

Convective Transport in Nanofluids

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Nanofluids are engineered colloids made of a base fluid and nanoparticles (1–100 nm). Nanofluids have higher thermal conductivity and single-phase heat transfer coefficients than their base fluids. In particular, the heat transfer coefficient increases appear to go beyond the mere thermal-conductivity effect, and cannot be predicted by traditional pure-fluid correlations such as Dittus-Boelter's. In the nanofluid literature this behavior is generally attributed to thermal dispersion and intensified turbulence, brought about by nanoparticle motion. To test the validity of this assumption, we have considered seven slip mechanisms that can produce a relative velocity between the nanoparticles and the base fluid. These are inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity. We concluded that, of these seven, only Brownian diffusion and thermophoresis are important slip mechanisms in nanofluids. Based on this finding, we developed a two-component four-equation nonhomogeneous equilibrium model for mass, momentum, and heat transport in nanofluids. A nondimensional analysis of the equations suggests that energy transfer by nanoparticle dispersion is negligible, and thus cannot explain the abnormal heat transfer coefficient increases. Furthermore, a comparison of the nanoparticle and turbulent eddy time and length scales clearly indicates that the nanoparticles move homogeneously with the fluid in the presence of turbulent eddies, so an effect on turbulence intensity is also doubtful. Thus, we propose an alternative explanation for the abnormal heat transfer coefficient increases: the nanofluid properties may vary significantly within the boundary layer because of the effect of the temperature gradient and thermophoresis. For a heated fluid, these effects can result in a significant decrease of viscosity within the boundary layer, thus leading to heat transfer enhancement. A correlation structure that captures these effects is proposed.

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1 Introduction

The earliest observations of thermal conductivity enhancement in liquid dispersions of submicronic solid particles (i.e., nanoparticles) were reported in 1993 by Masuda et al. [1]. The term “nanofluid” was first proposed by Choi about a decade ago [2], to indicate engineered colloids composed of nanoparticles dispersed in a base fluid. However, only recently have nanoparticles become sufficiently inexpensive and widely available to warrant their consideration for practical applications. For example, at the Massachusetts Institute of Technology (MIT) we are exploring the possibility of using nanofluids as coolants for advanced nuclear systems [3]. Contrary to the milli- and micro-sized particle slurries explored in the past, nanoparticles are relatively close in size to the molecules of the base fluid, and thus can realize very stable suspensions with little gravitational settling over long periods of time. Materials commonly used for nanoparticles include oxides such as alumina, silica, titania and copper oxide, and metals such as copper and gold. Carbon nanotubes and diamond nanoparticles have also been used to realize nanofluids. Popular base fluids include water and organic fluids such as ethanol and ethylene glycol. The volumetric fraction of the nanoparticles, ϕ , is usually below 5%.

The following nanofluid behavior has been observed consistently by different researchers at different organizations and with different nanofluids:

- (1) Abnormal thermal conductivity increase relative to the base

fluid. Eastman et al. [4] reported a 40% increase in the thermal conductivity of ethylene-glycol with 0.3 vol % copper nanoparticles of 10 nm diameter. Das et al. [5] have observed increases of 10–25% in water with 1–4 vol % alumina nanoparticles. Also, it appears that thermal conductivity of nanofluids is a strongly increasing function of temperature, much more so than that of pure liquids [5,6].

- (2) Abnormal viscosity increase relative to the base fluid. Pak and Cho [7] measured the viscosity of alumina/water and titania/water nanofluids at 1–10 vol %, and found it to be much higher than that of pure water, well beyond the prediction of traditional viscosity models such as Brikman-Einstein's [8] or Batchelor's [9]. The same conclusion was reached by Maïga et al. [10] while correlating the viscosity data of Lee et al. [11] and Wang et al. [12].
- (3) Abnormal single-phase convective heat-transfer coefficient increase relative to the base fluid. Pak and Cho [7] reported heat transfer data for turbulent flow of alumina/water and titania/water nanofluids in circular tubes. Their data show Nusselt numbers up to about 30% higher than predicted by the pure fluid correlation (Dittus-Boelter), even though the measured nanofluid properties were used in defining the dimensionless groups in the correlation (Fig. 1(a)). Nusselt numbers over 30% higher than the Dittus-Boelter correlation were also reported by Xuan and Li [13] for turbulent flow of copper/water nanofluids (Fig. 1(b)).

Despite several attempts, a satisfactory explanation for the abnormal increase of the thermal conductivity and viscosity in nanofluids is yet to be found, as emphasized by Eastman et al. [14] in their recent comprehensive review of the nanofluid literature. However, in this paper we do not intend to develop an ex-

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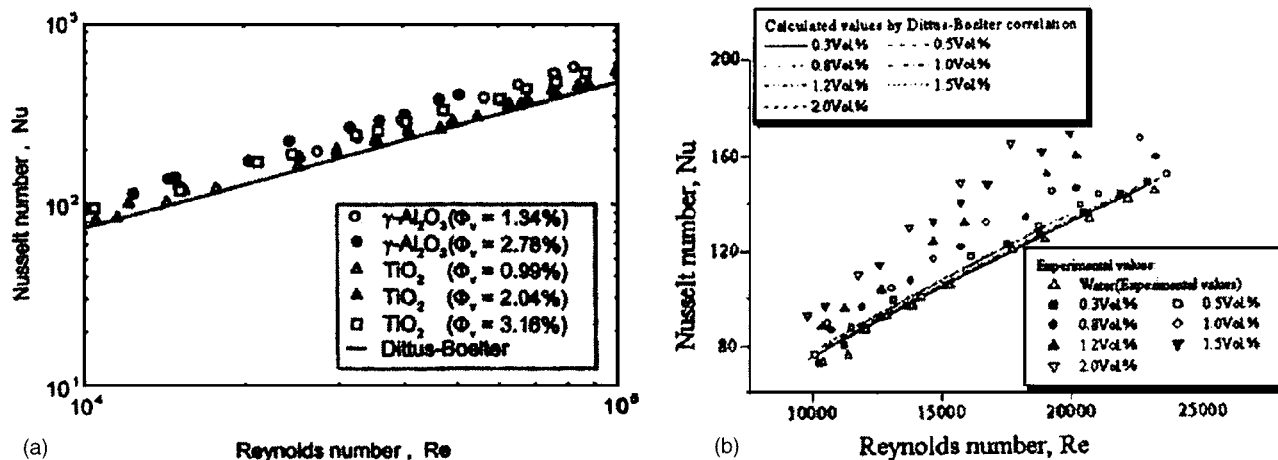


Fig. 1 Heat transfer data for water-based nanofluids with oxide nanoparticles [7] and metal nanoparticles [13]. Note the deviation from the Dittus-Boelter correlation, despite the fact that the nanofluid properties were used in defining Nu, Re, and Pr in both studies: (a) alumina and titania nanoparticles; and (b) copper nanoparticles.

planation for the effect of the nanoparticles on the thermophysical properties. Hereafter the thermophysical properties of nanofluids will be considered as given functions of the nanoparticle volumetric fraction (see Appendix A). Instead, we want to focus on explaining the further heat transfer enhancement observed in convective situations.

Pak and Cho [7], Xuan and Roetzel [15], and Xuan and Li [13] assumed that convective heat transfer enhancement is due mainly to dispersion of the suspended nanoparticles. However, our mechanistic description of particle dispersion suggests that this effect is very small in nanofluids, and thus cannot explain the observed heat transfer enhancement (see Secs. 3 and 4 below).

Xuan and Li [13] proposed that the enhancement could also come from intensification of turbulence due to the presence of the nanoparticles. However, pressure drop measurements by Xuan and Li [13] and Pak and Cho [7] clearly show that turbulent friction factors in their nanofluids can be very well predicted by the traditional friction factor correlations for pure fluids, if the measured nanofluid viscosity is used. This suggests that, beyond the obvious viscosity effect, turbulence is not affected by the presence of the nanoparticles. This conclusion is corroborated by a comparison of the time and length scales for the nanoparticles and the turbulent eddies (see Sec. 2.1 and Appendix B).

