a) $n^2 - 10n + 2 = O(n^2)$

$\rightarrow n^2 - 10n + 2 \leq 3n^2$

$$+\ \begin{array}{ll} n^2 \leq n^2 & \forall\ n \geq 1 \\ 2 \leq n^2 & \forall\ n \geq \sqrt{2} \\ & n \geq 2 \end{array}$$

$n^2 + 2 \leq 2n^2 \quad \forall\ n \geq 2$

$n^2 - 10n + 2 \leq 2n^2 \quad \forall\ n \geq 2$

Para $C = 2$ y $n_0 = 2$

b) $\lceil \frac{n}{3} \rceil = O(n)$

$\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \lceil \frac{n}{3} \rceil \leq cn$

$\lceil \frac{n}{3} \rceil \leq \lceil \frac{3n}{3} \rceil = \lceil n \rceil \leq n \Longrightarrow \lceil \frac{n}{3} \rceil \leq n \ ; \forall\ n \geq 1$

$C = 1$

c) $\lg n = O(\lg_{10} n)$

$\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \lg n \leq c \lg_{10} n$

$\cancel{\lg n} \leq C \dfrac{\cancel{\lg_2 n}}{\lg_2 10}$

$\lg 10 \leq C$

$C = 4$

Para $C = 4$ y $n_0 = 1$

$\forall\ n \geq n_0$

$\lg n \leq 4 \lg_{10} n \quad \forall\ n \geq n_0$

(d) $n = O(2^n)$

$\Leftrightarrow \exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \rightarrow n \leq c2^n$

Para $c = 1$ y $n_0 = 1$ cumple que : $n \leq 2^n$

lg

Porque $\lg(n) \leq \lg(2)^n$

$\lg(n) \leq n \rightarrow$ Es trivial

true

e) $\lg n$ no es $\Omega(n)$

$\Leftrightarrow$

$\lg n$ es $\Omega(n)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \rightarrow cn \leq \lg n$

$c \leq \frac{\lg n}{n}$

$\lim_{n \to \infty} \frac{\lg n}{n} = 0$ $\qquad c \leq 0$

$\hookrightarrow$

F) $\frac{n}{100}$ No es $O(1)$

$\hookrightarrow \frac{n}{100}$ es $O(1)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \rightarrow \frac{n}{100} \leq c$

$\hookrightarrow$ c depende de n

No sera constante

(h) $\frac{1}{3}(n+1)(n-2)-5=\Theta(n^2)$.

$$\frac{1}{3}(n^2-n-2)-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{2}{3}-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}=\Theta(n^2)$$

$$\exists\ c_1, c_2, n_0 > 0 \quad \forall\ n \geq n_0 \longrightarrow c_1 n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq c_2 n^2$$

Para big-O

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq c_2 n^2$$

si quito seguira siendo menor

$$\frac{1}{3}n^2 \leq n^2 \quad \forall\ n \geq 1 \ \wedge \ c_2 = 1$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq n^2 \quad \checkmark$$

Para Big-Ω:

torta

$$c_1 n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}$$

Los limitamos a mitad de torta

$$\frac{1}{\cancel{3}}\cancel{n} \leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^{\cancel{2}}\right) \longrightarrow 4 \leq n$$

$$\frac{17}{\cancel{3}} \leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^2\right) \longrightarrow \sqrt{17\times 4} \leq n$$

$8,24 \leq n$

$\hookrightarrow 9 \leq n$

Para $n \geq 9$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \geq \frac{4}{4}\frac{1}{3}n^2-\frac{1}{4}\left(\frac{1}{3}n^2\right)-\frac{1}{4}\left(\frac{1}{3}n^2\right)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \geq \frac{4n^2-n^2-n^2}{12} = \frac{\cancel{2}n^2}{\cancel{12}_{6}} = \frac{1}{6}n^2$$

c_1

q.d. Para $c_1=\frac{1}{6}$ $\wedge$ $c_2 = 1$ y $n_0 = 9$

$$\Longrightarrow \frac{1}{6}n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3} \leq n^2 \Longrightarrow \frac{1}{3}(n+1)(n-2)-5=\Theta(n^2) \text{ D.}$$

