# Interpretations of Quantum Mechanics

## Overview

1. Introduction
2. The Copenhagen Interpretation
3. The Pilot-wave theory
4. The many-worlds Interpretation
5. Spontaneous Collapse theory
6. Bell's Theorem

literature :

* Travis Norsen, Foundations of Quantum Mechanics
(an exploration of the physical meaning of QM)

other books :

* John Bell, The speakable and unspeakable in QM

* Jean Bricmont, Making sense of QM

* Heisenberg, Philosophy of physics
Der Teil und das Ganze

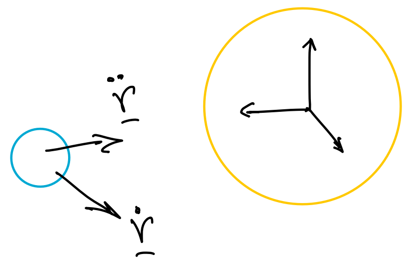
* Zeilinger, The message of the quantum

# 1. Introduction

## 1.1. Historical remarks

- Earth not a special place!

  → Galilean relativity



- $\underline{F}(\underline{r}) = m\ddot{\underline{r}}$ Newton

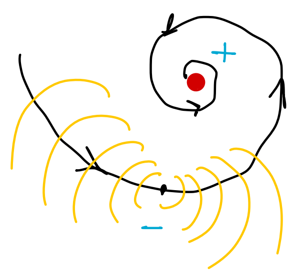
  Force

having $\left\{ \begin{array}{l} \ddot{\underline{r}} \\ \underline{r}(0) \\ \dot{\underline{r}}(0) \end{array} \right.$ we can calculate $\left\{ \begin{array}{l} \text{position } \underline{r}(t) \\ \text{momentum } \underline{p} = m\dot{\underline{r}} \end{array} \right.$

- Discovery of atoms

prediction of classical physics (mechanics & ED):

electron in atom emits light while accelerating in nucleus's field.

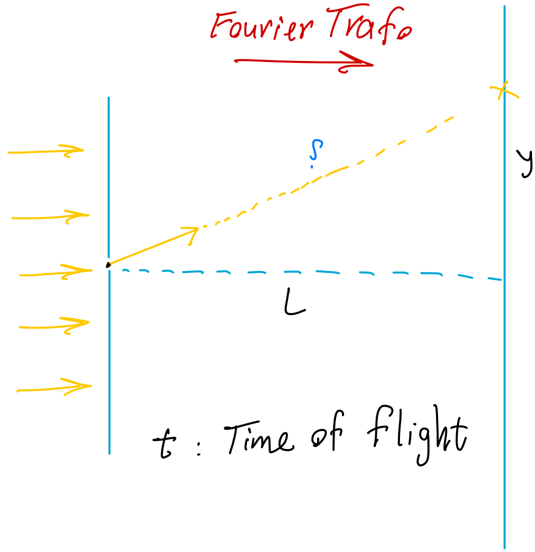


→ Atoms are not stable!

## * Heisenberg's uncertainty Principle

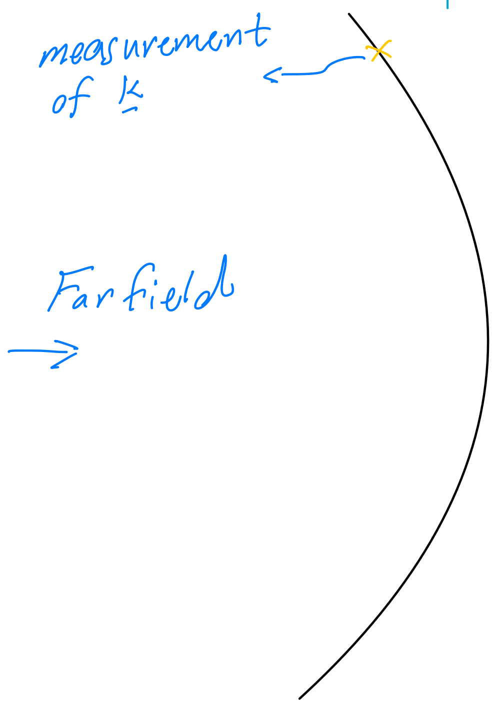
Simultaneous position and momentum of a particle cannot be adequately measured. How?

$$\Delta x \, \Delta p \geq \frac{h}{2\pi}$$




- how one measures momentum?