In his experiments with micro-sized particles (40–100 μm) suspended in water and glycerine, Ahuja [16] demonstrated that heat transfer enhancement may occur from particle rotation. That is, under the effect of the shear stress, the suspended particles rotate about an axis perpendicular to the main flow direction, creating a three-dimensional hydrodynamic boundary layer which increases the fluid flow towards the wall. To evaluate the importance of this effect with respect to heat conduction, Ahuja [16] proposed the use of a “rotational” Peclet number, Pe_r :

$$Pe_r \equiv (\tau_w/\mu) \cdot (d_p^2/\alpha) \quad (1)$$

where τ_w is the shear stress at the wall, d_p is the particle diameter, μ is the fluid viscosity, and $\alpha = k/\rho c$ is the fluid thermal diffusivity. In Eq. (1) the term (d_p^2/α) represents the time constant for conduction heat transfer, while the term (τ_w/μ) represents the angular velocity of the particle, which Ahuja recommends assuming to be of the same order of magnitude of the shear rate at the wall. So, if Pe_r is large, heat transfer enhancement by particle rotation is possible. Using the definition of the Darcy’s friction factor, f , the shear stress can be expanded as follows:

$$\tau_w \equiv \frac{f}{8} \rho \bar{V}^2 = \frac{C}{8} \frac{\mu^2}{\rho D^2} Re^{2-n} \quad (2)$$

In Eq. (2), \bar{V} is the mean axial velocity in the channel, ρ is the fluid density, D is the channel diameter, $Re = (\rho \bar{V} D)/\mu$ is the Reynolds number, and $f = C/Re^n$ is the friction factor correlation. For conditions typical of alumina/water nanofluids at room temperature ($\mu \sim 1 \text{ mPa s}$, $\rho \sim 1 \text{ g/cm}^3$, $\alpha \sim 2 \times 10^{-7} \text{ m}^2/\text{s}$, $Re \sim 50,000$, $D \sim 1 \text{ cm}$, $C = 0.184$, $n = 0.2$), and for nanoparticles with $d_p < 100 \text{ nm}$, one gets $Pe_r < 3 \times 10^{-4}$; thus nanoparticle rotation can also be discarded as a heat transfer enhancement mechanism in nanofluids.

Several authors have attempted to develop convective transport models for nanofluids. The models proposed so far are of the following two types:

- (1) Homogeneous flow models. The conventional transport equations for pure fluids are directly extended to the nanofluids. This means that all traditional heat transfer correlations (e.g., Dittus-Boelter) could be used also for a nanofluid, provided that the nanofluid thermophysical properties were used in the calculations. Therefore, heat transfer enhancement is assumed to come only from the higher thermal conductivity. This approach was initially adopted by Choi [2] and more recently by Maïga et al. [10].
- (2) Dispersion models. This approach is based on the assumption that the convective heat transfer enhancement in nanofluids comes from two factors, (i) the higher thermal conductivity, and (ii) the dispersion of the nanoparticles. In this approach, first proposed for nanofluids by Xuan and Roetzel [15], the effect of the nanoparticle/base fluid relative velocity is treated as a perturbation of the energy equation, and an empirical dispersion coefficient is introduced to describe the heat transfer enhancement.

The homogeneous flow models are in conflict with the experimental observation that pure-fluid correlations (such as Dittus-Boelter’s) tend to underpredict the nanofluid heat transfer coefficient systematically, e.g., see Fig. 1 or the comparison of analytical predictions and experimental results in Maïga et al. [10]. As far as the dispersion models are concerned, we will show in Sec. 4 that heat transfer enhancement from nanoparticle dispersion is completely negligible in nanofluids. Therefore, in this paper we develop an alternative model that eliminates the shortcom-

ings of the homogeneous and dispersion models. In this new model the effect of the nanoparticle/base-fluid relative velocity is described more mechanistically than in the dispersion models.

2 Nanoparticle/Fluid Slip

The nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative (slip) velocity. In order to develop a realistic two-component model for transport phenomena in nanofluids, it is important to understand the mechanisms by which the nanoparticles can develop a slip velocity with respect to the base fluid. The slip mechanisms mentioned in the literature [15,17–19] and analyzed here are seven: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity settling.

For the purpose of our studies, we will assume that the fluid around the nanoparticles can be regarded as a continuum. To assess the accuracy of this assumption, we need to calculate the Knudsen number, Kn , which is defined as the ratio of the water molecule mean free path, λ , to the nanoparticle diameter, d_p :

$$Kn = \frac{\lambda}{d_p} \quad (3)$$

The water molecule effective size and mean free path in liquid water are both of the order of 3 \AA (0.3 nm). Therefore, for the nanoparticle range of interest ($1\text{--}100 \text{ nm}$), the Knudsen number is relatively small ($Kn < 0.3$), and the continuum assumption is reasonable.

2.1 Inertia. Due to its inertia, a particle suspended in a fluid could develop a slip velocity in the presence of turbulent eddies. The nanoparticle/fluid slip velocity due to the turbulent eddies, V_e , can be obtained from the equation of motion:

$$\frac{\pi}{6} d_p^3 \rho_p \frac{dV_e}{dt} = -3\pi d_p \mu V_e \Rightarrow V_e = V_{eo} e^{-t/\tau_p} \text{ with } \tau_p = \frac{\rho_p d_p^2}{18\mu} \quad (4)$$

where Stokes Law was used for the viscous resistance, and its use is justified by the continuum assumption. In Eq. (4), ρ_p is the mass density of the nanoparticles, μ is the viscosity of the fluid, V_{eo} is the velocity of the turbulent eddies, and τ_p is the so-called “relaxation time” of the nanoparticles. Assuming typical values for water/alumina nanofluids ($\mu \sim 1 \text{ mPa s}$, $\rho_p \sim 4 \text{ g/cm}^3$) and $d_p < 100 \text{ nm}$, the relaxation time is $< 2 \text{ ns}$, which is negligible compared with the typical fluctuation time scale of the turbulent eddies (see Appendix B). Therefore, the nanoparticles are readily entrained by the fluid turbulent eddies, and the slip velocity is negligible. This is a direct consequence of the very small size and inertia of the nanoparticles.

The nanoparticle stopping distance, S_p , is defined as the distance a nanoparticle travels by inertia after the turbulent eddy that was carrying it comes to a complete stop. The stopping distance can be obtained by integrating Eq. (4)

$$S_p = \frac{\rho_p d_p^2}{18\mu} V_{eo} \quad (5)$$

where V_{eo} is of the order of the “shear velocity.”

$$V_{eo} \sim \sqrt{\tau_w/\rho} = \bar{V} \sqrt{f/8} = \frac{\mu}{\rho D} \sqrt{C/8} \text{Re}^{1-n/2} \quad (6)$$

Assuming the same conditions of Eq. (2), V_{eo} is about 0.3 m/s , and $S_p < 1 \text{ nm}$. Therefore, S_p is much smaller than the length scale of the eddies (see Appendix B). This result is important, because it ensures that the nanoparticles move homogeneously with the fluid in the presence of turbulent eddies.

Note also that the nanoparticles cannot penetrate the laminar sublayer near the wall by virtue of their inertia. The thickness of the laminar sublayer near the wall, δ_v , is of the order of 5 in dimensionless units:

$$\delta_v \sim 5 \frac{\mu/\rho}{\sqrt{\tau_w/\rho}} \quad (7)$$

which is about $20 \text{ }\mu\text{m}$ for the flow conditions assumed. Thus, $S_p \ll \delta_v$.

