 3
          Node *r = nullptr;
 4
          int cnt = 0;
 5
          int cntnew = 0;
 6
     };
 7
 8
     Node *add(int 1, int r, int x, int isnew) {
 9
          Node *t = new Node;
10
          t\rightarrow cnt = 1;
11
          t->cntnew = isnew;
12
          if (r - 1 == 1) {
13
               return t;
14
          }
15
          int m = (1 + r) / 2;
16
          if (x < m) {
17
               t->1 = add(1, m, x, isnew);
18
          } else {
19
               t->r = add(m, r, x, isnew);
20
21
          return t;
22
     }
23
24
     struct Info {
25
          Node *t = nullptr;
26
           int psum = 0;
27
          bool rev = false;
28
     };
29
30
     void pull(Node *t) {
31
          t \rightarrow cnt = (t \rightarrow 1 ? t \rightarrow 1 \rightarrow cnt : 0) + (t \rightarrow r ? t \rightarrow r \rightarrow cnt : 0);
32
          t \rightarrow cntnew = (t \rightarrow l ? t \rightarrow l \rightarrow cntnew : 0) + (t \rightarrow r ? t \rightarrow r \rightarrow cntnew : 0);
33
     }
34
35
     std::pair<Node *, Node *> split(Node *t, int 1, int r, int x, bool rev) {
36
          if (!t) {
37
               return {t, t};
38
39
          if (x == 0) {
40
               return {nullptr, t};
41
          }
42
          if (x == t\rightarrow cnt) {
43
               return {t, nullptr};
44
45
          if (r - 1 == 1) {
46
               Node *t2 = new Node;
47
               t2\rightarrow cnt = t\rightarrow cnt - x;
48
               t\rightarrow cnt = x;
```

```
49
             return {t, t2};
50
51
         Node *t2 = new Node;
52
         int m = (1 + r) / 2;
53
         if (!rev) {
54
             if (t->1 && x <= t->1->cnt) {
55
                 std::tie(t\rightarrow 1, t2\rightarrow 1) = split(t\rightarrow 1, 1, m, x, rev);
56
                 t2->r = t->r;
57
                 t->r = nullptr;
58
             } else {
59
                 std::tie(t->r, t2->r) = split(t->r, m, r, x - (t->l ? t->l->cnt : 0),
     rev);
60
             }
61
         } else {
62
             if (t->r && x <= t->r->cnt) {
63
                 std::tie(t->r, t2->r) = split(t->r, m, r, x, rev);
64
                 t2->1 = t->1;
65
                 t->l = nullptr;
66
             } else {
67
                 std::tie(t->1, t2->1) = split(t->1, 1, m, x - (t->r? t->r->cnt : 0),
     rev);
68
69
70
         pull(t);
71
         pull(t2);
72
         return {t, t2};
73
74
75
     Node *merge(Node *t1, Node *t2, int 1, int r) {
76
         if (!t1) {
77
             return t2;
78
         }
79
         if (!t2) {
80
             return t1;
81
82
         if (r - 1 == 1) {
83
             t1->cnt += t2->cnt;
84
             t1->cntnew += t2->cntnew;
85
             delete t2;
86
             return t1;
87
         }
88
         int m = (1 + r) / 2;
         t1->l = merge(t1->l, t2->l, l, m);
89
90
         t1->r = merge(t1->r, t2->r, m, r);
91
         delete t2;
92
         pull(t1);
93
         return t1;
94
    }
```

分数四则运算 (Frac) 4.8

```
1
    template<class T>
 2
    struct Frac {
 3
        T num;
 4
        T den;
 5
         Frac(T num_, T den_) : num(num_), den(den_) {
 6
             if (den < 0) {
 7
                 den = -den;
 8
                 num = -num;
 9
             }
10
11
         Frac() : Frac(0, 1) {}
```

