a) $n^2 - 10n + 2 = O(n^2)$

$\rightarrow n^2 - 10n + 2 \leq 3n^2$

$$+\begin{array}{ll} n^2 \leq n^2 & \forall\ n \geq 1 \\ 2 \leq n^2 & \forall\ n \geq \sqrt{2} \\ \hline & n \geq 2 \end{array}$$

$n^2 + 2 \leq 2n^2 \quad \forall\ n \geq 2$

$n^2 - 10n + 2 \leq 2n^2 \quad \forall\ n \geq 2$

Para $C = 2$ y $n_0 = 2$

b) $\lceil \frac{n}{3} \rceil = O(n)$

$\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \lceil \frac{n}{3} \rceil \leq cn$

$\lceil \frac{n}{3} \rceil \leq \lceil \frac{3n}{3} \rceil = \lceil n \rceil \leq n \implies \lceil \frac{n}{3} \rceil \leq n \ ; \forall\ n \geq 1$

$C = 1$

c) $\lg n = O(\lg_{10} n)$

$\exists\ c, n_0 > 0$ tq $\forall\ n \geq n_0 \longrightarrow \lg n \leq c \lg_{10} n$

$\cancel{\lg n} \leq C \dfrac{\cancel{\lg_2 n}}{\lg_2 10}$

$\lg 10 \leq C$

$\rightarrow C = 4$

Para $C = 4$ y $n_0 = 1$

$\forall\ n \geq n_0$

$\lg n \leq 4 \lg_{10} n \quad \forall\ n \geq n_0$

(d) $n = O(2^n)$

$\exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \rightarrow \quad n \leq c\,2^n$

Para $C=1$ y $n_0 = 1$ cumple que : $\quad n \leq 2^n$

$\lg$

Porque $\quad \lg(n) \leq \lg(2)^n$

$\lg(n) \leq n \rightarrow$ Es trivial

true

e) $\lg n$ no es $\Omega(n)$

$\lg n$ es $\Omega(n)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall\, n \geq n_0 \rightarrow \quad cn \leq \lg n$

$c \leq \dfrac{\lg n}{n}$

$\lim\limits_{n \to \infty} \dfrac{\lg n}{n} = 0 \qquad c \leq 0$

$\hookrightarrow$

F) $\frac{n}{100}$ No es $O(1)$

$\hookrightarrow \frac{n}{100}$ es $O(1)$

$\hookrightarrow \exists\, c, n_0 > 0 \quad$ tq $\forall\; n \geq n_0 \rightarrow \frac{n}{100} \leq C$

$\hookrightarrow$ C depende de n

No sera constante

(h) $\frac{1}{3}(n+1)(n-2)-5=\Theta(n^2)$.

$$\frac{1}{3}(n^2-n-2)-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{2}{3}-5=\Theta(n^2)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}=\Theta(n^2)$$

$$\exists\ c_1, c_2, n_0>0 \quad \forall\ n\geq n_0 \longrightarrow c_1 n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq c_2 n^2$$

Para big-O

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq c_2 n^2$$

$$\frac{1}{3}n^2\leq n^2 \qquad \forall\ n\geq 1 \ \wedge \ c_2=1$$

si quito seguirá siendo menor

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq n^2 \quad ✓$$

Para Big-Ω:

torta

$$c_1 n^2 \leq \frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}$$

Los limitamos a mitad de torta

$$\frac{1}{\cancel{3}}\cancel{n}\leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^{\cancel{2}}\right)\longrightarrow 4\leq n$$

$$\frac{17}{\cancel{3}}\leq \frac{1}{4}\left(\frac{1}{\cancel{3}}n^2\right)\longrightarrow \sqrt{17\times 4}\leq n$$

$$8,24\leq n$$

$$\hookrightarrow 9\leq n$$

Para $n\geq 9$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\geq\frac{4}{4}\frac{1}{3}n^2-\frac{1}{4}\left(\frac{1}{3}n^2\right)-\frac{1}{4}\left(\frac{1}{3}n^2\right)$$

$$\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\geq\frac{4n^2-n^2-n^2}{12}=\frac{\cancel{2}n^2}{\cancel{12}_{6}}=\frac{1}{6}n^2$$

c_1

o.d. Para $c_1=\frac{1}{6}\ \wedge\ c_2=1$ y $n_0=9$

$$\Rightarrow \frac{1}{6}n^2\leq\frac{1}{3}n^2-\frac{1}{3}n-\frac{17}{3}\leq n^2 \Rightarrow \frac{1}{3}(n+1)(n-2)-5=\Theta(n^2)\ \text{D}$$