2.2 Brownian Diffusion. The random motion of nanoparticles within the base fluid is called Brownian motion, and results from continuous collisions between the nanoparticles and the molecules of the base fluid. The nanoparticles themselves can be viewed effectively as large molecules, with an average kinetic energy equal to that of the fluid molecules ($\frac{1}{2} k_B T$), and thus with a considerable lower velocity. Here k_B is the Boltzmann’s constant and T is the nanofluid temperature. Brownian motion is described by the Brownian diffusion coefficient, D_B , which is given by the Einstein-Stokes’s equation:

$$D_B = \frac{k_B T}{3\pi\mu d_p} \quad (8)$$

For water nanofluids at room temperature with nanoparticles of $1\text{--}100 \text{ nm}$ diameter, the Brownian diffusion coefficient ranges from 4×10^{-10} to $4 \times 10^{-12} \text{ m}^2/\text{s}$. We will show in Sec. 2.8 that Brownian diffusion may become important as a slip mechanism in the absence of turbulent eddies. The nanoparticle mass flux due to Brownian diffusion, $\mathbf{j}_{p,B}$ ($\text{kg}/\text{m}^2 \text{ s}$) can be calculated as:

$$\mathbf{j}_{p,B} = -\rho_p D_B \nabla \phi \quad (9)$$

We will use this flux in developing the general transport model for nanofluids (see Sec. 3).

2.3 Thermophoresis. Particles can diffuse under the effect of a temperature gradient. This phenomenon is called thermophoresis, and is the “particle” equivalent of the well-known Soret effect for gaseous or liquid mixtures. The thermophoretic velocity, \mathbf{V}_T , can be found as:

$$\mathbf{V}_T = -\beta \frac{\mu}{\rho} \cdot \frac{\nabla T}{T}, \quad (10)$$

where an expression for the proportionality factor, β , is given by McNab and Meisen [20]:

$$\beta = 0.26 \frac{k}{2k + k_p} \quad (11)$$

In Eq. (11), k and k_p are the thermal conductivity of the fluid and particle materials, respectively. Equation (11) is based on data for relatively large particles ($1 \text{ }\mu\text{m}$) in water and n -hexane, and is recommended also by Lister [18] and, more recently, by Müller-Steinhagen [21]. Unfortunately, thermophoretic data for nanoparticles are not available at this time, so in the interim Eq. (11) will be used for the nanoparticles as well. The negative sign in Eq. (10) means that the particles move down the temperature gradient, i.e., from hot to cold. For alumina nanoparticles in water at room temperature, $k \sim 1 \text{ W/m K}$, $k_p \sim 40 \text{ W/m K}$, and assuming a temperature gradient of 10^5 K/m (corresponding to 100 kW/m^2 heat flux), one gets $V_T \sim 2 \times 10^{-6} \text{ m/s}$. We will show in Sec. 2.8 that thermophoresis may become important as a slip mechanism in the absence of turbulent eddies.

The nanoparticle mass flux due to thermophoretic effects, $\mathbf{j}_{p,T}$, can be calculated as

$$\mathbf{j}_{p,T} = \rho_p \phi \mathbf{V}_T = -\rho_p D_T \frac{\nabla T}{T} \text{ with } D_T \equiv \beta \frac{\mu}{\rho} \phi \quad (12)$$

The coefficient D_T is sometimes referred to as the “thermal diffusion” coefficient. We will use Eq. (12) in developing the general transport model for nanofluids (see Sec. 3).

2.4 Diffusiophoresis. A particle suspended in a solution in which a concentration gradient exists, is subjected to a net force acting in the direction opposite to that gradient. This phenomenon

is known as diffusiophoresis, and is caused by the impact of the particle with the diffusing species. However, the base fluid of nanofluids is usually a one-component substance with no concentration gradients, thus nanoparticle diffusiophoresis does not occur.

2.5 Magnus Effect. Under the effect of the shear stress, a particle rotates about an axis perpendicular to the main flow direction. If a relative axial velocity exists between the particle and the fluid, a force perpendicular to the main flow direction will arise. This is known as the Magnus effect, or lift force, and is due to the pressure gradient around the particle, created by its rotation. For large particles ($>1 \mu\text{m}$) moving in the laminar sublayer of a turbulent flow, it is commonly assumed that the relative axial velocity is significant and of the order of the turbulent eddy velocity [18]. This is based on the assumption that the particles are carried by turbulent eddies to the edge of the laminar sublayer and injected into the sublayer by inertia. However, we have shown in Sec. 2.1 that the nanoparticles cannot be injected into the laminar sublayer because their inertia is extremely low, and thus also the relative axial velocity is expected to be very low. Then it can be concluded that for nanoparticles the Magnus effect should be negligible.

2.6 Fluid Drainage. As a particle approaches the wall, there is a resistance caused by the pressure in the draining fluid film between the two approaching surfaces. This effect becomes important when the distance between the particle and the wall is of the order of the particle diameter. Therefore, for nanoparticles of 1–100 nm diameter, this effect is relevant only over a very small fraction of the laminar sublayer near the wall, and can be safely neglected.

2.7 Gravity. The nanoparticle settling velocity due to gravity, V_g , can be calculated from a balance of the buoyancy and viscous forces:

$$\frac{\pi}{6} d_p^3 (\rho_p - \rho) g = 3\pi d_p \mu V_g \Rightarrow V_g = \frac{d_p^2 (\rho_p - \rho) g}{18\mu} \quad (13)$$

where again Stokes Law was used for the viscous resistance. In the nanoparticle size range of interest ($<100 \text{ nm}$), V_g is then $<1.6 \times 10^{-8} \text{ m/s}$.

2.8 Relative Importance of the Nanoparticle Transport Mechanisms. To estimate the relative importance of a certain nanoparticle transport mechanism, we compute the time a nanoparticle takes to diffuse a length equal to its diameter under the effect of that mechanism. So, with reference to a nanoparticle of 100 nm diameter, one has $d_p/V_{eo} \sim 3 \times 10^{-7} \text{ s}$ for turbulent transport, $d_p^2/D_B \sim 0.002 \text{ s}$ for Brownian diffusion, $d_p/V_T \sim 0.05 \text{ s}$ for thermophoresis, and $d_p/V_g \sim 6 \text{ s}$ for gravity. Therefore, in the presence of turbulent eddies, turbulent transport of the nanoparticles dominates, i.e., the nanoparticles are carried by the turbulent eddies and other diffusion mechanisms are negligible. Note that turbulent transport occurs without slip, as per the discussion in Sec. 2.1. When turbulent effects are not important (e.g., in the laminar sublayer near the wall), Brownian diffusion and thermophoresis may become important as slip mechanisms. Gravity settling is negligible. These conclusions hold well for nanoparticles of any material and size.

3 Conservation Equations for Nanofluids

Now that the slip mechanisms for the nanoparticles have been clarified, we can develop a complete transport model for the nanofluids. We will treat the nanofluid as a two-component mixture (base fluid+nanoparticles) with the following assumptions:

- (1) incompressible flow,
- (2) no chemical reactions,
- (3) negligible external forces,

- (4) dilute mixture ($\phi \ll 1$),
- (5) negligible viscous dissipation,
- (6) negligible radiative heat transfer,
- (7) nanoparticles and base fluid locally in thermal equilibrium.

Assumptions (1) and (6) are accurate for liquid nanofluids. Assumption (2) is accurate because nanoparticle materials are chosen for their chemical inertness with the base fluid. Assumption (3) is justified in light of the discussion in Sec. 2.8. Assumption (4) is valid for most nanofluid studies published so far, in which $\phi < 0.05$. Assumption (5) is often made in heat transfer problems, in which the heat flux at the surface is the dominant energy source in the system. To justify assumption (7), one has to calculate the heat transfer time constants for heat conduction within the nanoparticles, and within the base fluid in the vicinity of the nanoparticles. These time constants can be estimated as d_p^2/α_p and d_p^2/α_{bf} , respectively, where $\alpha_p = k_p/\rho_p c_p$ and $\alpha_{bf} = k_{bf}/\rho_{bf} c_{bf}$ are the thermal diffusivity of the nanoparticle material and base fluid, respectively. For alumina nanoparticles of 100 nm diameter in water, α_p and α_{bf} are about $10^{-5} \text{ m}^2/\text{s}$ and $10^{-7} \text{ m}^2/\text{s}$, respectively, and the heat transfer time constants are about 1 ns and 100 ns, respectively, which are much smaller than the time constants calculated in Sec. 2.8 for Brownian diffusion and thermophoresis, and also smaller than the turbulent eddy time scale calculated in Appendix B. Therefore, as the nanoparticles move in the surrounding fluid, they achieve thermal equilibrium with it very rapidly, which justifies assumption (7).

