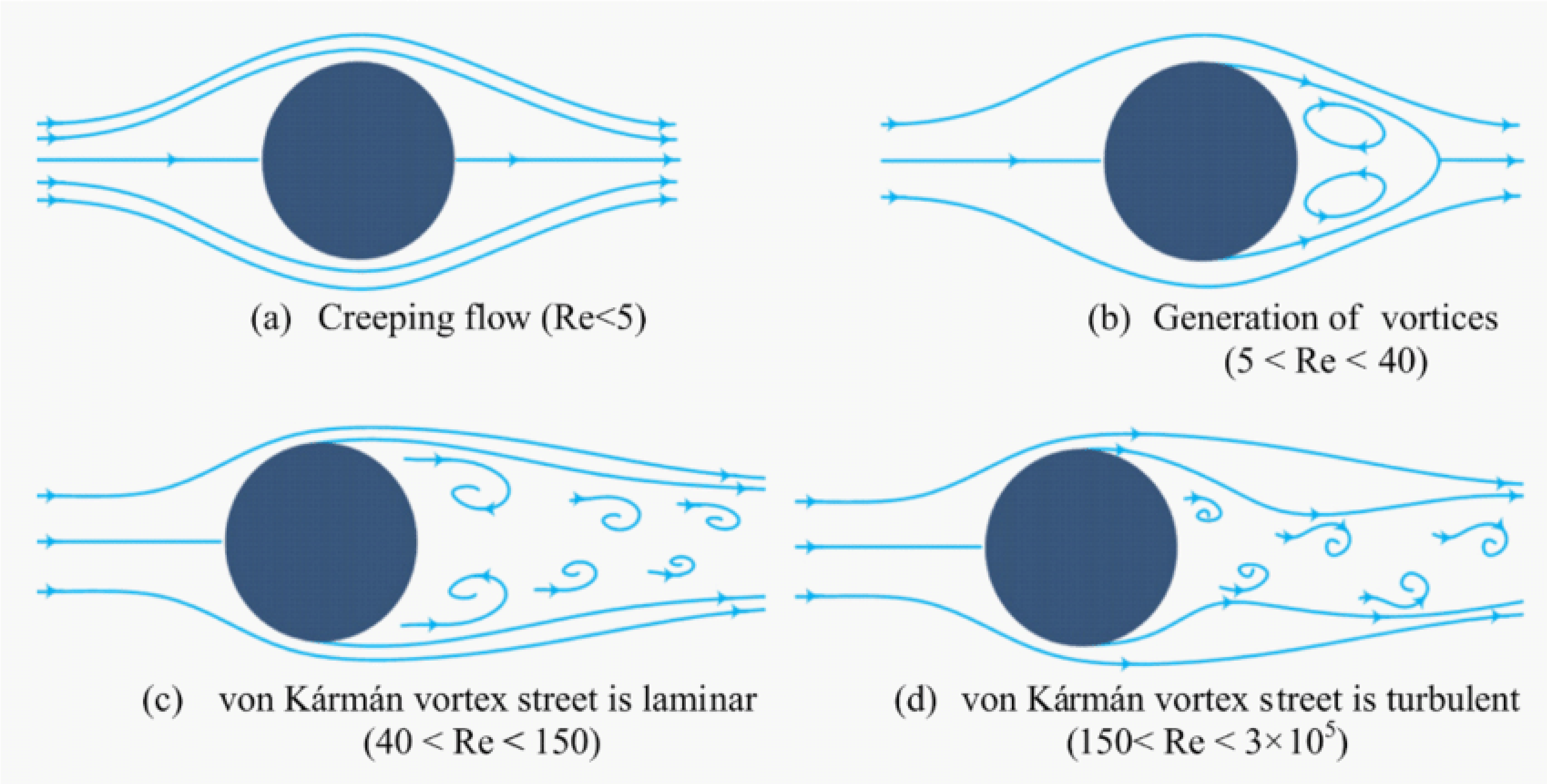
Overview

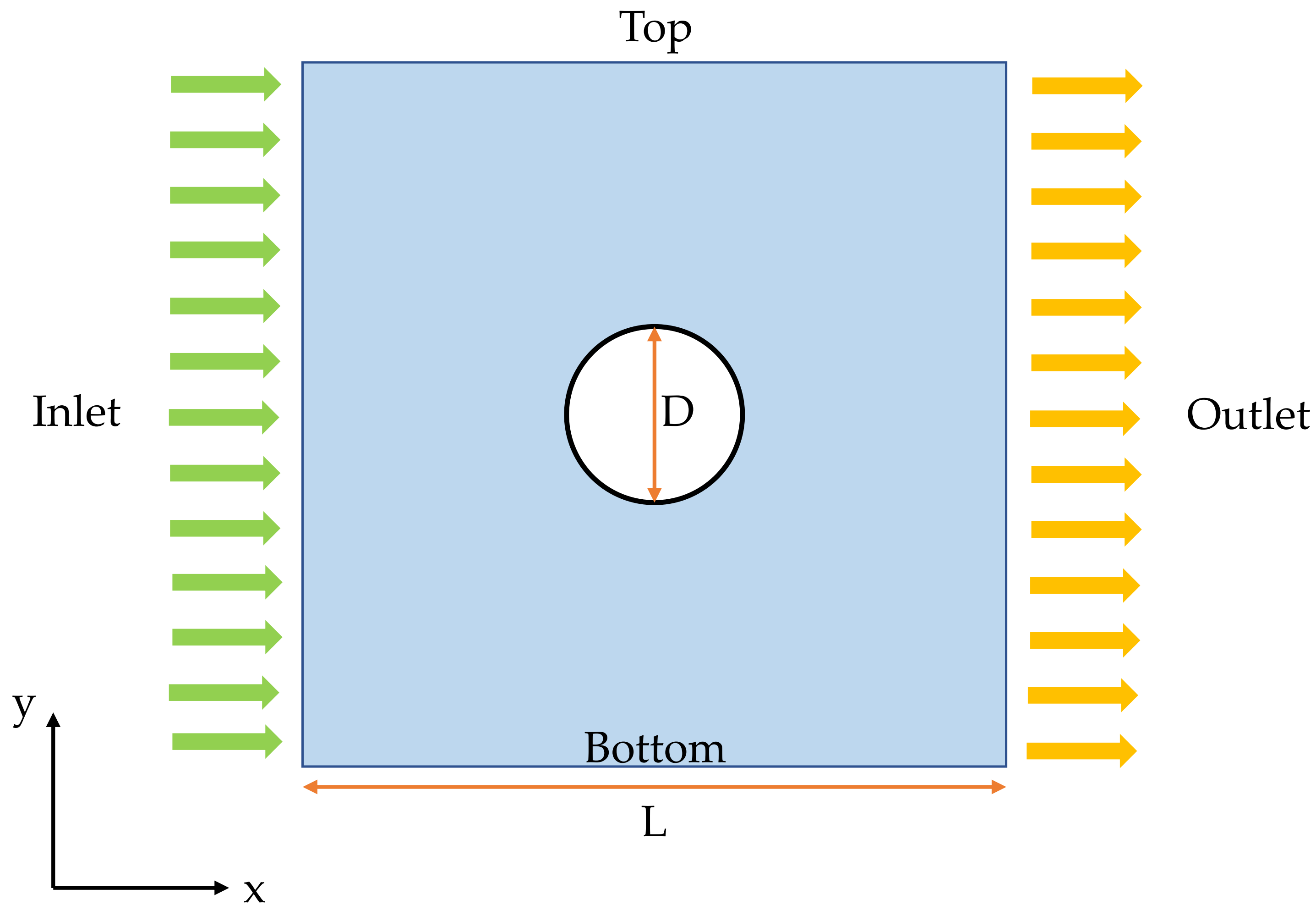
1. Compressible
2. Energy equation

Streamlines patterns of flow over a cylinder



Reynolds number for external flow $Re = \dfrac{\rho U_{\infty} D}{\mu}$

Example 4: External flow over Cylinder



Dimension and Properties

Dimension

Name	Value
L	$100D$
D	$1\ m$

Air properties

Name	Value
ρ	1.161
μ	18.57×10^{-6}
U_∞	From Re

Assumptions and governing equations

Assumptions: Laminar, incompressible, unsteady, ignore gravity

Mass conservation

$$\nabla . \vec{V} = 0$$

Momentum conservation

$$\rho \frac{\delta \vec{V}}{\delta t} + \rho \nabla . \left(\vec{V} \times \vec{V} \right) = -\nabla P + \nabla . \left(\mu \nabla \vec{V} \right)$$

Symbols

$\vec{V}$: Velocity vector $(\frac{m}{s})$

P: Pressure (Pa)

ρ: Density $(\frac{kg}{m^3})$

μ: Dynamic viscosity $(\frac{kg}{m.s})$

t: Time (s)

Boundary conditions

Abbreviations

BC: Boundary conditions

