Lecture 9: Placement

ECE201A

Some notes adopted from
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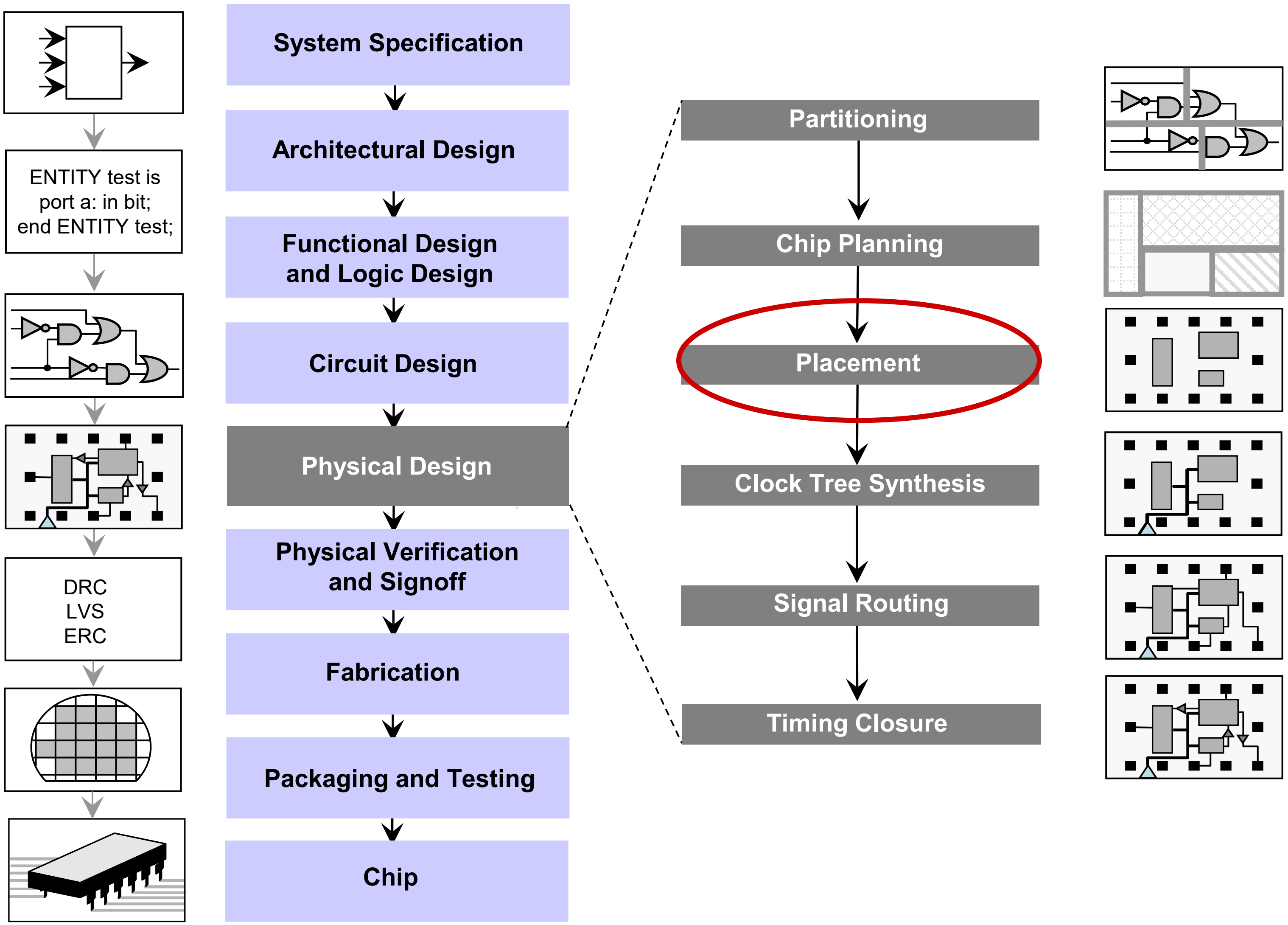
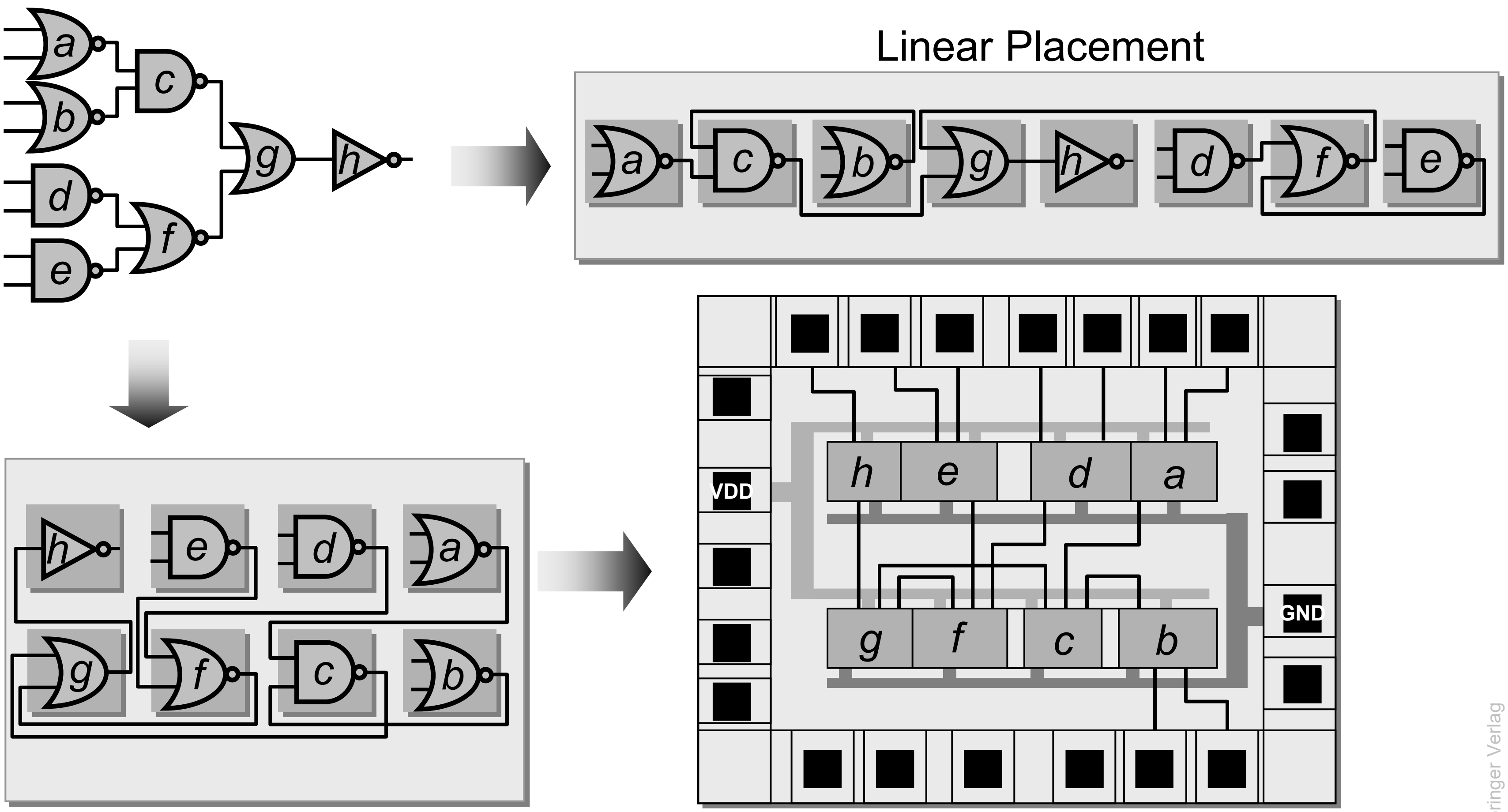
Introduction