The conservation equations for a two-component mixture can be found in most transport phenomena books. For our nanofluid application we will adopt the formalism of Bird, Stewart, and Lightfoot's classic textbook [22], which is referred to as BSL in the following discussion. To capture the effect of the nanoparticle/base fluid slip, we will use a four-equation approach (two mass equations, one momentum equation, and one energy equation).

From Assumption (1), the continuity equation for the nanofluid is (BSL, Eq. (18.1-9)):

$$\nabla \cdot \mathbf{v} = 0 \quad (14)$$

where \mathbf{v} is the nanofluid velocity. Equation (14) is identical to the continuity equation for a pure incompressible fluid. The continuity equation for the nanoparticles in the absence of chemical reactions (Assumption 2) is (BSL, Eq. (18.3-4)):

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = -\frac{1}{\rho_p} \nabla \cdot \mathbf{j}_p \quad (15)$$

where t is time, \mathbf{j}_p is the diffusion mass flux for the nanoparticles ($\text{kg}/\text{m}^2 \text{ s}$), and represents the nanoparticle flux relative to the nanofluid velocity \mathbf{v} . If the external forces are negligible (assumption (3)), \mathbf{j}_p can be written as the sum of only two diffusion terms, i.e., Brownian diffusion and thermophoresis:

$$\mathbf{j}_p = \mathbf{j}_{p,B} + \mathbf{j}_{p,T} = -\rho_p D_B \nabla \phi - \rho_p D_T \frac{\nabla T}{T} \quad (16)$$

The diffusion coefficients D_B and D_T can be calculated from Eqs. (8) and (12), respectively. Substituting Eq. (16) in Eq. (15), one gets:

$$\frac{\partial \phi}{\partial t} + \mathbf{v} \cdot \nabla \phi = \nabla \cdot \left[D_B \nabla \phi + D_T \frac{\nabla T}{T} \right] \quad (17)$$

Equation (17) states that the nanoparticles can move homogeneously with the fluid (second term of the left-hand side), but they also possess a slip velocity relative to the fluid (right-hand side), which is due to Brownian diffusion and thermophoresis.

The momentum equation for the nanofluid with negligible external forces is (BSL, Eq. (18.3-2)):

$$\rho \left[\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right] = -\nabla P - \nabla \cdot \boldsymbol{\tau} \quad (18)$$

where P is pressure. Note that Eq. (18) is identical to the momentum equation for a pure fluid. The stress tensor, $\boldsymbol{\tau}$, can be expanded assuming Newtonian behavior and incompressible flow:

$$\boldsymbol{\tau} = -\mu[\nabla \mathbf{v} + (\nabla \mathbf{v})^t] \quad (19)$$

where the superscript t indicates the transpose of $\nabla \mathbf{v}$. If the viscosity μ is constant, Eq. (18) becomes the usual Navier-Stokes equation. However, μ strongly depends on ϕ for a nanofluid. Therefore, Eqs. (18) and (17) are coupled.

The energy equation for the nanofluid can be written as (BSL, Table 18.3-1, Eq. (F)):

$$\rho c \left[\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right] = -\nabla \cdot \mathbf{q} + h_p \nabla \cdot \mathbf{j}_p \quad (20)$$

where assumptions (1), (2), (3), (4), and (5) were used. In Eq. (20), c is the nanofluid specific heat, T is the nanofluid temperature, h_p is the specific enthalpy of the nanoparticle material (J/kg), and \mathbf{q} is the energy flux relative to the nanofluid velocity \mathbf{v} . Neglecting radiative heat transfer (assumption (6)), \mathbf{q} can be calculated as the sum of the conduction heat flux and the heat flux due to nanoparticle diffusion (BSL, Eq. (18.4-2)):

$$\mathbf{q} = -k \nabla T + h_p \mathbf{j}_p \quad (21)$$

where k is the nanofluid thermal conductivity. Substituting Eq. (21) in Eq. (20), recognizing that $\nabla \cdot (h_p \mathbf{j}_p) \equiv h_p \nabla \cdot \mathbf{j}_p + \mathbf{j}_p \cdot \nabla h_p$, and indicating the nanoparticle specific heat with c_p , one gets:

$$\rho c \left[\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right] = \nabla \cdot k \nabla T - c_p \mathbf{j}_p \cdot \nabla T \quad (22)$$

where ∇h_p has been set equal to $c_p \nabla T$, which follows from assumption (7). Note that if \mathbf{j}_p is zero, Eq. (22) becomes the familiar energy equation for a pure fluid. Substituting Eq. (16) in Eq. (22), the final form of the energy equation for the nanofluid is found:

$$\rho c \left[\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T \right] = \nabla \cdot k \nabla T + \rho_p c_p \left[D_B \nabla \phi \cdot \nabla T + D_T \frac{\nabla T \cdot \nabla T}{T} \right] \quad (23)$$

Equation (23) states that heat can be transported in a nanofluid by convection (second term on the left-hand side), by conduction (first term on the right-hand side), and also by virtue of nanoparticle diffusion (second and third terms on the right-hand side). It is important to emphasize that ρc is the heat capacity of the nanofluid, and thus already accounts for the sensible heat of the nanoparticles as they move homogeneously with the fluid. Therefore, the last two terms on the right-hand side truly account for the additional contribution associated with the nanoparticle motion relative to the fluid.

In summary, Eq. (14) (nanofluid continuity), Eq. (17) (nanoparticle continuity), Eq. (18) (nanofluid momentum), and Eq. (23) (nanofluid energy) constitute a complete set of equations from which $\mathbf{v}(\mathbf{r}, t)$, $P(\mathbf{r}, t)$, $\phi(\mathbf{r}, t)$, and $T(\mathbf{r}, t)$ can be calculated, once the boundary and initial conditions are known, and the nanofluid transport coefficients ($\rho, c, \mu, k, D_B, D_T$) are known as functions of ϕ and temperature. To draw from the terminology of two-phase flow analysis, the nanofluid model we developed can be characterized as a two-“fluid” (nanoparticles+base fluid), four-equation (2 mass+1 momentum+1 energy), nonhomogeneous (nanoparticle/fluid slip velocity allowed) equilibrium (nanoparticle/fluid temperature differences not allowed) model.

Note that the conservation equations are strongly coupled. That is, \mathbf{v} depends on ϕ via viscosity; ϕ depends on T mostly because of thermophoresis; T depends on ϕ via thermal conductivity and also via the Brownian and thermophoretic terms in the energy

equation; ϕ and T obviously depend on \mathbf{v} because of the convection terms in the nanoparticle continuity and energy equations, respectively.

4 Discussion

To assess the relative importance of the various transport mechanisms in nanofluids, it is useful to make the conservation equations nondimensional. For this purpose, we make use of the following transformations:

$$\begin{aligned} \mathbf{V} &\equiv \frac{\mathbf{v}}{\bar{V}}; & \Phi &\equiv \frac{\phi}{\phi_b}; & \theta &\equiv \frac{T - T_b}{\Delta T}; \\ \wp &\equiv \frac{P}{\rho \bar{V}^2}; & \mathbf{R} &\equiv \frac{\mathbf{r}}{D}; & \xi &\equiv \frac{t}{(D/\bar{V})} \end{aligned} \quad (24)$$

where \bar{V} , ϕ_b , T_b , ΔT , and D are the reference velocity, nanoparticle volumetric fraction, temperature, temperature difference, and length, respectively. For example, for a heat transfer problem with internal flow in a round tube, \bar{V} , ϕ_b , T_b , ΔT , and D would be the mean axial velocity of the nanofluid, the nominal nanoparticle volumetric fraction, the bulk temperature, the film temperature drop, and the tube diameter, respectively. Introducing the above transformations into the conservation equations, assuming constant properties (which is appropriate for order-of-magnitude estimates), and recognizing that $\Delta T/T_b \ll 1$, Eqs. (14), (17), (18), and (23) become, respectively:

$$\nabla \cdot \mathbf{V} = 0 \quad (25)$$

$$\frac{\partial \Phi}{\partial \xi} + \mathbf{V} \cdot \nabla \Phi = \frac{1}{\text{Re} \cdot \text{Sc}} \left[\nabla^2 \Phi + \frac{\nabla^2 \theta}{N_{BT}} \right] \quad (26)$$

$$\frac{\partial \mathbf{V}}{\partial \xi} + \mathbf{V} \cdot \nabla \mathbf{V} = -\nabla \wp + \frac{\nabla^2 \mathbf{V}}{\text{Re}} \quad (27)$$

$$\left[\frac{\partial \theta}{\partial \xi} + \mathbf{V} \cdot \nabla \theta \right] = \frac{1}{\text{Re} \cdot \text{Pr}} \left[\nabla^2 \theta + \frac{\nabla \Phi \cdot \nabla \theta}{\text{Le}} + \frac{\nabla \theta \cdot \nabla \theta}{\text{Le} \cdot N_{BT}} \right] \quad (28)$$

