Quantum Decoherence of a Central Spin Systems

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Quantum Decoherence

Quantum Decoherence

Formally speaking, decoherence is a dynamical development of quantum correlations between the central system and its environment.

Quantum Decoherence

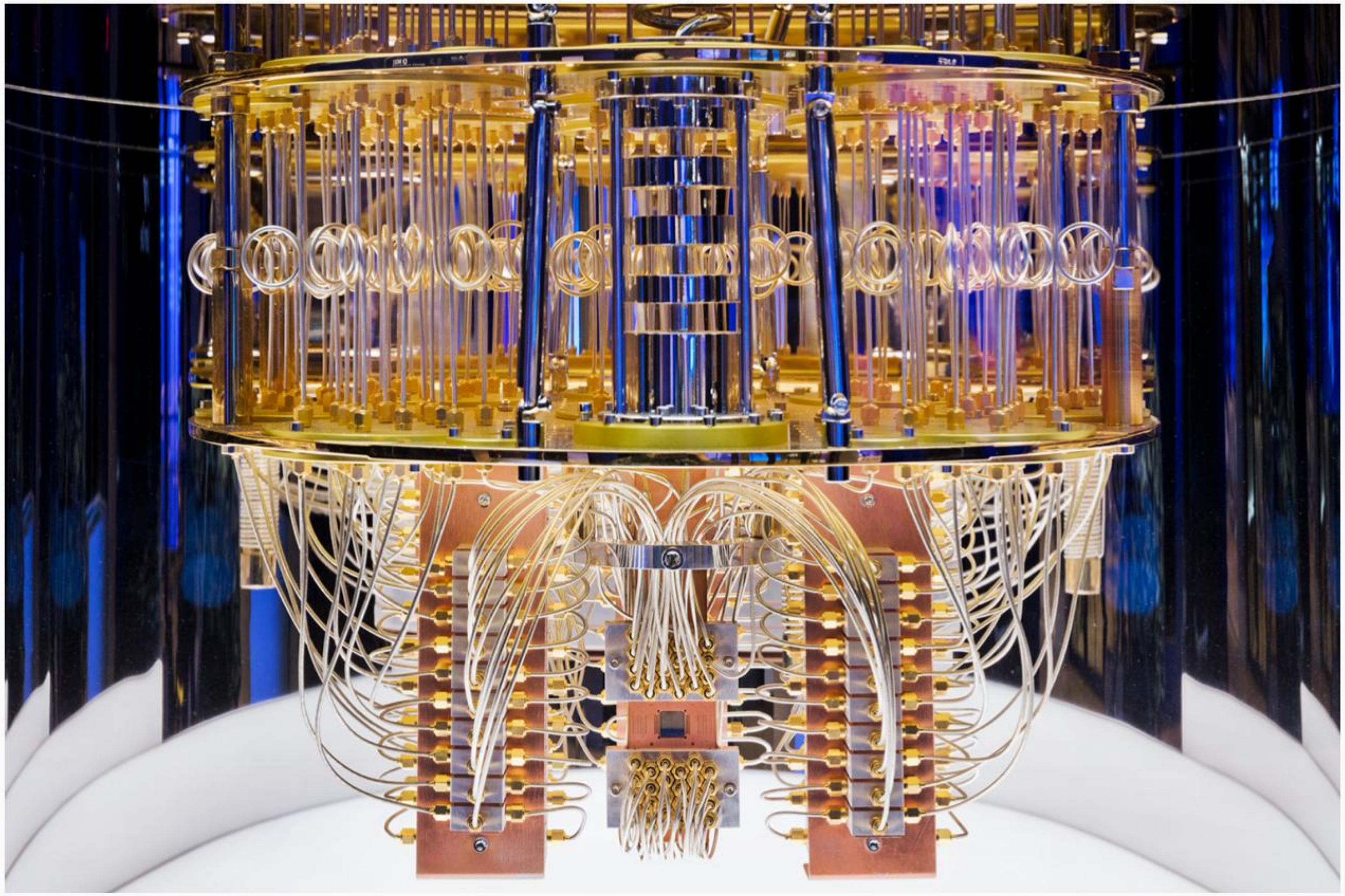


