233 Project 1 Overview

Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

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Slides adapted from Benjamin Domae’s presentation on 2021

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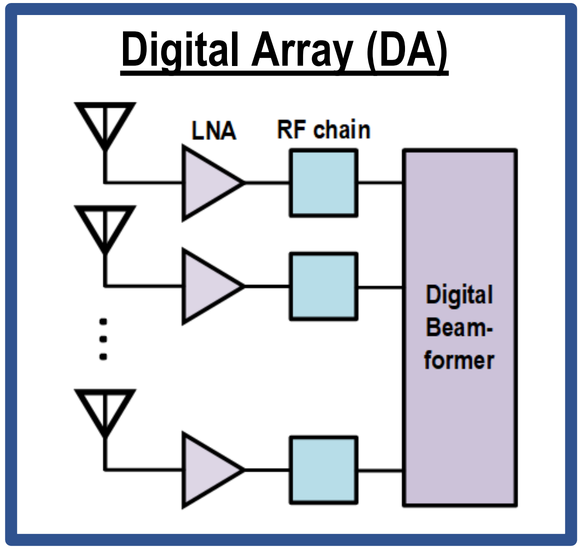
What the paper discussed

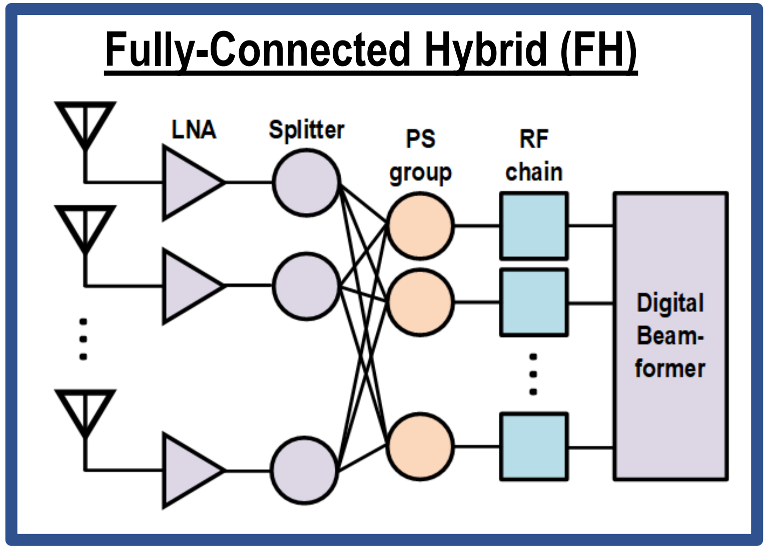
- **Authors present algorithms for hybrid MIMO precoding/combining**
  - Iterative algorithms for optimization
  - Aim for optimal precoding/combining (fully digital array solution)
  - Simple adjustment to handle PS quantization

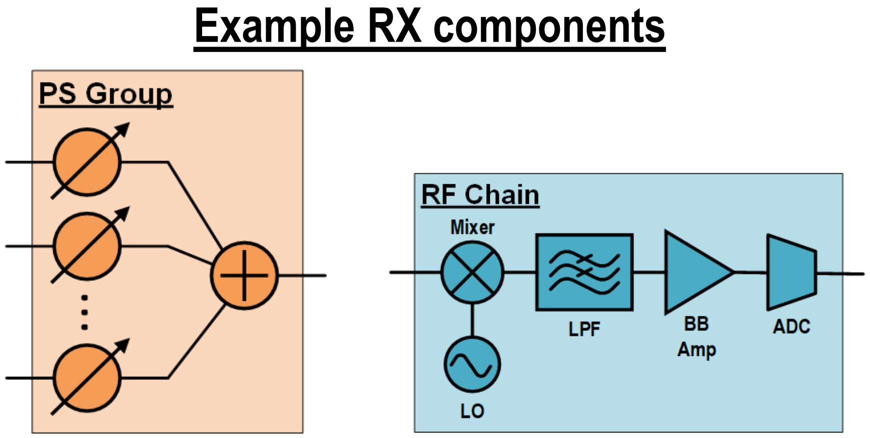
- **Three scenarios depicted:**
  1. Large number of streams ($N^{RF} \geq 2d$) – Proof of optimal performance
  2. Point-to-point SU-MIMO ($N^{RF} < 2d$) – Simulated near-optimal sum rate
  3. MU-MISO ($N^{RF} < 2d$) – Simulated near-optimal sum rate
  - Simulated performance with PS quantization close to optimal (exhaustive search)

Motivation

- **Hybrid antenna arrays can reduce power consumption and cost**
- **Analog processing limits MIMO precoding/combining**
  - Phase-only weights for the analog stages = sub-optimal solutions
  - Traditional MIMO algorithms designed for fully digital arrays







System Model Block Diagram

- **Multi-user (MU) MIMO downlink:**
  - Base station (BS) transmitting to K users (UEs)
  - Both BS and UEs have hybrid arrays
  - $N_t^{RF} \geq N_s$ and $N_r^{RF} \geq d$

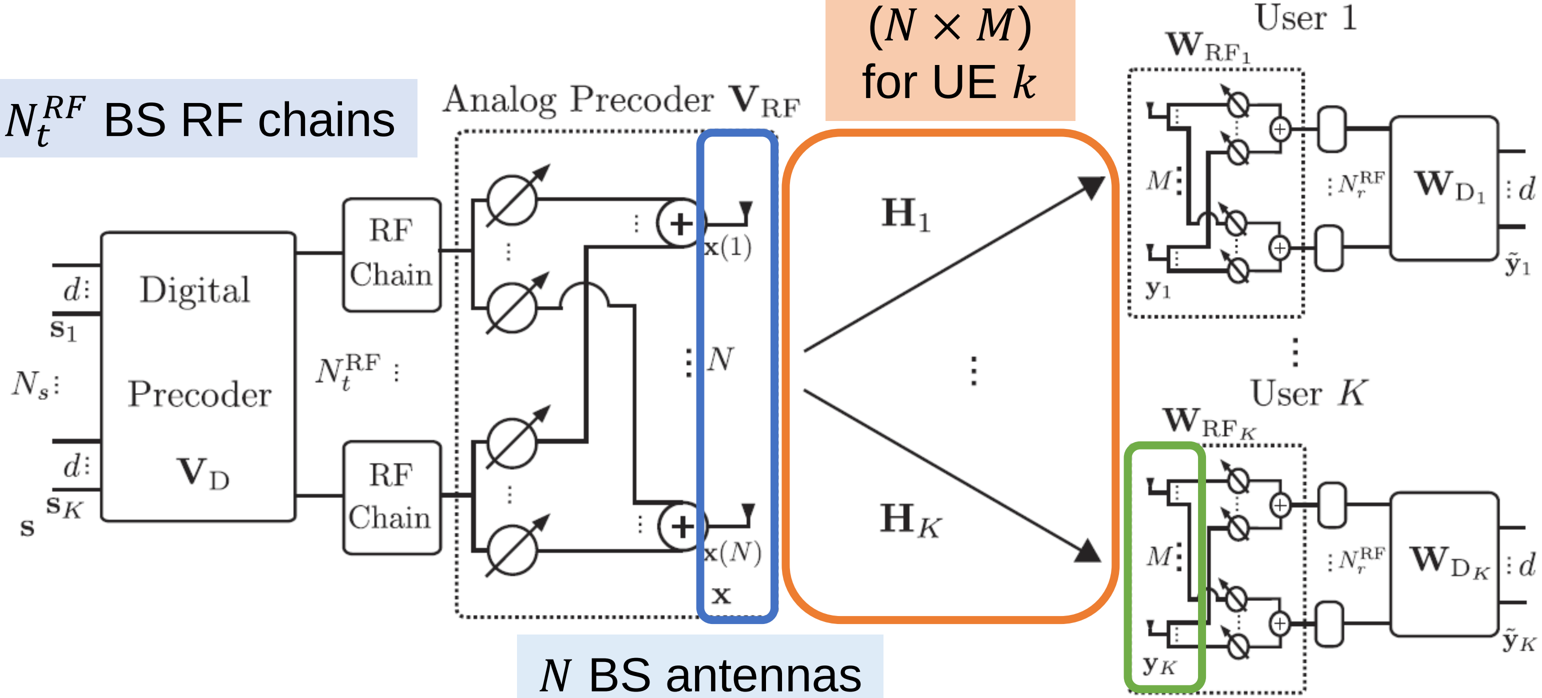
$\boldsymbol{H}_k$ MIMO channel $(N \times M)$ for UE k

N_r^{RF} UE RF chains

N_t^{RF} BS RF chains

N_s total TX streams

d streams per UE
$N_s = dK$ total streams



N BS antennas

M UE antennas

System Model Equations

General MU-MIMO downlink received symbols

$$\tilde{\mathbf{y}}_k = \underbrace{\mathbf{W}_{t_k}^H}_{\text{Hybrid Combiner}} \left(\underbrace{\mathbf{H}_k}_{\text{SU-MIMO Channel}} \underbrace{\mathbf{V}}_{\text{Total Hybrid Precoder}} \mathbf{s} + \underbrace{\mathbf{z}_k}_{\text{AWGN (per antenna)}}\right)$$

$$\boxed{\mathbf{W}_{t_k} = \mathbf{W}_{RF_k} \mathbf{W}_{D_k}}$$

$$\boxed{\mathbf{V} = \textstyle\sum_{k=1}^{K} \mathbf{V}_{t_k} = \sum_{k=1}^{K} \mathbf{V}_{RF} \mathbf{V}_{D_k}}$$

$$\boxed{\tilde{\mathbf{y}}_k = \underbrace{\mathbf{W}_{t_k}^H \mathbf{H}_k \mathbf{V}_{t_k} \mathbf{s}_k}_{\text{Desired Signals}} + \underbrace{\mathbf{W}_{t_k}^H \mathbf{H}_k \sum_{\ell \neq k} \mathbf{V}_{t_\ell} \mathbf{s}_\ell}_{\text{Inter-user Interference}} + \underbrace{\mathbf{W}_{t_k}^H \mathbf{z}_k}_{\text{Spatially-correlated Noise}}}$$

Transmitted symbols

$$\mathbf{x} = \mathbf{V}\mathbf{s} = \mathbf{V}_{t_k} \mathbf{s}_k + \sum_{\ell \neq k} \mathbf{V}_{t_\ell} \mathbf{s}_\ell$$

Variable	Dimension
$\tilde{\mathbf{y}}_k$	N_s
$\mathbf{W}_{RF_k}$	$N_s \times N_t^{RF}$
$\mathbf{W}_{D_k}$	$N_t^{RF} \times N$
$\mathbf{H}_k$	$N \times M$
$\mathbf{V}_{RF}$	$\mathrm{M} \times N_r^{RF}$
$\mathbf{V}_{D_k}$	$N_t^{RF} \times d$
$\mathbf{s}_k$	d
$\mathbf{z}_k$	N

Mathematical Objective

- **Rate (spectral efficiency) for user k:**

$$R_k = \log_2 \left| \mathbf{I}_M + \mathbf{W}_{t_k} \mathbf{C}_k^{-1} \mathbf{W}_{t_k}^H \mathbf{H}_k \mathbf{V}_{t_k} \mathbf{V}_{t_k}^H \mathbf{H}_k^H \right|$$

Inter-user interference + noise covariance

$$\mathbf{C}_k = \mathbf{W}_{t_k}^H \mathbf{H}_k \left(\sum_{\ell \neq k} \mathbf{V}_{t_\ell} \mathbf{V}_{t_\ell}^H \right) \mathbf{H}_k^H \mathbf{W}_{t_k} + \sigma^2 \mathbf{W}_{t_k}^H \mathbf{W}_{t_k}$$

- **Optimization problem:**

$$\begin{aligned} \underset{\mathbf{V}_{\mathrm{RF}}, \mathbf{V}_{\mathrm{D}}, \mathbf{W}_{\mathrm{RF}}, \mathbf{W}_{\mathrm{D}}}{\text{maximize}} \quad & \sum_{k=1}^{K} \beta_k R_k \\ \text{subject to} \quad & \mathrm{Tr}(\mathbf{V}_{\mathrm{RF}} \mathbf{V}_{\mathrm{D}} \mathbf{V}_{\mathrm{D}}^H \mathbf{V}_{\mathrm{RF}}^H) \leq P \\ & |\mathbf{V}_{\mathrm{RF}}(i,j)|^2 = 1, \forall i, j \\ & |\mathbf{W}_{\mathrm{RF}_k}(i,j)|^2 = 1, \forall i, j, k, \end{aligned}$$

Sum-rate weighted by user priority β_k

Total power constraint P

Analog hardware constraints

Project Requirement

- **Solve the problem for single-user case:**
  - Implement hybrid precoding and combining algorithms (SU case only)
  - Implement quantized design algorithm of analog precoder (SU case only)
  - Explore the case when you have more RF chains than the number of stream

- **Optimization problem:**
  - Spectral efficiency for single user case:

$$R = \log_2 \left| \mathbf{I}_M + \frac{1}{\sigma^2} \mathbf{W}_{\mathrm{t}} (\mathbf{W}_{\mathrm{t}}^H \mathbf{W}_{\mathrm{t}})^{-1} \mathbf{W}_{\mathrm{t}}^H \mathbf{H} \mathbf{V}_{\mathrm{t}} \mathbf{V}_{\mathrm{t}}^H \mathbf{H}^H \right|. \quad (8)$$

where $\mathbf{V}_{\mathrm{t}} = \mathbf{V}_{\mathrm{RF}} \mathbf{V}_{\mathrm{D}}$ and $\mathbf{W}_{\mathrm{t}} = \mathbf{W}_{\mathrm{RF}} \mathbf{W}_{\mathrm{D}}$.