In Eqs. (25)–(28), the following dimensionless groups have been used:

$$\text{Re} \equiv \frac{\rho \bar{V} D}{\mu} \quad (\text{Reynolds number} = \text{inertial forces/viscous forces}) \quad (29)$$

$$\text{Sc} \equiv \frac{\mu}{\rho D_B} \quad (\text{Schmidt number} = \text{momentum diffusivity/Brownian diffusivity}) \quad (30)$$

$$N_{BT} \equiv \frac{\phi_b D_B T_b}{D_T \Delta T} = \frac{D_B T_b \rho}{\beta \mu \Delta T} \quad (= \text{Brownian diffusivity/thermophoretic diffusivity}) \quad (31)$$

$$\text{Pr} \equiv \frac{c \mu}{k} \quad (\text{Prandtl} = \text{momentum diffusivity/thermal diffusivity}) \quad (32)$$

$$\text{Le} \equiv \frac{k}{\rho_p c_p D_B \phi_b} \quad (\text{Lewis} = \text{thermal diffusivity/Brownian diffusivity}) \quad (33)$$

and Eq. (12) was used to simplify the expression for N_{BT} .

Let us consider the following conditions, which are typical of the alumina/water (copper/water) nanofluid studies published in

the literature: $\phi_b \sim 0.01$, $T_b \sim 300$ K, $\Delta T \sim 10$ K, $\mu \sim 1$ mPa s, $\rho \sim 1$ g/cm³, $k \sim 1$ W/m K, $\rho_p c_p \sim 3.1$ MJ/m³ (~ 3.4 MJ/m³ for copper nanoparticles), $\beta \sim 0.006$ (~ 0.0006 for copper nanoparticles), $D_B \sim 4 \times 10^{-11}$ m²/s (calculated from Eq. (8) for $d_p = 10$ nm). With these values, $Le \sim 8 \times 10^5$, $N_{BT} \sim 0.2$ and $Le \cdot N_{BT} \sim 1.6 \times 10^5$ for alumina nanoparticles, and $Le \sim 7 \times 10^5$, $N_{BT} \sim 2$ and $Le \cdot N_{BT} \sim 1.4 \times 10^6$ for copper nanoparticles. Because $Le \gg 1$ and $Le \cdot N_{BT} \gg 1$, Eq. (28) suggests that heat transfer associated with nanoparticle dispersion is negligible compared with heat conduction and convection. It follows that, in solving nanofluid heat transfer problems, the second and third terms on the right-hand side of Eq. (23) can be neglected, which makes the energy equation for a nanofluid formally identical to that of a pure fluid. This also implies that, contrary to what is commonly stated in the literature, the nanoparticles indeed affect convective heat transfer in nanofluids only via the thermophysical properties. However, it is important to recognize that a spatial distribution of the nanofluid thermophysical properties (especially viscosity and thermal conductivity) is expected, because such properties do depend on the nanoparticle distribution within the channel (i.e., they strongly depend on ϕ), as well as temperature. Therefore, the nanoparticle continuity equation (Eq. (17)) must always be solved in conjunction with the energy equation, which explains why pure-fluid correlations fail to reproduce the heat transfer data. The importance of coupling a nanoparticle continuity equation with the energy equation was also emphasized by Wen and Ding [23]. About Eq. (17), one should also notice that for $d_p = 1-100$ nm, N_{BT} ranges from 2 to 0.02 for alumina nanoparticles, and from 20 to 0.2 for copper nanoparticles, which suggests that thermophoretic effects are of the same order or more important than Brownian diffusion effects for alumina nanoparticles over the whole nanoparticle range, but in general less important for copper nanoparticles, especially at the lower end of the size range. This stems from the higher thermal conductivity of copper, which makes the thermal diffusion coefficient, D_T , lower (see Eq. (11)).

5 Nanofluid Heat Transfer in Turbulent Flow

In the presence of turbulence the conservation equations have to be modified to include the effect of the momentum, energy and particle eddy diffusivities. If we focus on the region near the wall of a round tube at steady state, Eqs. (17), (18), and (23) can be rewritten, respectively, as:

$$\frac{d}{dy} \left[(D_B + \varepsilon_p) \frac{d\phi}{dy} + \frac{D_T}{T} \frac{dT}{dy} \right] = 0 \quad (34)$$

$$\frac{d}{dy} \left[(\mu + \rho \varepsilon_M) \frac{dv}{dy} \right] = 0 \quad (35)$$

$$\frac{d}{dy} \left[(k + \rho c \varepsilon_H) \frac{dT}{dy} \right] = 0 \quad (36)$$

where y is the radial coordinate measuring the distance from the wall, v is the axial component of the velocity, and ε_M , ε_H , and ε_p are the momentum, energy, and particle eddy diffusivities, respectively. In Eqs. (34)–(36) the typical boundary-layer approximations for fully-developed internal flow were made, i.e.:

- curvature effects are negligible,
- axial transport terms are small compared with radial transport terms.

Integration of the above equations yields:

$$(D_B + \varepsilon_p) \frac{d\phi}{dy} + \frac{D_T}{T} \frac{dT}{dy} = 0 \quad (37)$$

$$(\mu + \rho \varepsilon_M) \frac{dv}{dy} = \tau_w \quad (38)$$

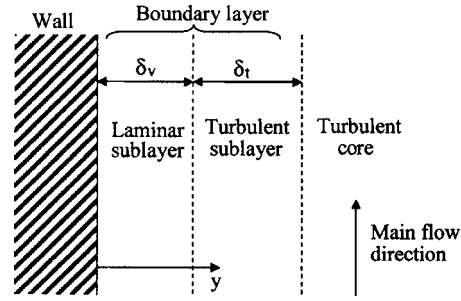


Fig. 2 Flow structure near the wall

$$(k + \rho c \varepsilon_H) \frac{dT}{dy} = -q_w \quad (39)$$

where τ_w is the wall shear stress, q_w is the wall heat flux, and also it was assumed that the net nanoparticle flux at the wall is zero, i.e., the nanoparticles neither deposit on nor are created at the wall.

Let us now assume that the wall layer is composed of two regions (Fig. 2): a laminar sublayer in which $D_B \gg \varepsilon_p$, $\mu \gg \varepsilon_M$, and $k \gg \varepsilon_H$, and a turbulent sublayer, in which $D_B \ll \varepsilon_p$, $\mu \ll \varepsilon_M$, $k \ll \varepsilon_H$. This approach was first proposed by Prandtl [24]. We can then solve Eqs. (37)–(39) with the boundary conditions: $\phi = \phi_b$, $v = \bar{V}$, $T = T_b$ at $y = \delta_v + \delta_t$, and $v = 0$, $T = T_w$ at $y = 0$, where the subscript b refers to bulk and w to wall.

5.1 Turbulent Sublayer. Because the nanoparticles move homogeneously with the turbulent eddies, it is reasonable to assume that $\varepsilon_p \sim \varepsilon_M$. In Sec. 2.8 we demonstrated that turbulent transport is the dominant nanoparticle transport mechanism in the presence of turbulent eddies, thus we can assume that $\phi \sim \phi_b$ in the turbulent sublayer. This conclusion is also corroborated by order-of-magnitude estimates of the eddy diffusivity and thermophoresis terms in Eq. (37). Integrating the momentum and energy equations in the turbulent sublayer, taking the ratio q_w/τ_w and using the Reynolds analogy ($\varepsilon_H \sim \varepsilon_M$), one gets:

$$\frac{q_w}{\tau_w} = c \frac{T_i - T_b}{\bar{V} - V_i} \quad (40)$$

where c is the nanofluid specific heat, and T_i and V_i are temperature and velocity at the interface of the laminar and turbulent sublayers.

