a) $n^2 - 10n + 2 = O(n^2)$

$\rightarrow n^2 - 10n + 2 \leq 3n^2$

$$+\begin{array}{ll} n^2 \leq n^2 & \forall\ n \geq 1 \\ 2 \leq n^2 & \forall\ n \geq \sqrt{2} \\ & \ \ n \geq 2 \end{array}$$

$n^2 + 2 \leq 2n^2 \quad \forall\ n \geq 2$

$n^2 - 10n + 2 \leq 2n^2 \quad \forall\ n \geq 2$

Para $C = 2$ y $n_0 = 2$

b) $\lceil \frac{n}{3} \rceil = O(n)$

$\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \lceil \frac{n}{3} \rceil \leq cn$

$\lceil \frac{n}{3} \rceil \leq \lceil \frac{3n}{3} \rceil = \lceil n \rceil \leq n \Longrightarrow \lceil \frac{n}{3} \rceil \leq n \ ; \forall\ n \geq 1$

$C = 1$

c) $\lg n = O(\lg_{10} n)$

$\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \lg n \leq c \lg_{10} n$

$\cancel{\lg n} \leq C \dfrac{\cancel{\lg_2 n}}{\lg_2 10}$

$\lg 10 \leq C$

$\rightarrow C = 4$

Para $C = 4$ y $n_0 = 1$

$\forall\ n \geq n_0$

$\lg n \leq 4 \lg_{10} n \quad \forall\ n \geq n_0$

(d) $n = O(2^n)$

$\exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow n \leq c\,2^n$

Para $c=1$ y $n_0=1$ cumple que: $n \leq 2^n$

lg

Porque $\lg(n) \leq \lg(2)^n$

$\lg(n) \leq n \longrightarrow$ Es trivial true

e) $\lg n$ no es $\Omega(n)$

$\lg n$ es $\Omega(n)$

$\exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow cn \leq \lg n$

$c \leq \dfrac{\lg n}{n}$

$\lim_{n\to\infty} \dfrac{\lg n}{n} = 0$ $\qquad c \leq 0$

f) $\dfrac{n}{100}$ No es $O(1)$

$\dfrac{n}{100}$ es $O(1)$

$\exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow \dfrac{n}{100} \leq c$

c depende de n

No sera constante

(h) $\frac{1}{3}(n+1)(n-2)-5=\Theta(n^2)$.

$$\frac{1}{3}(n^2-n-2)-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{2}{3}-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}=\Theta(n^2)$$

$$\exists\ c_1,c_2,n_0>0\quad \forall\ n\geq n_0 \longrightarrow c_1n^2\leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq c_2n^2$$

Para big-O

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq c_2n^2$$

si quito seguirá siendo menor

$$\frac{1}{3}n^2\leq n^2 \qquad \forall\ n\geq 1 \ \wedge\ c_2=1$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq n^2 \quad ✓$$

Para Big-Ω:

torta

$$c_1n^2\leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}$$

Los limitamos a mitad de torta

$$\frac{1}{\cancel{3}}\cancel{n}\leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^{\cancel{2}}\right)\longrightarrow 4\leq n$$

$$\frac{17}{\cancel{3}}\leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^2\right)\longrightarrow \sqrt{17\times 4}\leq n$$

$$8{,}24\leq n$$

$$\hookrightarrow 9\leq n$$

Para $n\geq 9$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\geq \frac{4}{4}\frac{1}{3}n^2-\frac{1}{4}\left(\frac{1}{3}n^2\right)-\frac{1}{4}\left(\frac{1}{3}n^2\right)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\geq \frac{4n^2-n^2-n^2}{12}=\frac{\cancel{2}n^2}{\cancel{12}_6}=\frac{1}{6}n^2$$

c_1

q.e.d. Para $c_1=\frac{1}{6}$ ∧ $c_2=1$ y $n_0=9$

$$\Rightarrow \frac{1}{6}n^2\leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq n^2 \Rightarrow \frac{1}{3}(n+1)(n-2)-5=\Theta(n^2)\ \text{D}

(i) $n \lg n - \lceil 2n/3 \rceil - \lg n + 4$ es $\Omega(2n \lg n)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow c\, 2n\lg n \leq n\lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4$

tarta

$c\, 2n\lg n \leq n\lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4$

acotamos a mitad de tarta

1° $\lceil \frac{2n}{3} \rceil \leq \frac{1}{4} n \lg n$

2° $\lg n \leq \frac{1}{4} n \lg n$

1° $\lceil \frac{2n}{3} \rceil \leq \lceil \frac{3n}{3} \rceil = \cancel{n} \leq \frac{1}{4}\cancel{n}\lg n$