  - Geometric channel model:

$$\mathbf{H}_k = \sqrt{\frac{NM}{L}} \sum_{\ell=1}^{L} \alpha_k^\ell \mathbf{a}_r(\phi_{r_k}^\ell) \mathbf{a}_t(\phi_{t_k}^\ell)^H, \quad (33)$$

Point-to-Point SU-MIMO ($N^{RF} < 2d$)

- **Nested joint optimization problem:**
  - Optimizing 4 total matrices ($\boldsymbol{V}_{RF}, \boldsymbol{V}_D, \boldsymbol{W}_{RF}, \boldsymbol{W}_D$)

- **Two stage procedure overall:**
  - Initially assume a fixed optimal combiner structure
  1. Design the precoders
  2. Design the combiners using the computed precoders
- **Precoders and combiners are each designed in two stages as well:**
  - Initially assume a fixed digital precoder/combiner structure
  1. Iteratively solve for the RF precoder/combiner
  2. Solve for the optimal digital precoder/combiner

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Iterative Analog Precoder Design

- **Iteratively maximize the contribution of the j^{th} column to the rate**
  - Note: $\bar{V}^{j}_{RF}$ is V_{RF} with the j^{th} column removed
- **Combiner design similar: $\gamma^2 \rightarrow \frac{1}{M}$ and $F_1 \rightarrow F_2 = HV_tV_t^HH^H$**

<u>Parameters</u>
$F_1 = \mathrm{H}^{\mathrm{H}}\mathrm{H}$ = Channel covariance matrix
γ^2 = Average power per antenna
σ^2 = Noise variance (per antenna)

<u>Solve simplified optimization problem:</u>

$$\max_{\mathbf{V}_{\mathrm{RF}}} \quad \log_2 \left| \mathbf{I} + \frac{\gamma^2}{\sigma^2} \mathbf{V}_{\mathrm{RF}}^H \mathbf{F}_1 \mathbf{V}_{\mathrm{RF}} \right| \qquad (12a)$$

$$\text{s.t.} \quad |\mathbf{V}_{\mathrm{RF}}(i,j)|^2 = 1, \forall i,j, \qquad (12b)$$

Algorithm 1. Design of $\mathbf{V}_{\mathrm{RF}}$ by solving (12)

Given: $\mathbf{F}_1$, γ^2, σ^2
1: Initialize $\mathbf{V}_{\mathrm{RF}} = \mathbf{1}_{N \times N^{\mathrm{RF}}}$.
2: **for** $j = 1 \rightarrow N^{\mathrm{RF}}$ **do**
3: Calculate $\mathbf{C}_j = \mathbf{I} + \frac{\gamma^2}{\sigma^2} (\bar{\mathbf{V}}_{\mathrm{RF}}^j)^H \mathbf{F}_1 \bar{\mathbf{V}}_{\mathrm{RF}}^j$.
4: Calculate $\mathbf{G}_j = \frac{\gamma^2}{\sigma^2} \mathbf{F}_1 - \frac{\gamma^4}{\sigma^4} \mathbf{F}_1 \bar{\mathbf{V}}_{\mathrm{RF}}^j \mathbf{C}_j^{-1} (\bar{\mathbf{V}}_{\mathrm{RF}}^j)^H \mathbf{F}_1$.
5: **for** $i = 1 \rightarrow N$
6: Find $\eta_{ij} = \sum_{\ell \neq i} \mathbf{G}_j(i,\ell) \mathbf{V}_{\mathrm{RF}}(\ell, j)$.
7: $\mathbf{V}_{\mathrm{RF}}(i,j) = \begin{cases} 1, & \text{if } \eta_{ij} = 0, \\ \frac{\eta_{ij}}{|\eta_{ij}|}, & \text{otherwise.} \end{cases}$
8: **end for**
9: **end for**
10: Check convergence. If yes, stop; if not go to Step 2.

Coordinate descent (greedy updates based on [34] [41])

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[34] Sohrabi and Yu, "Hybrid beamforming with finite-resolution phase shifters for large-scale MIMO systems," pg 3, SPAWC 2015
[41] Pi. "Optimal Transmitter Beamforming with Per-antenna Power Constraints," pg 4, IEEE ICC 2012

Point-to-Point SU-MIMO Algorithm

1. RF precoder (V_{RF}): Iterative optimization
2. Digital precoder (V_D): Water-filling solution
3. RF combiner (W_{RF}): Iterative optimization
4. Digital combiner (W_D): MMSE solution

Algorithm 2. Design of Hybrid Beamformers for Point-to-Point MIMO systems

Given: σ^2, P

1: Assuming $\mathbf{V}_{\mathrm{D}}\mathbf{V}_{\mathrm{D}}^H = \gamma^2\mathbf{I}$ where $\gamma = \sqrt{P/(NN^{\mathrm{RF}})}$, find $\mathbf{V}_{\mathrm{RF}}$ by solving the problem in (12) using Algorithm 1.

2: Calculate $\mathbf{V}_{\mathrm{D}} = (\mathbf{V}_{\mathrm{RF}}^H\mathbf{V}_{\mathrm{RF}})^{-1/2}\mathbf{U}_e\mathbf{\Gamma}_e$ where $\mathbf{U}_e$ and $\mathbf{\Gamma}_e$ are defined as following (11).

3: Find $\mathbf{W}_{\mathrm{RF}}$ by solving the problem in (16) using Algorithm 1.

4: Calculate $\mathbf{W}_{\mathrm{D}} = \mathbf{J}^{-1}\mathbf{W}_{\mathrm{RF}}^H\mathbf{H}\mathbf{V}_{\mathrm{RF}}\mathbf{V}_{\mathrm{D}}$ where $\mathbf{J} = \mathbf{W}_{\mathrm{RF}}^H\mathbf{H}\mathbf{V}_{\mathrm{RF}}\mathbf{V}_{\mathrm{D}}\mathbf{V}_{\mathrm{D}}^H\mathbf{V}_{\mathrm{RF}}^H\mathbf{H}^H\mathbf{W}_{\mathrm{RF}} + \sigma^2\mathbf{W}_{\mathrm{RF}}^H\mathbf{W}_{\mathrm{RF}}$.

Phase Shifter (PS) Quantization

- **Simple integrations into the algorithms**
  - n_{PS} possible phases (steps of $\omega = e^{j2\pi/n_{PS}}$) for $\boldsymbol{V}_{RF}$ and $\boldsymbol{W}_{RF}$ $\longrightarrow$ $\mathcal{F} = \{1, \omega, \omega^2, \dots, \omega^{n_{PS}-1}\}$

Algorithm 1. Design of $\mathbf{V}_{\mathrm{RF}}$ by solving (12)

Given: $\mathbf{F}_1$, γ^2, σ^2

1: Initialize $\mathbf{V}_{\mathrm{RF}} = \mathbf{1}_{N \times N^{\mathrm{RF}}}$.

2: **for** $j = 1 \to N^{\mathrm{RF}}$ **do**

3: Calculate $\mathbf{C}_j = \mathbf{I} + \frac{\gamma^2}{\sigma^2}(\bar{\mathbf{V}}_{\mathrm{RF}}^j)^H \mathbf{F}_1 \bar{\mathbf{V}}_{\mathrm{RF}}^j$.

4: Calculate $\mathbf{G}_j = \frac{\gamma^2}{\sigma^2}\mathbf{F}_1 - \frac{\gamma^4}{\sigma^4}\mathbf{F}_1 \bar{\mathbf{V}}_{\mathrm{RF}}^j \mathbf{C}_j^{-1} (\bar{\mathbf{V}}_{\mathrm{RF}}^j)^H \mathbf{F}_1$.

5: **for** $i = 1 \to N$

6: Find $\eta_{ij} = \sum_{\ell \neq i} \mathbf{G}_j(i, \ell) \mathbf{V}_{\mathrm{RF}}(\ell, j)$.

7: $\boxed{\mathbf{V}_{\mathrm{RF}}(i,j) = \begin{cases} 1, & \text{if } \eta_{ij} = 0, \\ \frac{\eta_{ij}}{|\eta_{ij}|}, & \text{otherwise.} \end{cases}}$

8: **end for**

9: **end for**

10: Check convergence. If yes, stop; if not go to Step 2.

Quantized Version:

$$\mathbf{V}_{RF}(i,j) = Q\left(\frac{\eta_{ij}}{|\eta_{ij}|}\right) = \underset{\mathbf{V}_{RF}(i,j) \in \mathcal{F}}{\arg\min} \left| \mathbf{V}_{RF}(i,j) - \frac{\eta_{ij}}{|\eta_{ij}|} \right|^2$$