(i) $n \lg n - \lceil 2n/3 \rceil - \lg n + 4$ es $\Omega(2n \lg n)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow c\, 2n \lg n \leq n \lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4$

tarta

$c\, 2n \lg n \leq n \lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4$

acotamos a mitad de tarta

1° $\lceil \frac{2n}{3} \rceil \leq \frac{1}{4} n \lg n$

2° $\lg n \leq \frac{1}{4} n \lg n$

1° $\lceil \frac{2n}{3} \rceil \leq \lceil \frac{3n}{3} \rceil = \cancel{n} \leq \frac{1}{4} \cancel{n} \lg n$

$4 \leq \lg n$

$2^4 \leq n$

2° $\cancel{\lg n} \leq \frac{1}{4} n \cancel{\lg n}$

$4 \leq n$

$\Rightarrow n_0 = 2^4$

$$n \lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4 \geq n \lg n - \frac{1}{4} n \lg n - \frac{1}{4} n \lg n = \frac{1}{2} n \lg n$$

$$= \frac{1}{4} \cdot 2n \lg n$$

Para $c = \frac{1}{4}$ ∧ $n_0 = 2^4$

$\therefore \Rightarrow n \lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4 \geq \frac{1}{4} 2n \lg n$

$\therefore$

$$n \lg n - \lceil \frac{2n}{3} \rceil - \lg n + 4 = \Omega(2n \lg n)$$

(j) $\lg n!$ es $\Omega(n \lg n)$

$$\lg n! \Rightarrow \underbrace{\lg n + \lg(n-1) + \lg(n-2) + \dots \lg(1)}_{\sum_{i=1}^{n} \lg(i)}$$

$$\sum_{i=1}^{n} \lg(i) = \Omega \; n \lg n$$

$$\Rightarrow \exists \; c, n_0 > 0 \text{ tq } \forall \; n \geq n_0 \rightarrow c n \lg n \leq \sum_{i=1}^{n} \lg(i)$$

$$\frac{1}{c} \sum$$

$$\sum_{i=4}^{10}$$

$$\sum_{i=1}^{n} \lg(i) = \sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=1}^{n} \lg(i) \qquad 2x$$

$$10 - 4 + 1$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=\frac{n}{2}+1}^{n} \lg(n) \leq \sum_{i=1}^{n} \lg(i)$$

$$n - \frac{n}{2} \cancel{+1} + 1$$

$$\frac{n}{2}$$

$$\frac{n}{2} \lg n \leq \lg n + \lg(n-1)$$

$$\frac{n}{2}(\lg(n)) - \lg n \leq \lg(n-1)$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(n) + \sum_{i=1}^{\frac{n}{2}} \lg(\leq$$

$$2^{\frac{n}{2} \lg n} \leq 2$$

$$\sqrt{2^{n \lg n}} \leq$$

Ejercicio 2: Demostrar ...

(a) $\lg\sqrt{n} = O(\lg n)$

$\Rightarrow \exists\, c, n_0 > 0$, tq $\forall\, n \geq n_0 \longrightarrow (\lg\sqrt{n}) \leq c \lg n$

$\frac{1}{2}\lg n \leq c\lg n$

Es verdad para $c = 1$ y $n_0 = 1$

ya que tendremos: $\frac{1}{2}\lg n \leq \lg n \longrightarrow$ Esto es verdad ya que la función logarítmica es creciente

(b) Si $f(n) = O(g(n))$ y $g(n) = O(h(n))$ entonces $f(n) = O(h(n))$

$f(n) = O(g(n))$

$\hookrightarrow f(n) \leq c_1 g(n)$

$g(n) = O(h(n))$

$\hookrightarrow (g(n) \leq c_2(h(n))) \times c_1$

$c_1 g(n) \leq \underbrace{c_1 \times c_2}\, h(n)$

$f(n) \leq c_1 g(n) \leq c\, h(n)$

$f(n) \leq c\,h(n) \Longrightarrow \underline{f(n) = O(h(n))}$

(c) Si $f(n) = O(g(n))$ y $g(n) = \Theta(h(n))$ entonces $f(n) = \Theta(h(n))$

Si $f(n) = O(g(n))$ $\wedge$ $g(n) = \Theta(h(n))$

$\hookrightarrow f(n) \leq c_1 g(n)$ $\quad c_2 h(n) \leq g(n) \leq c_3 h(n)$ $\Rightarrow$ ¿$f(n) = \Theta(h(n))$?