Figure 1: IBM's Quantum Computer

Quantum Decoherence

System initial state is

$$|\Psi(t=0)\rangle = |\psi_0\rangle \otimes |\chi_0\rangle \tag{1}$$

Density matrix

$$\rho_S(t) = \mathrm{Tr}_B\, |\Psi(t)\rangle\langle\Psi(t)| \tag{2}$$

Quantum Decoherence

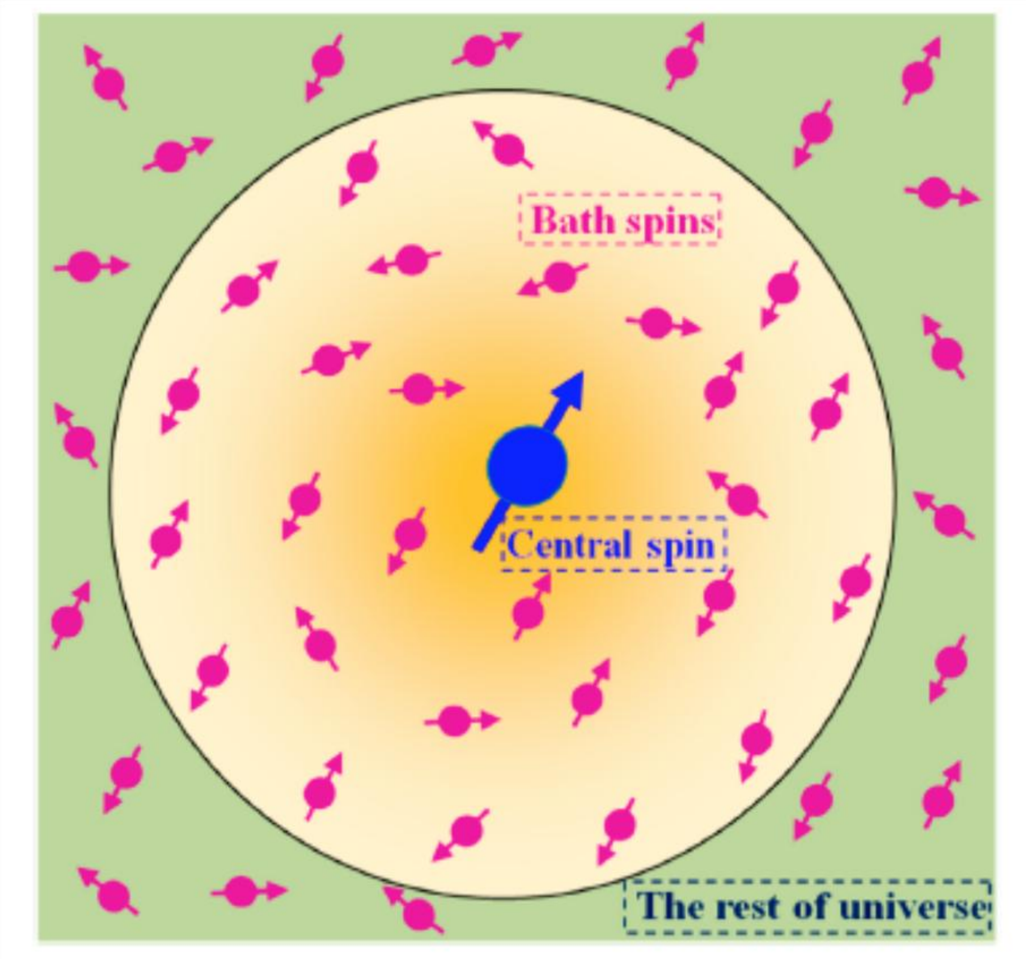


Figure 2: Central spin System[3]