```
Frac(T num_) : Frac(num_, 1) {}
13
         explicit operator double() const {
14
             return 1. * num / den;
15
16
         Frac &operator+=(const Frac &rhs) {
17
             num = num * rhs.den + rhs.num * den;
18
             den *= rhs.den;
19
             return *this;
20
21
         Frac &operator-=(const Frac &rhs) {
22
             num = num * rhs.den - rhs.num * den;
23
             den *= rhs.den;
24
             return *this;
25
26
         Frac &operator*=(const Frac &rhs) {
27
             num *= rhs.num;
28
             den *= rhs.den;
29
             return *this;
30
31
         Frac &operator/=(const Frac &rhs) {
32
             num *= rhs.den;
33
             den *= rhs.num;
34
             if (den < 0) {
35
                 num = -num;
36
                 den = -den;
37
38
             return *this;
39
40
         friend Frac operator+(Frac lhs, const Frac &rhs) {
41
             return lhs += rhs;
42
43
         friend Frac operator-(Frac lhs, const Frac &rhs) {
44
             return lhs -= rhs;
45
46
         friend Frac operator*(Frac lhs, const Frac &rhs) {
47
             return lhs *= rhs;
48
49
         friend Frac operator/(Frac lhs, const Frac &rhs) {
50
             return lhs /= rhs;
51
52
         friend Frac operator-(const Frac &a) {
53
            return Frac(-a.num, a.den);
54
55
         friend bool operator == (const Frac &lhs, const Frac &rhs) {
56
             return lhs.num * rhs.den == rhs.num * lhs.den;
57
58
         friend bool operator!=(const Frac &lhs, const Frac &rhs) {
59
             return lhs.num * rhs.den != rhs.num * lhs.den;
60
61
         friend bool operator<(const Frac &lhs, const Frac &rhs) {
62
             return lhs.num * rhs.den < rhs.num * lhs.den;</pre>
63
64
         friend bool operator>(const Frac &lhs, const Frac &rhs) {
65
             return lhs.num * rhs.den > rhs.num * lhs.den;
66
67
         friend bool operator<=(const Frac &lhs, const Frac &rhs) {
68
             return lhs.num * rhs.den <= rhs.num * lhs.den;</pre>
69
70
         friend bool operator>=(const Frac &lhs, const Frac &rhs) {
71
             return lhs.num * rhs.den >= rhs.num * lhs.den;
72
73
         friend std::ostream &operator<<(std::ostream &os, Frac x) {</pre>
74
             T g = std::gcd(x.num, x.den);
75
             if (x.den == g) {
```

4.9 线性基 (Basis)

```
1
    struct Basis {
 2
        int a[20] {};
 3
        int t[20] {};
 4
 5
        Basis() {
 6
            std::fill(t, t + 20, -1);
 7
 8
 9
        void add(int x, int y = 1E9) {
10
            for (int i = 0; i < 20; i++) {
11
                if (x >> i & 1) {
12
                     if (y > t[i]) {
13
                         std::swap(a[i], x);
14
                         std::swap(t[i], y);
15
                    }
16
                    x ^= a[i];
17
                }
18
            }
19
        }
20
        bool query(int x, int y = 0) {
21
22
            for (int i = 0; i < 20; i++) {
23
                if ((x >> i & 1) & t[i] >= y) {
24
                    x ^= a[i];
25
26
27
            return x == 0;
28
        }
29 };
```

/END/

5 字符串

5.1 马拉车 (Manacher 新版)

```
1
    std::vector<int> manacher(std::vector<int> s) {
 2
         std::vector<int> t{0};
 3
         for (auto c : s) {
 4
             t.push_back(c);
 5
             t.push_back(0);
 6
 7
         int n = t.size();
 8
         std::vector<int> r(n);
 9
         for (int i = 0, j = 0; i < n; i++) {
10
             if (2 * j - i >= 0 && j + r[j] > i) {
11
                 r[i] = std::min(r[2 * j - i], j + r[j] - i);
12
             }
13
             while (i - r[i] >= 0 \&\& i + r[i] < n \&\& t[i - r[i]] == t[i + r[i]]) {
14
                 r[i] += 1;
15
16
             if (i + r[i] > j + r[j]) {
17
                 j = i;
18
             }
19
         }
20
         return r;
21
   }
```

5.2 Z函数

```
std::vector<int> zFunction(std::string s) {
 2
         int n = s.size();
 3
         std::vector<int> z(n + 1);
 4
         z[0] = n;
 5
         for (int i = 1, j = 1; i < n; i++) {
 6
             z[i] = std::max(0, std::min(j + z[j] - i, z[i - j]));
 7
             while (i + z[i] < n \&\& s[z[i]] == s[i + z[i]]) {
 8
 9
             }
10
             if (i + z[i] > j + z[j]) {
11
                 j = i;
12
13
14
         return z;
15
```

5.3 后缀数组 (SA)

