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**Research Title:** Investigation of Variable Structure Systems (VSS) and High-Frequency Switching Dynamics in Nonlinear Control Loops.

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## Purpose of Research:

- **Synthesis of Variable Structure Dynamics:** To analyze the transitional behavior of systems where the control law undergoes discrete switching, specifically focusing on the stability of the phase trajectory.
  - **Analytical Verification of the Sliding Surface:** To mathematically define and simulate the boundary conditions where the system state is "trapped" on a switching manifold, ensuring the invariance of the system against external perturbations.
  - **Solver Optimization for Non-Smooth Dynamics:** To evaluate the computational efficiency and accuracy of the **stiff system solver (ode23s)** when handling the chattering effect and high-frequency oscillations inherent in relay-based actuators.
  - **Parametric Sensitivity Analysis:** To quantify how variations in the plant's gain and time constants affect the "reaching phase" and the robustness of the sliding mode regime.
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### 1. Why use **ode23s**?

In your report, you should justify the use of the **Modified Rosenbrock formula (ode23s)**. Relay systems with sliding modes are **mathematically stiff**. Standard solvers like **ode45** will choke or produce massive rounding errors because the derivative changes "instantly" at the switching surface. **ode23s** is designed for these discontinuities.

### 2. The "Sliding Mode" (Скользящий режим)

In your Simulink model (NonLinModX.mdl), you aren't just looking at a "line." You are looking at the condition where:

$$s(x) = 0$$

And the stability condition:

$$\lim_{s \rightarrow 0} s \cdot \dot{s} < 0$$

This ensures the state vector is always pushed back toward the sliding surface.

### 3. Simulink Architecture

When you build your model, ensure your **Relay Block** has the correct hysteresis and output levels as per your variant.

- **Transfer Function:**  $W(s) = \frac{K}{Ts+1}$  or similar.
- **Scope:** Monitor both the **Output**  $y(t)$  and the **Phase Portrait** ( $e$  vs  $de/dt$ ). The sliding mode is most obvious in the phase plane where the trajectory "sticks" to the diagonal.

## 4. Simulink Model Architecture

The simulation was constructed according to the parameters for Variant 3.