(i) $n \lg n - \lceil 2n/3 \rceil - \lg n + 4$ es $\Omega(2n \lg n)$

$\hookrightarrow \exists\, c, n_0 > 0$ tq $\forall n \geq n_0 \longrightarrow c\, 2n \lg n \leq n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4$

tarta

$c\, 2n \lg n \leq n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4$

acotamos a mitad de tarta

1° $\left\lceil \frac{2n}{3} \right\rceil \leq \frac{1}{4} n \lg n$

2° $\lg n \leq \frac{1}{4} n \lg n$

1° $\left\lceil \frac{2n}{3} \right\rceil \leq \left\lceil \frac{3n}{3} \right\rceil = \cancel{n} \leq \frac{1}{4} \cancel{n} \lg n$

$4 \leq \lg n$

$()^2 \quad 2^4 \leq n$

2° $\cancel{\lg n} \leq \frac{1}{4} n \cancel{\lg n}$

$4 \leq n$

$\Rightarrow n_0 = 2^4$

$n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4 \geq n \lg n - \frac{1}{4} n \lg n - \frac{1}{4} n \lg n = \frac{1}{2} n \lg n$

$= \frac{1}{4} \cdot 2n \lg n$

Para $c = \frac{1}{4}$ ∧ $n_0 = 2^4$

$\therefore \Rightarrow n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4 \geq \frac{1}{4} 2n \lg n$

$\therefore$

$n \lg n - \left\lceil \frac{2n}{3} \right\rceil - \lg n + 4 = \Omega(2n \lg n)$

(j) $\lg n!$ es $\Omega(n \lg n)$

$$\lg n! \Rightarrow \underbrace{\lg n + \lg(n-1) + \lg(n-2) + \dots \lg(1)}_{\sum_{i=1}^{n} \lg(i)}$$

$$\sum_{i=1}^{n} \lg(i) = \Omega\, n \lg n$$

$$\Rightarrow \exists\, c, n_0 > 0 \text{ tq } \forall\, n \geq n_0 \rightarrow c\, n \lg n \leq \sum_{i=1}^{n} \lg(i)$$

$$\frac{1}{c} \sum$$

$$\sum_{i=4}^{10}$$

$$10 - 4 + 1$$

$$\sum_{i=1}^{n} \lg(i) = \sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=1}^{n} \lg(i) \qquad 2\times$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(i) \leq \sum_{i=\frac{n}{2}+1}^{n} \lg(n) \leq \sum_{i=1}^{n} \lg(i)$$

$$n - \frac{n}{2} \cancel{+1} + 1$$

$$\frac{n}{2}$$

$$\frac{n}{2} \lg n \leq \lg n + \lg(n-1)$$

$$\frac{n}{2} \left(\lg(n)\right) - \lg n \leq \lg(n-1)$$

$$\sum_{i=\frac{n}{2}+1}^{n} \lg(n) + \sum_{i=1}^{\frac{n}{2}} \lg(\leq$$

$$2^{\frac{n}{2} \lg n} \leq 2$$

$$\sqrt{2^{n \lg n}} \leq$$

Ejercicio 2: Demostrar q

(a) $\lg \sqrt{n} = O(\lg n)$

$\Rightarrow \exists\, c, n_0 > 0$, tq $\forall\, n \geq n_0 \longrightarrow (\lg\sqrt{n}) \leq c \lg n$

$\frac{1}{2}\lg n \leq c \lg n$

Es verdad para $c=1$ y $n_0 = 1$

ya que tendremos: $\frac{1}{2}\lg n \leq \lg n \longrightarrow$ Esto es verdad ya que la función logarítmica es creciente

(b) Si $f(n) = O(g(n))$ y $g(n) = O(h(n))$ entonces $f(n) = O(h(n))$

$f(n) = O(g(n))$

$\hookrightarrow f(n) \leq c_1 g(n)$

$g(n) = O(h(n))$

$\hookrightarrow (g(n) \leq c_2(h(n))) \times c_1$

$c_1 g(n) \leq \underbrace{c_1 \times c_2}\, h(n)$

$f(n) \leq c_1 g(n) \leq c\, h(n)$

$f(n) \leq c\, h(n) \Longrightarrow f(n) = O(h(n))$

(c) Si $f(n) = O(g(n))$ y $g(n) = \Theta(h(n))$ entonces $f(n) = \Theta(h(n))$

Si $f(n) = O(g(n)) \wedge g(n) = \Theta(h(n))$

$\hookrightarrow f(n) \leq c_1 g(n)$ $\quad c_2 h(n) \leq g(n) \leq c_3 h(n) \Rightarrow$ ¿$f(n) = \Theta(h(n))$?

$n^2 - 3n \leq n^2$ $\quad \frac{1}{2}(n^2+2) \leq n^2 \leq n^2+2$ $\quad n^3 \leq n \leq n^3$

$\cancel{n}^2+2 \leq \cancel{n}^2 - 3n$