```
1
    struct SuffixArray {
 2
        int n;
 3
         std::vector<int> sa, rk, lc;
 4
         SuffixArray(const std::string &s) {
 5
             n = s.length();
 6
             sa.resize(n);
 7
             lc.resize(n - 1);
 8
             rk.resize(n);
 9
             std::iota(sa.begin(), sa.end(), 0);
10
             std::sort(sa.begin(), sa.end(), [&](int a, int b) {return s[a] < s[b];});</pre>
11
             rk[sa[0]] = 0;
12
             for (int i = 1; i < n; ++i)
13
                 rk[sa[i]] = rk[sa[i - 1]] + (s[sa[i]] != s[sa[i - 1]]);
```

```
14
             int k = 1;
15
             std::vector<int> tmp, cnt(n);
16
             tmp.reserve(n);
17
             while (rk[sa[n-1]] < n-1) {
18
                 tmp.clear();
19
                 for (int i = 0; i < k; ++i)
20
                     tmp.push_back(n - k + i);
21
                 for (auto i : sa)
22
                     if (i >= k)
23
                         tmp.push back(i - k);
24
                 std::fill(cnt.begin(), cnt.end(), 0);
25
                 for (int i = 0; i < n; ++i)
26
                     ++cnt[rk[i]];
27
                 for (int i = 1; i < n; ++i)
28
                     cnt[i] += cnt[i - 1];
29
                 for (int i = n - 1; i >= 0; --i)
30
                     sa[--cnt[rk[tmp[i]]]] = tmp[i];
31
                 std::swap(rk, tmp);
32
                 rk[sa[0]] = 0;
33
                 for (int i = 1; i < n; ++i)
34
                     rk[sa[i]] = rk[sa[i - 1]] + (tmp[sa[i - 1]] < tmp[sa[i]] |  |  sa[i - 1]|
    1] + k == n | tmp[sa[i - 1] + k] < tmp[sa[i] + k]);
35
                 k *= 2;
36
             }
37
             for (int i = 0, j = 0; i < n; ++i) {
38
                 if (rk[i] == 0) {
39
                     j = 0;
40
                 } else {
41
                     for (j -= j > 0; i + j < n \&\& sa[rk[i] - 1] + j < n \&\& s[i + j] ==
    s[sa[rk[i] - 1] + j];)
42
                         ++j;
43
                     lc[rk[i] - 1] = j;
44
                 }
45
             }
46
        }
47
    };
```

5.4 后缀自动机

5.4.1 后缀自动机 (SuffixAutomaton 旧版)

```
1
    struct SuffixAutomaton {
 2
         static constexpr int ALPHABET_SIZE = 26, N = 5e5;
 3
         struct Node {
 4
             int len;
 5
             int link;
 6
             int next[ALPHABET SIZE];
 7
             Node() : len(0), link(0), next{} {}
 8
         } t[2 * N];
 9
         int cntNodes;
10
         SuffixAutomaton() {
11
             cntNodes = 1;
12
             std::fill(t[0].next, t[0].next + ALPHABET SIZE, 1);
13
             t[0].len = -1;
14
15
         int extend(int p, int c) {
16
             if (t[p].next[c]) {
17
                 int q = t[p].next[c];
18
                 if (t[q].len == t[p].len + 1)
19
                     return q;
20
                 int r = ++cntNodes;
21
                 t[r].len = t[p].len + 1;
```

```
22
                 t[r].link = t[q].link;
23
                 std::copy(t[q].next, t[q].next + ALPHABET_SIZE, t[r].next);
24
                 t[q].link = r;
25
                 while (t[p].next[c] == q) {
26
                     t[p].next[c] = r;
27
                     p = t[p].link;
28
                 }
29
                 return r;
30
31
            int cur = ++cntNodes;
32
            t[cur].len = t[p].len + 1;
33
            while (!t[p].next[c]) {
34
                 t[p].next[c] = cur;
35
                 p = t[p].link;
36
             }
37
            t[cur].link = extend(p, c);
38
             return cur;
39
        }
40
   };
```

5.4.2 后缀自动机 (SAM 新版)