$n^2 - 3n \leq n^2$ $\quad \frac{1}{2}(n^2+2) \leq n^2 \leq n^2+2$ $\quad n^3 \leq n \leq n^3$

$\cancel{n^2}+2 \leq \cancel{n^2}-3n$

(d) Si $f(n) = O(g(n))$ entonces $2^{f(n)} = O(2^{g(n)})$

Esto va a cumplir para algun $c \geq 2$

$2 \lg n \leq c \lg n \longrightarrow ¿2^{2\lg n} \leq 2^{\lg n}?$

$n^2 \leq n$ $(\rightarrow\leftarrow)$ D

(f) $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$, para funciones no negativas $f(n)$ y $g(n)$.

Es cierto que $f(n) \leq f(n) + g(n) \wedge g(n) \leq f(n) + g(n)$

$\therefore$ $\max(f(n), g(n)) \leq f(n) + g(n)$

$\hookrightarrow \max(f(n), g(n)) = O(f(n) + g(n))$

Es cierto que $f(n) + g(n) \leq 2\max(f(n), g(n))$

$\frac{1}{2}(f(n) + g(n)) \leq \max(f(n), g(n))$

$\Rightarrow \max(f(n), g(n)) = \Omega(f(n), g(n))$

$\therefore$ $\max(f(n), g(n)) = \Theta(f(n), g(n))$ $\triangle$

(g) $(n+a)^b = \Theta(n^b)$, donde $a, b \in \mathbb{R}$ y $b > 0$.

$$c_1 n^b \leq n^b + \dots + a^b \leq c_2 n^b$$

$c_1 = 1$ cumple

Big-O:

$$\underbrace{n^b + a^b + \dots}_{b+1 \text{ terminos}} \leq c_2 n^b$$

$$+\left\downarrow \quad b+1 \text{ terminos} \begin{cases} n^b \leq n^b & \forall n \geq 1 \\ a^b \leq n^b & \forall n \geq a \\ \vdots & \\ \leq n^b & \end{cases} \right.$$

Para algun $n \geq X$ siendo X el maximo valor de cada cota que puede tomar n

$$(n+a)^b \leq (b+1) n^b$$

Para un $c_2 = b+1$

$$\hookrightarrow (n+a)^b \leq (b+1) n^b$$

así $(n+a)^b = \Theta(n^b)$

(h) $\sqrt{n} = O(\lg^2 n)$.

(F)

Contradicción:

$\exists\, c, n > 0 \rightarrow$ tq $\forall\, n \geq n_0$

$$\left(\frac{\sqrt{n}}{\lg n}\right)' = \left[\frac{\frac{1}{2\sqrt{n}}}{\frac{1}{n\cdot\ln 2}}\right]$$

$$\lg n = \frac{1}{\ln 2\, n} \qquad \frac{n \ln 2}{2\sqrt{n}}$$

$$\frac{\sqrt{n}\,\ln 2}{2} = \infty$$

$2^{n+1}-1$

1 $\quad n^{2}$

(i) $\sum_{k=1}^{n} k^{99} = \Theta(n^{100})$

$\underbrace{1+2^{99}+\dots n^{99}}$

$\exists\, c_1, c_2, n_0 > 0$ tq $\forall\, n \geq n_0 \longrightarrow c_1 n^{100} \leq \sum_{k=1}^{n} k^{99} \leq c_2 n^{100}$

(j) Suponga que $\lg(g(n)) \geq 1$ y que $f(n) \geq 1$ para todo n suficientemente grande. En ese caso, si $f(n) = O(g(n))$ entonces $\lg(f(n)) = O(\lg(g(n)))$.

$\lg(g(n)) \geq 1$ | $f(n) \geq 1$

$2^{\lg(g(n))} \geq 2^1$ | $\lg(f(n)) \geq 0$

$g(n) \geq 2$

Si:

$\Rightarrow f(n) = O(g(n)) \longrightarrow$ ¿$\lg(f(n)) = O(\lg(g(n)))$?