- principally momentum is the Fourier-transform of position.




# Schrödinger's wave mechanics

$$i\hbar\,\partial_t \Psi(\underline{r},t) = \left(-\frac{\hbar^2}{2m}\Delta + V(x)\right)\Psi(\underline{r},t)$$

electron as a standing wave in the nuclei potential:

$$V(x) = k\frac{q}{r}$$

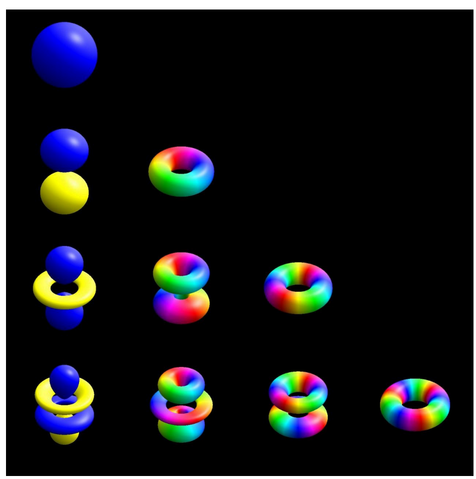
motion without trajectory (Path)!

probability of finding the electron at $\underline{r}, t$

$$P(\underline{r},t) = |\Psi(\underline{r},t)|^2 = \Psi(\underline{r},t)\Psi^*(\underline{r},t)$$

* quantised Angular Momentum. $\hat{L} = \hat{r} \times \hat{p}$

- electronic Orbitals instead of orbits.



# Einstein-Heisenberg debate over the path of the particle.

* Der Teil und Das Ganze W. H

if we can see the path of the electrons in cloudchamber, why can't they have paths inside atoms as well?

Einstein to Heisenberg
Berlin, 1926

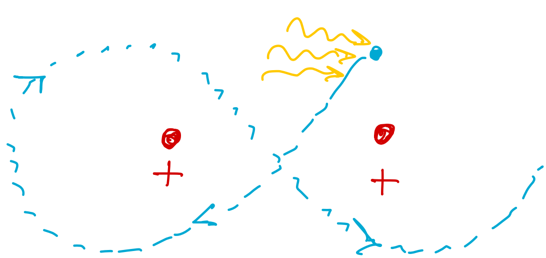
Exercise: think about a possible answer!
more to this in Chapter 2

debatable:

* Can one force the electron in e.g. $He^+$ to move on a classical path?

Yes! using ultra-short pulsed laser.

Thomson



# Schrödinger-Bohr debate over Quantum Jumps

S. How is the electron's behaviour during a transition between one quantised state to another in atom?

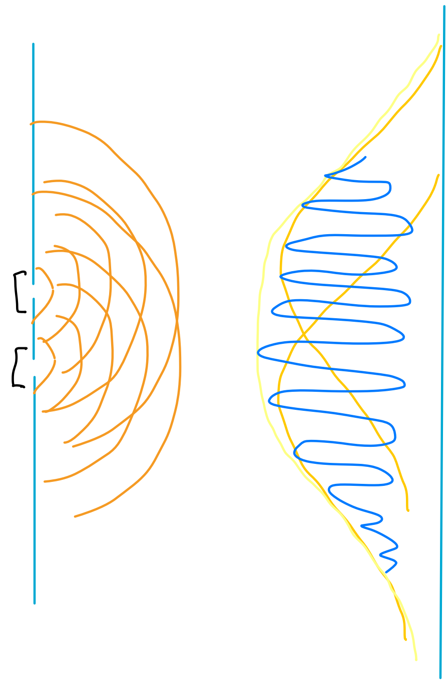
"If one has to stick to this damned quantum jumping, then I regret having ever been involved in QM!" Schrödinger to Bohr
München Seminar 1926

Exercise: think about a possible answer!