Model

Model

We consider the following class of models[1]. There is a central system made of M coupled spins S_m

$$S_m = 1/2, \quad m = 1, \ldots, M \tag{3}$$

The spins S_m interact with a bath consisting of N enviromental spins I_n

$$I_n = 1/2, \quad n = 1, \ldots, N \tag{4}$$

The Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{V} = \mathcal{H}_S + \mathcal{H}_B + \mathcal{V} \tag{5}$$

Model

$$\begin{aligned}\mathcal{H}_S &= \sum_{\langle m,m'\rangle}\sum_{\alpha=x,y,z} J^{\alpha}_{mm'} S^{\alpha}_m S^{\alpha}_{m'} + \sum_{m}\sum_{\alpha=x,y,z} H^{\alpha}_m S^{\alpha}_m,\\ \mathcal{H}_B &= \sum_{\langle n,n'\rangle}\sum_{\alpha=x,y,z} \Gamma^{\alpha}_{nn'} I^{\alpha}_n I^{\alpha}_{n'} + \sum_{n}\sum_{\alpha=x,y,z} H^{\alpha}_n I^{\alpha}_n,\\ \mathcal{V} &= \sum_{\langle m,n\rangle}\sum_{\alpha=x,y,z} A^{\alpha}_{mn} S^{\alpha}_m I^{\alpha}_n.\end{aligned} \tag{6}$$

Model

Schrödinger equation

$$id\Psi(t)/dt = \mathcal{H}\Psi(t) \tag{7}$$

For a time-independent Hamiltonian

$$\Psi(t) = \exp(-it\mathcal{H})\Psi_0 = U(t)\Psi_0, \tag{8}$$

Chebyshev Polynomials

$$T_n(x) = \cos(n \cos^{-1} x), \quad n \in [-1, 1] \tag{9}$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n = 1, 2, \ldots \tag{10}$$

Chebyshev Polynomials

$$f(x) = \sum_{n=0}^{\infty} c_n T_n(x), \quad x \in [-1, 1] \tag{11}$$

where

$$c_0 = \frac{1}{\pi} \int_{-1}^{1} \frac{f(x) T_n(x)}{\sqrt{1 - x^2}} dx, \tag{12}$$

and

$$c_n = \frac{2}{\pi} \int_{-1}^{1} \frac{f(x) T_n(x)}{\sqrt{1 - x^2}} dx, \quad n = 1, 2, 3, \ldots \tag{13}$$

Model

$$
\begin{aligned}
E_c = \tfrac{1}{2}\left(E_{\mathrm{max}} + E_{\mathrm{min}}\right), \quad E_0 = E_{\mathrm{max}} - E_{\mathrm{min}} \\
E_{\mathrm{min}} = \min\langle\mathcal{H}\rangle = \min_{\langle\Phi|\Phi\rangle=1}\langle\Phi|\mathcal{H}|\Phi\rangle \\
E_{\mathrm{max}} = \max\langle\mathcal{H}\rangle = \max_{\langle\Phi|\Phi\rangle=1}\langle\Phi|\mathcal{H}|\Phi\rangle
\end{aligned}
\tag{14}
$$

The rescaled operator

$$
\mathcal{G} = 2(\mathcal{H} - E_c)/E_0 \tag{15}
$$

Model

The evolution operator

$$U(t) = \exp(-i\tau\mathcal{G}) = \sum_{k=0}^{\infty} c_k T_k(\mathcal{G}), \tag{16}$$

where $\tau = E_0 t/2$,

$$c_k = \frac{a_k}{\pi}\int_{-1}^{1} \frac{T_k(x)\exp(-ix\tau)}{\sqrt{1-x^2}}dx = a_k(-i)^k J_k(\tau), \tag{17}$$

$$a_k = \begin{cases} 1, & \text{for } \; k = 0 \\ 2, & \text{for } \; k \geq 1 \end{cases}$$

and

$$T_{k+1}(\mathcal{G}) = 2\mathcal{G}T_k(\mathcal{G}) - T_{k-1}(\mathcal{G}), \quad T_0(\mathcal{G}) = 1, T_1(\mathcal{G}) = \mathcal{G} \tag{18}$$

