Assignment 2

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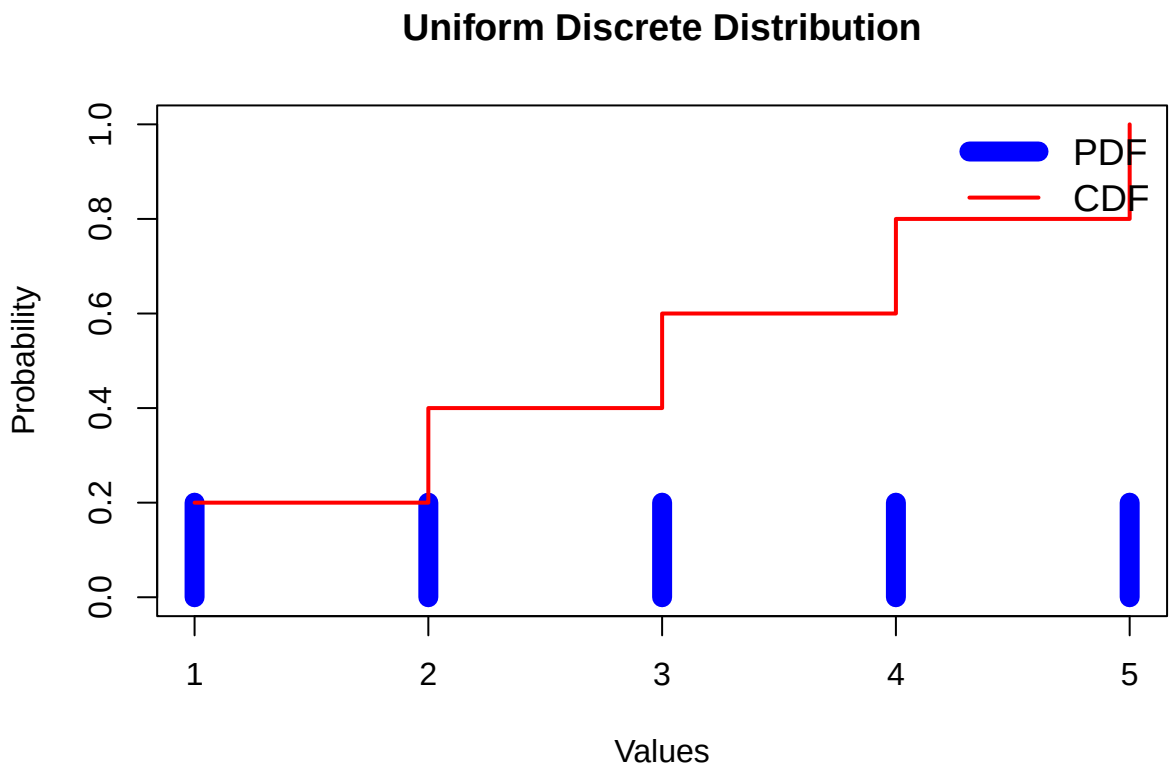
Exercise 1: Discrete Random Variables

```
x = 1:5
min_x = 1
max_x = 6

pdf = dunif(x ,  min = min_x , max = max_x)
cdf = punif(x, min = min_x-1 , max = max_x-1)
```

1)

```
plot(x, pdf, type = "h", col = "blue", lwd = 10, ylim = c(0, max(pdf, cdf)),
     ylab = "Probability", xlab = "Values", main = "Uniform Discrete Distribution")
lines(x, cdf, type = "s", col = "red", lwd = 2)
legend("topright", c("PDF", "CDF"), lty = c(1, 1), col = c("blue", "red"),
       lwd = c(10, 2), bty = "n", cex = 1.2)
```