```
struct SAM {
 2
         static constexpr int ALPHABET_SIZE = 26;
 3
         struct Node {
 4
             int len;
 5
             int link;
 6
             std::array<int, ALPHABET_SIZE> next;
 7
             Node() : len{}, link{}, next{} {}
 8
         };
 9
         std::vector<Node> t;
10
         SAM() {
11
             init();
12
13
         void init() {
14
             t.assign(2, Node());
15
             t[0].next.fill(1);
16
             t[0].len = -1;
17
         }
18
         int newNode() {
19
             t.emplace_back();
20
             return t.size() - 1;
21
22
         int extend(int p, int c) {
23
             if (t[p].next[c]) {
24
                 int q = t[p].next[c];
25
                 if (t[q].len == t[p].len + 1) {
26
                     return q;
27
                 }
28
                 int r = newNode();
29
                 t[r].len = t[p].len + 1;
30
                 t[r].link = t[q].link;
31
                 t[r].next = t[q].next;
32
                 t[q].link = r;
33
                 while (t[p].next[c] == q) {
34
                     t[p].next[c] = r;
35
                     p = t[p].link;
36
                 }
37
                 return r;
38
39
             int cur = newNode();
40
             t[cur].len = t[p].len + 1;
```

```
41
            while (!t[p].next[c]) {
42
                 t[p].next[c] = cur;
43
                 p = t[p].link;
44
45
            t[cur].link = extend(p, c);
46
             return cur;
47
48
         int extend(int p, char c, char offset = 'a') {
49
             return extend(p, c - offset);
50
        }
51
52
         int next(int p, int x) {
53
            return t[p].next[x];
54
         }
55
56
        int next(int p, char c, char offset = 'a') {
57
             return next(p, c - 'a');
58
         }
59
60
        int link(int p) {
61
            return t[p].link;
62
63
64
        int len(int p) {
65
             return t[p].len;
66
67
68
        int size() {
69
             return t.size();
70
71
    };
```

回文自动机 (PAM) 5.5

```
1
    struct PAM {
 2
        static constexpr int ALPHABET_SIZE = 28;
 3
         struct Node {
 4
             int len;
 5
             int link;
 6
             int cnt;
 7
             std::array<int, ALPHABET_SIZE> next;
 8
             Node() : len{}, link{}, cnt{}, next{} {}
 9
        };
10
         std::vector<Node> t;
11
         int suff;
12
         std::string s;
13
        PAM() {
14
             init();
15
16
         void init() {
             t.assign(2, Node());
17
18
             t[0].len = -1;
19
             suff = 1;
20
             s.clear();
21
         }
22
         int newNode() {
23
             t.emplace_back();
24
             return t.size() - 1;
25
         }
26
27
         bool add(char c, char offset = 'a') {
28
             int pos = s.size();
```

```
29
             s += c;
30
             int let = c - offset;
31
             int cur = suff, curlen = 0;
32
33
             while (true) {
34
                 curlen = t[cur].len;
35
                 if (pos - 1 - curlen >= 0 \& s[pos - 1 - curlen] == s[pos])
36
                     break;
37
                 cur = t[cur].link;
38
             }
39
             if (t[cur].next[let]) {
40
                 suff = t[cur].next[let];
41
                 return false;
42
             }
43
44
             int num = newNode();
45
             suff = num;
46
             t[num].len = t[cur].len + 2;
47
             t[cur].next[let] = num;
48
49
             if (t[num].len == 1) {
50
                 t[num].link = 1;
51
                 t[num].cnt = 1;
52
                 return true;
53
             }
54
55
             while (true) {
56
                 cur = t[cur].link;
57
                 curlen = t[cur].len;
58
                 if (pos - 1 - curlen >= 0 && s[pos - 1 - curlen] == s[pos]) {
59
                     t[num].link = t[cur].next[let];
60
                     break;
61
                 }
62
             }
63
64
            t[num].cnt = 1 + t[t[num].link].cnt;
65
66
            return true;
67
        }
68
    };
69
70
   PAM pam;
```

AC自动机 5.6

AC自动机 (AC 旧版) 5.6.1