Simulation Results

Simulation Results

Hamiltonian

$$\mathcal{H}_S = J\mathbf{S}_1 \cdot \mathbf{S}_2, \quad \mathcal{H}_B = 0, \quad \mathcal{V} = \sum A_n \left(\mathbf{S}_1 + \mathbf{S}_2\right) \cdot \mathbf{I}_n \tag{19}$$

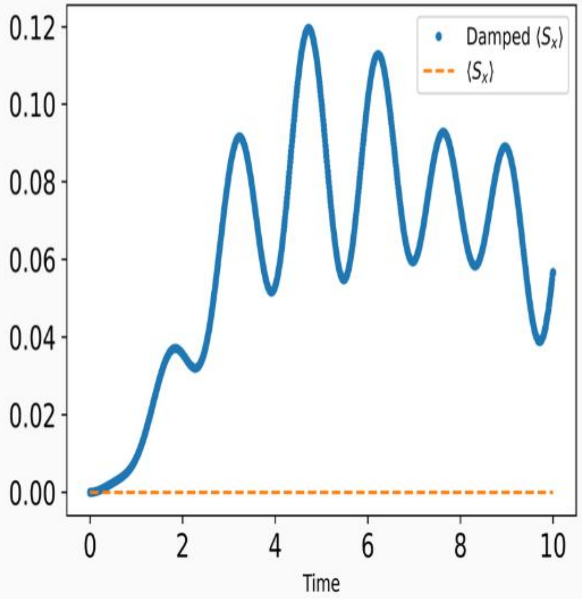
where

$$J = 1, \quad A_n = [-1, 0]. \tag{20}$$

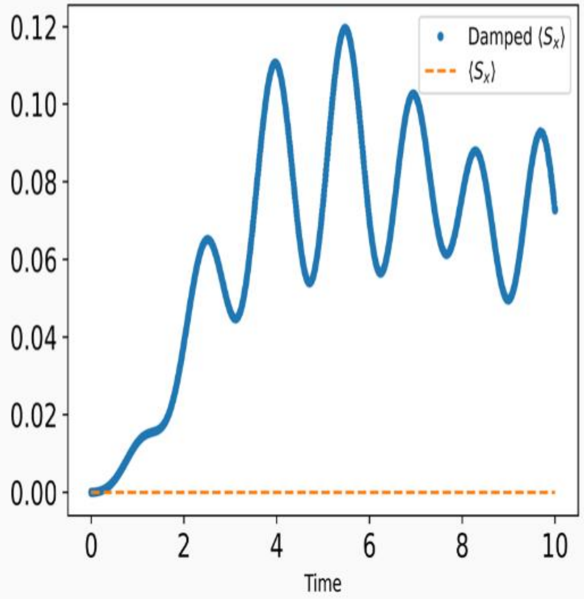
System's initial state

$$|\Psi(t)\rangle = |\psi_0\rangle \otimes |\chi_0\rangle \tag{21}$$

Simulation Results



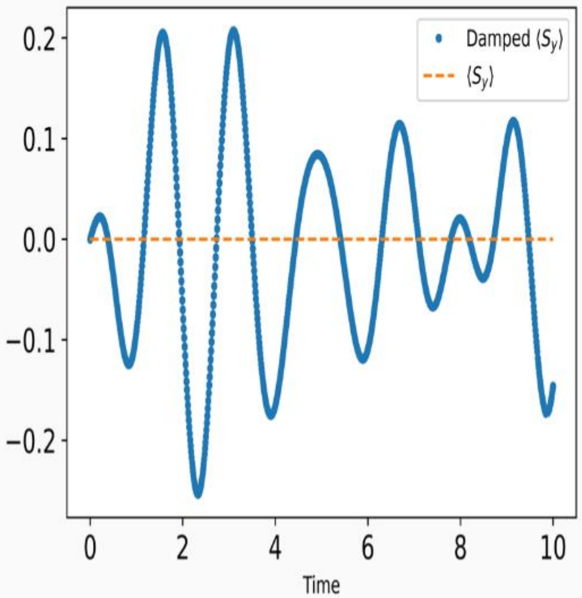