$\exists\, c, n_0 > 0 \;;\; \forall\, n \geq n_0$

$\hookrightarrow f(n) \leq c\, g(n)$

$\lg()\{$

$\lg(f(n)) \leq \lg(c\, g(n))$

$\lg(f(n)) \leq \lg c + \lg(g(n))$ Queremos que

c

$\lg(f(n)) \leq c + \lg(g(n))$ $\quad c \leq \lg(g(n))$

¿o? $\quad 2^c \leq g(n)$

$\lg(f(n)) \leq c + \lg(g(n))$

$\leq \lg(g(n)) + \lg(g(n))$ ¿o? $g(n) \geq 2^c$

$\lg(f(n)) \leq 2\lg(g(n)) \longrightarrow$ Para un $g(n) \geq 2^c$

$2^n + n^{20} = O(3^n)$

$2^n \leq 3^n$ ✓

$n^{20} \leq 3^n$

Para un $n = 100$

$10^{40} \leq 3^{100}$

$2n^2 + n\lg n = \theta(n^2)$

$$\begin{aligned} &2n^2 \leq 2n^2 \quad \forall\, n \geq 1 \\ + \quad &(\lg n \leq n) \times n \\ &n\lg n \leq n^2 \quad \forall\, n \geq 1 \end{aligned}$$

$2n^2 + n\lg n \leq 3n^2 \quad \forall\, \underline{n \geq 1} \quad \wedge \quad \underline{c = 3}$

$n^{1+\frac{2}{\sqrt{\lg n}}} = O(n\lg n)$ $\quad \frac{2}{\sqrt{\lg n}} \to 0$ $\quad$ (V)

$\exists\, c, n_0 > 0$ tq $\forall\, n \geq n_0 \longrightarrow n^{1+\frac{2}{\sqrt{\lg n}}} \leq c\, n\lg n$

$(\sqrt{\lg n})^2 \geq 2^2$

$\lg n \geq 4$ $\qquad n^{1+\frac{2}{\sqrt{\lg n}}} = O(n\lg n)$

$n \geq 16$

Para un $n \geq 2^{256}$ $\qquad n^{2/\sqrt{\lg n}} \leq \lg n$

$n^{1+\;2}$

$n^{0,3} < n^{0,5}$ ✓

$C\sqrt{\frac{n}{2}}$ — $C\sqrt{n+26}$ — $\sqrt[3]{n+1}$ — $\sqrt[3]{2n}$

Por contradicción

↳ $\sqrt[3]{n+1} = \Omega(\sqrt{n-26})$

↳ $\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \sqrt[3]{n+1} \geq c\sqrt{n-26} \geq 0$

la idea es la siguiente

Enunciamos el contraejemplo

↳ Sea $n = n_0 + 2c$

$$C\frac{1}{2}\sqrt{n} \leq C\sqrt{\frac{1}{2}}\sqrt{n} = C\sqrt{\frac{n}{2}} \leq C\sqrt{n-26} \leq \sqrt[3]{n+1} \leq \sqrt[3]{2n} \leq 2\sqrt[3]{n}$$

$\frac{n}{2} \leq n-26$

$n \leq 2n - 52$

$n \geq 52$

$$C\frac{1}{2}\sqrt{n} \leq 2\sqrt[3]{n}$$

$$\left(\frac{C}{4}\sqrt{n} \leq \sqrt[3]{n}\right)^6$$

$$\left(\frac{C}{4}\right)^6 n^{3} \leq \cancel{n^2}$$

$$n \leq \left(\frac{4}{C}\right)^6$$

$n^{1.001} + n\lg n = O(n\lg n)$ $\Rightarrow$ Por limites

Si: $\lim_{n\to\infty} \frac{F(n)}{S(n)} = 0$ true

sino: FALSE

Sacando Limites esto es falso

$\rightarrow$ Por contradicción

$\exists\ n_0, c > 0$ tq. $\forall\ n \geq n_0 \quad 0 < n^{1.001} + n\lg n \leq c\,n\lg n$

$$\frac{n^{0.001}}{\lg n} + 1 \leq C$$

Para un $\boxed{n = 2^{\frac{c}{0.001}}}$

$$\frac{2^{\frac{c}{0.001}\cdot 0.001}}{\lg 2^{\frac{c}{0.001}}} \Rightarrow \frac{2^c}{\frac{c}{0.001}} + 1 \leq C$$

$$¿\ \frac{2^c}{10^3 c} + 1 \leq C\ ?$$

$\to \left(\sqrt{n+20}\right)\left(\sqrt{n}-24\right) = \Theta(0.2024n)$

$\exists\, c_1, c_2, n_0$ tq $\forall\, n \geq n_0$

$$\left(\sqrt{n+20}\right)\left(\sqrt{n}-24\right) \leq c_2 (0,2024) n$$

$$\sqrt{n+20} \leq \frac{1}{0,2024} 0.2024 \sqrt{2n}$$

$$\sqrt{n} - 24 \leq \sqrt{2n} \qquad \forall\, n \geq 25^2$$

$$\left(\sqrt{n+20}\right)\left(\sqrt{n}-24\right) \leq \frac{1}{0,2024} \cdot 0,2024 \sqrt{4n^2}$$

$$\left(\sqrt{n+20}\right)\left(\sqrt{n}-24\right) \leq \underbrace{\frac{2}{0,2024}}_{C} \cdot 0,2024 n$$