2)

```
mean_ = mean(x , prob = pdf)
variance_ = var(x)

print(mean_)
```

3)

```
## [1] 3
```

```
print(variance_)
```

```
## [1] 2.5
```

4) first we define the function:

```
func = function(k){k * (6 - k)}
```

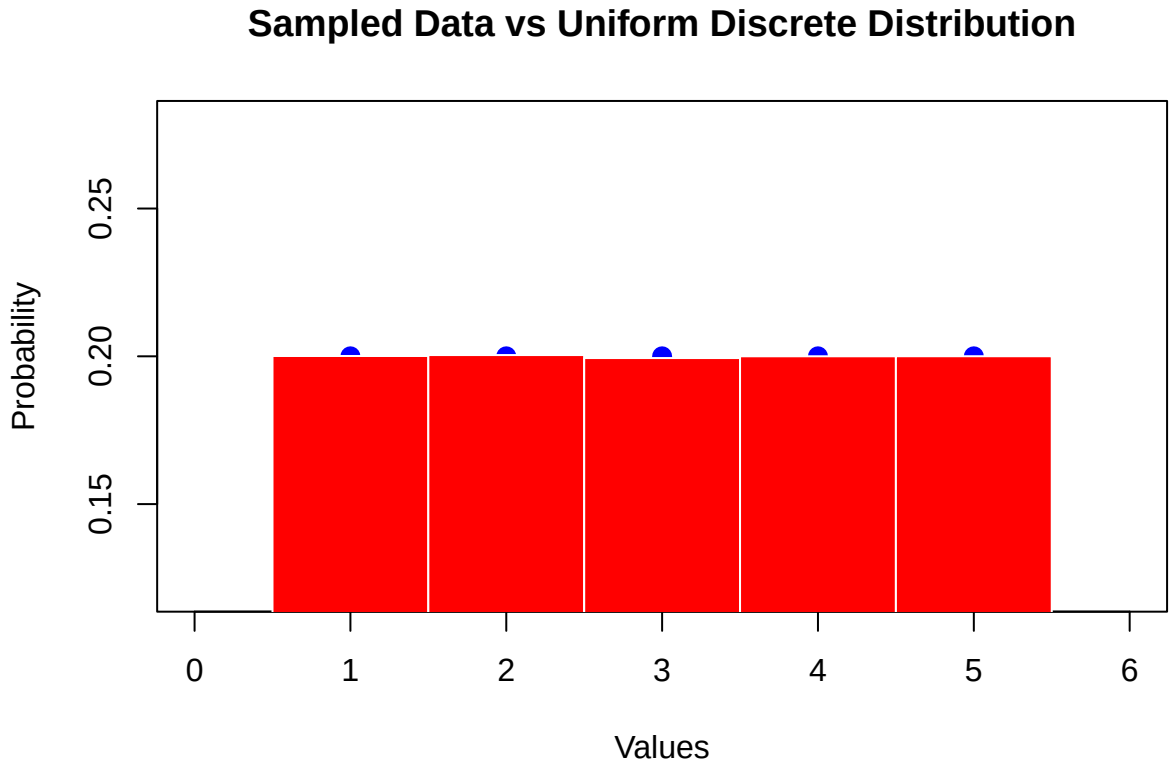
```
expected_value = mean(func(x))
expected_value
```

```
## [1] 7
```

```r
samples = sample(x, size = 1000000, replace = TRUE, prob = pdf)
```

5)

```r
plot(x, pdf, type = "h", col = "blue", lwd = 10, xlim = c(min_x - 1, max_x),
     ylab = "Probability", xlab = "Values", main = "Sampled Data vs Uniform Discrete Distribution")
hist(samples, breaks = seq(min_x - 0.5, max_x + 0.5, by = 1), col = "red", border = "white" , freq = FA
```



6)

Exercise 2: Continuous random variable

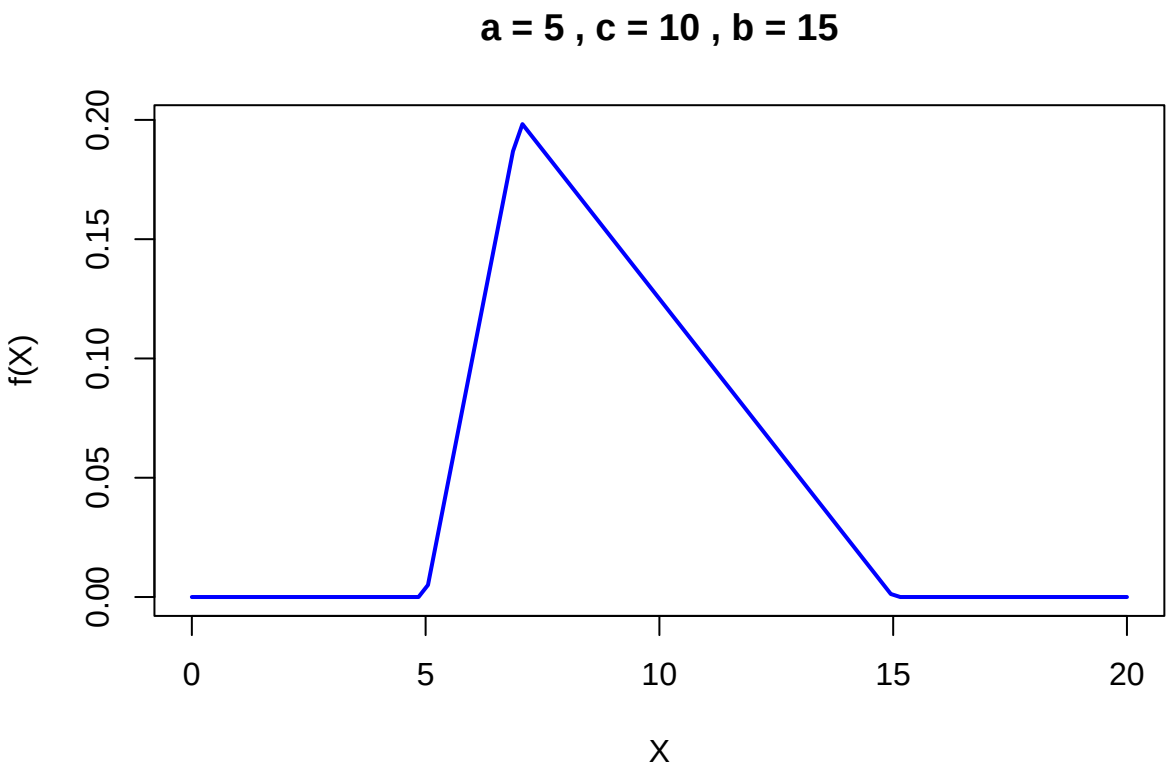
a) we first define the function f(X) and assign some values to a,b and c parameters:

```r
f = function(x,a,b,c){ifelse(x<a|x>b, 0, ifelse(x<c, 2*(x-a)/((b-a)*(c-a)) , 2 * (b-x)/((b-a)*(b-c))))}

a = 5
b = 15
c = 7
```