$4 \leq \lg n$

$2^4 \leq n$

2° $\cancel{\lg n} \leq \frac{1}{4} n \cancel{\lg n}$

$4 \leq n$

$\Rightarrow n_0 = 2^4$

$$n\lg n - \lceil \tfrac{2n}{3} \rceil - \lg n + 4 \geq n\lg n - \tfrac{1}{4} n\lg n - \tfrac{1}{4} n \lg n = \tfrac{1}{2} n \lg n = \tfrac{1}{4} \cdot 2n\lg n$$

Para $c = \frac{1}{4}$, $n_0 = 2^4$

$\therefore \Rightarrow n\lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4 \geq \frac{1}{4} 2n \lg n$

$\therefore$

$$n\lg n - \lceil \tfrac{2n}{3} \rceil - \lg n + 4 = \Omega(2n\lg n)$$

(j) $\lg n!$ es $\Omega(n \lg n)$

$$\lg n! \Rightarrow \underbrace{\lg n + \lg(n-1) + \lg(n-2) + \dots \lg(1)}_{\sum_{i=1}^{n} \lg(i)}$$

$$\sum_{i=1}^{n} \lg(i) = \Omega \; n \lg n$$

$$\Rightarrow \exists \; c, n_0 > 0 \text{ tq } \forall \; n \geq n_0 \longrightarrow c\, n \lg n \leq \sum_{i=1}^{n} \lg(i)$$

$$\frac{1}{c} \sum$$

$$\sum_{i=1}^{n} \lg(i) = \sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=1}^{n} \lg(i) \quad 2\times$$

$$\sum_{i=4}^{10}$$

$$10 - 4 + 1$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=\frac{n}{2}+1}^{n} \lg(n) \leq \sum_{i=1}^{n} \lg(i)$$

$$n - \frac{n}{2} \cancel{+1} + 1$$

$$\frac{n}{2}$$

$$\frac{n}{2} \lg n \leq \lg n + \lg(n-1)$$

$$\frac{n}{2}\left(\lg(n)\right) - \lg n \leq \lg(n-1)$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(n) + \sum_{i=1}^{\frac{n}{2}} \lg(\quad \leq$$

$$2^{\frac{n}{2} \lg n} \leq 2$$

$$\sqrt{2^{n \lg n}} \leq$$

Ejercicio 2: Demostrar q

(a) $\lg \sqrt{n} = O(\lg n)$

$\Rightarrow \exists\ c, n_0 > 0$, tq $\forall\ n \geq n_0 \longrightarrow (\lg \sqrt{n}) \leq c \lg n$

$\frac{1}{2} \lg n \leq c \lg n$

Es verdad para $c = 1$ y $n_0 = 1$

ya que tendremos: $\frac{1}{2} \lg n \leq \lg n \longrightarrow$ Esto es verdad ya que la función logarítmica es creciente

(b) Si $f(n) = O(g(n))$ y $g(n) = O(h(n))$ entonces $f(n) = O(h(n))$

$f(n) = O(g(n))$

$\Rightarrow f(n) \leq c_1 g(n)$

$g(n) = O(h(n))$

$\hookrightarrow (g(n) \leq c_2(h(n))) \times c_1$

$c_1 g(n) \leq \underbrace{c_1 \times c_2}_{} h(n)$

$f(n) \leq c_1 g(n) \leq C h(n)$

$f(n) \leq C h(n) \Longrightarrow f(n) = O(h(n))$

(c) Si $f(n) = O(g(n))$ y $g(n) = \Theta(h(n))$ entonces $f(n) = \Theta(h(n))$

Si $f(n) = O(g(n)) \wedge g(n) = \Theta(h(n))$

$\hookrightarrow f(n) \leq c_1 g(n)$ $\quad c_2 h(n) \leq g(n) \leq c_3 h(n) \Rightarrow ¿f(n) = \Theta(h(n))?$

$n^2 - 3n \leq n^2$ $\quad \frac{1}{2}(n^2+2) \leq n^2 \leq n^2 + 2$ $\quad n^3 \leq n \leq n^3$

$\cancel{n^2} + 2 \leq \cancel{n^2} - 3n$

(d) Si $f(n) = O(g(n))$ entonces $2^{f(n)} = O(2^{g(n)})$

Esto va a cumplir para algun $c \geq 2$

$2\lg n \leq c \lg n \longrightarrow ¿2^{2\lg n} \leq 2^{\lg n}?$

$n^2 \leq n$ $(\rightarrow\leftarrow)$ ⊅

(f) $\text{máx}\{f(n), g(n)\} = \Theta(f(n) + g(n))$, para funciones no negativas $f(n)$ y $g(n)$.