B.Cs of Velocity

	Inlet	Outlet	Top and bottom	Cylinder
Type	Uniform	Hydrodynamically developed	Symmetry	No slip
Value	$\vec{V}.\hat{n} = U_\infty$	$\nabla\vec{V}.\hat{n} = 0$	$\vec{V}.n_y = 0$	$\vec{V}$=0

B.Cs of Pressure

	Inlet	Outlet	Top and bottom	Cylinder
Type	Developed	Atmosphere	Symmetry	Zero gradient
Value	$\nabla P.\hat{n} = 0$	$P = 0$	$\nabla P.n_y = 0$	$\nabla P.\hat{n} = 0$

Assumptions and governing equations

Assumptions: Laminar, compressible, unsteady

Mass conservation

$$\frac{d\rho}{dt} + \nabla.\left(\rho\vec{V}\right) = 0$$

Momentum conservation

$$\frac{D\left(\rho\vec{V}\right)}{Dt} = \nabla.\vec{\sigma}$$

$$\vec{\sigma} = -p\vec{I} + \vec{\tau} = -p\vec{I} + \mu\left[\left(\nabla.\vec{V}\right) + \left(\nabla.\vec{V}\right)^{T}\right] - \frac{2\mu}{3}\left(\nabla.\vec{V}\right)\vec{I}$$

Energy conservation

$$\frac{D(\rho e)}{Dt} + \nabla.\left(\rho\vec{V}\vec{V} + p\vec{I}\right) = \nabla.(k\nabla T)$$