Einstein-Bohr debate in Chapter 2

Copenhagen Interpretation

# Double Slit experiment



- wave fronts exert the slits in phase.

→ Interference happens

- Wave-particle duality:

There is a particle associated with the wave.

- particles go throw both slits.

- maybe particles scatter or interact ?

- send particles one by one : Interference keeps happening

→ Interference isn't a many particle effect.

→ Each particle interferes with itself!

Possible interpretation:

Particle seems to go through both Slits at once!

Introducing a new concept:

$\boxed{\text{Particle turns into a superposition of going through both slits.}}$

* Superposition: An entirely new kind of existence?

Nobody can observe a particle exiting both slits simultaneously.

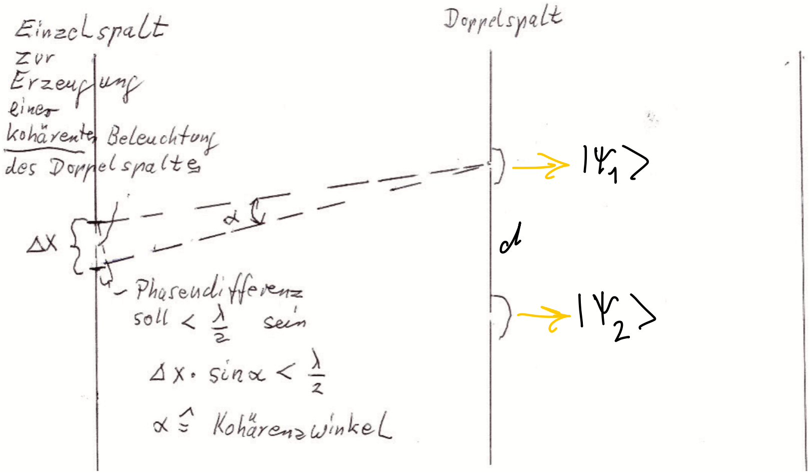
- but mathematically it's not a weird concept:

$\boxed{\text{If } \psi_i \text{ is a possible state of a system, e.g. a particle then any superposition } \psi = \sum c_i \psi_i \text{ also possible.}}$

- when the which-way information not available in the double-slit setup:

→ Superposition of both ways ⟶ Interference

# – Superposition in Double-Slit experiment:




Skript Atomphysik 1 Dörner 2021

$$|\Psi\rangle = |\Psi_1\rangle + |\Psi_2\rangle$$ in Hilbert space

$$\Psi_{\underline{k}}(\underline{r},t) = \left( A_1(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} + \frac{d}{2} e_y)} + A_2(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} - \frac{d}{2} e_y)} \right) \cdot e^{-i\omega t}$$

– Calculate $|\Psi(\underline{r},t)|^2 = \Psi(\underline{r})\,\Psi^*(\underline{r})$

on the detection screen: Intensity of particles on the screen.

$$= \left( A_1(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} + \frac{d}{2})} + A_2(\underline{r})\, e^{i\underline{k}\cdot(\underline{r} - \frac{d}{2})} \right)\left( A_1^*(\underline{r})\, e^{-i\underline{k}\cdot(\underline{r} + \frac{d}{2})} + A_2^*(\underline{r})\, e^{-i\underline{k}\cdot(\underline{r} - \frac{d}{2})} \right)$$

$$= |A_1(\underline{r})|^2 + |A_2(\underline{r})|^2 + \quad ,,, \quad \text{Exercise} \quad ,,,$$

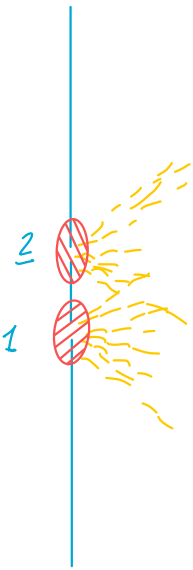
* How to know through which slit (which way?) did the particles actually go?