```
1
    constexpr int N = 3e5 + 30, A = 26;
 2
 3
    struct Node {
 4
        int fail;
 5
        int sum;
 6
         int next[A];
 7
        Node() : fail(-1), sum(0) {
 8
             std::memset(next, -1, sizeof(next));
 9
        }
10
    } node[N];
11
12
    int cnt = 0;
13
    int bin[N];
14
    int nBin = 0;
15
```

```
16
     int newNode() {
17
         int p = nBin > 0 ? bin[--nBin] : cnt++;
18
         node[p] = Node();
19
         return p;
20
    }
21
22
     struct AC {
23
         std::vector<int> x;
24
         AC(AC \&\&a) : x(std::move(a.x)) {}
25
         AC(std::vector<std::string> s, std::vector<int> w) {
26
             x = {newNode(), newNode()};
27
             std::fill(node[x[0]].next, node[x[0]].next + A, x[1]);
28
             node[x[1]].fail = x[0];
29
30
             for (int i = 0; i < int(s.size()); i++) {
31
                 int p = x[1];
32
                 for (int j = 0; j < int(s[i].length()); j++) {</pre>
33
                      int c = s[i][j] - 'a';
34
                      if (node[p].next[c] == -1) {
35
                          int u = newNode();
36
                          x.push_back(u);
37
                          node[p].next[c] = u;
38
39
                      p = node[p].next[c];
40
41
                 node[p].sum += w[i];
42
             }
43
44
             std::queue<int> que;
45
             que.push(x[1]);
46
             while (!que.empty()) {
47
                 int u = que.front();
48
                 que.pop();
49
                 node[u].sum += node[node[u].fail].sum;
50
                 for (int c = 0; c < A; c++) {
51
                      if (node[u].next[c] == -1) {
52
                          node[u].next[c] = node[node[u].fail].next[c];
53
                      } else {
54
                          node[node[u].next[c]].fail = node[node[u].fail].next[c];
55
                          que.push(node[u].next[c]);
56
                      }
57
                 }
58
             }
59
         }
60
         ~AC() {
61
             for (auto p : x) {
62
                 bin[nBin++] = p;
63
             }
64
65
         i64 query(const std::string &s) const {
66
             i64 \text{ ans} = 0;
67
             int p = x[1];
68
             for (int i = 0; i < int(s.length()); i++) {</pre>
69
                 int c = s[i] - 'a';
70
                 p = node[p].next[c];
71
                 ans += node[p].sum;
72
73
             return ans;
74
         }
75
    };
```

5.6.2 AC自动机 (AhoCorasick 新新版)

```
1
    struct AhoCorasick {
 2
         static constexpr int ALPHABET = 26;
 3
         struct Node {
 4
             int len;
 5
             int link;
 6
             std::array<int, ALPHABET> next;
 7
             Node() : len{0}, link{0}, next{} {}
 8
        };
 9
10
         std::vector<Node> t;
11
12
         AhoCorasick() {
13
             init();
14
15
16
         void init() {
17
             t.assign(2, Node());
18
             t[0].next.fill(1);
19
             t[0].len = -1;
20
         }
21
22
         int newNode() {
23
             t.emplace_back();
24
             return t.size() - 1;
25
26
27
         int add(const std::string &a) {
28
             int p = 1;
29
             for (auto c : a) {
30
                 int x = c - 'a';
31
                 if (t[p].next[x] == 0) {
32
                     t[p].next[x] = newNode();
33
                     t[t[p].next[x]].len = t[p].len + 1;
34
                 }
35
                 p = t[p].next[x];
36
             }
37
             return p;
38
         }
39
40
         void work() {
41
             std::queue<int> q;
42
             q.push(1);
43
44
             while (!q.empty()) {
45
                 int x = q.front();
46
                 q.pop();
47
48
                 for (int i = 0; i < ALPHABET; i++) {
49
                     if (t[x].next[i] == 0) {
50
                          t[x].next[i] = t[t[x].link].next[i];
51
                     } else {
52
                         t[t[x].next[i]].link = t[t[x].link].next[i];
53
                          q.push(t[x].next[i]);
54
                     }
55
                 }
56
             }
57
58
59
         int next(int p, int x) {
60
             return t[p].next[x];
61
```