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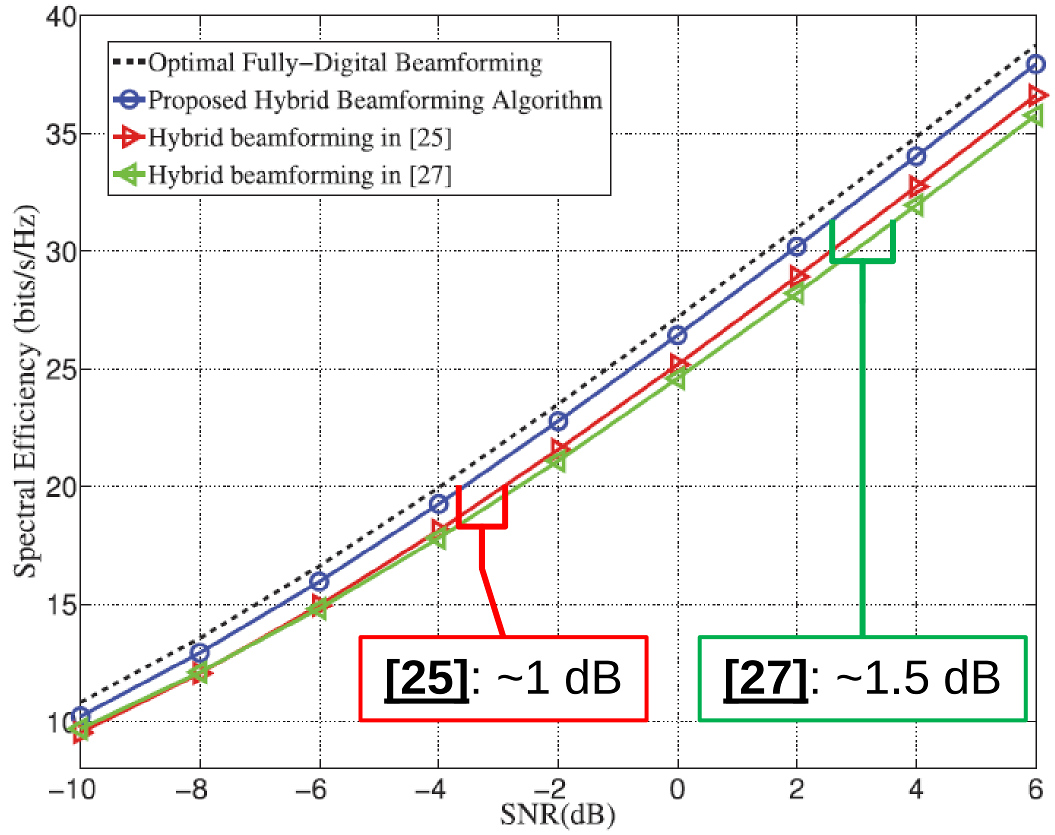
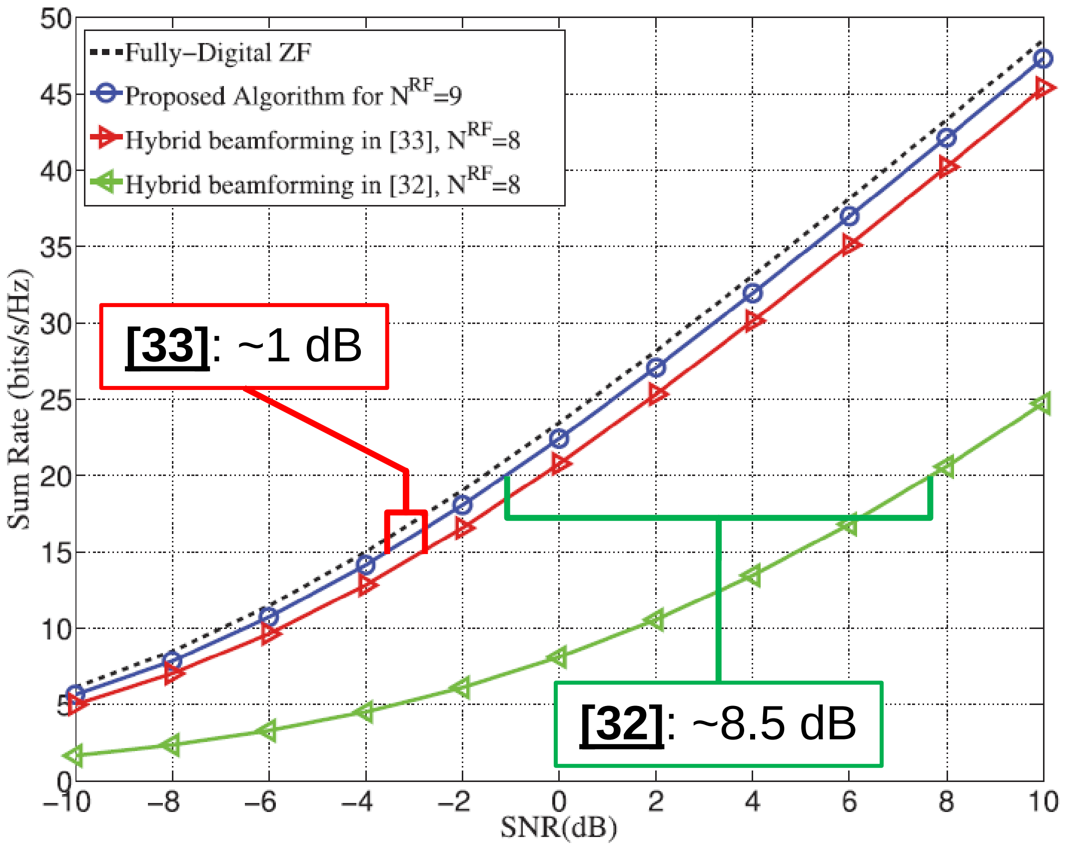
Simulation Results (Perfect PS)

- **Random geometric channels – 15 paths per link**
  - Average spectral efficiency from 100 trials

Case 2: 64 x 16 MIMO, $N_{RF} = N_s = 6$



Case 3: 64 x 8, 8-user MISO, $N_{RF} = 8$ *or* 9



Projects on True-Time-Delay (TTD) Arrays

Project 4: Beam Training with Analog TTD Arrays

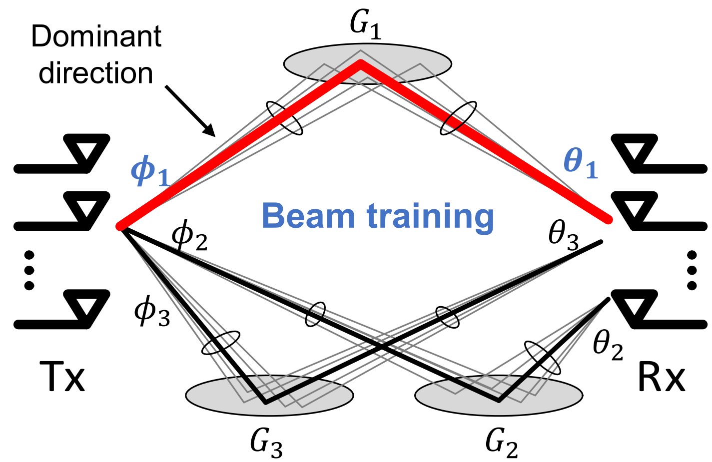
Project 2: Flexible Frequency-Dependent beamforming with Analog TTD arrays

Project 4: Beam Training w/ Analog TTD

Beam Training: Link establishment procedure to estimate the AoA of the strongest path

- **Partial channel knowledge** –angles (AoA) corresponding to the *dominant propagation path*
- Enables beam-steering in data communication

- **Low training overhead –** *compared to channel estimation*



Spatially clustered multipath: few spatial clusters reach the receiver

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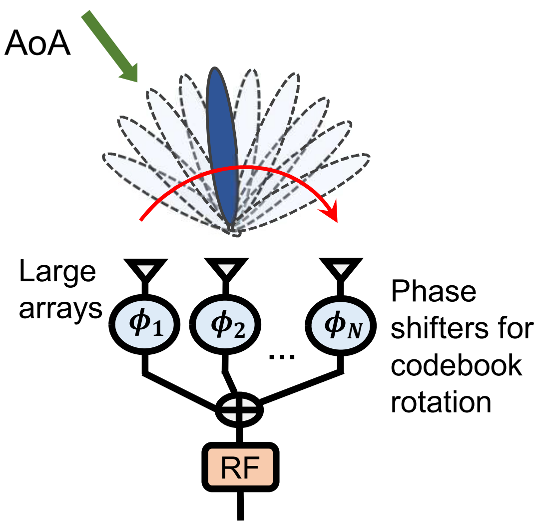
Project 4: Beam Training w/ Analog TTD

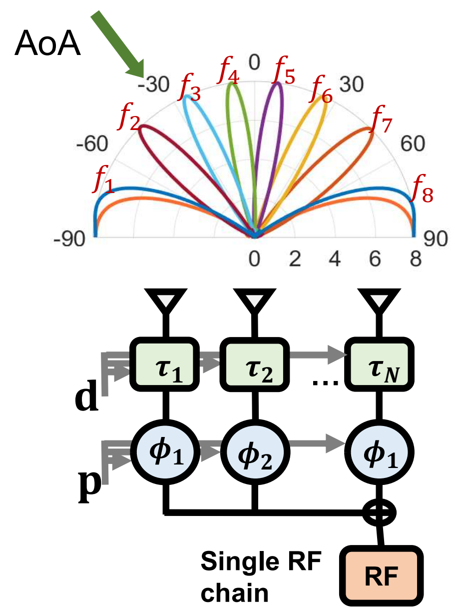
Beam Training with Analog PS vs TTD array:

Analog PS-based

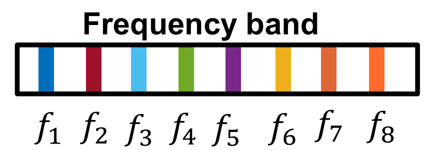
Exhaustive beam sweep to estimate AoA



Analog TTD



Single shot AoA estimation



Upto M_{tot} beams using 1 OFDM symbol

$$\Delta\tau = \frac{R}{BW}$$

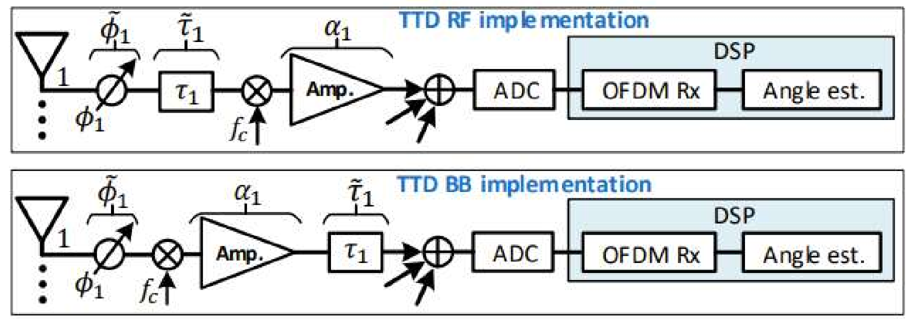
Project 4: Beam Training w/ Analog TTD

System Model for Rx Beam Training:

- $N_R \times 1$ Analog TTD array
- Freq. dependent TTD combiner

RF $[\mathbf{w}_{\mathrm{RF}}[m]]_n = \alpha_n \exp\left[-j\left(2\pi f_m \tilde{\tau}_n + \tilde{\phi}_n\right)\right]$

BB $[\mathbf{w}_{\mathrm{BB}}[m]]_n = \alpha_n \exp\left[-j\left(2\pi (f_m - f_c) \tilde{\tau}_n + \tilde{\phi}_n\right)\right]$

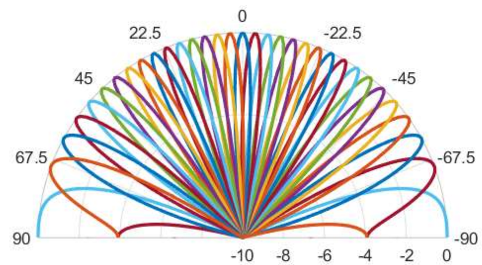


Hardware impairments

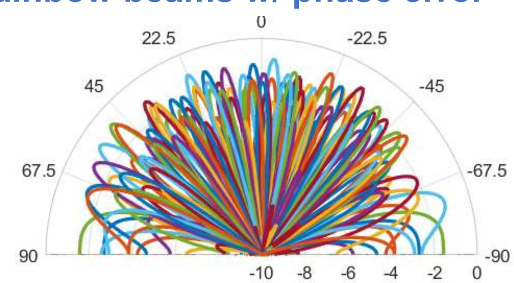
Cause distortions in TTD Rx beam patterns

- Delay error: $\tilde{\tau}_n \sim \mathcal{N}(\tau_n, \sigma_T^2)$
- Phase error: $\tilde{\phi}_n \sim \mathcal{N}(\phi_n, \sigma_P^2)$
- Gain error: $10\log_{10}\alpha_n \sim \mathcal{N}(0, \sigma_A^2)$

Rainbow beams w/o error



Rainbow beams w/ phase error



[1] V. Boljanovic, H. Yan, E. Ghaderi, D. Heo, S. Gupta and D. Cabric, "Design of Millimeter-Wave Single-Shot Beam Training for True-Time-Delay Array," 2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), pp. 1-5, 2020

Project 4: Beam Training w/ Analog TTD

Channel Model: wideband frequency selective [Sec II]

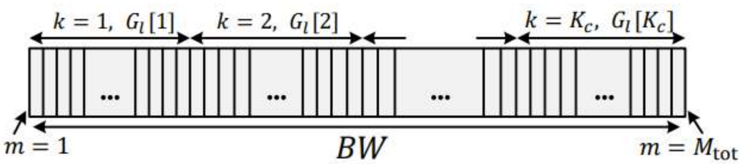


Fig. 1. The illustration of sub-bands and their corresponding channel gains for the l-th cluster.

$$\mathbf{H}[k]=\sum_{l=1}^{L} G_l[k]\mathbf{a}_{\mathrm{R}}(\theta_l^{(\mathrm{R})})\mathbf{a}_{\mathrm{T}}^{\mathrm{H}}(\theta_l^{(\mathrm{T})}),$$

$$\mathbb{E}\left(G_{l_1}[k_1]G_{l_2}^*[k_2]\right)=\begin{cases}\sigma_{l_1}^2, & \text{if } l_1=l_2, k_1=k_2\\ 0, & \text{otherwise.}\end{cases}$$

TTD codebook design [Sec III]

$$[\mathbf{w}[m]]_n=\exp\left[-j(2\pi f_m \tau_n+\phi_n)\right],$$

$$\tau_n=(n-1)R/\mathrm{BW}, \quad \phi_n=(n-1)[\mathrm{sgn}(\psi)\pi-\psi], \quad (9)$$

where $\psi=\mathrm{mod}(2\pi R(f_c-\mathrm{BW}/2)/\mathrm{BW}+\pi, 2\pi)-\pi$. sgn()