5.2 Laminar Sublayer. Using Eq. (39), the temperature gradient can be eliminated from Eq. (37). Thus:

$$D_B \frac{d\phi}{dy} - \frac{D_T q_w}{T k} = 0 \quad (41)$$

Equation (41) suggests that for a heated fluid ($q_w > 0$), the effect of thermophoresis is to reduce ϕ in the laminar sublayer. Using the definition of D_T (Eq. (12)) and neglecting the temperature dependencies, Eq. (41) can be readily integrated to yield:

$$\phi = \phi_b e^{(1/N_{BT})(1-y/\delta_v)} \quad (42)$$

with the following definition of the N_{BT} number:

$$N_{BT} \equiv \frac{D_B T_b \rho}{\beta \mu \left[\frac{q_w \delta_v}{k} \right]} \quad (43)$$

Equation (42) is plotted in Fig. 3 with N_{BT} as a parameter. Thus,

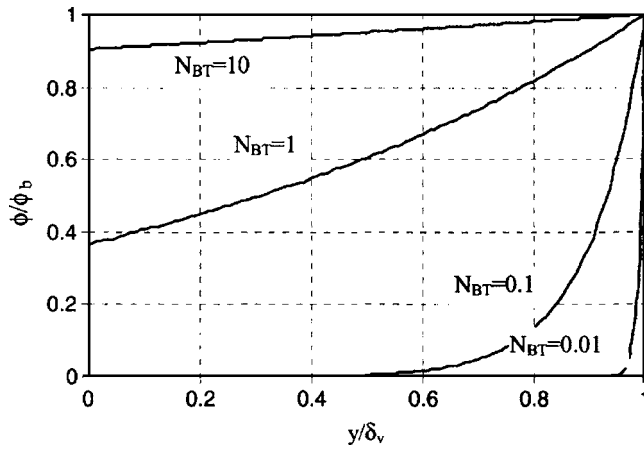


Fig. 3 Variation of the nanoparticle volumetric fraction within the laminar sublayer

for relatively low values of N_{BT} (e.g., large nanoparticles, high heat fluxes), the nanoparticle volumetric fraction near the wall can be significantly lower than in the bulk.

The average nanoparticle volumetric fraction in the laminar sublayer, $\phi_v (\equiv 1/\delta_v \int_0^{\delta_v} \phi dy)$, can be obtained from Eq. (42):

$$\frac{\phi_v}{\phi_b} = N_{BT}(1 - e^{-(1/N_{BT})}) \quad (44)$$

The momentum and energy equations in the laminar sublayer can be readily integrated to yield:

$$\frac{q_w}{\tau_w} \approx \frac{k_v}{\mu_v} \frac{T_w - T_i}{V_i} \quad (45)$$

where k_v and μ_v are the nanofluid thermal conductivity and viscosity corresponding to ϕ_v , respectively. Equation (45) is approximate because in reality both thermal conductivity and viscosity vary within the laminar sublayer, due to the variation of ϕ .

5.3 Heat Transfer Coefficient. Eliminating T_i from Eqs. (40) and (45), and introducing the definition of heat transfer coefficient, $h \equiv q_w/(T_w - T_b)$, one gets:

$$h = \frac{\tau_w}{\frac{\mu_v V_i}{k_v} + \frac{\bar{V} - V_i}{c}} \quad (46)$$

Using the definition of friction factor (Eq. (2)) and rearranging the terms, Eq. (46) can be rewritten as:

$$Nu_b = \frac{\frac{f}{8} Re_b Pr_b}{1 + \frac{V_i}{\bar{V}}(Pr_v - 1)} \quad (47)$$

where the subscript b indicates the use of properties corresponding to ϕ_b , while the subscript v indicates the use of properties corresponding to ϕ_v . In the derivation of Eq. (47), it was assumed that the ratio $c_b/c_v \sim 1$, which is true at low nanoparticle concentrations. Assuming that velocity varies linearly within the laminar sublayer, the ratio V_i/\bar{V} becomes equal to $\delta_v^+ \sqrt{f}/8$, where δ_v^+ is the thickness of the laminar sublayer in dimensionless units. In Prandtl's original derivation δ_v^+ was assumed to be 8.7 [24]. Thus, Eq. (47) becomes:

$$Nu_b = \frac{\frac{f}{8} Re_b Pr_b}{1 + 8.7 \sqrt{\frac{f}{8}} (Pr_v - 1)} \quad (48)$$

which is the well-known Prandtl correlation for a pure fluid, except that the Prandtl number at the denominator is not evaluated with the bulk properties, but with the laminar sublayer properties. Equation (48) can be used to explain the convective heat transfer enhancements in nanofluids and their deviation from a pure-fluid correlation, as follows. For a heated fluid ($T_w > T_b$), one has $Pr_v < Pr_b$ because of two effects:

- (1) $\mu_v < \mu_b$ and $k_v > k_b$. While this effect is present also in pure liquids, it is more pronounced for nanofluids, because viscosity and especially thermal conductivity are much stronger functions of temperature for nanofluids than for pure liquids [5,25].
- (2) $\phi_v < \phi_b$. Both μ and k increase with ϕ . However, the μ dependence on ϕ is stronger [7,10,13] (see also Appendix A). Therefore, the net effect of a ϕ reduction is a reduction of the Prandtl number in the boundary layer.

With reference to Eq. (48), the ratio

$$\frac{1 + 8.7 \sqrt{\frac{f}{8}} (Pr_b - 1)}{1 + 8.7 \sqrt{\frac{f}{8}} (Pr_v - 1)}$$

is a measurement of the nanofluid heat transfer deviation from the predictions of a pure-fluid correlation. Clearly, if $Pr_v < Pr_b$, this ratio is greater than unity. We will demonstrate this conclusion quantitatively in Sec. 5.4. In the meantime, it should be noted that the Prandtl correlation is known to be valid only for Prandtl numbers around unity, which is consistent with the use of the Reynolds analogy in its derivation. However, nanofluids can have Prandtl numbers significantly higher than unity (> 10). A correlation that has a physical basis similar to Eq. (48), but is valid within a much broader range of the parameters ($0.5 < Pr < 2000$, $2300 < Re < 5 \times 10^6$) is that of Gnielinski [26], recommended in most heat transfer handbooks [27,28]:

$$Nu = \frac{\frac{f}{8} (Re - 1000) Pr}{1 + 12.7 \sqrt{\frac{f}{8}} (Pr^{2/3} - 1)} \quad (49)$$

Thus, we adopt the following correlation structure for nanofluid heat transfer in turbulent flow:

$$Nu_b = \frac{\frac{f}{8} (Re_b - 1000) Pr_b}{1 + \delta_v^+ \sqrt{\frac{f}{8}} (Pr_v^{2/3} - 1)} \quad (50)$$

where δ_v^+ is now a constant to be determined empirically. Once δ_v^+ is given, Eq. (50) can be used as follows:

- (i) Calculate the friction factor (and the shear stress) from a traditional friction factor correlation for turbulent flow, e.g., McAdams' or Karman-Nikuradse's.
- (ii) Guess a value for ϕ_v .
- (iii) Calculate the thickness of the laminar sublayer, $\delta_v \sim \delta_v^+ (\mu_v/\rho)/\sqrt{\tau_w/\rho}$