- Make the which-way information principally available (leakage of information) by relating the slits differently to an intrinsic property of the particle.

example : Putting perpendicular polarisers on slits.

:= putting which-way mark on particle.

result: No Interference!

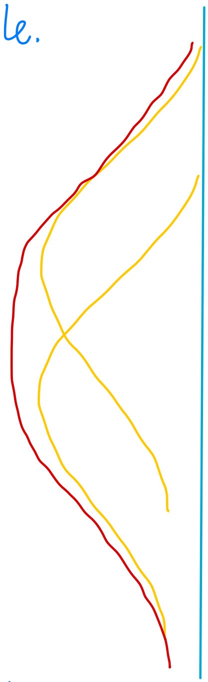



passed particle

Polarisation determined

if 45° : went through (1)

if 135° : went through (2)



We don't necessarily need to know the polarisation, it just needs to be principally possible to know in the new setup

* Did we just measure the which-way slit?

- Yes, $|\psi\rangle = |\psi_1\rangle + |\psi_2\rangle$ reduced to $|\psi_1\rangle$ or $|\psi_2\rangle$

Particles go through one of the slits each time: no superposition?

- Measurement in QM:

$|\psi\rangle = \sum c_i \psi_i$

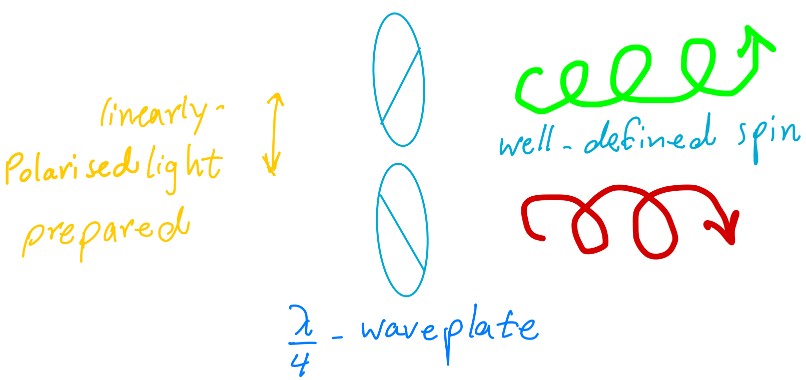
Collapse or reduction of the state (: wave-function) to one of the eigenstates. $|\psi\rangle \to |\psi_i\rangle$

with probability: $P_i = |c_i|^2$

- not necessarily: Explaining Measurement in terms of Entangled Superposition of the Particle-Apparatus

- more about Entanglement next page.

* Another example of which-way marker:




which-way mark can be put with a coherent change in the state.

# * quantum entanglement (crash cource)

Consider the simplest quantum system:

spin $\frac{1}{2}$ particle: two possible states $\uparrow$ $\downarrow$

naming them $|0\rangle$ and $|1\rangle$ in 2d Hilbert space.

A system of two particles:

not. Entangled

A          B

each can be in superposition of $|0\rangle$ and $|1\rangle$

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A + |1\rangle_A\right) \qquad |B\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_B + |1\rangle_B\right)$$

The wave function of both particles is:

$$|AB\rangle = \frac{1}{2}\left(|0\rangle_A + |1\rangle_A\right) \otimes \left(|0\rangle_B + |1\rangle_B\right)$$

$$= \frac{1}{2}\left(|0\rangle_A|0\rangle_B + |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

or $:= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$

- Entangled superposition:

Using the Conservation of Spin

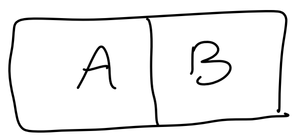
if the total spin of particles A and B is zero: $\underset{A}{\uparrow}$ $\underset{B}{\downarrow}$ or $\underset{A}{\downarrow}$ $\underset{B}{\uparrow}$

→ Entangled Stat: $|AB\rangle = \frac{1}{2}(|01\rangle + |10\rangle)$

cannot be written as product-State:

$$|AB\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

Entangled state




if A and B entangled, we can't write a state for each subsystem alone.

compare with:

$$|AB\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |00\rangle)$$

$$= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes (|1\rangle_B + |0\rangle_B))$$

$$= |0\rangle_A \otimes \frac{1}{\sqrt{2}}(|1\rangle_B + |0\rangle_B)$$

product state

→ not entangled

Entangled superposition doesn't exhibit Interference!

Example: Particle's wave function $|\psi\rangle$ going through the double-Slit DS, which-way information available

$\psi(x) = \langle x|\psi\rangle$

$$|DS,\psi\rangle = \underbrace{N}_{\text{normalising factor}}\left(|L\rangle|x\rangle + |R\rangle|y\rangle\right)$$

$L, R$ Left - Right Slit

$x, y$ horizontal, Perpendicular polarisation

$$\langle DS,\psi|DS,\psi\rangle = |N|^2\left(\langle x|\langle L| + \langle y|\langle R|\right)\left(|L\rangle|x\rangle + |R\rangle|y\rangle\right)$$

$$= |N|^2\left(\overbrace{\langle x|x\rangle}^{|\psi_1(\underline{r})|^2}\underbrace{\langle L|L\rangle}_{=1} + \overbrace{\langle y|y\rangle}^{|\psi_2(\underline{r})|^2}\underbrace{\langle R|R\rangle}_{=1} + \langle x|y\rangle\underbrace{\langle L|R\rangle}_{=0} + \langle y|x\rangle\underbrace{\langle R|L\rangle}_{=0}\right)$$

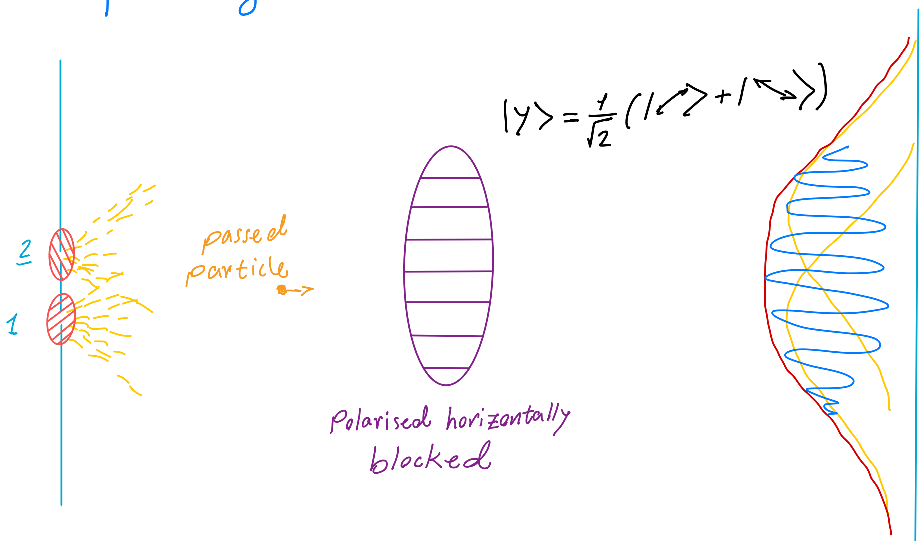
$$= |N|^2\left(\underbrace{|\psi_1(\underline{r})|^2 + |\psi_2(\underline{r})|^2}_{\text{Sums of the individual intensities}}\right)$$

no interference terms

* Can we bring back the lost interference?

