a) $n^2 - 10n + 2 = O(n^2)$

$\to \underline{n^2} - 10n \underline{+2} \leq 3n^2$

$+ \; n^2 \leq n^2 \quad \forall \; n \geq 1$

$\underline{2 \leq n^2} \quad \forall \; n \geq \sqrt{2}$

$n \geq 2$

$n^2 + 2 \leq 2n^2 \quad \forall \; n \geq 2$

$n^2 - 10n + 2 \leq 2n^2 \quad \forall \; n \geq 2$

Para $\underline{C = 2 \text{ y } n_0 = 2}$

b) $\lceil \frac{n}{3} \rceil = O(n)$

$\exists \; c, n_0 > 0 \text{ tq } \forall \; n \geq n_0 \longrightarrow \lceil \frac{n}{3} \rceil \leq cn$

$\lceil \frac{n}{3} \rceil \leq \lceil \frac{3n}{3} \rceil = \lceil n \rceil \leq n \Longrightarrow \lceil \frac{n}{3} \rceil \leq n \; ; \forall n \geq 1$

$C = 1$

c) $\lg n = O(\lg_{10} n)$

$\exists \; c, n_0 > 0 \text{ tq } \forall \; n \geq n_0 \longrightarrow \lg n \leq c \lg_{10} n$

$\cancel{\lg n} \leq C \frac{\cancel{\lg_2 n}}{\lg_2 10}$

$\lg 10 \leq C$

$C = 4$

Para $C = 4$ y $n_0 = 1$

$\forall \; n \geq n_0$

$\lg n \leq 4 \lg_{10} n \quad \forall \; n \geq n_0$

(d) $n = O(2^n)$

$\exists\ c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow \quad n \leq c 2^n$

Para $c = 1$ y $n_0 = 1$ cumple que : $\quad n \leq 2^n$

$\lg \quad \lg$

Porque $\quad \lg(n) \leq \lg(2)^n$

$\lg(n) \leq n \longrightarrow$ Es trivial

true

e) $\lg n$ no es $\Omega(n)$

$\lg n$ es $\Omega(n)$

$\hookrightarrow \exists\ c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow \quad cn \leq \lg n$

$c \leq \dfrac{\lg n}{n}$

$\lim\limits_{n \to \infty} \dfrac{\lg n}{n} = 0 \qquad c \leq 0$

$\hookrightarrow$

F) $\frac{n}{100}$ No es $O(1)$

$\hookrightarrow \frac{n}{100}$ es $O(1)$

$\hookrightarrow \exists\ c, n_0 > 0$ tq $\forall n \geq n_0 \rightarrow \frac{n}{100} \leq c$

$\hookrightarrow c$ depende de n

No sera constante

(h) $\frac{1}{3}(n+1)(n-2)-5=\Theta(n^2)$.

$$\frac{1}{3}(n^2-n-2)-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{2}{3}-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}=\Theta(n^2)$$

$$\exists\ c_1, c_2, n_0 > 0 \quad \forall\ n \geq n_0 \longrightarrow c_1 n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq c_2 n^2$$

Para big-O

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq c_2 n^2$$

$$\frac{1}{3}n^2 \leq n^2 \qquad \forall\ n \geq 1 \ \wedge \ c_2 = 1$$

si quito seguira siendo menor

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq n^2 \quad \checkmark$$

Para Big-Ω:

torta

$$c_1 n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}$$

Los limitamos a mitad de torta

$$\frac{1}{\cancel{3}}n \leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^{\cancel{2}}\right) \longrightarrow 4 \leq n$$

$$\frac{17}{\cancel{3}} \leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^2\right) \longrightarrow \sqrt{17\times 4} \leq n$$

$8,24 \leq n$

$\hookrightarrow 9 \leq n$

Para $n \geq 9$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \geq \frac{4}{4}\frac{1}{3}n^2-\frac{1}{4}\left(\frac{1}{3}n^2\right)-\frac{1}{4}\left(\frac{1}{3}n^2\right)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \geq \frac{4n^2-n^2-n^2}{12} = \frac{\cancel{2}n^2}{\cancel{12}\,6} = \frac{1}{6}n^2$$

c_1

dd Para $c_1=\frac{1}{6} \wedge c_2=1$ y $n_0=9$

$$\Rightarrow \frac{1}{6}n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq n^2 \Rightarrow \frac{1}{3}(n+1)(n-2)-5=\Theta(n^2) \text{ D}$$