```
62
63
        int link(int p) {
64
            return t[p].link;
65
        }
66
67
        int len(int p) {
68
             return t[p].len;
69
70
71
        int size() {
72
            return t.size();
73
         }
74 };
```

随机生成模底 字符串哈希 (例题) 5.7

```
1
    #include <bits/stdc++.h>
 3
    using i64 = long long;
 4
 5
    bool isprime(int n) {
 6
        if (n <= 1) {
            return false;
 8
        }
 9
        for (int i = 2; i * i <= n; i++) {
10
            if (n % i == 0) {
11
                 return false;
12
13
14
        return true;
15
16
17
    int findPrime(int n) {
18
        while (!isprime(n)) {
19
            n++;
20
        }
21
        return n;
22
23
24
    using Hash = std::array<int, 2>;
25
26
    int main() {
27
         std::ios::sync_with_stdio(false);
28
         std::cin.tie(nullptr);
29
30
        std::mt19937 rng(std::chrono::steady_clock::now().time_since_epoch().count());
31
32
        const int P = findPrime(rng() % 900000000 + 100000000);
33
34
        std::string s, x;
35
        std::cin >> s >> x;
36
37
        int n = s.length();
38
        int m = x.length();
39
40
        std::vector<int> h(n + 1), p(n + 1);
41
         for (int i = 0; i < n; i++) {
42
            h[i + 1] = (10LL * h[i] + s[i] - '0') % P;
43
44
        p[0] = 1;
45
         for (int i = 0; i < n; i++) {
46
            p[i + 1] = 10LL * p[i] % P;
```

```
47
         }
 48
 49
          auto get = [&](int 1, int r) {
 50
              return (h[r] + 1LL * (P - h[1]) * p[r - 1]) % P;
 51
         };
 52
 53
         int px = 0;
 54
          for (auto c : x) {
 55
              px = (10LL * px + c - '0') % P;
 56
          }
 57
 58
         for (int i = 0; i \le n - 2 * (m - 1); i++) {
 59
              if ((get(i, i + m - 1) + get(i + m - 1, i + 2 * m - 2)) \% P == px) {
 60
                  std::cout << i + 1 << " " << i + m - 1 << "\n";
                  std::cout << i + m << " " << i + 2 * m - 2 << "\n";
 61
 62
                  return 0;
 63
              }
 64
          }
 65
 66
         std::vector<int> z(m + 1), f(n + 1);
 67
         z[0] = m;
 68
 69
         for (int i = 1, j = -1; i < m; i++) {
 70
              if (j != -1) {
 71
                  z[i] = std::max(0, std::min(j + z[j] - i, z[i - j]));
 72
 73
              while (z[i] + i < m \&\& x[z[i]] == x[z[i] + i]) {
 74
                  z[i]++;
 75
              }
 76
              if (j == -1 || i + z[i] > j + z[j]) {
 77
                  j = i;
 78
              }
 79
         }
 80
         for (int i = 0, j = -1; i < n; i++) {
 81
              if (j != -1) {
 82
                  f[i] = std::max(0, std::min(j + f[j] - i, z[i - j]));
 83
 84
              while (f[i] + i < n \&\& f[i] < m \&\& x[f[i]] == s[f[i] + i]) {
 85
                  f[i]++;
 86
 87
              if (j == -1 || i + f[i])  {
 88
                  j = i;
 89
              }
 90
         }
 91
 92
         for (int i = 0; i + m <= n; i++) {
 93
              int l = std::min(m, f[i]);
 94
 95
              for (auto j : \{ m - 1, m - 1 - 1 \}) {
 96
                  if (j <= 0) {
 97
                      continue;
 98
                  }
99
                  if (j \le i \&\& (get(i - j, i) + get(i, i + m)) \% P == px) {
100
                      std::cout << i - j + 1 << " " << i << "\n";
                      std::cout << i + 1 << " " << i + m << "\n";
101
102
                      return 0;
103
104
                  if (i + m + j \le n \&\& (get(i, i + m) + get(i + m, i + m + j)) % P ==
     px) {
105
                      std::cout << i + 1 << " " << i + m << "\n";
106
                      std::cout << i + m + 1 << " " << i + m + j << "\n";
107
                      return 0;
108
                  }
109
```

```
110 }
111 |
112 | return 0;
113 }
```

Edited by *Wida*

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