Placement



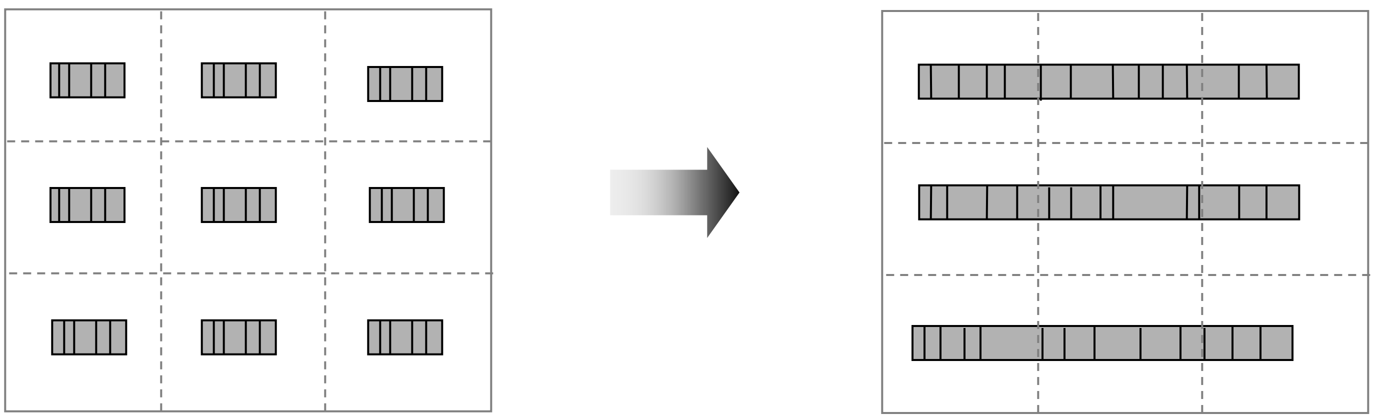
2D Placement

Placement and Routing with Standard Cells

Introduction

Global Placement

Detailed Placement



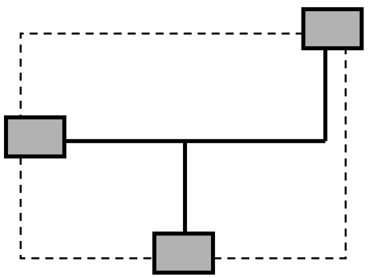
Optimization Objectives

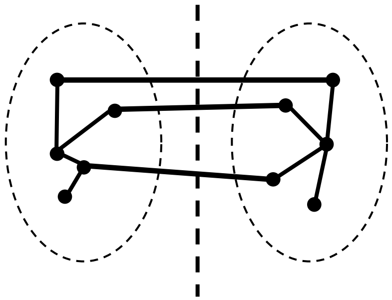
Total Wirelength

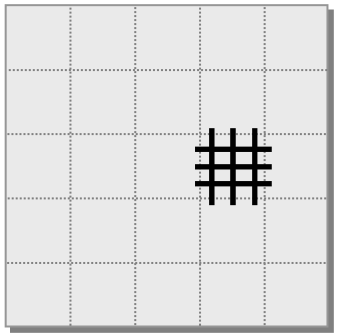
Number of Cut Nets

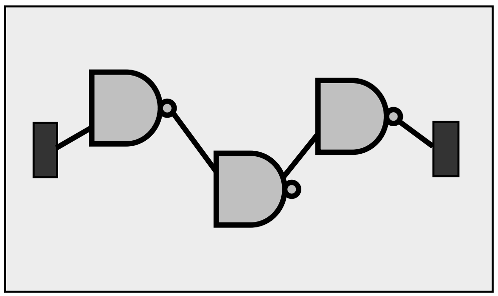
Wire Congestion

Signal Delay

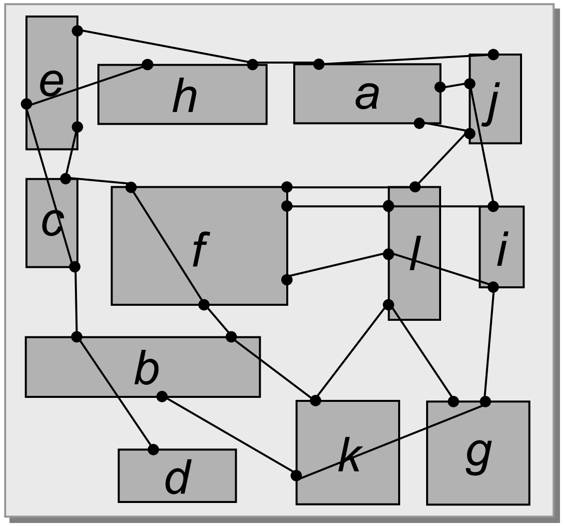


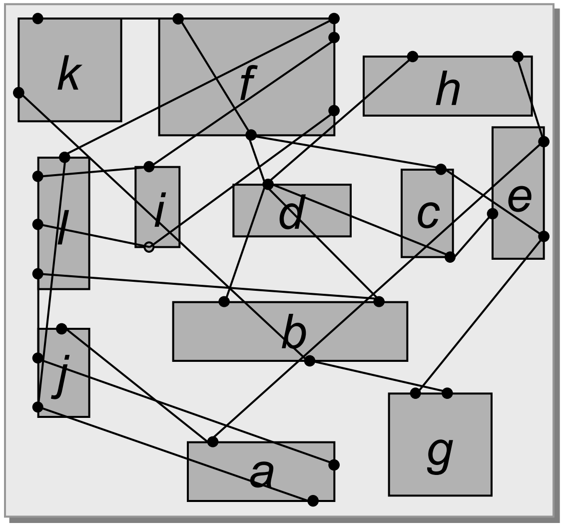




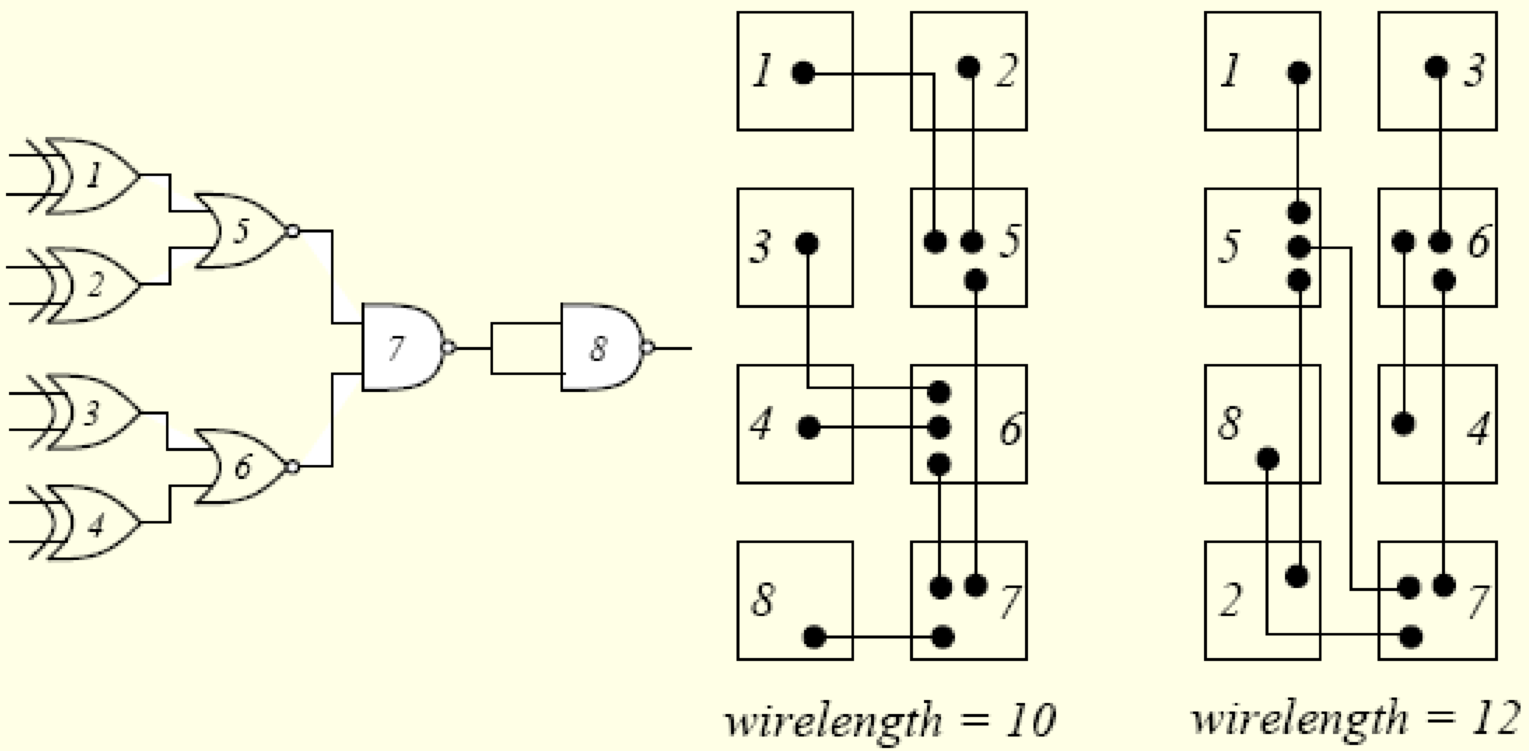


Optimization Objectives – Total Wirelength





Wirelength = Manhattan (L_1) Metric



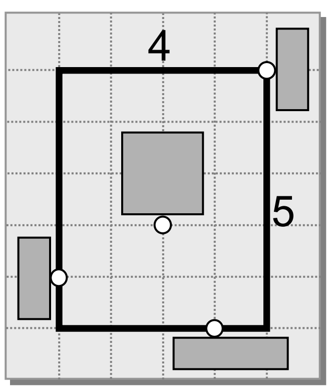
L_p norm: $((\Delta x)^p + (\Delta y)^p)^{1/p}$

p = 1 (Manhattan); p = 2 (Euclidean);
p = ∞ (max, Chebyshev)

How to measure WL ?