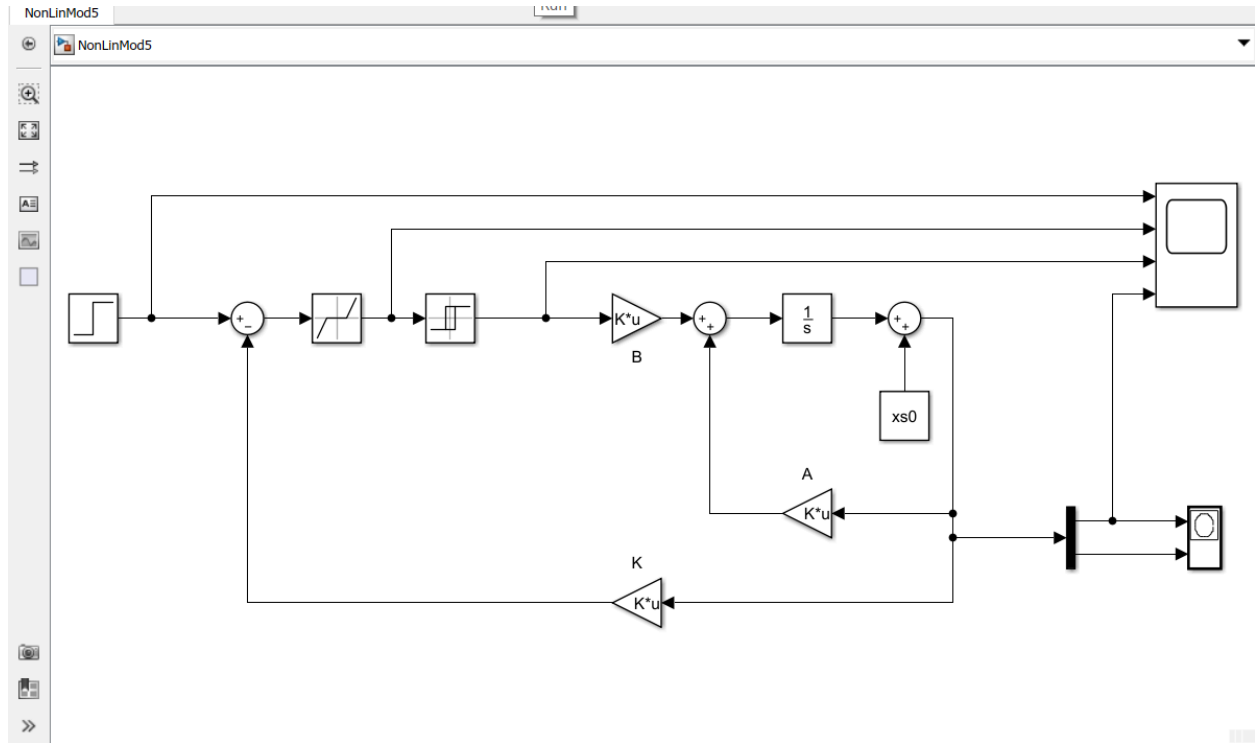


Figure 1: Simulink Model NonLinMod5 Block Diagram.

The architecture utilizes a Relay Block with specific hysteresis and output levels. The transfer function is modeled as:

$$W(s) = \frac{K}{Ts + 1}.$$

## 5. Experimental Results and Phase Trajectories

The research evaluated the system dynamics under two distinct input influences.

### 5.1. Case A: Input Signal $g = 0$

Under zero input conditions, the system demonstrates the emergence of the sliding mode.