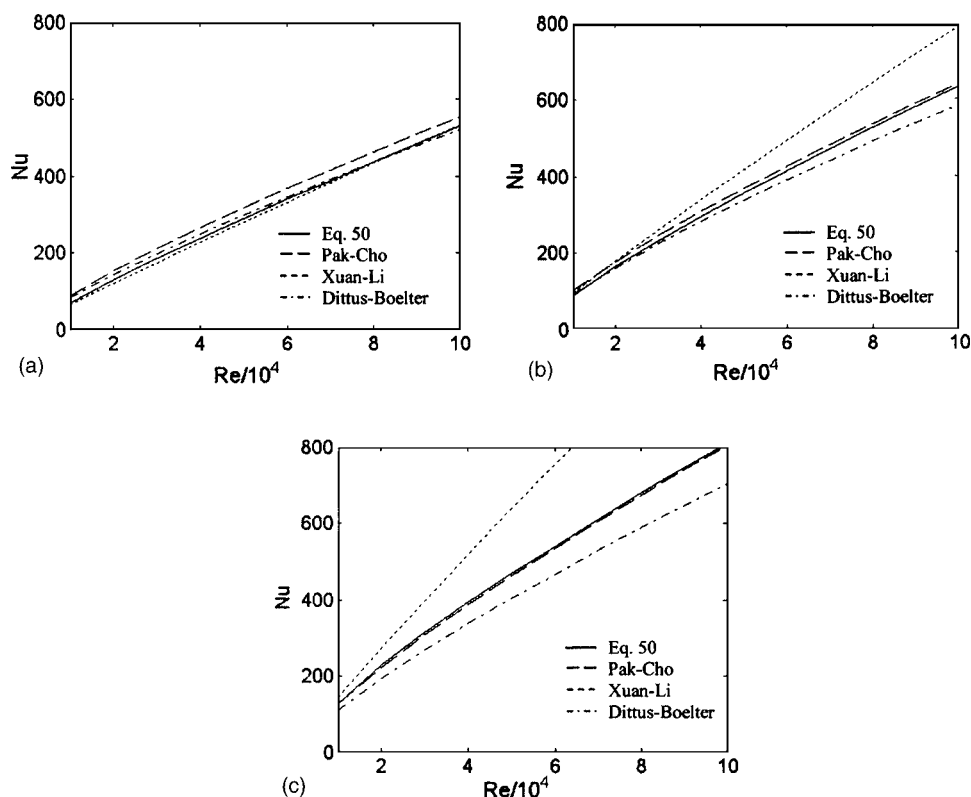


Fig. 4 Heat transfer in alumina/water nanofluids: (a) $\phi_b=0$; (b) $\phi_b=0.01$; and (c) $\phi_b=0.03$

- (iv) Calculate N_{BT} from Eq. (43).
- (v) Calculate ϕ_v from Eq. (44).
- (vi) Repeat steps (iii)–(v) until ϕ_v converges.
- (vii) Calculate Pr_v for $\phi = \phi_v$ and $T = (T_w + T_b)/2$.
- (viii) Calculate the heat transfer coefficient from Eq. (50).
- (ix) Calculate T_w from Newton's law of cooling.

For a problem with fixed q_w , these steps need to be repeated until T_w converges.

5.4 Performance of the New Heat Transfer Correlation for Nanofluids. The two most comprehensive sets of data for single-phase turbulent heat transfer in nanofluids are those of Pak and Cho [7] and Xuan and Li [13]. These databases are reproduced well by the following two correlations, respectively:

$$Nu_b = 0.021 \cdot Re_b^{0.8} Pr_b^{0.5} \quad (51)$$

$$Nu_b = 0.0059 \cdot \left[1 + 7.6286 \cdot \phi_b^{0.6886} \left(Re_b Pr_b \frac{d_p}{D} \right)^{0.001} \right] Re_b^{0.9238} Pr_b^{0.4} \quad (52)$$

To test the performance of our correlation, we use Pak and Cho's data [7], represented by Eq. (51). The Nusselt number for their water-based nanofluids with three different loadings of alumina nanoparticles (13 nm) is plotted in Fig. 4, as a function of the Reynolds number. There are four curves in this figure: the data (Pak and Cho's correlation, Eq. (51)) and three different correlations, i.e., Eq. (50), Xuan-Li (Eq. (52)) and Dittus-Boelter. It can be seen that all correlations are in good agreement for $\phi_b=0$ (pure water), while for $\phi_b>0$ Dittus-Boelter and Xuan-Li tend to significantly under- and overestimate the Nusselt number, respectively. On the other hand, Eq. (50) is in good agreement with the data. The Nusselt number for water-based nanofluids with titania nanoparticles (27 nm) is plotted in Fig. 5, and the same qualitative behavior is seen. Thus, we conclude that our correlation is reason-

able. In generating the curves of Figs. 4 and 5, the following assumptions were made: $\delta_v^* \sim 15.5$, $T_b=20^\circ\text{C}$, and $q_w=50\text{ kW/m}^2$. The values of the bulk temperature and wall heat flux are consistent with Pak and Cho's experimental conditions.

Unfortunately, Xuan and Li's copper/water nanofluid data cannot be used to test the performance of our correlation further, because the size of their nanoparticles is not reported in [13]. Moreover, Xuan and Li did not measure the temperature dependence of the thermophysical properties, but this dependence is expected to be very strong especially for copper and copper-oxide nanoparticles [5,25]. From the rated power of Xuan and Li's power supply and the values they report for the heat transfer coefficient, we estimate that in their experiments the temperature drop across the boundary layer could have been as high as 15°C , which according to Jang and Choi's model [25] could result in a thermal conductivity enhancement of about 30% at the wall. The combination of this effect and the effect of reduced viscosity in the boundary layer would explain the very significant deviation from the Dittus-Boelter correlation in their experiments.

Note that Pak and Cho's correlation is completely empirical, while Xuan and Li's is based on the dispersion model, but needed five empirical coefficients to match the data. In comparison, Eq. (50) is physically based, has only one empirical coefficient (i.e., δ_v^*), and has the following two desirable features:

- (1) It accounts for the effect of thermophoresis and temperature on the thermophysical properties of the nanofluid.
- (2) It reduces to a reliable pure-fluid correlation for $\phi_b=0$.

Note that the new correlation implies that the heat transfer coefficient in nanofluids depends on the wall heat flux (via the N_{BT} number and the dependence of Pr_v on T_w). Moreover, if our explanation of the heat transfer enhancement in flowing nanofluids is correct, the Nusselt number measured in a cooling experiment ($T_w < T_b$) should be lower than that predicted by a pure-fluid correlation. To test our conclusions, we will soon measure the heat

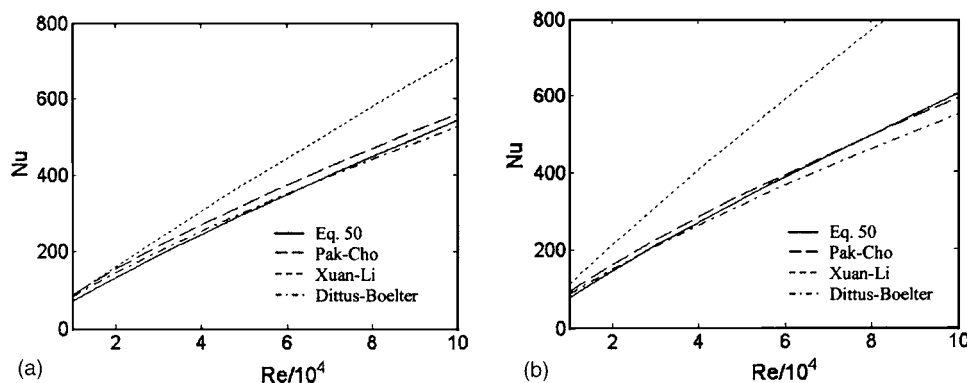


Fig. 5 Heat transfer in titania/water nanofluids: (a) $\phi_b=0.01$; and (b) $\phi_b=0.03$

transfer coefficient for various nanofluids and for both heating and cooling situations, in a flow loop under construction at MIT.