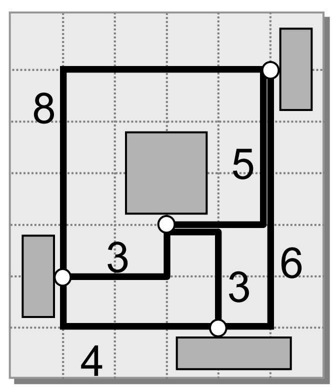
Wirelength estimation for a given placement

Half-perimeter wirelength (HPWL)



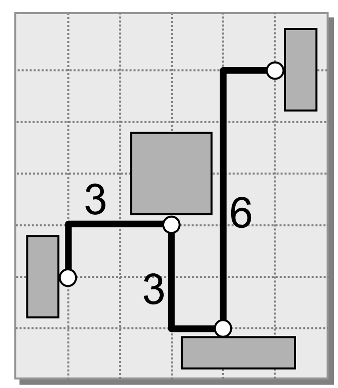
HPWL = 9
Exact for #pins = 2, 3

Complete graph (clique)



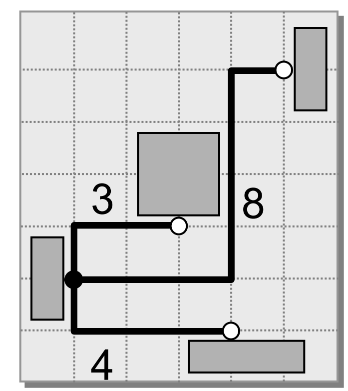
Clique Length = $(2/p)\Sigma_{e \in \text{clique}} d_M(e)$ = 14.5

Monotone chain



Chain Length = 12

Star model



Star Length = 15

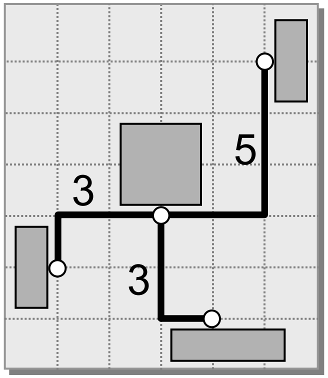
How to measure WL ?

Wirelength estimation for a given placement (cont'd.)

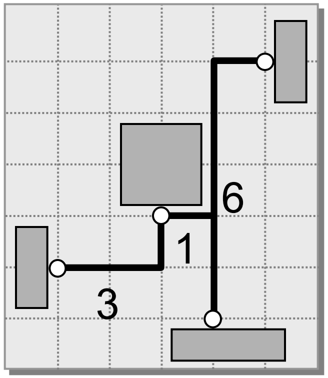
Rectilinear minimum spanning tree (RMST)

Rectilinear Steiner minimum tree (RSMT)

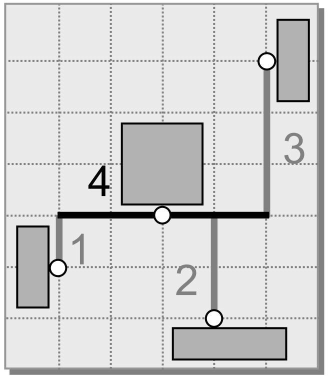
Single-trunk Steiner tree (STST)



RMST Length = 11
O(n log n)



RSMT Length = 10
NP-hard optimization
True WL



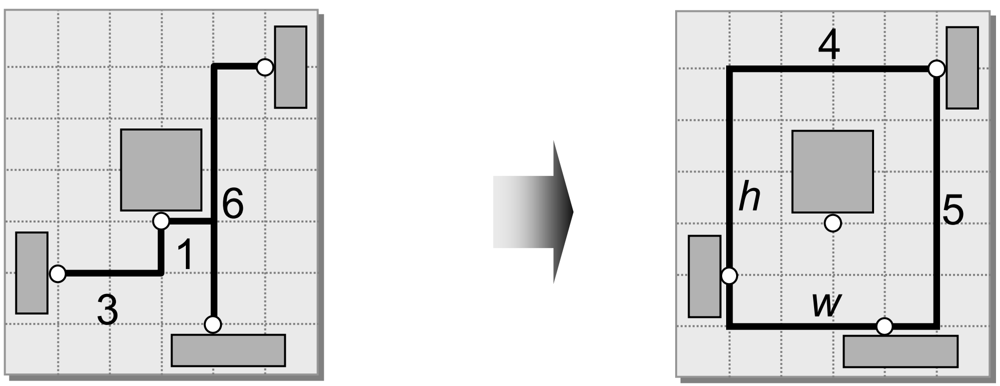
STST Length = 10

Optimization Objectives – Total Wirelength

Wirelength estimation for a given placement (cont'd.)

Preferred method: Half-perimeter wirelength (HPWL)

- Fast (order of magnitude faster than RSMT)
- Equal to length of RSMT for 2- and 3-pin nets
- Margin of error for real circuits approx. 8%



$$L_{\mathrm{HPWL}} = w + h$$

RSMT Length = 10

HPWL = 9

Optimization Objectives – Total Wirelength

Total wirelength with net weights (weighted wirelength)

- For a placement P, an estimate of total weighted wirelength is

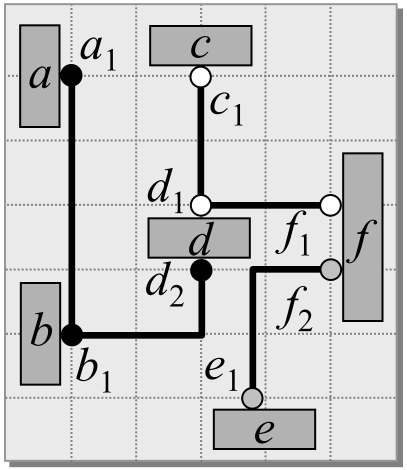
$$L(P) = \sum_{net \in P} w(net) \cdot L(net)$$

where $w(net)$ is the weight of net, and $L(net)$ is the estimated wirelength of net.

Weight could be timing criticality

- Example:

Nets	Weights
$N_1 = (a_1, b_1, d_2)$	$w(N_1) = 2$
$N_2 = (c_1, d_1, f_1)$	$w(N_2) = 4$
$N_3 = (e_1, f_2)$	$w(N_3) = 1$



$$L(P) = \sum_{net \in P} w(net) \cdot L(net) = 2 \cdot 7 + 4 \cdot 4 + 1 \cdot 3 = 33$$

Optimization Objectives – Number of Cut Nets

Cut sizes of a placement

- To improve total wirelength of a placement *P*, separately calculate the number of crossings of global vertical and horizontal cutlines, and minimize

$$L(P) = \sum_{v \in V_P} \psi_P(v) + \sum_{h \in H_P} \psi_P(h)$$

where $\Psi_P(cut)$ be the set of nets cut by a cutline *cut*

- Net-cut cost = number of external nets between different global bins
  - Asymptotically, net-cut cost = wire length
  - Minimizing net-cut in global placement tends to put highly connected cells close to each other.

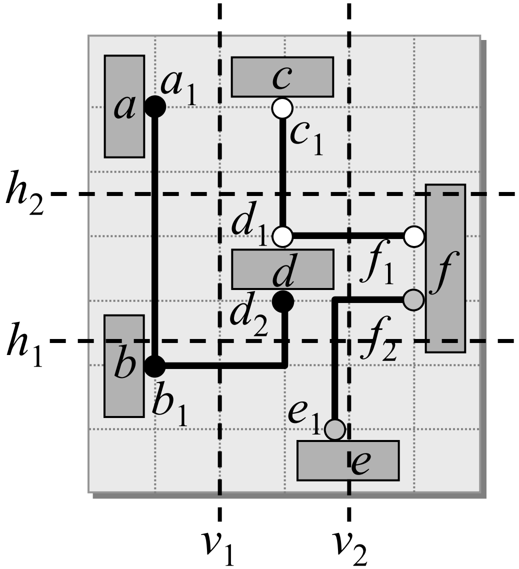
Optimization Objectives – Number of Cut Nets

Cut sizes of a placement

- Example:

  Nets

  $N_1 = (a_1, b_1, d_2)$
  $N_2 = (c_1, d_1, f_1)$
  $N_3 = (e_1, f_2)$



- Cut values for each global cutline

  $\psi_P(v_1) = 1 \quad \psi_P(v_2) = 2$

  $\psi_P(h_1) = 3 \quad \psi_P(h_2) = 2$

- Total number of crossings in P

  $\psi_P(v_1) + \psi_P(v_2) + \psi_P(h_1) + \psi_P(h_2) = 1 + 2 + 3 + 2 = 8$

- Cut sizes

  $X(P) = \max(\psi_P(v_1),\psi_P(v_2)) = \max(1,2) = 2$

  $Y(P) = \max(\psi_P(h_1),\psi_P(h_2)) = \max(3,2) = 3$

Optimization Objectives – Wire Congestion

Routing congestion of a placement

- Formally, the local wire density $\varphi_P(e)$ of an edge e between two neighboring grid cells is

$$\varphi_P(e) = \frac{\eta_P(e)}{\sigma_P(e)}$$

where $\eta_P(e)$ is the estimated number of nets that cross e and
$\sigma_P(e)$ is the maximum number of nets that can cross e

- If $\varphi_P(e) > 1$, then too many nets are estimated to cross e, making P more likely to be unroutable.
- The wire density of P is $\Phi(P) = \max_{e \in E}\left(\varphi_P(e)\right)$

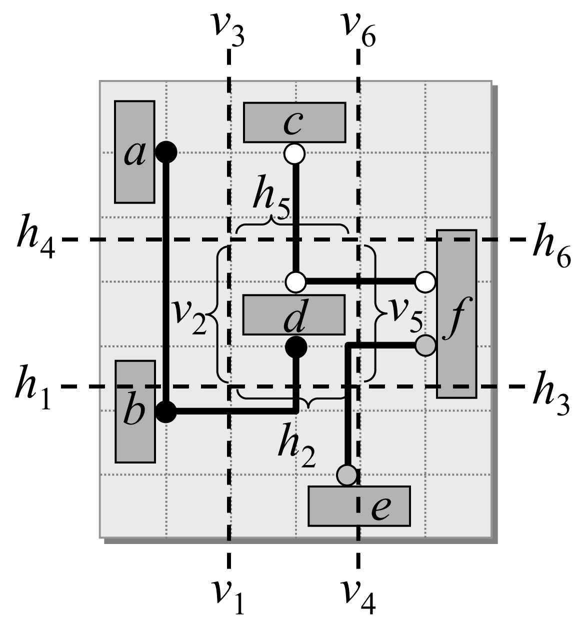
where E is the set of all edges

- If $\Phi(P) < 1$, then the design is estimated to be fully routable, otherwise routing will need to detour some nets through less-congested edges

Optimization Objectives – Wire Congestion

Wire Density of a placement

$\eta_P(h_1) = 1$ $\quad$ $\eta_P(v_1) = 1$
$\eta_P(h_2) = 2$ $\quad$ $\eta_P(v_2) = 0$
$\eta_P(h_3) = 0$ $\quad$ $\eta_P(v_3) = 0$
$\eta_P(h_4) = 1$ $\quad$ $\eta_P(v_4) = 0$
$\eta_P(h_5) = 1$ $\quad$ $\eta_P(v_5) = 2$
$\eta_P(h_6) = 0$ $\quad$ $\eta_P(v_6) = 0$



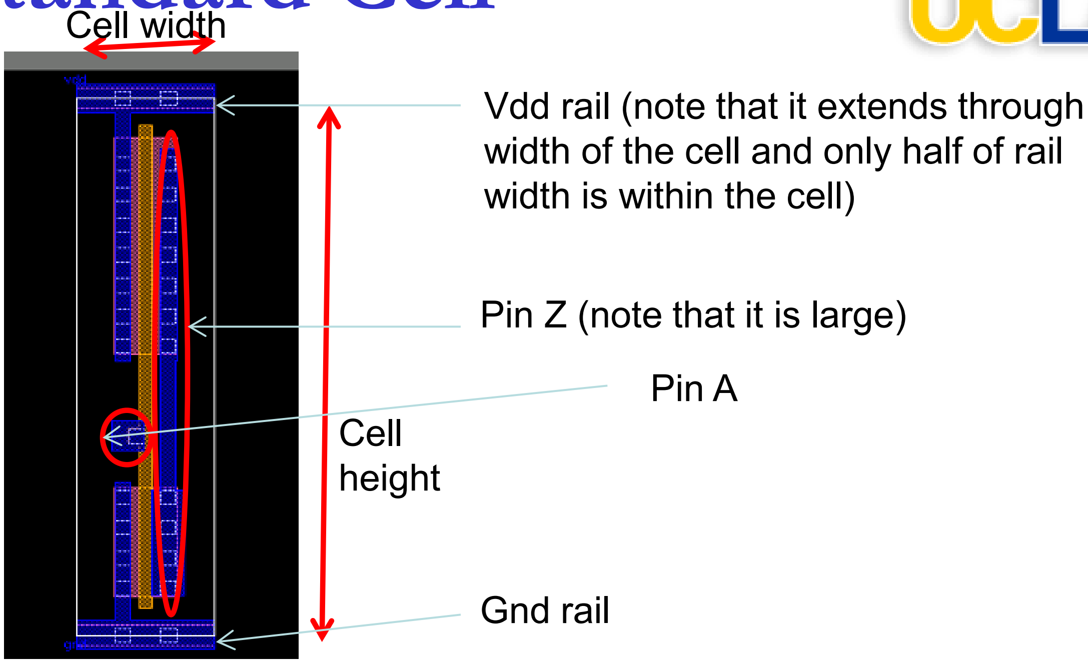
Maximum: $\eta_P(e) = 2$

$$\boxed{\Phi(P) = \frac{\eta_P(e)}{\sigma_P(e)} = \frac{2}{3}}$$

Routable

The Standard Cell

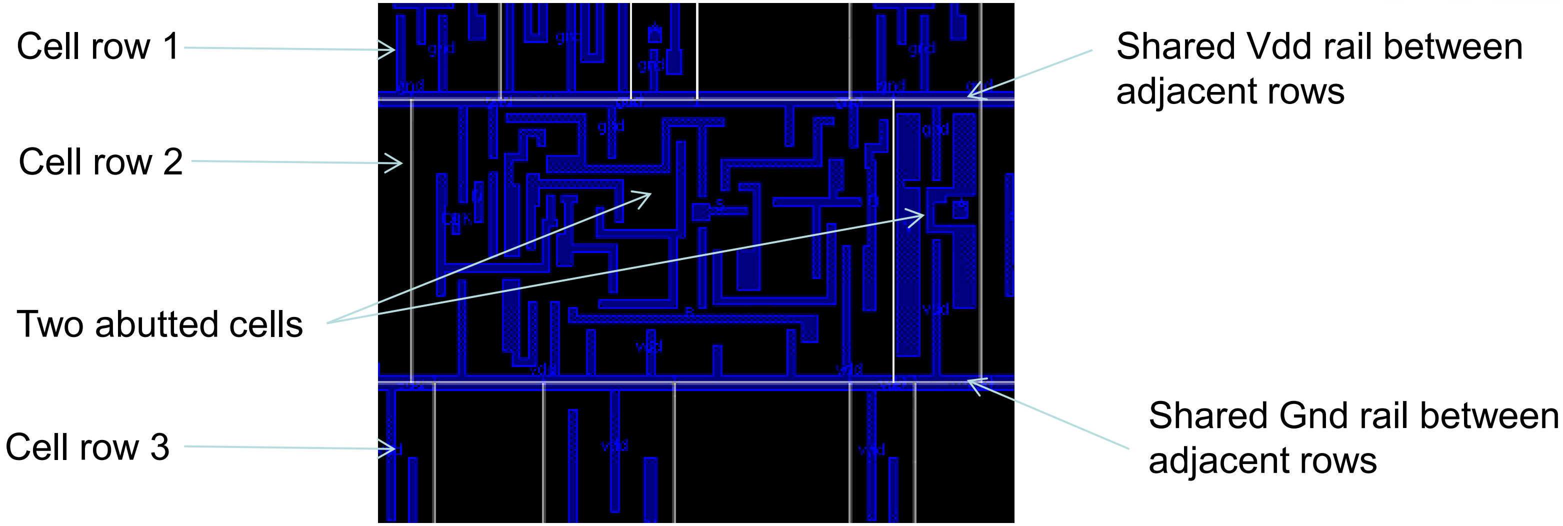
Cell layouts are front-end only: typically use only up to M1 (sometimes M2)



An INV Cell

- Almost all modern digital designs are cell-based.
  - Cells are typically small gates (NAND, FF, etc) containing 2-100 transistors.
- There are ~1000 cells in a library
  - Typically one design uses one library only.
- Cell width and height are quantized (measured in #metal tracks)
  - Cell height usually fixed for a library (8-track, 10-track, etc)
  - Cell width a multiple of minimum sized inverter (again multiple of min. metal pitch)
  - Constraint ensures that when cells are placed abutted, all pins fall on metal track intersections → easier for router to connect to them.