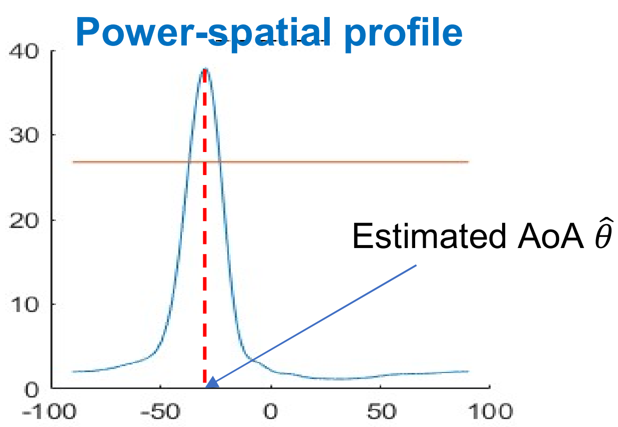
[1] V. Boljanovic, H. Yan, E. Ghaderi, D. Heo, S. Gupta and D. Cabric, “Design of Millimeter-Wave Single-Shot Beam Training for True-Time-Delay Array," 2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), pp. 1-5, 2020

Project 4: Beam Training w/ Analog TTD

Beam Training Algorithm [Sec IV]

Received signal: $Y[m] = M^{-1/2}\mathbf{w}^{\mathrm{H}}[m]\mathbf{H}[k]\mathbf{v} + \mathbf{w}^{\mathrm{H}}[m]\mathbf{n}[m],\ m \in \mathcal{M}.$

Projection onto Dictionary matrix: $\mathbf{B} \in \mathbb{R}^{D\times Q}, \quad [\mathbf{B}]_{d,q} = \left|\mathbf{f}_d^{\mathrm{H}}\mathbf{a}_{\mathrm{R}}(\xi_q)\right|^2.$



Find the RMSE

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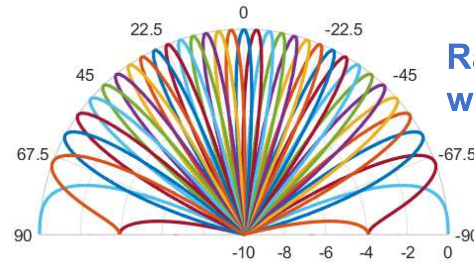
[1] V. Boljanovic, H. Yan, E. Ghaderi, D. Heo, S. Gupta and D. Cabric, "Design of Millimeter-Wave Single-Shot Beam Training for True-Time-Delay Array," 2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), pp. 1-5, 2020

Project 4: Beam Training w/ Analog TTD

Problem statements:



Rainbow beams w/o error

1. **Plot the beam patterns of designed codebook for analog TTD arrays.**

2. **Plot the RMSE of angle estimation vs SNR (-20 dB to 20 dB)**
   - In absence of hardware impairments
   - For $R = 1,2,4$

3. **RMSE of angle estimation vs standard deviation of gain error σ_A (0 dB to 4.5 dB)**

4. **[Bonus]: RMSE of angle estimation vs standard deviation of phase error σ_P $(0° \text{ to } 50°)$**

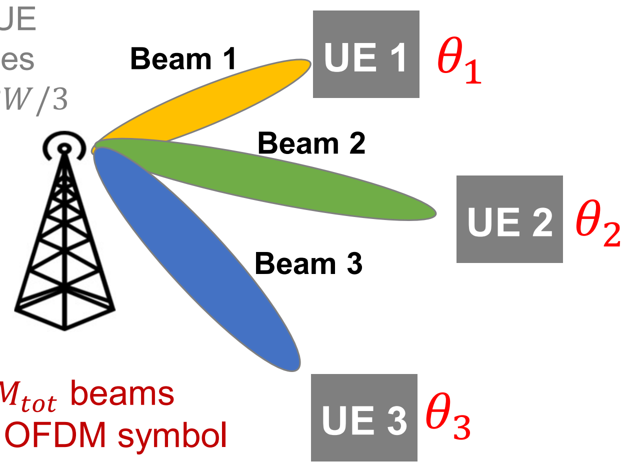
[1] V. Boljanovic, H. Yan, E. Ghaderi, D. Heo, S. Gupta and D. Cabric, “Design of Millimeter-Wave Single-Shot Beam Training for True-Time-Delay Array," 2020 IEEE 21st International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), pp. 1-5, 2020

Project 2: Flexible beamforming w/ TTD

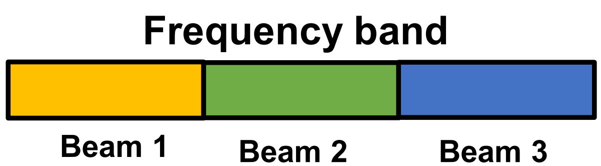
Sub-band-specific beams for simultaneous multi-user data communication

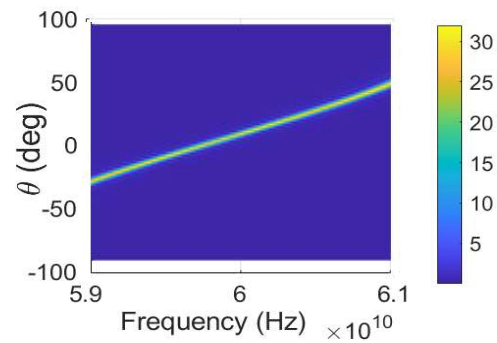
Each UE operates over $BW/3$

K=3 users



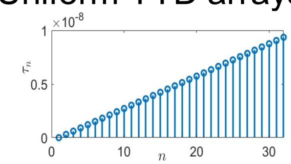
$N_{UE} \ll M_{tot}$ beams using 1 OFDM symbol

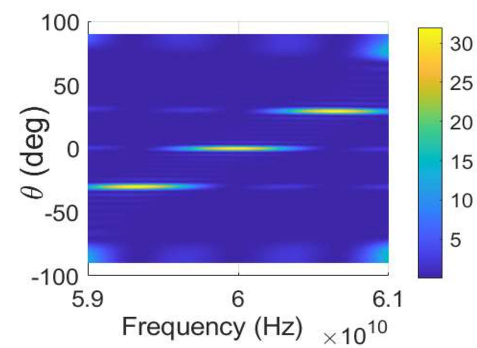




Rainbow beams: complete frequency-angle dispersion

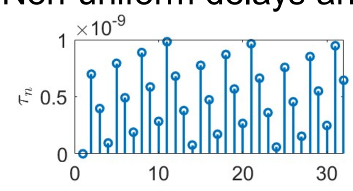
- Fast beam Training
- Uniform TTD arrays





Quantized sub-band specific dispersion[1]

- Multi-user data communication
- Non-uniform delays and phases

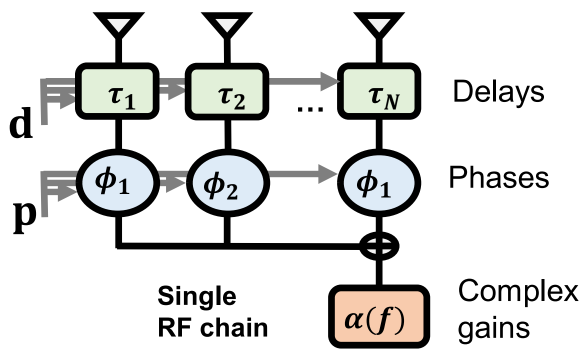


[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.

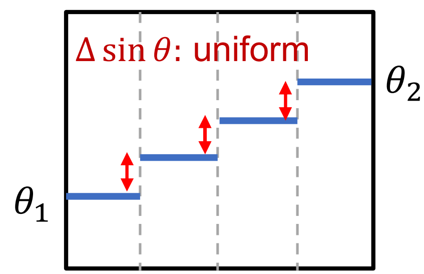
Project 2: Flexible beamforming w/ TTD

Problem statement: How to design TTD codebook to implement sub-band beams with single RF chain?

TTD array architecture



Target beam pattern



***K* users in *K* sub-bands**

Given:

Target freq. angle map: $\Theta(f)$

Target beam:

$$\mathbf{b}_f = \frac{1}{\sqrt{N_T}}\exp(-j\pi \frac{f_m}{f_c}\sin\Theta(f))$$

TTD combiner: $\mathbf{w}(f) = \frac{1}{\sqrt{N_T}}\alpha(f).exp(-j(2\pi f_m \boldsymbol{\tau} - \boldsymbol{\Phi})) \quad \forall f = 1, ..., M_{tot}$

How to design τ_n , ϕ_n $\forall\, n \in \{1, \ldots, N_R\}$, $\alpha(f)$

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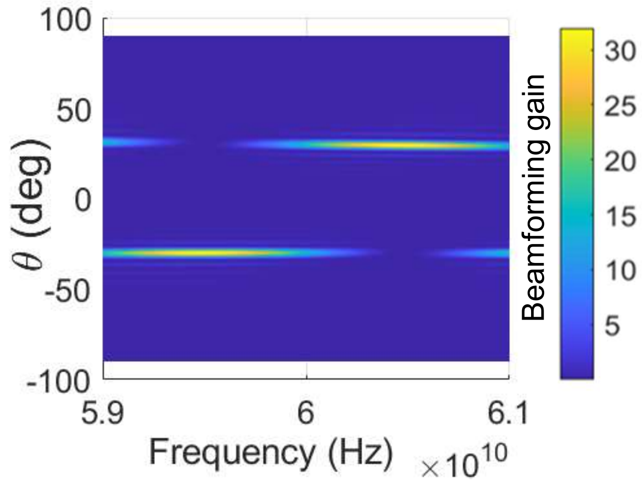
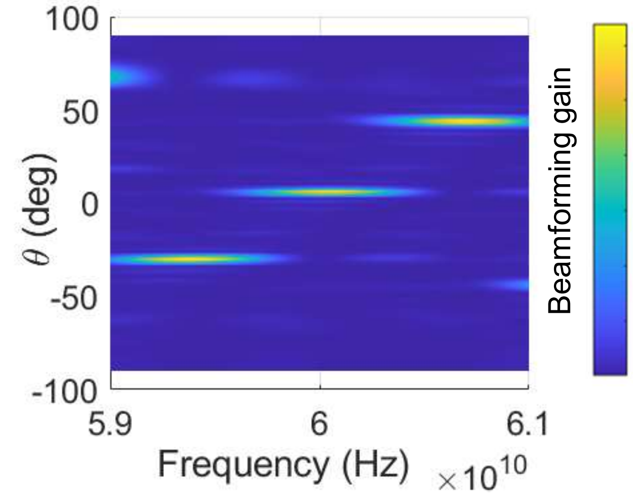
[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.
[3] I. K. Jain, R. R. Vennam, R. Subbaraman, and D. Bharadia, “mmflexible: Flexible directional frequency multiplexing for multi-user mmwave networks,” arXiv preprint arXiv:2301.10950, 2023.

Project 2: Flexible beamforming w/ TTD

Examples of sub-band angle maps





$$K = 2;\ \Theta(f) = \begin{cases} -30^\circ\ ; f < f_c \\ 30^\circ\ ;\ \ f \geq f_c \end{cases}$$

$$K = 3;\ \Theta(f) = \begin{cases} -30^\circ & ; f < f_c - BW/6 \\ 5^\circ\ ; & f_c - \frac{BW}{6} \leq f < f_c + BW/6 \\ 45^\circ & ; f \geq f_c + BW/6 \end{cases}$$

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[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.
[3] I. K. Jain, R. R. Vennam, R. Subbaraman, and D. Bharadia, “mmflexible: Flexible directional frequency multiplexing for multi-user mmwave networks,” arXiv preprint arXiv:2301.10950, 2023.

Project 2: Flexible beamforming w/ TTD

Algorithms: to design TTD codebook to implement required sub-band-angle mapping

Algo 1: **Alternating minimization** [2, Sec V, Algo 1]

Given: **Target beam:**

$$\mathbf{b}_f = \frac{1}{\sqrt{N_T}} \exp(-j\pi \frac{f_m}{f_c} \sin\Theta(f))$$

Determine: τ_n , ϕ_n $\forall\, n \in \{1, \ldots, N_R\}$, $\alpha(f)$

- Iterative adaptation of τ_n: [2, eqn 9 – *simplify to obtain closed form expression*]
- Iterative adaptation of ϕ_n: [2, eqn 8b]
- Iterative adaptation of $\alpha(f)$: [2, eqn 11]

Algo 2: **mmFlexible** [3, Appendix A, Theorem 2]

Given: **Target angle mapping:** $\Theta(f)$

Determine: τ_n , ϕ_n $\forall\, n \in \{1, \ldots, N_R\}$

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[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.
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Project 2: Flexible beamforming w/ TTD

Goodness of fit: measure of how well the designed beams achieve the target angle mapping

Goodness of fit: The goodness of fit of combiner $\mathbf{w}(f) \in \mathbb{C}^{N_T \times 1}$ with respect to the target beam $\mathbf{b}_f$ is defined as $\frac{1}{M_{tot}} \sum_f |\mathbf{w}^H(f)\mathbf{b}_f|$.