Yes! → Quantum Eraser

:= erasing which-way information after putting which-way mark on particle.




- Can we bring back the which-way info?

No! Erasure is irreversible.

- Can we erase the which-way info after particles hitting the screen?! Yes!

Delayed choice: using Entanglement (can be discussed) Q-Eraser

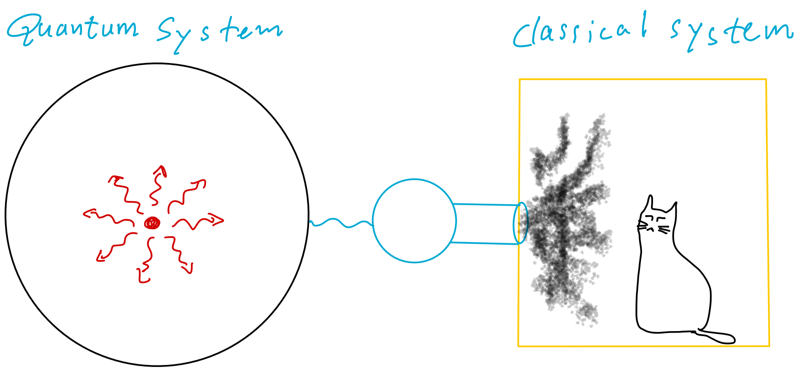
- How would you interpret this?

## 1.2. Measurement Problem

- The entangled superposition of the quantum object and the measuring apparatus, turns the apparatus into a superposition of all possible outcomes. i.e. a superposition of macroscopic objects.

This is ABSURD according to Schrödinger/Einstein.

- Schrödinger's Cat Gedankenexperiment:




- nobody has seen a blurry dead & alive mixture of cats. QM applied to macroscopic world

→ reductio ad absurdum

wrong:

$|\text{Not Decayed}\rangle + |\text{Decayed}\rangle \quad |\text{Alive}\rangle + |\text{Dead}\rangle$

Correct state: Entangled state

$$|\text{Not Decayed}\rangle |\text{Alive}\rangle + |\text{Decayed}\rangle |\text{Dead}\rangle$$

## 1.3. Interpretations of QM are different — approaches to measurement problem

### 1.3.1. Copenhagen Interpretation (orthodox QM)

- keeps the measurement problem unsolved.

  **measurement Pustulate in QM, Born's rule**

- **The wave function has all the Information.** about the q-system!

  $|\psi(t)\rangle$

- The quantum-System described with QM while the measuring apparatus described classically.

- No Theory for measurement process itself.

- No clear, consistent, unified, literal way to describe quantum phenomena is possible!

- Moral: Don't try to find better concepts than the already existing classical concepts.

- Complementarity, Uncertainty-relations

## 1.3.2. de Broglie-Bohm pilot-wave theory

- particle or wave $\longrightarrow$ particle and wave
- particles move on trajectories governed by the wavefunction.
- Introduces a so called Hidden Variable and completes the usual QM to a classical deterministic theory.

Hidden variable : $X(t)$ actual position of particles

$$\psi(\underline{r},t) = R(\underline{r},t)\, e^{iS(\underline{r},t)}$$

$S(\underline{r},t)$ Phase of $\psi$

$R(\underline{r},t)$ modulus of $\psi$

1. $i\hbar\,\partial_t \psi = \hat{H}\psi$
2. $\partial_t X(t) = \frac{\hbar}{m}\underline{\nabla} S(\underline{r},t)$

- No division of the world into classical/Quantum.

- Solves the measurement problem!