Big-Ω

$$c_1 0,2024 n \leq \left(\sqrt{n+20}\right)\left(\sqrt{n}-24\right)$$

$$0.2024\sqrt{n} \leq \sqrt{n} \leq \sqrt{n+20} \qquad \forall\, n \geq 1$$

$$\sqrt{n} \leq \sqrt{n} - 24 \qquad \forall\, n \geq 1$$

$$0.2024\sqrt{n} \leq \sqrt{n+20} \qquad \forall\, n \geq 1$$

$$\sqrt{n} \leq \sqrt{n} - 24 \qquad \forall\, n \geq 25$$

$$0.2024 n \leq \left(\sqrt{n+20}\right)\left(\sqrt{n}-24\right) \qquad \forall\, n \geq 1$$

$$2n^3 - 2023n^2 - 2023^2 n\lg n + 10^{10}\lg^2 n + 1 = O\left(\frac{1}{2}^{2023} n^3\right)$$

$$- \quad 10^{10}\lg^2 n \leq 10^{10}\cancel{n^2} \leq n^{\cancel{3}}$$

$$n \geq 10^{10}$$

$$\left.\begin{array}{l} n \geq 10^{10} \\ - \quad 1 \leq n^3 \quad \forall\, n \geq 1 \end{array}\right\} \quad n \geq 10^{10}$$

$$2n^3 - 2023n^2 - 2023^2 n\lg n + 10^{10}\lg^2 n + 1 \leq 2n^3 + n^3 + n^3$$

$$\leq 4n^3 = 4 \cdot 2^{2023} \cdot \frac{1}{2^{2023}}\, n^3$$

$$= 2^2 \cdot 2^{2023} \cdot \left(\frac{1}{2}\right)^{2023} n^3$$

$$2n^3 - 2023n^2 - 2023^2 n\lg n + 10^{10}\lg^2 n + 1 \leq 2^{2025}\left(\frac{1}{2}\right)^{2023} n^3$$

$\hookrightarrow$ Para un $C = 2^{2025}$ $\wedge$ $n_0 = 10^{10}$ //

(a) $n = \lfloor \frac{n}{3} \rfloor + \lfloor \frac{n+1}{3} \rfloor + \lfloor \frac{n+2}{3} \rfloor$ for all $n \in \mathbb{Z}$

1° $n = 3k \to k \in \mathbb{Z}$

$k + \lfloor \frac{1}{3} \rfloor$ $\quad$ $k + \lfloor \frac{2}{3} \rfloor$

$$\left\lfloor \frac{3k}{3} \right\rfloor + \left\lfloor \frac{3k}{3} + \frac{1}{3} \right\rfloor + \left\lfloor \frac{3k}{3} + \frac{2}{3} \right\rfloor$$

$k + k + k$

$3k = n$ ✓

2° $n = 3k+1$

$$\left\lfloor \frac{3k}{3} + \frac{1}{3} \right\rfloor + \left\lfloor \frac{3k}{3} + \frac{2}{3} \right\rfloor + \left\lfloor \frac{3k}{3} + \frac{3}{3} \right\rfloor$$

$k+0 + k+0 + k+1$

$3k+1 = n$ ✓

3° $n = 3k+2$

$$\left\lfloor \frac{3k}{3} + \frac{2}{3} \right\rfloor + \left\lfloor \frac{3k}{3} + \frac{3}{3} \right\rfloor + \left\lfloor \frac{3k}{3} + \frac{4}{3} \right\rfloor$$

$k+0 + k+1 + k+1$

$3k+2 = n$

$$\text{a}^{\text{o}}\text{d} \quad n = \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n+1}{3} \right\rfloor + \left\lfloor \frac{n+2}{3} \right\rfloor \quad \square$$