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[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.
[3] I. K. Jain, R. R. Vennam, R. Subbaraman, and D. Bharadia, "mmflexible: Flexible directional frequency multiplexing for multi-user mmwave networks," arXiv preprint arXiv:2301.10950, 2023.

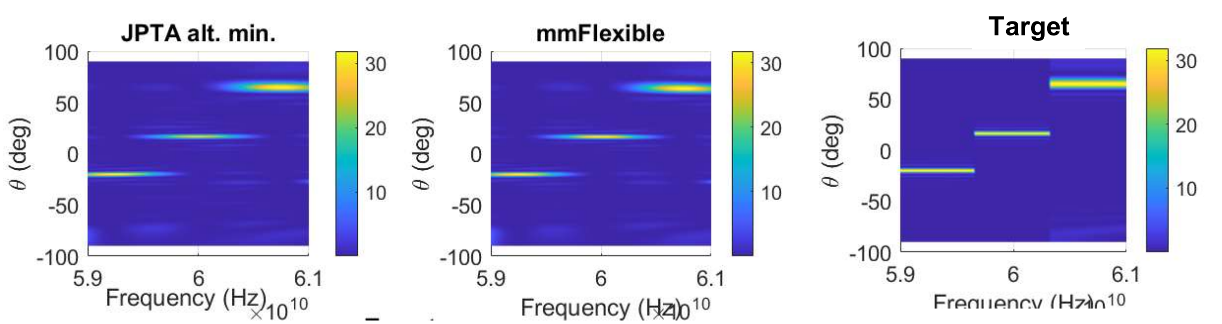
Project 2: Flexible beamforming w/ TTD

Required results:

1. **You are given three target frequency-angle mappings. For each case, show the beamforming gain as a function of frequency-angle obtained via**
   1. 20 iterations of alternating minimization
   2. mmFlexible closed form solution
   3. Target beam pattern

In each case, compute and report the goodness of fit of resulting beams

Example:



[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.

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Project 2: Flexible beamforming w/ TTD

Required results:

2. **For K=2 users, vary $\theta^{(1)}, \theta^{(2)}$ across 25 angles in the range of $[-60°, 60°]$. Compute goodness of fit for each of the resulting 625 angle-mappings, and plot its CDF for**
   i. 20 iterations of alternating minimization & mmFlexible closed form solution
   ii. Plot the CDFs for BW = 1% of f_c and 10% of f_c for each algorithm (4 CDF curves totally)

3. **For K=4 users, vary $\theta^{(i)} \forall i = 1, \ldots, 4$ across 10 angles in the range of $[-60°, 60°]$. Repeat steps in (2) for each of the resulting 10000 angle-mappings.**

4. **List three main observations about the proposed algorithms that you make based on the results in parts 1 to 3.** Provide adequate justification backed by evidence to substantiate your inferences.

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[2] Ratnam, Vishnu & Mo, Jianhua & Alammouri, Ahmad & Ng, Boon & Zhang, Jianzhong & Molisch, Andreas. (2022). Joint Phase-Time Arrays: A Paradigm for Frequency-Dependent Analog Beamforming in 6G. IEEE Access. 10. 1-1. 10.1109/ACCESS.2022.3190418.
[3] I. K. Jain, R. R. Vennam, R. Subbaraman, and D. Bharadia, “mmflexible: Flexible directional frequency multiplexing for multi-user mmwave networks,” arXiv preprint arXiv:2301.10950, 2023.

Project 3:

Compressive Estimation of Millimeter-Wave Channels

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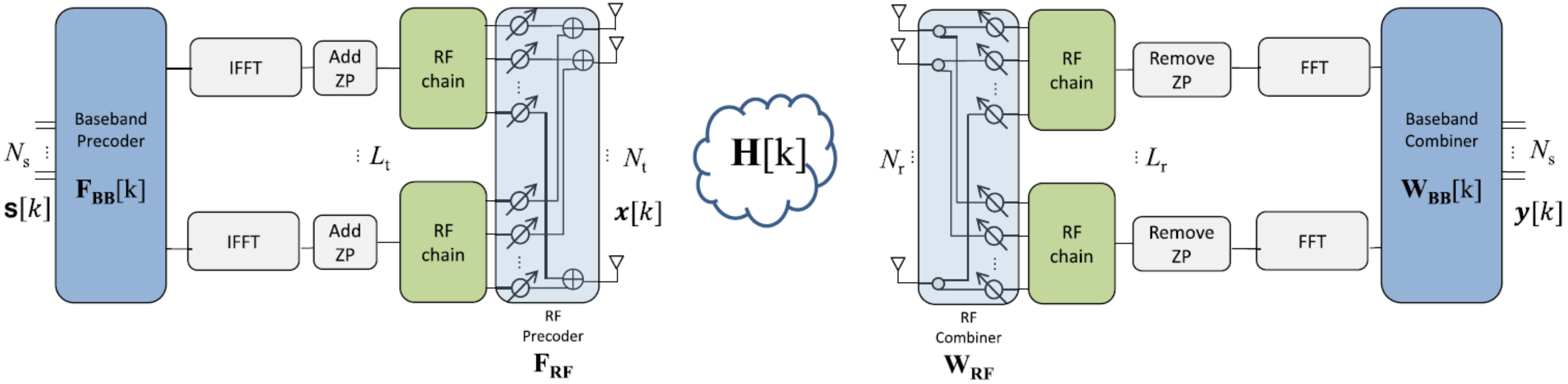
Hybrid MIMO Architecture



- Received signal model:
  - ➢ In training phase:

$$\mathbf{y}^{(m)}[k] = \mathbf{W}_{\mathrm{tr}}^{(m)^*}\mathbf{H}[k]\mathbf{F}_{\mathrm{tr}}^{(m)}\mathbf{q}^{(m)}\mathbf{t}^{(m)}[k] + \mathbf{n}_{\mathrm{c}}^{(m)}[k]$$

  - ➢ In data transmission phase

$$\mathbf{y}[k] = \mathbf{W}_{\mathrm{BB}}^*[k]\mathbf{W}_{\mathrm{RF}}^*\mathbf{H}[k]\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}[k]\mathbf{s}[k] + \mathbf{W}_{\mathrm{BB}}^*[k]\mathbf{W}_{\mathrm{RF}}^*\mathbf{n}[k]$$

[R-4] J. Rodríguez-Fernández, N. González-Prelcic, K. Venugopal and R. W. Heath, "Frequency-Domain Compressive Channel Estimation for Frequency-Selective Hybrid Millimeter Wave MIMO Systems," in *IEEE Transactions on Wireless Communications*, May 2018

Sparse Representation of the Channel

- Extended virtual representation for the channel model [ref 2 in R-4]

$$\mathbf{H}_d = \sqrt{\frac{N_\mathrm{t} N_\mathrm{r}}{L \rho_\mathrm{L}}} \sum_{\ell=1}^{L} \alpha_\ell p_\mathrm{rc}(dT_\mathrm{s} - \tau_\ell) \mathbf{a}_\mathrm{R}(\phi_\ell) \mathbf{a}_\mathrm{T}^*(\theta_\ell)$$

$$= \mathbf{A}_\mathrm{R} \mathbf{\Delta}_d \mathbf{A}_\mathrm{T}^* \qquad \mathbf{\Delta}_d \in \mathbb{C}^{L \times L}$$

$$\approx \tilde{\mathbf{A}}_\mathrm{R} \mathbf{\Delta}_d^v \tilde{\mathbf{A}}_\mathrm{T}^* \qquad \mathbf{\Delta}_d^v \in \mathbb{C}^{G_\mathrm{r} \times G_\mathrm{t}}$$

- The two matrices $\tilde{\mathbf{A}}_\mathrm{R}$, $\tilde{\mathbf{A}}_\mathrm{T}^*$, are formulated by array response with possible G_r, G_t AOD/AOAs

- $G_r, G_t \gg L$, therefore, $\mathbf{\Delta}_d^v$ becomes a sparse diagonal matrix

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[ref 2 in R-4] R. W. Heath, N. González-Prelcic, S. Rangan, W. Roh and A. M. Sayeed, "An Overview of Signal Processing Techniques for Millimeter Wave MIMO Systems," in *IEEE Journal of Selected Topics in Signal Processing*, April 2016

Sparse Reconstruction Problem Formulation

- Vectorization of all measurements and put it in matrix form:

$$\underbrace{\begin{bmatrix} \mathbf{y}^{(1)}[k] \\ \vdots \\ \mathbf{y}^{(M)}[k] \end{bmatrix}}_{\mathbf{y}[k]} = \underbrace{\begin{bmatrix} \mathbf{\Phi}^{(1)} \\ \vdots \\ \mathbf{\Phi}^{(M)} \end{bmatrix}^T}_{\mathbf{\Phi}} \mathbf{\Psi}\mathbf{h}^{\text{v}}[k] + \underbrace{\begin{bmatrix} \mathbf{n}_{\text{c}}^{(1)}[k] \\ \vdots \\ \mathbf{n}_{\text{c}}^{(M)}[k] \end{bmatrix}}_{\mathbf{n}_{\text{c}}[k]}, \qquad \mathbf{h}^{\text{v}}[k] = \text{vec}\{\mathbf{\Delta}[k]\}$$

- Sparse reconstruction problem

$$\min \|\mathbf{h}^{\text{v}}[k]\|_1 \quad \text{subject to } \|\mathbf{y}[k] - \mathbf{\Phi}\mathbf{\Psi}\mathbf{h}^{\text{v}}[k]\|_2^2 < \epsilon.$$

Simultaneous Weighted Orthogonal Matching Pursuit

Algorithm 1 Simultaneous Weighted Orthogonal Matching Pursuit (SW-OMP)

procedure SW-OMP($\mathbf{y}[k]$,$\mathbf{\Phi}$,$\mathbf{\Psi}$,ϵ)
  Compute the whitened equivalent observation matrix
    $\mathbf{\Upsilon}_{\mathrm{w}} = \mathbf{D}_{\mathrm{w}}^{-*}\mathbf{\Phi}\mathbf{\Psi}$
  Initialize the residual vectors to the input signal vectors and support estimate
    $\mathbf{y}_{\mathrm{w}}[k] = \mathbf{D}_{\mathrm{w}}^{-*}\mathbf{y}[k]$, $\mathbf{r}[k] = \mathbf{y}_{\mathrm{w}}[k]$, $k = 0, \ldots, K-1$, $\hat{\mathcal{T}} = \{\emptyset\}$
  while MSE $> \epsilon$ **do**
    Distributed Correlation
      $\mathbf{c}[k] = \mathbf{\Upsilon}_{\mathrm{w}}^{*}\mathbf{r}[k]$, $k = 0, \ldots, K-1$
    Find the maximum projection along the different spaces
      $p^{\star} = \arg\max_{p} \sum_{k=0}^{K-1} |\{\mathbf{c}[k]\}_p|$
    Update the current guess of the common support
      $\hat{\mathcal{T}} = \hat{\mathcal{T}} \cup p^{\star}$
    Project the input signal onto the subspace given by the support using WLS
      $\mathbf{x}_{\hat{\mathcal{T}}}[k] = \left([\mathbf{\Upsilon}_{\mathrm{w}}]_{:,\hat{\mathcal{T}}}\right)^{\dagger} \mathbf{y}_{\mathrm{w}}[k]$
      $k = 0, \ldots, K-1$
    Update residual
      $\mathbf{r}[k] = \mathbf{y}_{\mathrm{w}}[k] - [\mathbf{\Upsilon}_{\mathrm{w}}]_{:,\hat{\mathcal{T}}}\,\hat{\hat{\boldsymbol{\xi}}}[k]$
      where $\hat{\hat{\boldsymbol{\xi}}}[k] = \mathbf{x}_{\hat{\mathcal{T}}}[k]$, $k = 0, \ldots, K-1$
    Compute the current MSE
      $\mathrm{MSE} = \frac{1}{KML_{\mathrm{r}}} \sum_{k=0}^{K-1} \mathbf{r}^{*}[k]\mathbf{r}[k]$
  end while
end procedure

Brief summary

Step 0: Create the dictionary matrix and initialize the residual

In *While* iteration

- Step 1: Run the correlation with the dictionary and find the candidate
- Step 2: Put the candidate in the common support
- Step 3: Project the measurement to the common support
- Step 4: Update the residual
- Step 5: End while when power of residual is smaller than ϵ