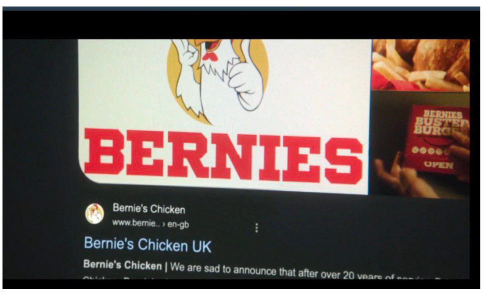
* Discussion of Contextuality Chapter $\underline{3}$

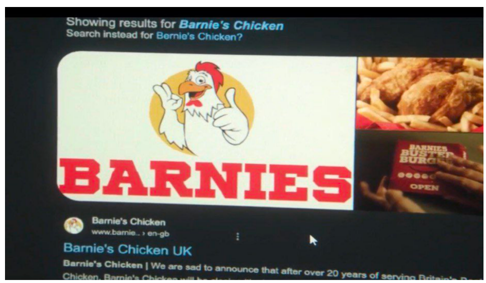
## 1.3.3. Interpretations that get rid of the measurement problem:

### * Many-worlds or Multiverse Theory: Chapter 4

- wave function gives complete information.
- No division of the world into classical/Quantum.
- No external Observation.
- Entanglement, Decoherence → Creation of parallel realities.

Everett




* Popular Science-fiction, Black mirror 2025

### * objective collapse of the $\Psi(\vec{r},t)$ Theories:

- GRW (Spontaneous Collapse) Chapter 5
- Gravitation's role in the Collapse of the $\Psi$

Penrose.

...

## 1.4. Ontology

Definition: What exists? What is real?

Question: Are superpositions real?
They exist, but we can't observe them!

Question: Is the wave function physically real?

Question: Are physical quantities e.g. spin real independent of our observation?

Question: Can we define physical reality?

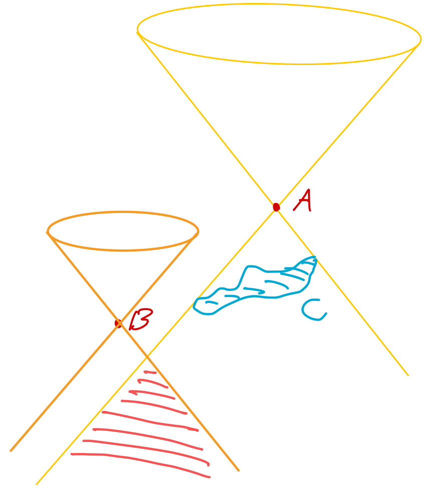
ERR: If the theory predicts it with 100% probability without in any way disturb/changing the system.

Three positions: Griffith

- realist
- Orthodox Quantum theory (Copenhagen)
- Agnostic

# 1.4. Locality  chapter 6

Special relativity

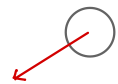



$$P(A|C,B) = P(A|C)$$

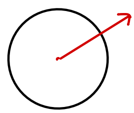
Maxwellian Electrodynamics: $\Box \underline{A}(\underline{r},t) = 0$

per construct local!

Newtonian Gravity:



non-local

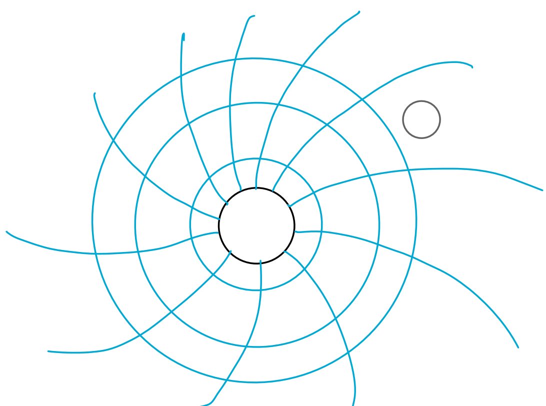


$$\underline{F} = G\frac{m_1 m_2}{r^2}\hat{\underline{r}}$$

Instantaneous

General relativity

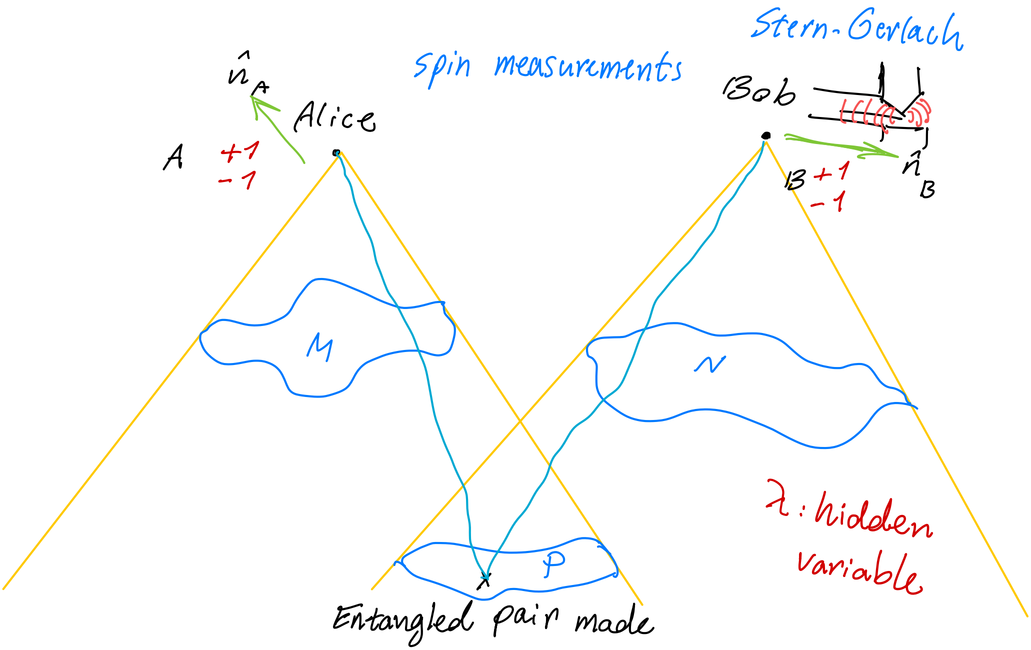
local



$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

# Einstein-Podolsky-Rosen Paradox (Bohm's version)

* had measurement outcomes existed prior to measurement ?




$$\psi_{AB} = \frac{1}{\sqrt{2}}\left(|0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B\right)$$

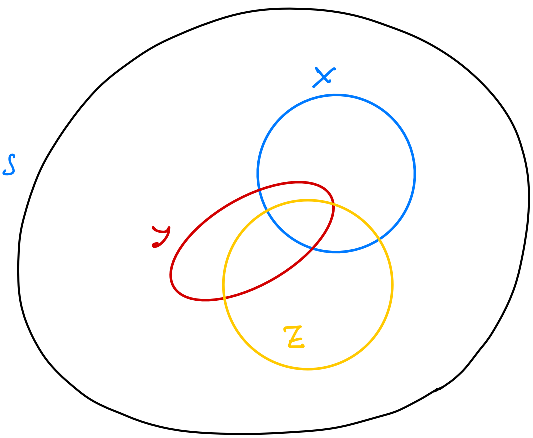
Subsystems A and B don't possess a definite state

Experimental way to settle the debate between Einstein and quantum mechanics over the reality of pre-existing properties.

Nobel prize 2022

## Bell inequality

$$P(x, y) \leq P(y, z) + P(x, \neg z)$$




- other similar inequalities can be written.

CHSH - inequality and others ...

will be discussed in Chapter 6.

# 1.5. Projects for students

- your own choosing!
- Quantum Zeno-effect
- Delayed choice quantum eraser
- Decoherence
- Gravitation as the cause of the Collapse of the wave-function, Penrose
- The Role of Consciousness in Collapse of the wave function, Wigner, von Neumann
- Time-Symmetric Interpretation (Aharonov, Vaidman) Weak measurement
- Super-determinism