Multi-agent Decision Making

COMP 4418 – Assignment 3

Due Date: 26 Oct. 2018, 15:00
Total Marks: 50
Late Penalty: 10 marks per day

Worth: 15% of the course

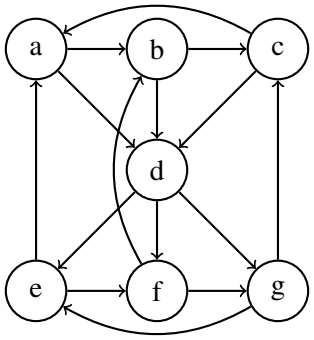

Figure 1: Tournament

Question 1 (10 marks) In the tournament in Figure 1, all the arcs missing from the figure are downward arcs. For the tournament in Figure 1, find the

(a) the uncovered set

(b) the top cycle

(c) the set of Copeland winners

(d) the set of Banks winners

(e) the set of Condorcet winners

and give arguments for your answers.

Question 2 (10 marks) Prove or disprove that the Condorcet winner always has the maximum Borda score among all the alternatives. Prove or disprove that the Condorcet winner has at least half of the Borda score of the Borda winner.

Question 3 (10 marks) Consider a Shapley-Scarf housing market with a set of agents $N = \{1,2,3,4,5\}$, a set of items $O = \{o_1, o_2, o_3, o_4, o_5\}$, an endowment function $\omega : N \to 2^O$ such that $\omega(i) = \{o_i\}$. The preferences of the agents are as follows from left to right in decreasing order of preference.

$$1 : o_5, o_2, o_1, o_3, o_4$$
$$2 : o_5, o_4, o_3, o_1, o_2$$
$$3 : o_4, o_2, o_3, o_5, o_1$$
$$4 : o_2, o_1, o_5, o_3, o_4$$
$$5 : o_2, o_4, o_1, o_5, o_3$$

Find the outcome of the TTC (top trading cycles) algorithm. Can agent 4 misport her preference to get a better a match? Prove or disprove that the outcome is individually rational.

Question 4 (10 marks) Consider the following school choice problem with five students 1, 2, 3, 4, 5 and five schools a, b, c, d, and e with each school having exactly one seat. The preferences of the students are as follows from left to right in decreasing order of preference.

$$1 : e, b, a, c, d$$
$$2 : b, a, c, d, e$$
$$3 : a, b, c, d, e$$
$$4 : a, b, c, d, e$$
$$5 : d, b, c, a, e$$

The priorities of the schools are as follows from left to right in decreasing order of priority.

$$a : 2, 4, 3, 5, 1$$
$$b : 3, 2, 4, 5, 1$$
$$c : 3, 2, 4, 5, 1$$
$$d : 5, 2, 4, 3, 1$$
$$e : 1, 2, 3, 4, 5$$

Find the outcome matching of the student proposing deferred acceptance algorithm and explain how you found the matching. Prove or disprove that the resultant matching is Pareto optimal for the students.

Question 5 (10 marks) Consider a resource allocation setting in which n agents have positive and additive utilities over m items. Recall that the algorithm of Lipton et al. (2004) takes time $O(n^3 m)$ to compute an EF1 (envy-free up to one item) allocation. Design an algorithm that takes time $O(nm)$ and computes an EF1 allocation. Prove the running time of the algorithm. Prove that the algorithm always returns an EF1 allocation. Discuss any advantage that the algorithm of Lipton et al. (2004) may have over the new algorithm.

SUBMISSION

- You will need to answer the questions in a file named `assn3.pdf`. Submit using the command:

```
give cs4418 assn3 assn3.pdf
```

- Your answers are to be submitted in a single PDF file.
- The deadline for this submission is 26 Oct. 2018, 15:00

Academic Honesty and Plagiarism

All work submitted for assessment must be your own work. Assignments must be completed individually. We regard copying of assignments, in whole or part, as a very serious offence. Be warned that:

- the submission of work derived from another person, or jointly written with someone else will, at the very least, result in automatic failure for COMP4418 with a mark of zero;
- allowing another student to copy from you will, at the very least, result in a mark of zero for your own assignment; and
- severe or second offences will result in automatic failure, exclusion from the University, and possibly other academic discipline.
- students are not allowed to derive solutions together as a group during such discussions. Students are also warned not to send solution fragments of the assignments to each other in any form (e.g. as email or listings).
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