Cell-Based Placement



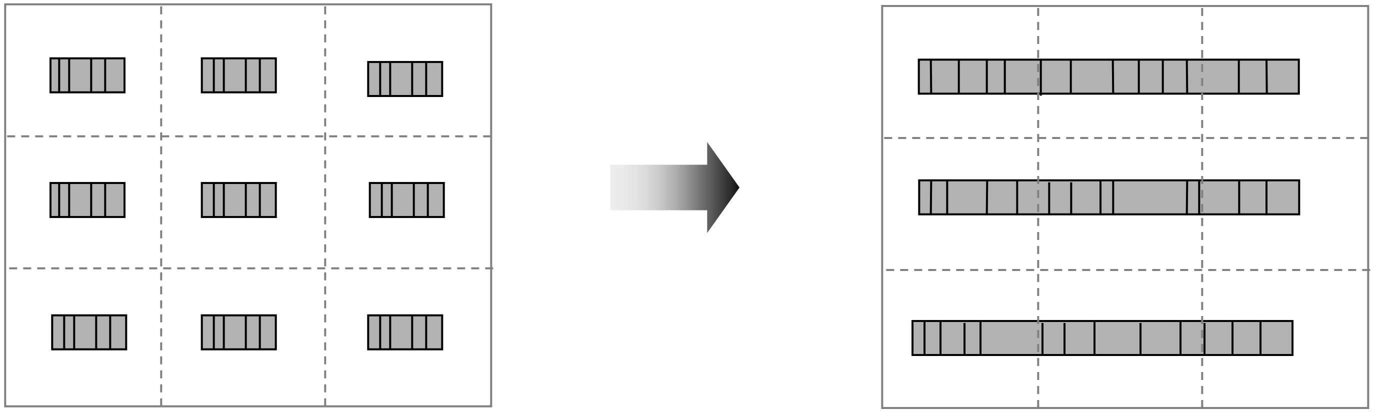
- Cells are placed in rows
- Almost all logic is automatically synthesized, placed using cell libraries
- Modern designs easily have 10M+ cells
- Goal of placement: timing correct, routable placement of all cells

5 min break

Introduction

Global Placement

Detailed Placement



Global Placement

- Partitioning-based algorithms:
  - The netlist and the layout are divided into smaller sub-netlists and sub-regions, respectively
  - Process is repeated until each sub-netlist and sub-region is small enough to be handled optimally
  - Detailed placement often performed by optimal solvers, facilitating a natural transition from global placement to detailed placement
  - Example: min-cut placement
- Analytic techniques:
  - Model the placement problem using an objective (cost) function, which can be optimized via numerical analysis
  - Examples: quadratic placement and force-directed placement
- Stochastic algorithms:
  - Randomized moves that allow hill-climbing are used to optimize the cost function
  - Example: simulated annealing

Global Placement

Partitioning-based

Analytic

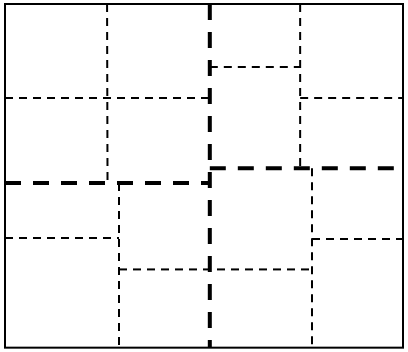
Stochastic

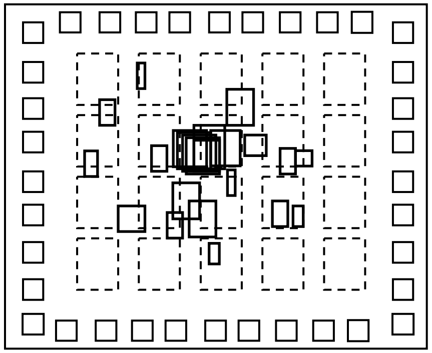
Min-cut placement

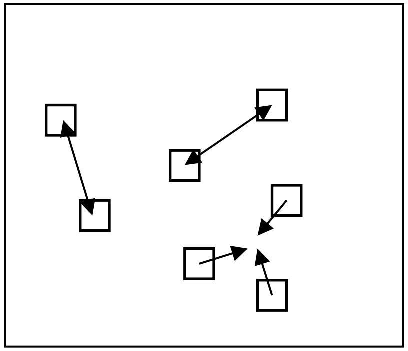
Quadratic placement

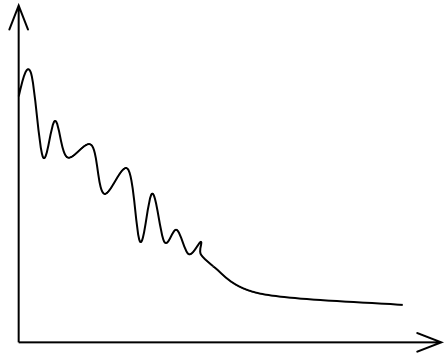
Force-directed placement

Simulated annealing

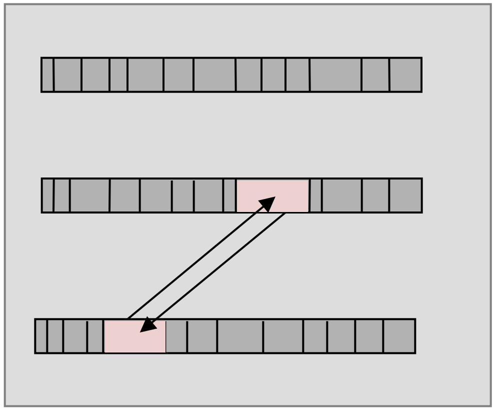


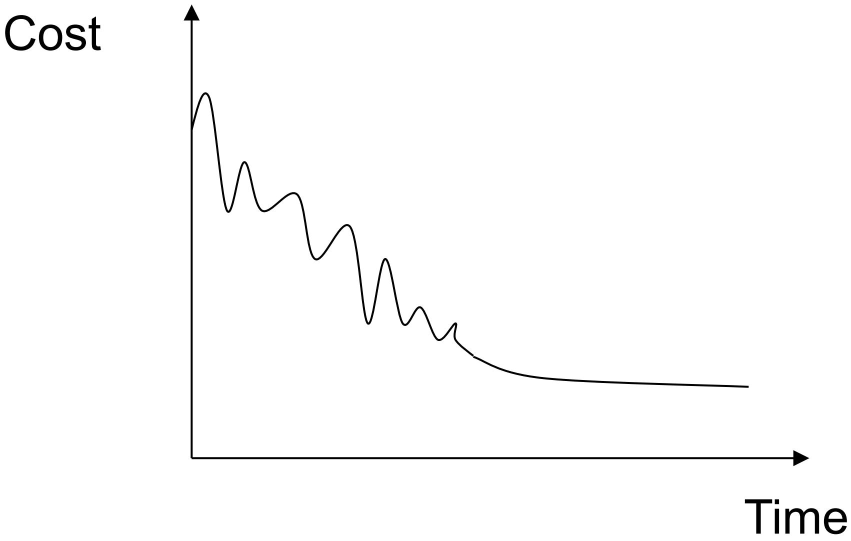






Simulated Annealing

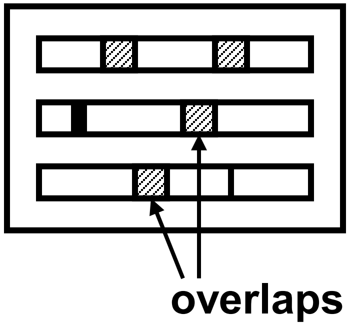




- Analogous to the physical annealing process
  - Melt metal and then slowly cool it
  - Result: energy-minimal crystal structure
- Modification of an initial configuration (placement) by moving/exchanging of randomly selected cells
  - Accept the new placement if it improves the objective function
  - If no improvement: Move/exchange is accepted with temperature-dependent (i.e., decreasing) probability

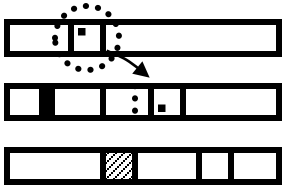
Neighborhood Structure for SA

Solution space: All arrangements of cells into rows, possibly with overlaps

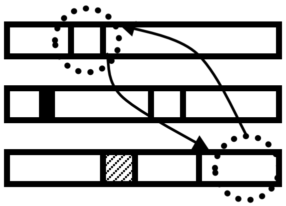


Three types of moves:

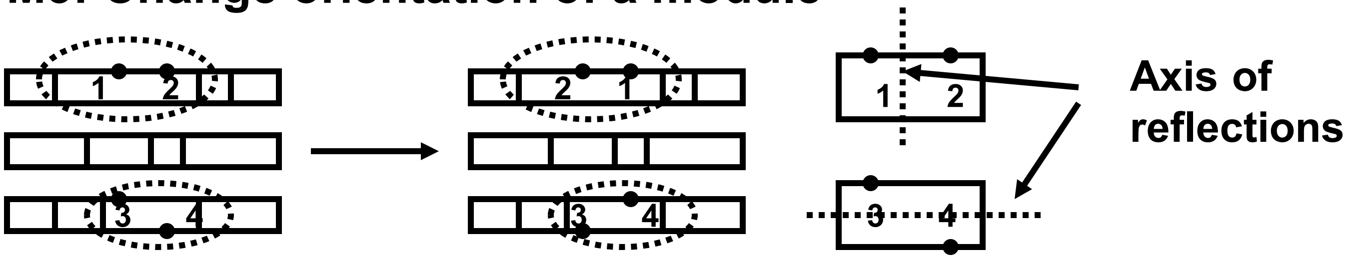
M1: Displace a module to a new location



M2: Interchange two modules



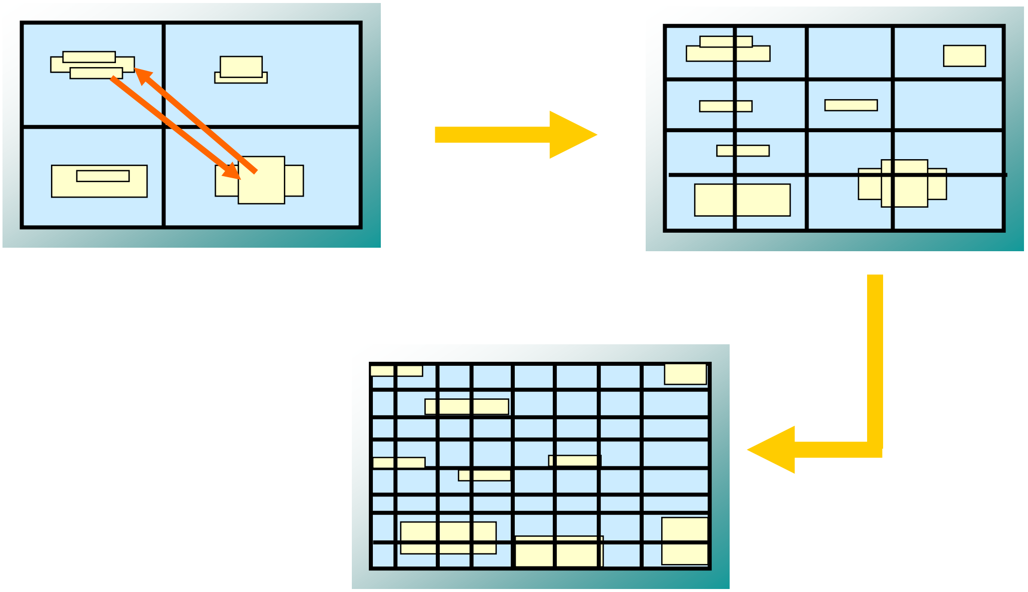
M3: Change orientation of a module



Simulated Annealing

- Advantages:
  - Can find global optimum (given sufficient time)
  - Well-suited for detailed placement
- Disadvantages:
  - Very slow
  - To achieve high-quality implementation, laborious parameter tuning is necessary
  - Randomized, chaotic algorithms - small changes in the input lead to large changes in the output
- Practical applications of SA:
  - Very small placement instances with complicated constraints
  - Detailed placement, where SA can be applied in small windows (not common anymore)
  - FPGA layout, where complicated constraints are becoming a norm

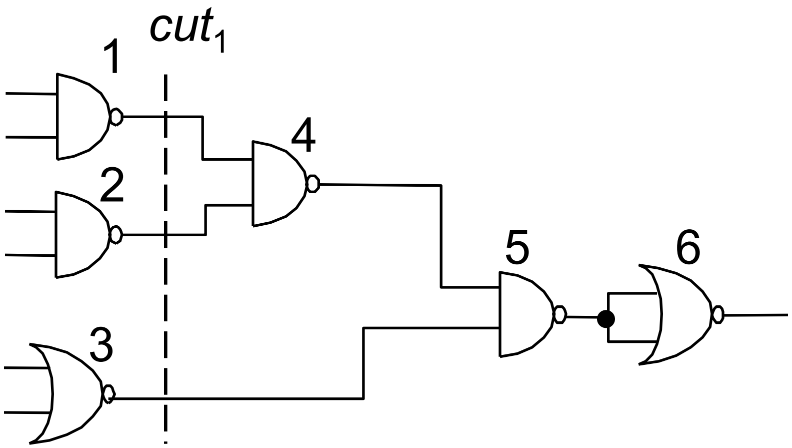
Min-Cut Placement



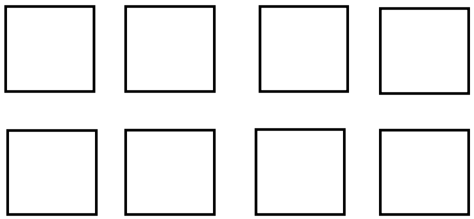
- Uses partitioning algorithms to divide (1) the netlist and (2) the layout region into smaller sub-netlists and sub-regions
- Conceptually, each sub-region is assigned a portion of the original netlist
- Each cut heuristically minimizes the number of cut nets using,
  - Fiduccia-Mattheyses (FM) algorithm

Min-Cut Placement – Example

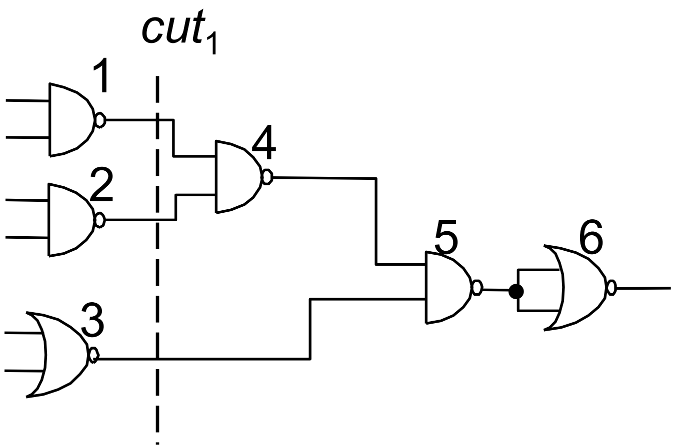
Given:



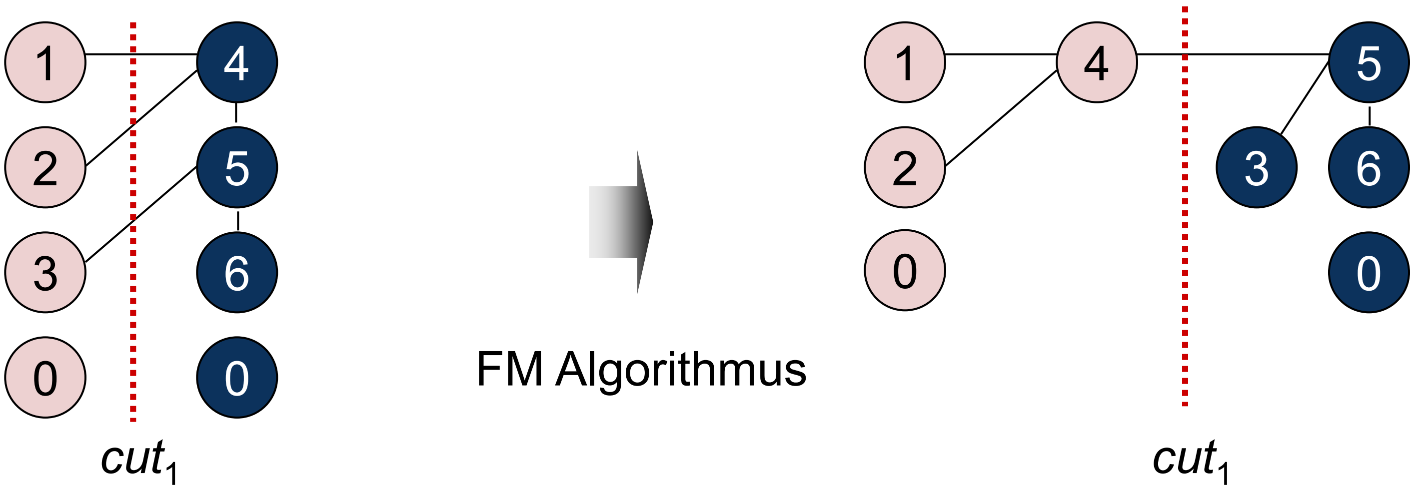
Task: 4 x 2 placement with minimum wirelength using alternative cutline directions and the FM algorithm