Performance Evaluation

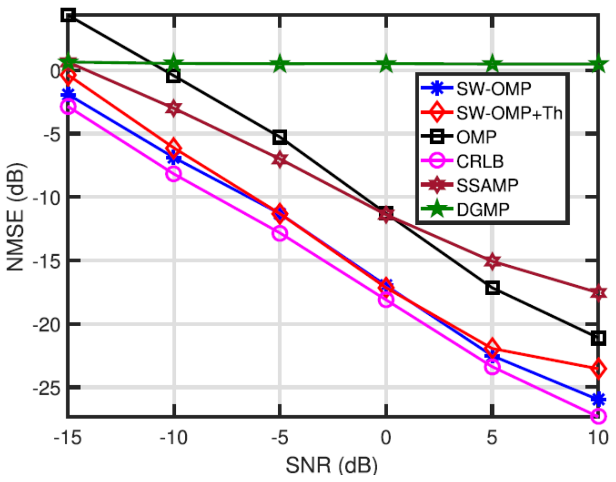
- NMSE of channel estimation result

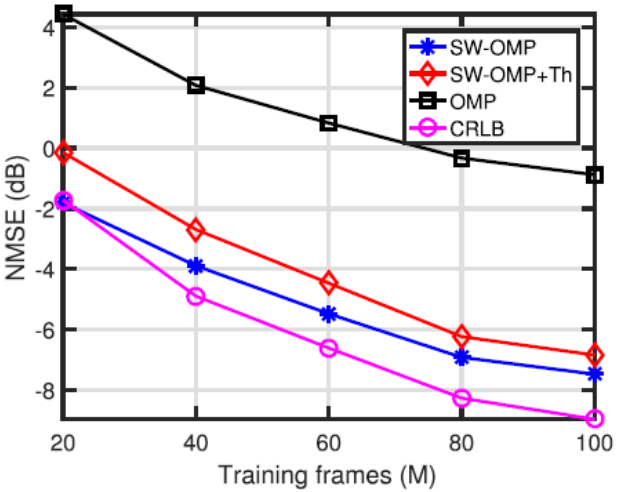
$$\mathrm{NMSE} = \frac{\sum_{k=0}^{K-1} \|\hat{\mathbf{H}}[k] - \mathbf{H}[k]\|_F^2}{\sum_{k=0}^{K-1} \|\mathbf{H}[k]\|_F^2}$$

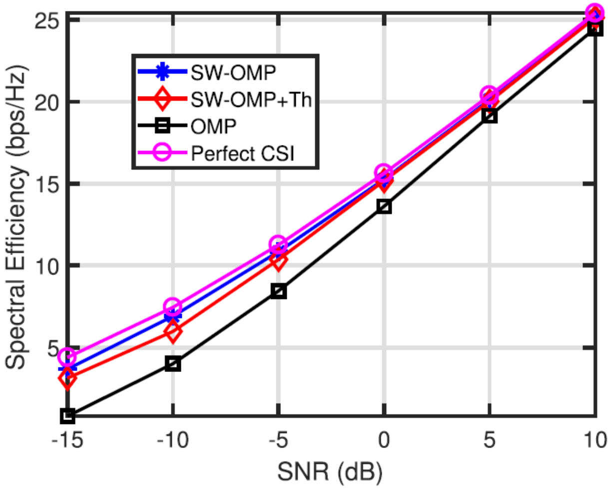
- Spectral efficiency

$$R = \frac{1}{K} \sum_{k=0}^{K-1} \sum_{n=1}^{N_s} \log\left(1 + \frac{\mathrm{SNR}}{N_s} \lambda_n (\mathbf{H}_{\mathrm{eff}}[k])^2\right)$$

- Results in the paper







UCLA Sam Schoo

ECE 233: Spring 2023

Project 5: Beam Training with Compressive Sensing and Machine Learning

Benjamin Domae

May 25, 2023

System Model

- **32 element array (uniform linear, 0.5λ spacing)**
- **Channels: 2 paths (-15 dB relative gain)**

$$y = \left| W^H B(hs + n) \right|^2$$

RSS Measurements

"Codebook" – stacked PN beamformers

Impairments –
$\boldsymbol{B} = diag(\boldsymbol{b})$
$\boldsymbol{b}_i = \left(1 + g_{err}(i)\right) e^{j p_{err}(i)}$

Channel – 2 AWVs added

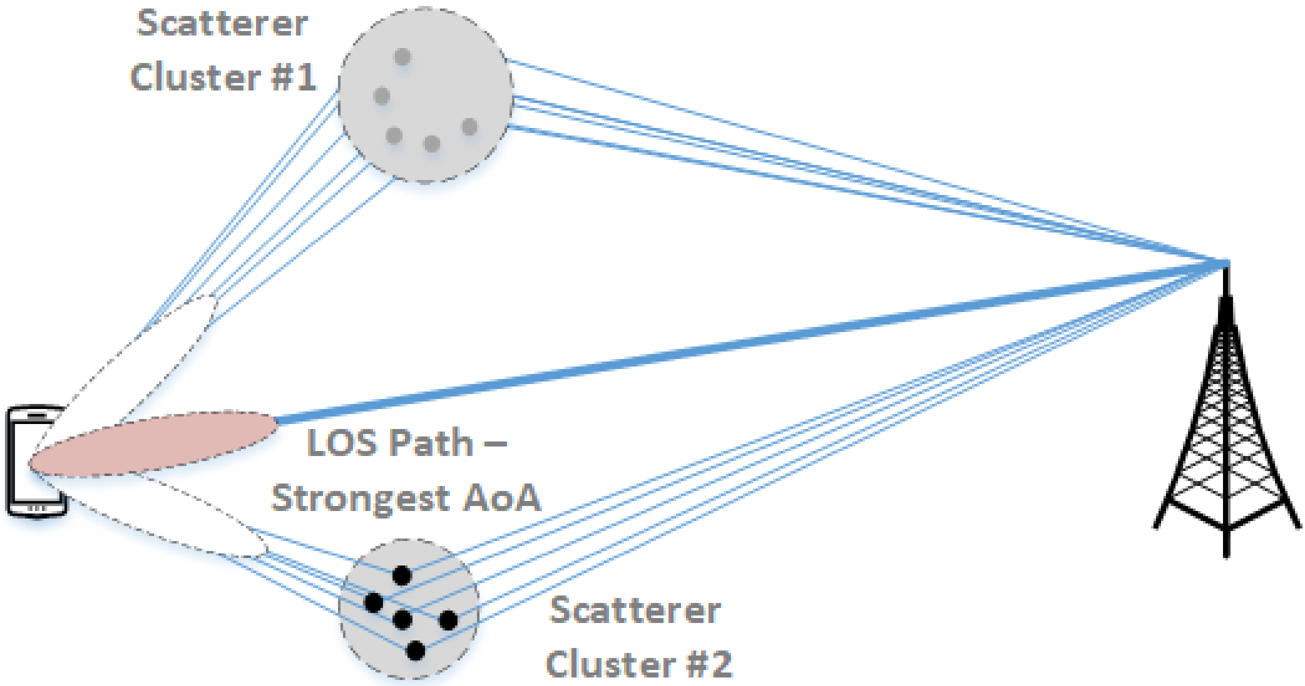
"Symbol" – 1

AWGN

Algorithm Objectives

- **Objective: Predict the strongest path**
  - Analog phased array -> only 1 probing/data stream at the time

- **mmW channels are sparse**
  - Typically, only a few dominant paths
- **Phase-less measurements:**
  - Only using received signal strength (RSS)
  - No raw IQ samples or phase measurements



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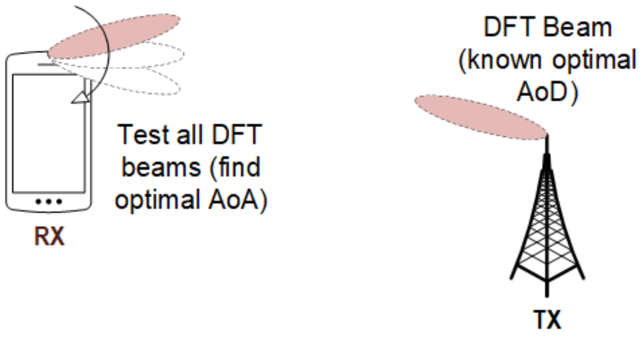
mmW Beam Alignment Codebooks

- **Codebook: set of antenna phases/weights**
  - Beam training: whole measurement/sounding codebook $\mathbf{W}_S$ (size M)
  - Data comm: one beam from the directional codebook $\mathbf{W}_D$ (size K)

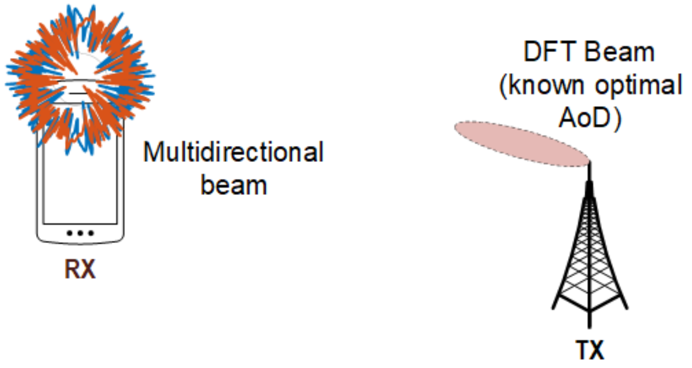
Traditional BA

- Exhaustive search
- Codebook: $\mathbf{W}_s = \mathbf{W}_D$
- Overhead $M = K \propto$ number of elements (N)



Compressed Sensing (CS) BA

- Pseudorandom noise (PN) beams as probing beams $\mathbf{W}_S$
- Overhead $M \propto \log(N)$
- Sensitive to array impairment

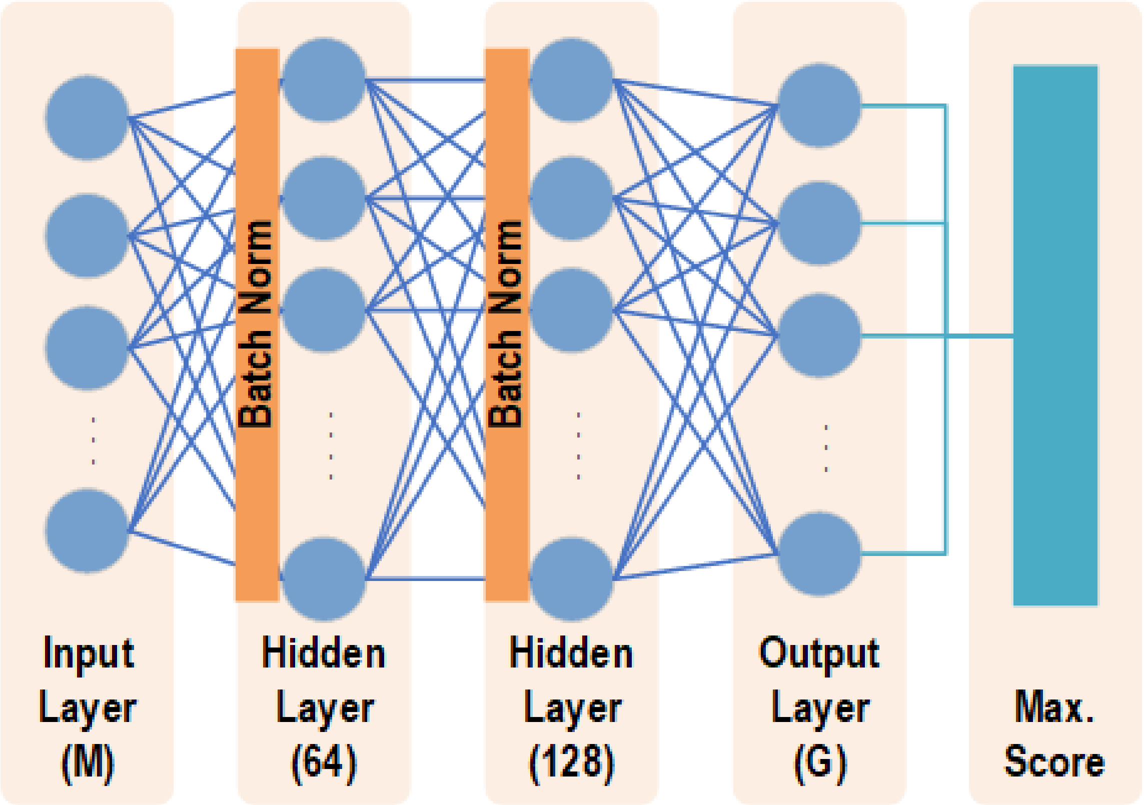


Combine CS and Machine Learning (ML)