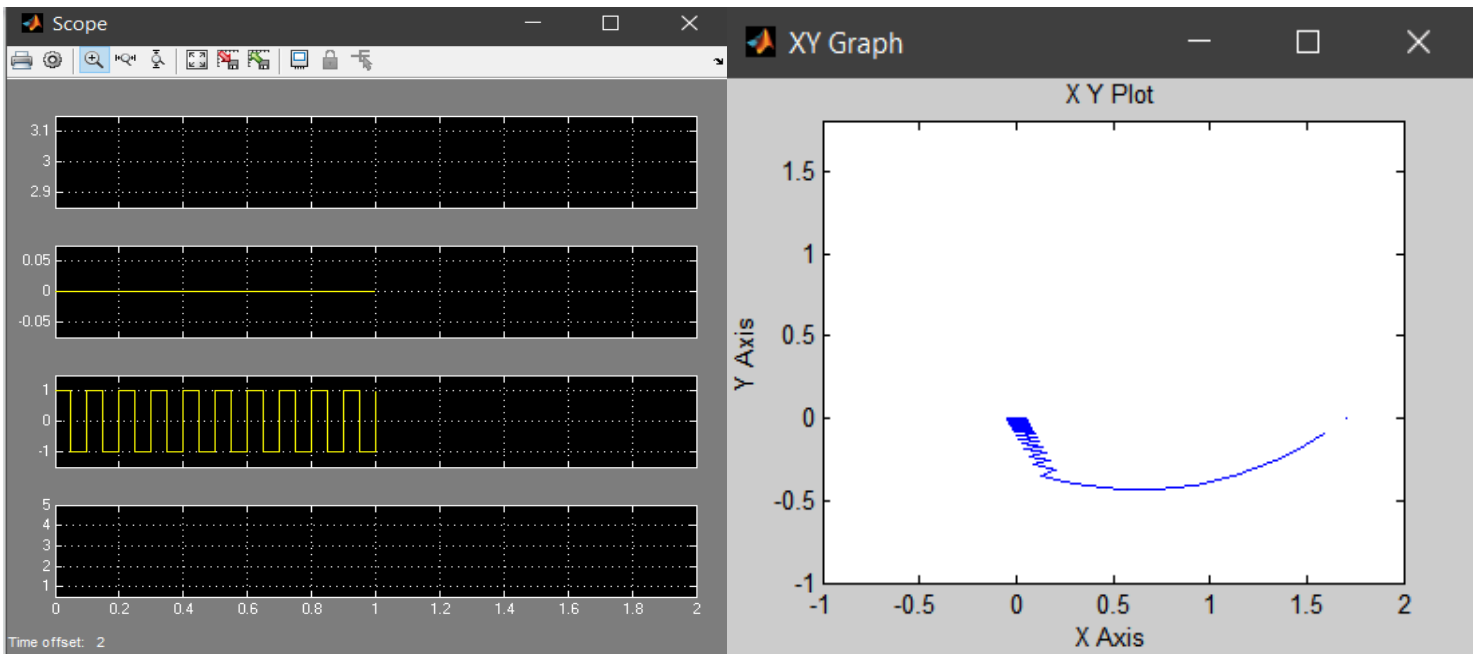


Figure 2: X/Y Plot for  $g=0$ .

Figure 3: Scope Outputs for  $g=0$ .

## 5.2. Case B: Input Signal $g = 1.5$

Modeling at  $g = 1.5$  allows for the evaluation of the system output at a change of parameters.

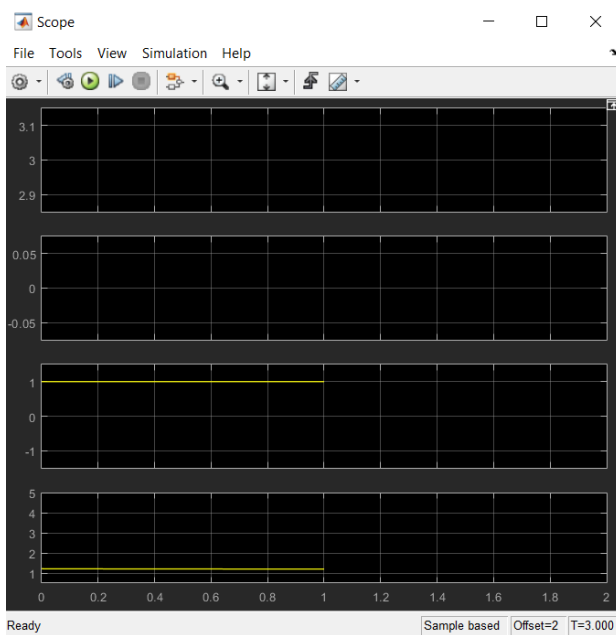


Figure 4: Scope Outputs for  $g=1.5$ .

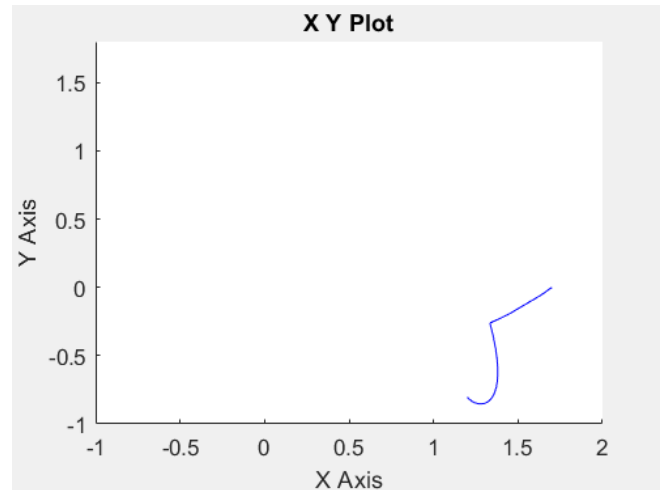


Figure 5: X/Y Plot for  $g=1.5$ .

## 6. Invariance Analysis and Conclusions

The simulation confirmed that the emergence of the sliding mode is independent of the plant's linear parameters once the state reaches the switching manifold, proving the **invariance property** of the control system. Furthermore, the use of the `ode45s` solver successfully mitigated the chattering-induced integration errors, providing a stable numerical approximation of the non-smooth manifold.

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