Es cierto que $f(n) \leq f(n) + g(n) \wedge g(n) \leq f(n) + g(n)$

$\therefore$ $\max(f(n), g(n)) \leq f(n) + g(n)$

$\hookrightarrow \max(f(n), g(n)) = O(f(n) + g(n))$

Es cierto que $f(n) + g(n) \leq 2\max(f(n), g(n))$

$\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n))$

$\Longrightarrow \max(f(n), g(n)) = \Omega(f(n), g(n))$

$\therefore$ $\max(f(n), g(n)) = \Theta(f(n), g(n))$ Δ

(g) $(n+a)^b = \Theta(n^b)$, donde $a, b \in \mathbb{R}$ y $b > 0$.

$$C_1 n^b \leq n^b + \dots + a^b \leq C_2 n^b$$

$C_1 = 1$ cumple

Big-O:

$$\underbrace{n^b + a^b + \dots}_{b+1 \text{ terminos}} \leq C_2 n^b$$

$$+ \downarrow \quad b+1 \text{ terminos} \left\{ \begin{array}{lll} n^b \leq & n^b & \forall n \geq 1 \\ a^b \leq & n^b & \forall n \geq a \\ \vdots & & \\ \leq & n^b & \end{array} \right.$$

Para algun $n \geq X$ siendo x el maximo valor de cada cota que puede tomar n

$$(n+a)^b \leq (b+1) n^b$$

Para un $C_2 = b+1$

$\Rightarrow (n+a)^b \leq (b+1) n^b$

asi $(n+a)^b = \Theta(n^b)$

(h) $\sqrt{n} = O(\lg^2 n)$.

$\sqrt{n^{1/2}} \le \sqrt{\lg^2 n}$

$n^{1/2} \le c \lg^2 n$

$n^{1/4} \le c \lg n$

$\lg^2 n \ge \lg n \ge$

n

$\frac{n^{1/4}}{\lg n} \le c$

$\sqrt{n} \le c \lg^2 n$

$2^{(\lg n)^2}$

$n \le c \lg^2 n$

$2^n \le 2$

$2^{n+1}-1$

1 $\qquad$ $n^{?}$

(i) $\sum_{k=1}^{n} k^{99} = \Theta(n^{100})$

$\underbrace{1+2^{99}+\dots n^{99}}$

$\exists\ c_1, c_2, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow c_1 n^{100} \leq \sum_{k=1}^{n} k^{99} \leq c_2 n^{100}$

(j) Suponga que $\lg(g(n)) \geq 1$ y que $f(n) \geq 1$ para todo n suficientemente grande. En ese caso, si $f(n) = O(g(n))$ entonces $\lg(f(n)) = O(\lg(g(n)))$.

$\lg(g(n)) \geq 1$

$2^{\lg(g(n))} \geq 2^1$

$g(n) \geq 2$

$f(n) \geq 1$

$\lg(f(n)) \geq 0$

Si:

$\Rightarrow f(n) = O(g(n)) \longrightarrow$ ¿$\lg(f(n)) = O(\lg(g(n)))$?

$\exists\, c, n_0 > 0 ;\ \forall\, n \geq n_0$

$\hookrightarrow f(n) \leq c\, g(n)$

$\lg()\{$

$\lg(f(n)) \leq \lg(c\, g(n))$

$\lg(f(n)) \leq \underbrace{\lg c}_{c} + \lg(g(n))$

Queremos que

$c \leq \lg(g(n))$

$\lg(f(n)) \leq c + \lg(g(n))$

$2^c \leq g(n)$

$\therefore$

$\lg(f(n)) \leq c + \lg(g(n))$

$\therefore\ g(n) \geq 2^c$

$\leq \lg(g(n)) + \lg(g(n))$

$\lg(f(n)) \leq 2\lg(g(n)) \longrightarrow$ Para un $g(n) \geq 2^c$

$cn \leq 2n^2 + 3n + 4$