- RSS from PN beams
  -> *Reduced measurement complexity*
- Neural Network for signal processing
  -> *Learns inherent hardware impairment*

mmRAPID BA Algorithm

- **K-level classification problem**
  - Features: RSS values from M PN Rx beams
  - Labels: Best DFT Rx beam in codebook $\mathbf{W}_{\mathrm{D}}$
- **Two stage usage:**
  - Training: Rx measures PN and DFT beams and feed into neural network (NN)
  - Operation: Use NN to predict DFT beam from just PN sounding
  - **Retrain for every number of meas. and impairments instance** $\mathrm{M} = \{4, 8, 12\}$



Plots Required

<u>All required plots</u>: w/ and w/o impairments results overlaid

1. Directional beam pattern
2. PN beam pattern
3. RMSE vs SNR (8 meas.)
4. RMSE vs # of meas. (20 dB SNR)

5. (Bonus) RMSE vs 2nd path gain
   (8 meas., 20 dB SNR, w/o impairments)

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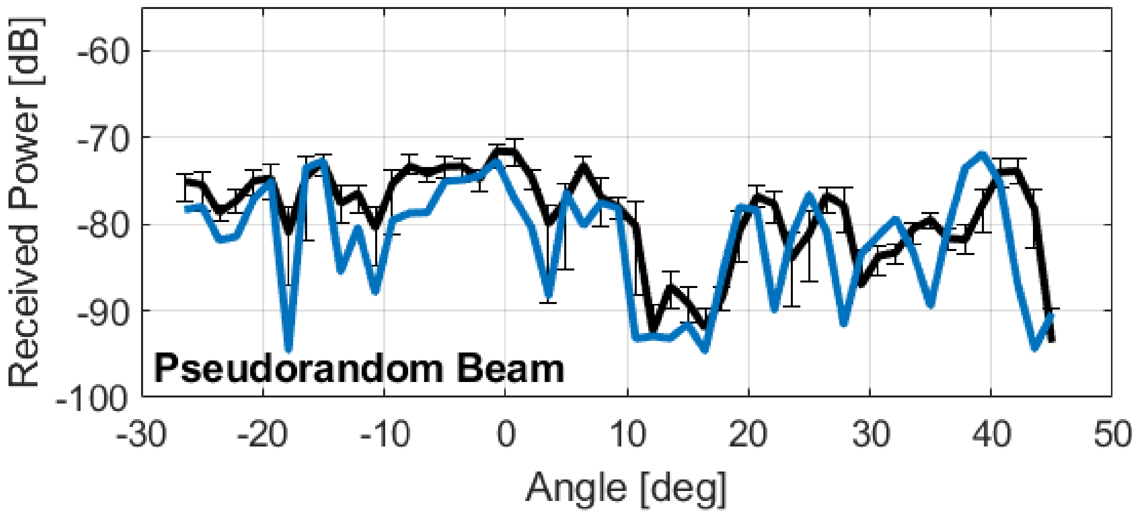
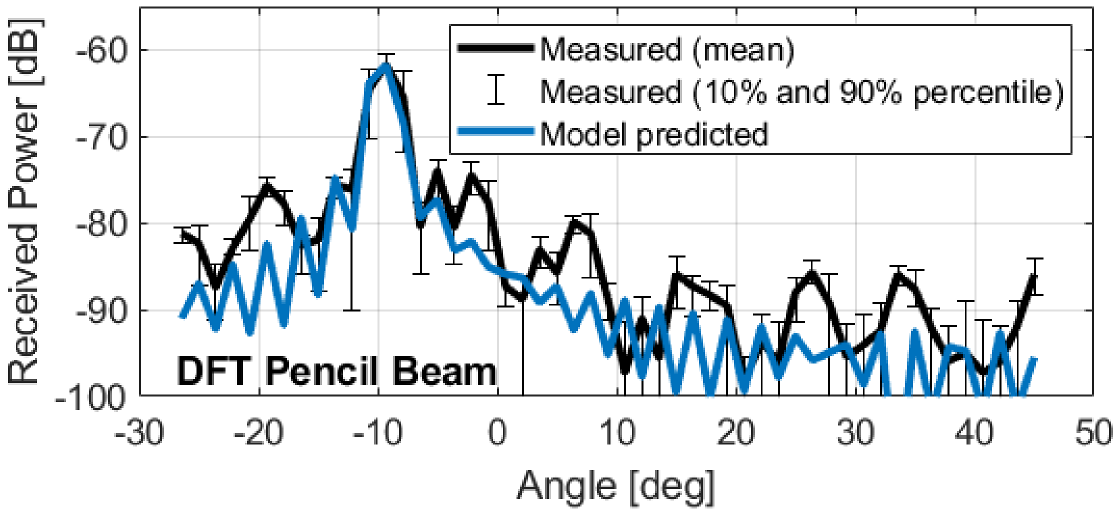
Measurement of PN Beam Mismatch

- **Demonstrated array impairment → beam design mismatch**
  - DFT beams aren't as sensitive
- **PN beam spatial fingerprint:**
  - Set of responses for each AoA for all measurement beams
  - Hardware impairment dependent (need to learn for each device)





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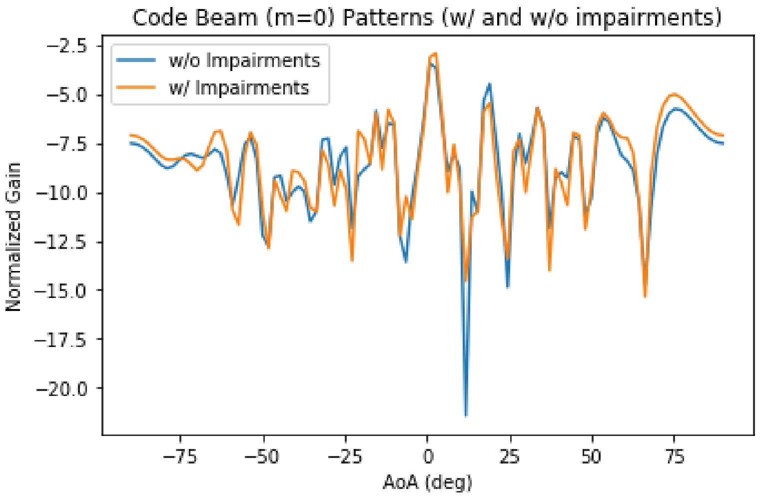
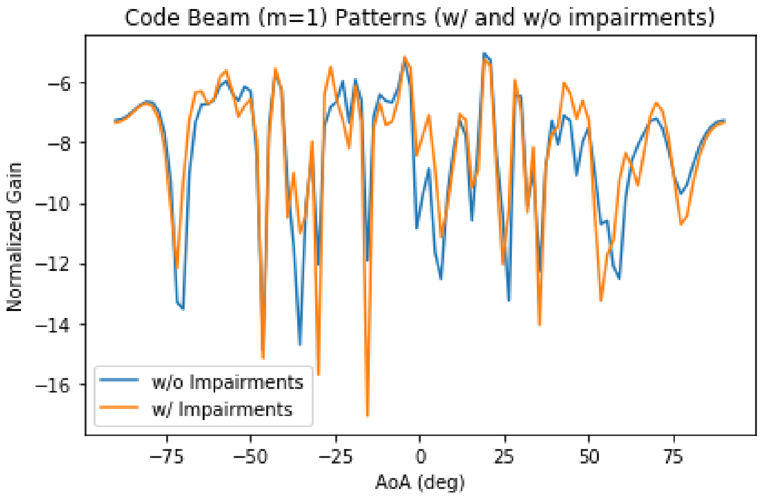
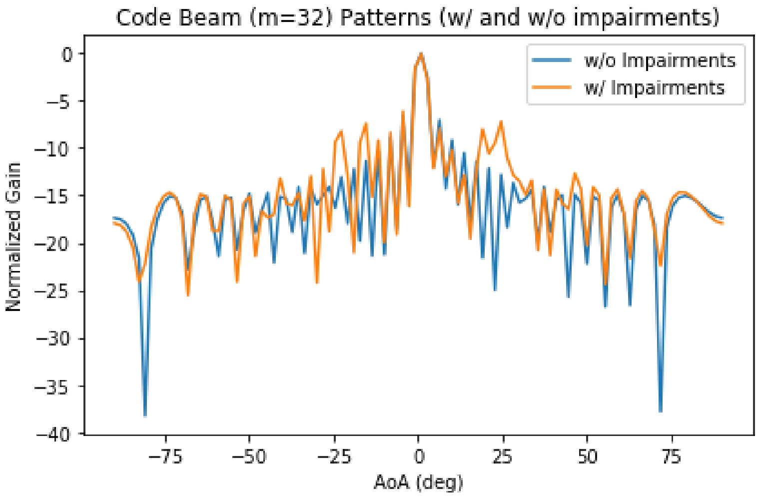
Beam Patterns w/ Impairments

- **2 PN beam examples**
- **1 DFT beam example**
- **Note: $B = I_{N_r}$ (identity matrix) when there are no impairments**







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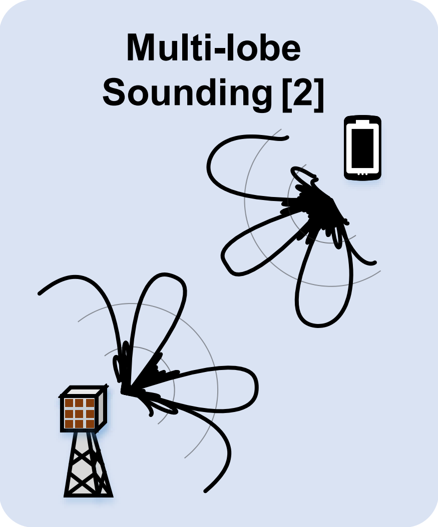
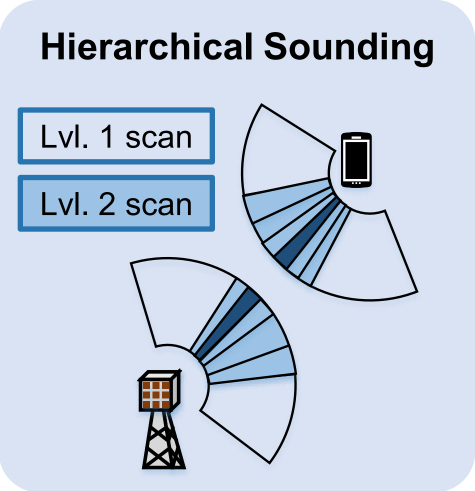
Extra Slides

Beam Alignment (BA) in mmW Systems

- **Key procedure: beam alignment**
  - Goal: find best beam steering directions
  - Challenge in mmW evolution
    - Larger Tx/Rx arrays will be used
    - Increased training overhead & complexity

- **Approaches are constrained by array**
  - Advanced array architecture – fast alignment but high power and complexity
    - Fully-digital array, true-time-delay array, etc.
  - **Phased array are preferred for mobile terminals**
    - **Beam alignment is a bottleneck**





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Project 6:

Scaling up MIMO: Massive MIMO Communication

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Downlink MU-MIMO System Model

M antennas

K UEs

- Received signal model:

$$\boldsymbol{x}_f = \boldsymbol{G}^T \boldsymbol{W} \boldsymbol{s} + \boldsymbol{n}$$

$$\boldsymbol{x}_f, \boldsymbol{n}, \boldsymbol{s} \in \mathbb{C}^{K\times 1}, \boldsymbol{G}^T \in \mathbb{C}^{K\times M}, \boldsymbol{W} \in \mathbb{C}^{M\times K}, M > K$$