(i) $n \lg n - \lceil 2n/3 \rceil - \lg n + 4$ es $\Omega(2n \lg n)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall\, n \geq n_0 \longrightarrow c\, 2n \lg n \leq n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4$

tarta

$c\, 2n \lg n \leq n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4$

acotamos a mitad de tarta

1° $\left\lceil \frac{2n}{3} \right\rceil \leq \left\lceil \frac{3n}{3} \right\rceil = \cancel{n} \leq \frac{1}{4} \cancel{n} \lg n$

$4 \leq \lg n$

$2^4 \leq n$

1° $\left\lceil \frac{2n}{3} \right\rceil \leq \frac{1}{4} n \lg n$

2° $\lg n \leq \frac{1}{4} n \lg n$

2° $\cancel{\lg n} \leq \frac{1}{4} n \cancel{\lg n}$

$4 \leq n$

$\Rightarrow n_0 = 2^4$

$$n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4 \geq n \lg n - \frac{1}{4} n \lg n - \frac{1}{4} n \lg n = \frac{1}{2} n \lg n$$

$$= \frac{1}{4} \cdot 2n \lg n$$

Para $c = \frac{1}{4}$ ∧ $n_0 = 2^4$

∴ $\Rightarrow n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4 \geq \frac{1}{4} 2n \lg n$

∴

$$n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4 = \Omega(2n \lg n)$$

(j) $\lg n!$ es $\Omega(n \lg n)$

$$\lg n! \Rightarrow \underbrace{\lg n + \lg(n-1) + \lg(n-2) + \dots \lg(1)}_{\sum_{i=1}^{n} \lg(i)}$$

$$\sum_{i=1}^{n} \lg(i) = \Omega\ n \lg n$$

$$\Rightarrow \exists\ c, n_0 > 0 \text{ tq } \forall\ n \geq n_0 \to c\, n \lg n \leq \sum_{i=1}^{n} \lg(i)$$

$$\frac{1}{c} \sum$$

$$\sum_{i=4}^{10}$$

$$\sum_{i=1}^{n} \lg(i) = \sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=1}^{n} \lg(i) \qquad 2\times$$

$$10 - 4 + 1$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=\frac{n}{2}+1}^{n} \lg(n) \leq \sum_{i=1}^{n} \lg(i)$$

$$n - \frac{n}{2} \cancel{+1} + 1$$

$$\frac{n}{2}$$

$$\frac{n}{2} \lg n \leq \lg n + \lg(n-1)$$

$$\frac{n}{2}\left(\lg(n)\right) - \lg n \leq \lg(n-1)$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(n) + \sum_{i=1}^{\frac{n}{2}} \lg(\leq$$

$$2^{\frac{n}{2}\lg n} \leq 2$$

$$\sqrt{2^{n \lg n}} \leq$$

Ejercicio 2. Demostrar o

(a) $\lg \sqrt{n} = O(\lg n)$

$\Rightarrow \exists\, c, n_0 > 0$, tq $\forall\, n \geq n_0 \longrightarrow (\lg \sqrt{n}) \leq c \lg n$

$\frac{1}{2} \lg n \leq c \lg n$

Es verdad para $c = 1$ y $n_0 = 1$

ya que tendremos: $\frac{1}{2} \lg n \leq \lg n \longrightarrow$ Esto es verdad ya que la función logarítmica es creciente

(b) Si $f(n) = O(g(n))$ y $g(n) = O(h(n))$ entonces $f(n) = O(h(n))$

$f(n) = O(g(n))$

$\hookrightarrow f(n) \leq c_1 g(n)$

$g(n) = O(h(n))$

$\hookrightarrow (g(n) \leq c_2(h(n))) \times c_1$

$c_1 g(n) \leq \underbrace{c_1 \times c_2}\, h(n)$

$f(n) \leq c_1 g(n) \leq c\, h(n)$

$f(n) \leq c\, h(n) \Rightarrow f(n) = O(h(n))$

(c) Si $f(n) = O(g(n))$ y $g(n) = \Theta(h(n))$ entonces $f(n) = \Theta(h(n))$

Si $f(n) = O(g(n)) \wedge g(n) = \Theta(h(n)) \Rightarrow$ ¿$f(n) = \Theta(h(n))$?