$$e = \frac{1}{2}\left|\vec{V}\right|^{2} + u$$

Symbols

$\vec{V}$: Velocity vector ($\frac{m}{s}$)

$\vec{\sigma}$: Stress Tensor ()

P: Pressure (Pa)

e: Total Energy ()

u: Internal Energy ()

T: Temperature (K)

ρ: Density ($\frac{kg}{m^3}$)

μ: Dynamic viscosity ($\frac{kg}{m.s}$)

k: Thermal conductivity ()

t: Time (s)

Boundary conditions

Abbreviations

BC: Boundary conditions

B.Cs of Velocity

	Inlet	Outlet	Top and bottom	Cylinder
Type	Uniform	Hydrodynamically developed	Symmetry	No slip
Value	$\vec{V}.\hat{n} = U_\infty$	$\nabla\vec{V}.\hat{n} = 0$	$\vec{V}.n_y = 0$	$\vec{V}$=0

B.Cs of Pressure

	Inlet	Outlet	Top and bottom	Cylinder
Type	Developed	Atmosphere	Symmetry	Zero gradient
Value	$\nabla P.\hat{n} = 0$	$P = 0$	$\nabla P.n_y = 0$	$\nabla P.\hat{n} = 0$

B.Cs of Energy

	Inlet	Outlet	Top and bottom	Cylinder
Type	Developed	Atmosphere	Symmetry	Uniform
Value	$T = 300\ K$	$\nabla T.\hat{n} = 0$	$\nabla P.n_y = 0$	$T = 310\ K$