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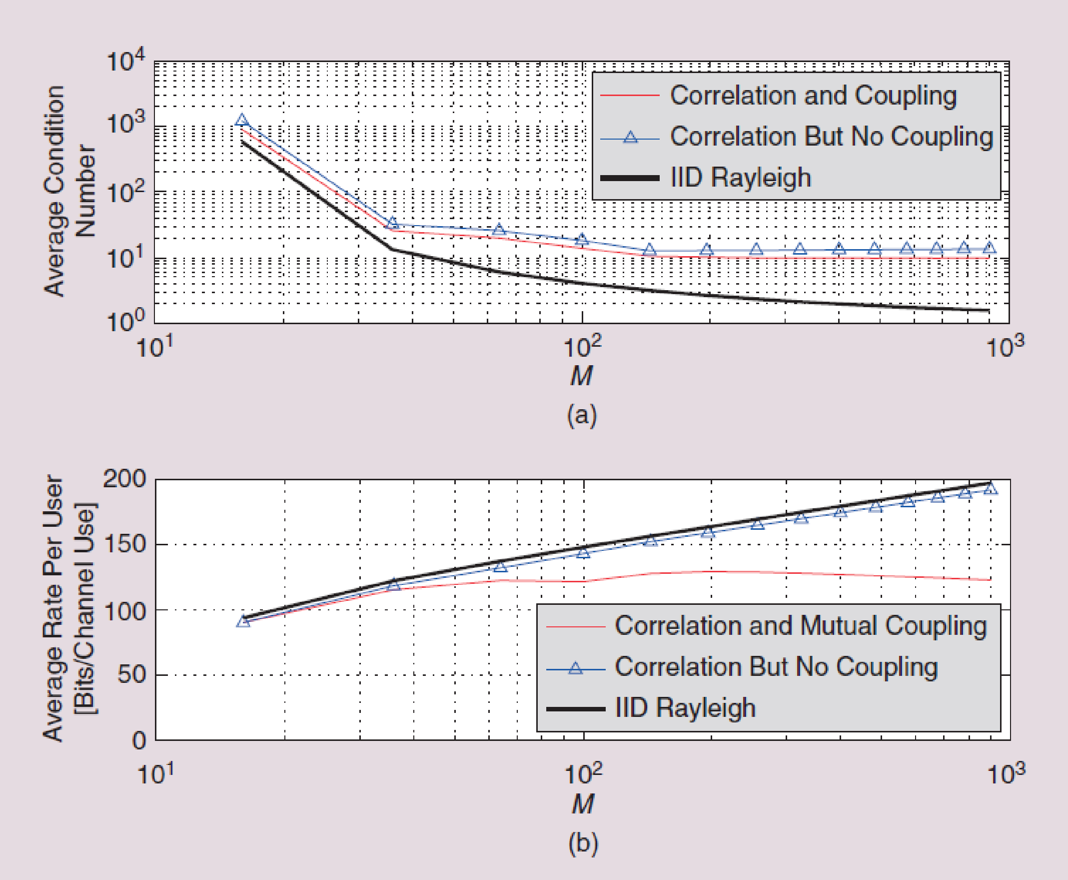
[1] F. Rusek et al., "Scaling Up MIMO: Opportunities and Challenges with Very Large Arrays," in IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 40-60, Jan. 2013

Benefit of Massive MIMO

- Channel condition and capacity [1] as number of antenna grows up

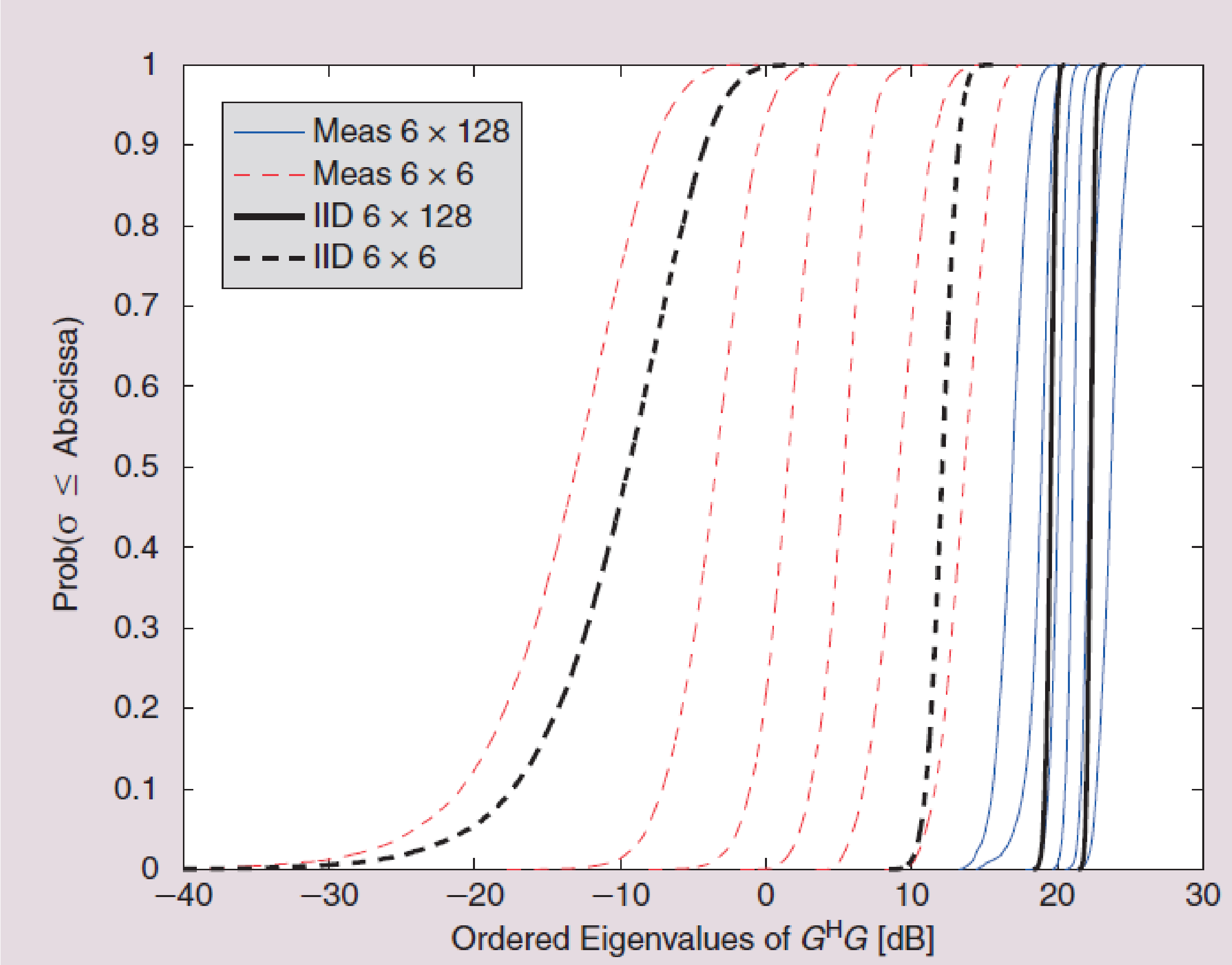
$$C_{\mathrm{sum\_r}} = \log_2 \det(I_K + \rho_{\mathrm{r}} G^{\mathrm{H}} G)$$



[1] F. Rusek et al., "Scaling Up MIMO: Opportunities and Challenges with Very Large Arrays," in IEEE Signal Processing Magazine, vol. 30, no. 1, pp. 40-60, Jan. 2013

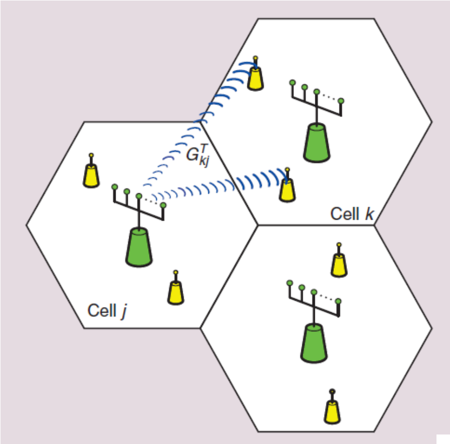
Benefit of Massive MIMO

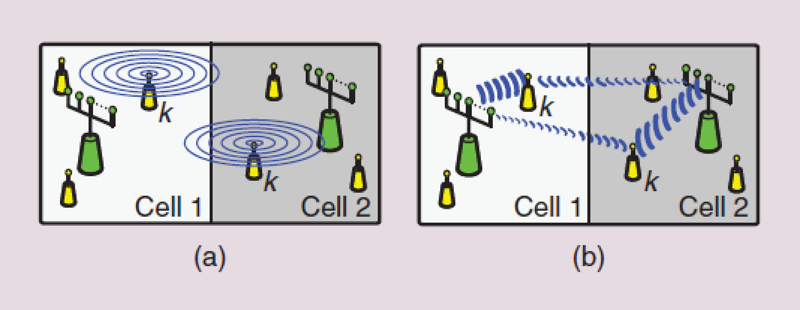
- Eigenvalue distribution [1] as number of antenna grows up



Issues of Massive MIMO

- When the number of antenna gets large, the channel estimation becomes challenging
- Number of orthogonal pilots are limited and will be reused by all cells
- Multi-cell DL MU-MIMO with pilot contamination [1]





$$\hat{G}^{\mathrm{T}}_{nn} = \sqrt{\rho_{\mathrm{p}}}\, G^{\mathrm{T}}_{nn} + \sqrt{\rho_{\mathrm{p}}} \sum_{i \neq n} G^{\mathrm{T}}_{in} + V^{\mathrm{T}}_{n}$$

$$x_{\mathrm{f}j} = \sqrt{\rho_{\mathrm{f}}} \sum_{n} G^{\mathrm{T}}_{jn} \hat{G}^{*}_{nn} q_{\mathrm{f}n} + w_{\mathrm{f}j}$$
$$= \sqrt{\rho_{\mathrm{f}}} \sum_{n} G^{\mathrm{T}}_{jn} \left[\sqrt{\rho_{\mathrm{p}}} \sum_{i} G^{\mathrm{T}}_{in} + V^{\mathrm{T}}_{n} \right]^{\mathrm{H}} q_{\mathrm{f}n} + w_{\mathrm{f}j}$$

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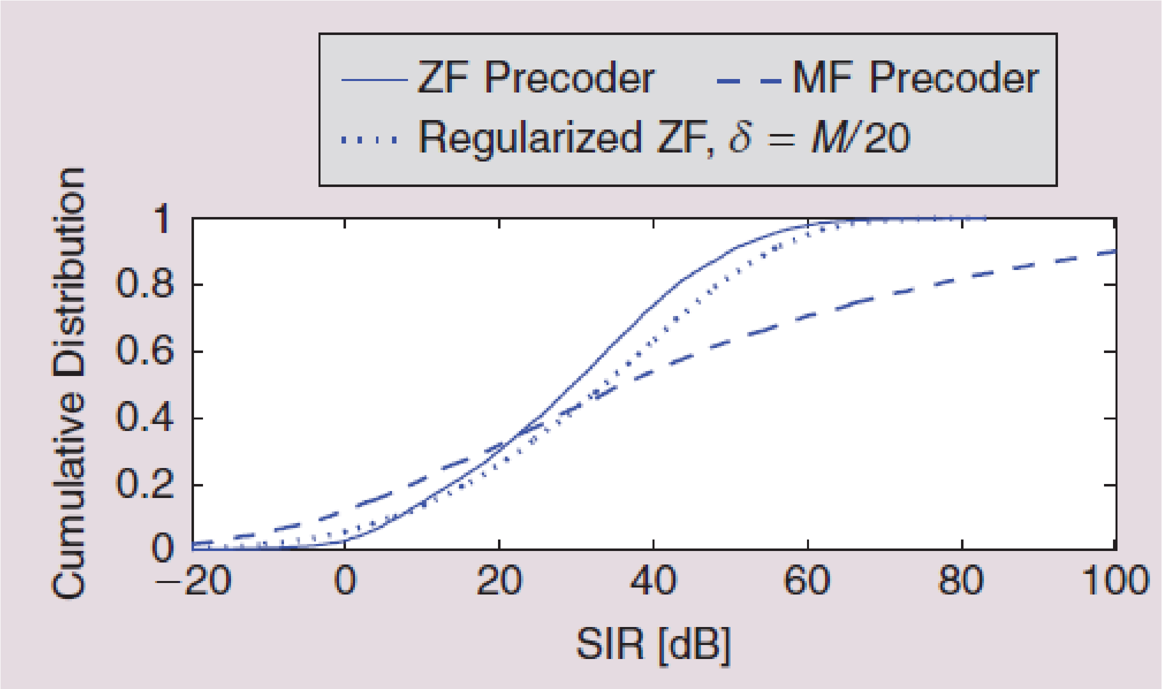
Issues of Massive MIMO

- Signal to Interference ratio (SIR) performance of different precoders [1]

$$\mathrm{SIR} = \frac{\beta_{jj\ell}^2 / \left(\sum_i \beta_{ij\ell} + \frac{1}{\rho_p}\right)^2}{\sum_{n \neq j} \beta_{jn\ell}^2 / \left(\sum_i \beta_{in\ell} + \frac{1}{\rho_p}\right)^2}$$



$$\mathrm{SIR} = \frac{\beta_{jj\ell}^2}{\sum_{n \neq j} \beta_{jn\ell}^2},$$