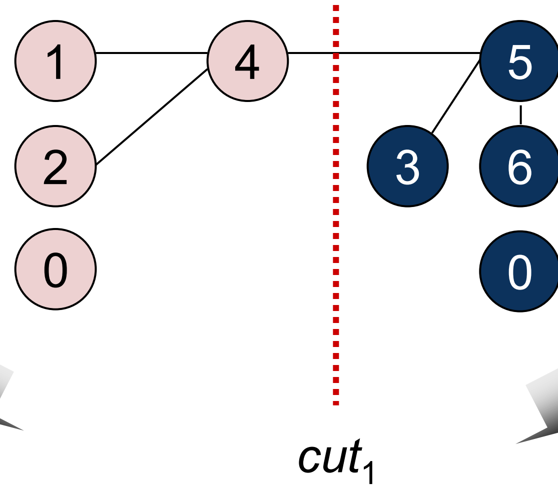


Min-Cut-Placement



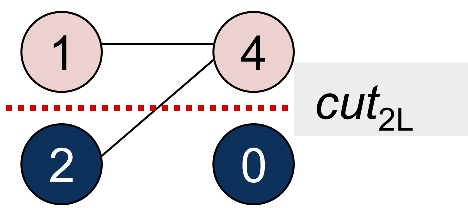
Vertical cut cut_1: L={1,2,3}, R={4,5,6}

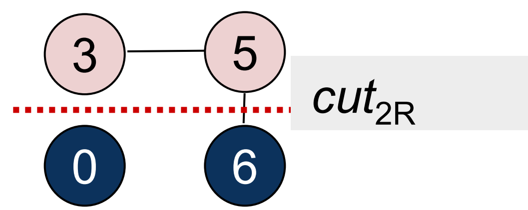


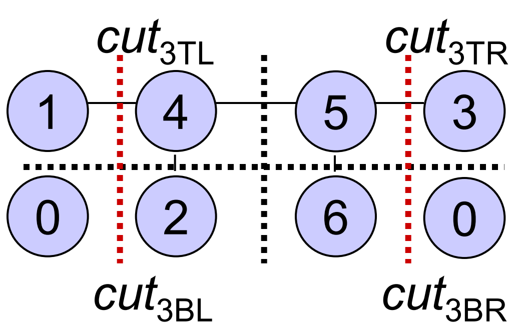


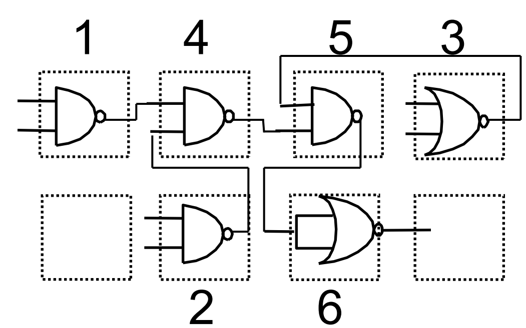
Horizontal cut cut_{2L}: T={1,4}, B={2,0}

Horizontal cut cut_{2R}: T={3,5}, B={6,0}

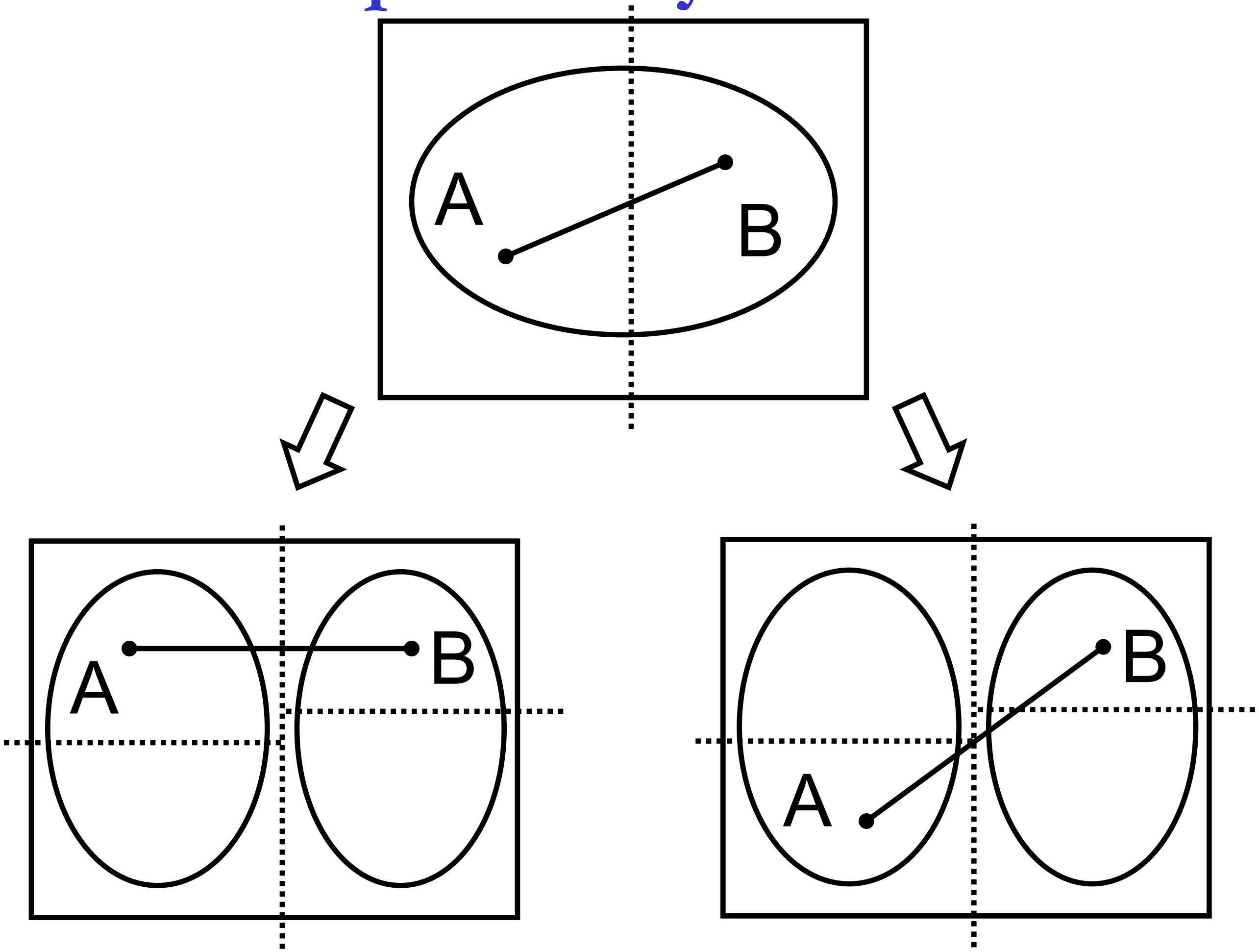






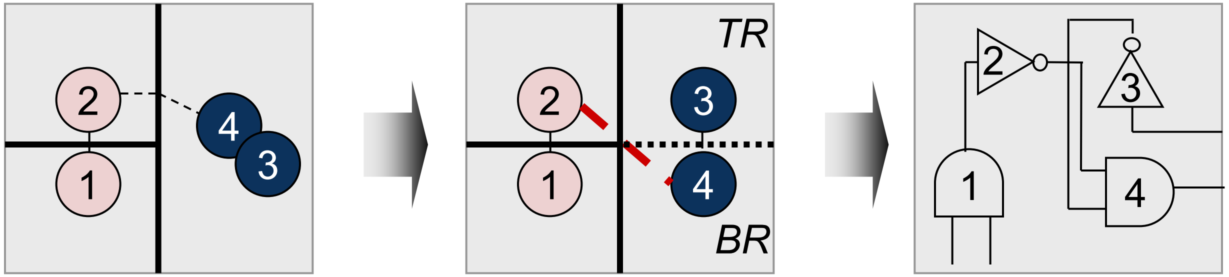


How to Partition Subcircuits "Independently"?