6 Conclusions

The objective of this work was to develop an explanation for the abnormal convective heat transfer enhancement observed in nanofluids. The major findings contained in this paper are as follows:

- Brownian diffusion and thermophoresis have been identified as the two most important nanoparticle/base-fluid slip mechanisms.
- A general two-component nonhomogeneous equilibrium model for transport phenomena in nanofluids has been developed, incorporating the effects of Brownian diffusion and thermophoresis.
- Order-of-magnitude estimates for the various terms of the energy equation suggest that energy transfer by nanoparticle dispersion is negligible, contrary to what is commonly stated in the literature.
- Application of our model to nanofluid heat transfer in the turbulent regime has highlighted the role of nanoparticle diffusion effects in the boundary layer near the wall. In particular, this paper represents the first attempt to describe the effect of thermophoresis in nanofluids mechanistically.
- Convective heat transfer enhancement can be explained mainly with a reduction of viscosity within and consequent thinning of the laminar sublayer.
- We have developed a new correlation structure (based on this explanation of the heat transfer enhancement) that can reproduce published nanofluid heat transfer data reasonably well. The correlation is given by Eq. (50).

Nomenclature

- c = nanofluid specific heat (J/kg K)
 c_{bf} = base fluid specific heat (J/kg K)
 c_p = Nanoparticle specific heat (J/kg K)
 C = friction factor correlation coefficient
 d_p = nanoparticle diameter (m)
 D = channel diameter (m)
 D_B = Brownian diffusion coefficient (m^2/s)
 D_T = thermal diffusion coefficient (m^2/s)
 f = friction factor
 g = acceleration of gravity (m/s^2)
 h = heat transfer coefficient ($\text{W}/\text{m}^2 \text{K}$)
 h_p = nanoparticle specific enthalpy (J/kg)
 \mathbf{j}_p = total nanoparticle mass flux ($\text{kg}/\text{m}^2 \text{s}$)

- $\mathbf{j}_{p,B}$ = nanoparticle mass flux due to Brownian diffusion ($\text{kg}/\text{m}^2 \text{s}$)
 $\mathbf{j}_{p,T}$ = nanoparticle mass flux to thermophoresis ($\text{kg}/\text{m}^2 \text{s}$)
 k = nanofluid thermal conductivity ($\text{W}/\text{m K}$)
 k_B = Boltzmann constant (J/K)
 k_p = nanoparticle thermal conductivity ($\text{W}/\text{m K}$)
 Kn = Knudsen number
 ℓ_o = large eddy length scale (m)
 ℓ_s = small eddy length scale (m)
 Le = Lewis number
 n = friction factor correlation exponent
 N_{BT} = ratio of Brownian and thermophoretic diffusivities
 P = pressure (Pa)
 φ = dimensionless pressure
 Pe_r = rotational Peclet number
 Pr = prandtl number
 \mathbf{q} = energy flux (W/m^2)
 q_w = heat flux at the wall (W/m^2)
 \mathbf{r} = position vector (m)
 \mathbf{R} = dimensionless position vector
 Re = Reynolds number
 S_p = nanoparticle stopping distance (m)
 Sc = Schmidt number
 t = time (s)
 T = nanofluid temperature (K)
 T_i = temperature at the laminar/turbulent sublayer interface (K)
 \mathbf{v} = nanofluid velocity (m/s)
 \mathbf{V} = dimensionless nanofluid velocity
 \bar{V} = mean axial velocity (m/s)
 V_e = nanoparticle slip velocity due to turbulent eddies (m/s)
 V_{eo} = turbulent eddy velocity (m/s)
 V_g = settling velocity due to gravity (m/s)
 V_T = thermophoretic velocity (m/s)
 V_i = velocity at the laminar/turbulent sublayer interface (m/s)
 y = radial coordinate (m)

Greek

- α = thermal diffusivity (m^2/s)
 β = thermophoretic coefficient
 δ_v = thickness of the laminar sublayer (m)
 δ_v^+ = dimensionless thickness of the laminar sublayer
 δ_t = thickness of the turbulent sublayer (m)
 ΔT = film temperature drop (K)
 ε_H = eddy diffusivity for heat (m^2/s)

ε_M = eddy diffusivity for momentum (m^2/s)
 ε_p = eddy diffusivity for nanoparticles (m^2/s)
 ϕ = nanoparticle volumetric fraction
 Φ = normalized nanoparticle volumetric fraction
 λ = water molecules mean free path (m)
 μ = viscosity (Pa s)
 θ = dimensionless temperature
 ρ = nanofluid density (kg/m^3)
 ρ_{bf} = base fluid density (kg/m^3)
 ρ_p = nanoparticle density (kg/m^3)
 τ = stress tensor (Pa)
 τ_o = large eddy time scale (s)
 τ_p = nanoparticle relaxation time (s)
 τ_s = small eddy time scale (s)
 τ_w = shear stress at the wall (Pa)
 ξ = dimensionless time

Subscript

b = bulk
 bf = base fluid
 i = interface
 p = nanoparticle
 v = laminar sublayer
 w = wall

Appendix A: Thermophysical Properties of Nanofluids

A.1 Density. The nanofluid density, ρ , is the average of the nanoparticle and base fluid densities:

$$\rho = \phi \rho_p + (1 - \phi) \rho_{\text{bf}} \quad (\text{A1})$$

where the subscripts p and bf refer to the nanoparticles and base fluid, respectively.

A.2 Specific Heat. Assuming that the nanoparticles and the base fluid are in thermal equilibrium, the nanofluid specific heat, c ($\text{J}/\text{kg K}$), should be calculated as follows:

$$c = \frac{\phi \rho_p c_p + (1 - \phi) \rho_{\text{bf}} c_{\text{bf}}}{\rho} \quad (\text{A2})$$

However, some authors [7,10,29] prefer to use a simpler (albeit incorrect) approach:

$$c = \phi c_p + (1 - \phi) c_{\text{bf}} \quad (\text{A3})$$

Note that Pak and Cho correlated their data with Eq. (A3). Therefore, to be consistent with them, we also made use of this equation in generating the curves of Fig. 4 and 5 in Sec. 5.4.

A.2.1 Viscosity. General and accurate models for prediction of the viscosity of a nanofluid, μ , are not available at this time. However, the room-temperature viscosity measured by Pak and Cho [7] can be correlated by means of the following equations:

$$\mu = \mu_{\text{bf}}(1 + 39.11\phi + 533.9\phi^2) \quad (\text{alumina nanoparticles}) \quad (\text{A4})$$

$$\mu = \mu_{\text{bf}}(1 + 5.45\phi + 108.2\phi^2) \quad (\text{titania nanoparticles}) \quad (\text{A5})$$

A.2.2 Thermal Conductivity. Models for prediction of the nanofluid thermal conductivity, k , are not available either. The thermal conductivity data used by Pak and Cho [7] can be correlated as follows:

$$k = k_{\text{bf}}(1 + 7.47\phi) \quad (\text{alumina nanoparticles}) \quad (\text{A6})$$

$$k = k_{\text{bf}}(1 + 2.92\phi - 11.99\phi^2) \quad (\text{titania nanoparticles}) \quad (\text{A7})$$

Appendix B: Turbulent Scales

An important concept in turbulent flow is that of energy cascade. That is, the kinetic energy fed to turbulence goes primarily into large eddies, from which it is transferred to smaller eddies, then to still smaller ones, until is dissipated (converted to heat) by viscous forces. Therefore, within a turbulent flow there exists a spectrum of turbulent eddies. The large eddies have length and time scales comparable with those of the flow itself [30]. For example, with reference to turbulent flow inside a round tube of diameter D and mean velocity \bar{V} , the length scale of the large eddies, ℓ_o , would be of the order of D , and their time scale, τ_o , of the order of D/\bar{V} . On the other hand, the smallest (dissipative) eddies have a length scale, ℓ_s , and a time scale, τ_s , that can be found by the Kolmogorov's scaling laws [30]:

$$\ell_s/\ell_o \sim \text{Re}^{-3/4} \quad (\text{B1})$$

$$\tau_s/\tau_o \sim \text{Re}^{-1/2} \quad (\text{B2})$$

Assuming typical flow conditions ($\text{Re} \sim 50,000$, $D \sim 1$ cm, $\bar{V} \sim 5$ m/s), one gets $\ell_o \sim 1$ cm, $\ell_s \sim 3$ μm , $\tau_o \sim 2$ ms, and $\tau_s \sim 10$ μs . Therefore, the length and time scales of the turbulent eddies are much larger than the nanoparticle size and relaxation time, respectively (see Sec. 2.1). This means that the nanoparticles are transported by the turbulent eddies very effectively.

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