$\hookrightarrow f(n) \leq c_1 g(n)$ $\quad$ $c_2 h(n) \leq g(n) \leq c_3 h(n)$

$n^2 - 3n \leq n^2$ $\quad$ $\frac{1}{2}(n^2+2) \leq n^2 \leq n^2+2$ $\quad$ $n^3 \leq n \leq n^3$

$\cancel{n}^2+2 \leq \cancel{n}^2 - 3n$

(d) Si $f(n) = O(g(n))$ entonces $2^{f(n)} = O(2^{g(n)})$

Esto va a cumplir para algun $c \geq 2$

$2 \lg n \leq c \lg n \longrightarrow ¿2^{2\lg n} \leq 2^{\lg n}?$

$n^2 \leq n$ $(\rightarrow\leftarrow)$

(f) $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$, para funciones no negativas $f(n)$ y $g(n)$.

Es cierto que $f(n) \leq f(n) + g(n) \wedge g(n) \leq f(n) + g(n)$

$\therefore$ $\max(f(n), g(n)) \leq f(n) + g(n)$

$\hookrightarrow \max(f(n), g(n)) = O(f(n) + g(n))$

Es cierto que $f(n) + g(n) \leq 2\max(f(n), g(n))$

$\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n))$

$\Rightarrow \max(f(n), g(n)) = \Omega(f(n), g(n))$

$\therefore$ $\max(f(n), g(n)) = \Theta(f(n), g(n))$ $\triangle$

(g) $(n+a)^b = \Theta(n^b)$, donde $a, b \in \mathbb{R}$ y $b > 0$.

$$C_1 n^b \leq n^b + \dots + a^b \leq C_2 n^b$$

$C_1 = 1$ cumplo

Big-O:

$$\underbrace{n^b + a^b + \dots}_{b+1 \text{ terminos}} \leq C_2 n^b$$

$$+ \downarrow \; b+1 \text{ terminos} \begin{cases} n^b \leq n^b & \forall n \geq 1 \\ a^b \leq n^b & \forall n \geq a \\ \vdots & \\ \leq n^b & \end{cases}$$

Para algun $n \geq X$ siendo x el maximo valor de cada cota que puede tomar n

$$(n+a)^b \leq (b+1) n^b$$

Para un $C_2 = b+1$

$\hookrightarrow \quad (n+a)^b \leq (b+1) n^b$

o sea $(n+a)^b = \Theta(n^b)$

(h) $\sqrt{n} = O(\lg^2 n)$.

$\sqrt{n^{1/2}} \leq \sqrt{\lg^2 n}$

$n^{1/2} \leq c \lg^2 n$

$n^{1/4} \leq c \lg n$

$\lg^2 n \geq \lg n \geq$

$\frac{n^{1/4}}{\lg n} \leq c$

n

$\sqrt{n} \leq c \lg^2 n$

$2^{(\lg n)^2}$

$n \leq c \lg^2 n$

$2^n \leq 2$

$2^{n+1}-1$

1 $\quad n^{?}$

(i) $\sum_{k=1}^{n} k^{99} = \Theta(n^{100})$

$\underbrace{1+2^{99}+\dots n^{99}}$

$\exists\, c_1, c_2, n_0 > 0$ tq $\forall\, n \geq n_0 \longrightarrow c_1 n^{100} \leq \sum_{k=1}^{n} k^{99} \leq c_2 n^{100}$

(j) Suponga que $\lg(g(n)) \geq 1$ y que $f(n) \geq 1$ para todo n suficientemente grande. En ese caso, si $f(n) = O(g(n))$ entonces $\lg(f(n)) = O(\lg(g(n)))$.

$\lg(g(n)) \geq 1$ | $f(n) \geq 1$

$2^{\lg(g(n))} \geq 2^1$ | $\lg(f(n)) \geq 0$

$g(n) \geq 2$

Si:

$\Rightarrow f(n) = O(g(n)) \longrightarrow$ ¿$\lg(f(n)) = O(\lg(g(n)))$?

$\exists\, c, n_0 > 0 \;;\; \forall n \geq n_0$

$\hookrightarrow f(n) \leq c\, g(n)$

$\lg(\,)\{$

$\lg(f(n)) \leq \lg(c\, g(n))$

$\lg(f(n)) \leq \underbrace{\lg c}_{c} + \lg(g(n))$ Queremos que

$\lg(f(n)) \leq c + \lg(g(n))$ $\quad c \leq \lg(g(n))$

$2^c \leq g(n)$

$\lg(f(n)) \leq c + \lg(g(n))$

$\leq \lg(g(n)) + \lg(g(n))$ $\quad$ o sea $g(n) \geq 2^c$

o sea

$\lg(f(n)) \leq 2\lg(g(n)) \longrightarrow$ Para un $g(n) \geq 2^c$

$cn \leq 2n^2 + 3n + 4$