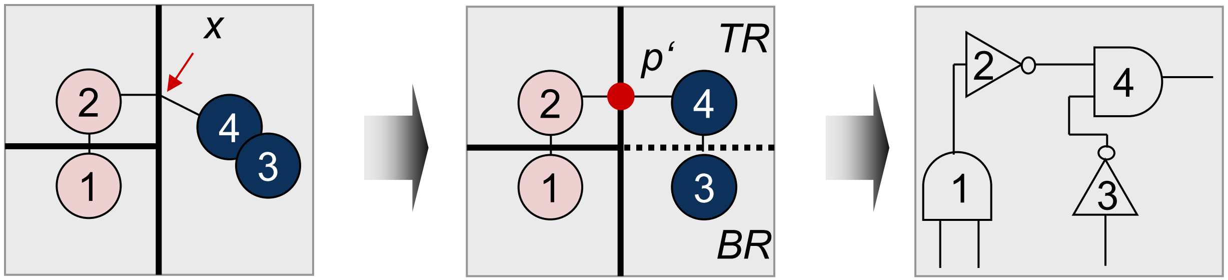


The costs of these two solutions are not the same

Min-Cut Placement – Terminal Propagation



- Terminal Propagation
  - External connections are represented by artificial connection points on the cutline
  - Dummy nodes in hypergraphs



Min-Cut Placement Summary

- Advantages:
  - Reasonable fast
  - Objective function can be adjusted, e.g., to perform timing-driven placement
  - Hierarchical strategy applicable to large circuits

- Disadvantages:
  - Randomized, chaotic algorithms – small changes in input lead to large changes in output
  - Optimizing one cutline at a time may result in routing congestion elsewhere

Analytic Placement – Quadratic Placement

- Objective function is quadratic; sum of (weighted) squared Euclidean distance represents placement objective function

$$L(P) = \frac{1}{2}\sum_{i,j=1}^{n} c_{ij}\left(\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2\right)$$

where *n* is the total number of cells, and *c*(*i*,*j*) is the connection cost between cells *i* and *j*.

- Only two-point-connections
- Minimize objective function by equating its derivative to zero which reduces to solving a system of linear equations

Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2}\sum_{i,j=1}^{n} c_{ij}\left((x_i - x_j)^2 + (y_i - y_j)^2\right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j.

- Each dimension can be considered independently:

$$L_x(P) = \sum_{i=1,\, j=1}^{n} c(i,j)(x_i - x_j)^2 \qquad L_y(P) = \sum_{i=1,\, j=1}^{n} c(i,j)(y_i - y_j)^2$$

- Convex quadratic optimization problem: any local minimum solution is also a global minimum
- Optimal x- and y-coordinates can be found by setting the partial derivatives of $L_x(P)$ and $L_y(P)$ to zero

Analytic Placement – Quadratic Placement

$$L(P) = \frac{1}{2}\sum_{i,j=1}^{n} c_{ij}\left(\left(x_i - x_j\right)^2 + \left(y_i - y_j\right)^2\right)$$

where n is the total number of cells, and $c(i,j)$ is the connection cost between cells i and j.

$$L_x(P) = \sum_{i=1, j=1}^{n} c(i,j)(x_i - x_j)^2$$

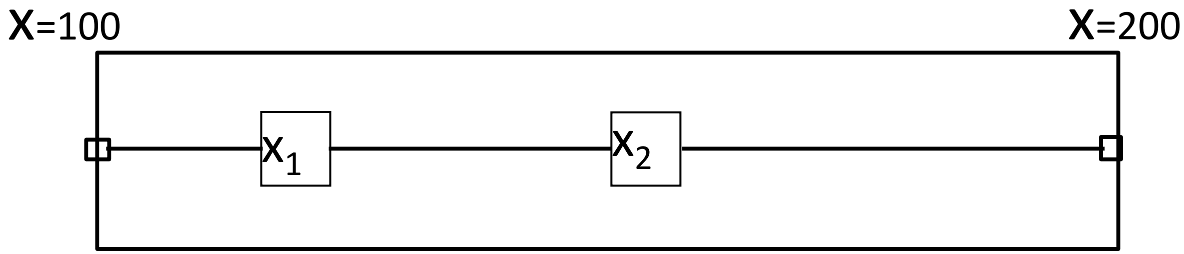
$$\frac{\partial L_x(P)}{\partial X} = AX - b_x = 0$$

$$L_y(P) = \sum_{i=1, j=1}^{n} c(i,j)(y_i - y_j)^2$$

$$\frac{\partial L_y(P)}{\partial Y} = AY - b_y = 0$$

- Each dimension can be considered independently

Analytical Placement Example



$$Cost = (x_1 - 100)^2 + (x_1 - x_2)^2 + (x_2 - 200)^2$$

$$\frac{\partial}{\partial x_1} Cost = 2(x_1 - 100) + 2(x_1 - x_2)$$

$$\frac{\partial}{\partial x_2} Cost = -2(x_1 - x_2) + 2(x_2 - 200)$$

Setting the partial derivatives $= 0$, we solve for the minimum Cost :

$$Ax = b$$

$$\begin{bmatrix} 4 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 200 \\ 400 \end{bmatrix}$$

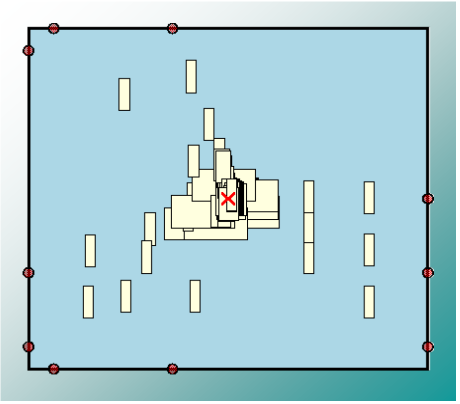
$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 200 \end{bmatrix}$$

$$x_1 = \frac{400}{3} \qquad x_2 = \frac{500}{3}$$

A_{ii} = degree of a node
A_{ij} = -(i-j) connectivity
b_i = sum of locations connected to cell i

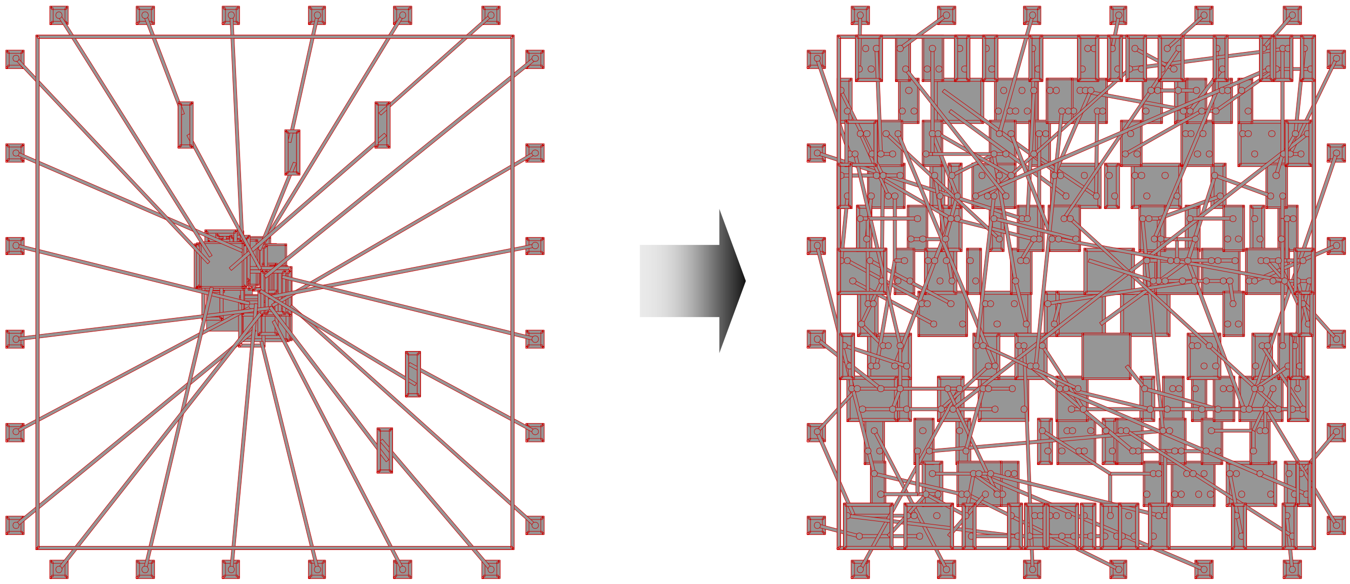
Why “Squared Wirelength” ?

- Because we can
- Because it is trivial to solve
- Because there is only one solution
- Because the solution is a global optimum
- Because the solution conveys “relative order” information
- (Because the solution conveys “global position” information)
- **Key issue: “spreading”**
  - What is the optimal Solution in previous case if No pin locations are there ?



Analytic Placement – Quadratic Placement

- Second stage of quadratic placers: cells are spread out to remove overlaps
- Methods:
  - Adding fake nets that pull cells away from dense regions toward anchors
  - Geometric sorting and scaling
  - Repulsion forces, etc.



Analytic Placement – Quadratic Placement

- Advantages:
  - Captures the placement problem concisely in mathematical terms
  - Leverages efficient algorithms from numerical analysis and available software
  - Can be applied to large circuits without netlist clustering (flat)
  - Stability: small changes in the input do not lead to large changes in the output
- Disadvantages:
  - Connections to fixed objects are necessary: I/O pads, pins of fixed macros, etc.