next we plot the function:

```r
curve(f(x,a,b,c) , from = 0 , to = 20 , n = 100 , ylab = 'f(X)' , main = 'a = 5 , c = 10 , b = 15' ,col
```



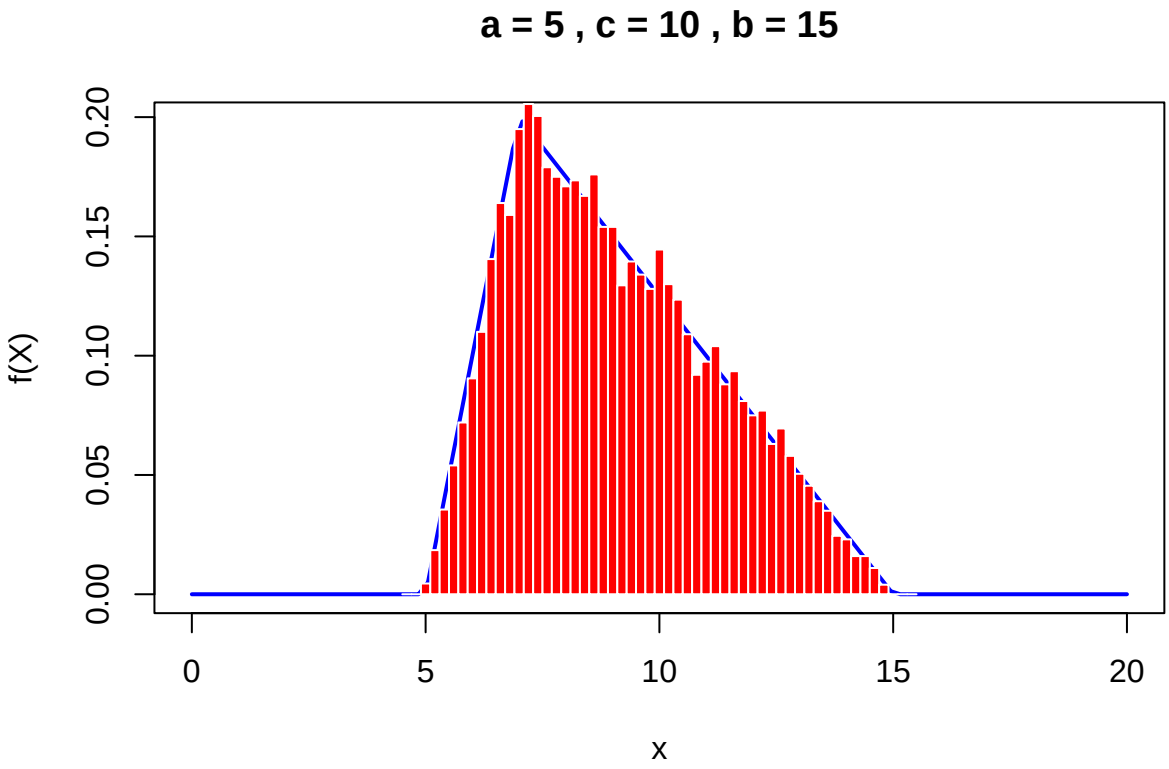
b) We can use the inverse transform sampling method to generate random numbers from the above distribution. The inverse of the cumulative distribution function is found to be:

```r
X = function(u, a , b , c){ifelse(u < (c - a)/(b - a) , sqrt(u * (b - a) * (c - a)) + a , -sqrt((b - a)
```

c) Now we just need to generate some numbers between 0 and 1 and use the above transformation:

```r
u = runif(10000)
Xs = X(u , a,b,c)

curve(f(x,a,b,c) , from = 0 , to = 20 , n = 100 , ylab = 'f(X)' , main = 'a = 5 , c = 10 , b = 15' ,col
hist(Xs, breaks = seq(a - 0.5, b + 0.5, by = 0.2), col = "red", border = "white" , freq = FALSE ,add =
```


a = 5 , c = 10 , b = 15
f(X)
0.00
0.05
0.10
0.15
0.20
0
5
10
15
20
x