(a) $S_1^x(t)$



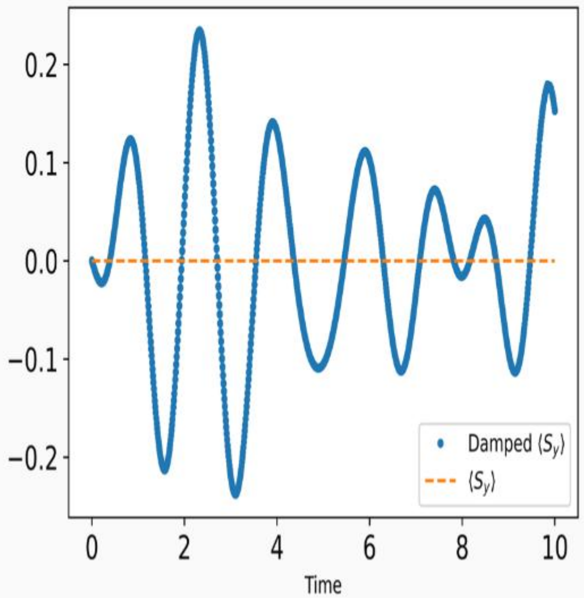
(b) $S_2^x(t)$

Figure 3

Simulation Results



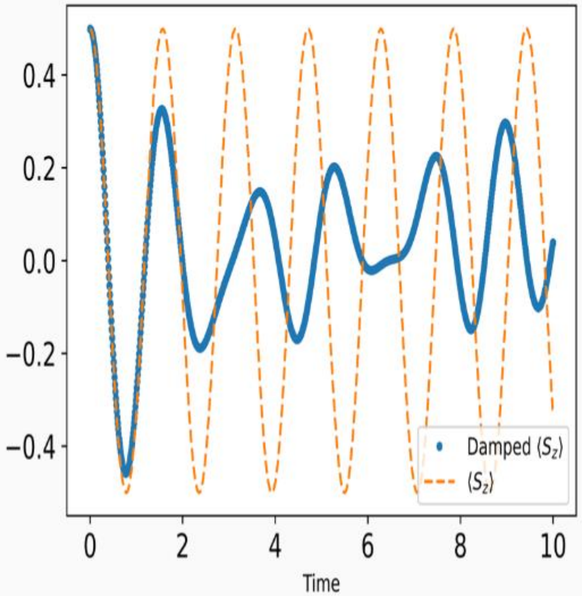
(a) $S_1^x(t)$



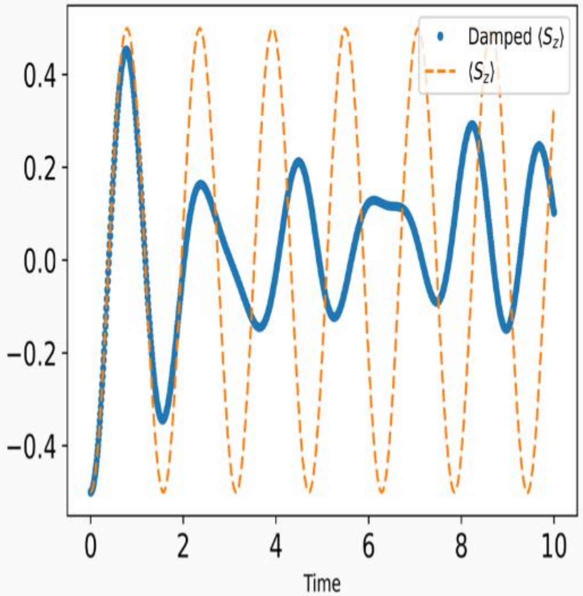
(b) $S_2^x(t)$

Figure 4

Simulation Results



(a) $S_1^x(t)$



(b) $S_2^x(t)$

Figure 5

Qutip

Qutip

QuTiP, short for the Quantum Toolbox in Python, is an open-source computational physics software library for simulating quantum systems, particularly open quantum systems.

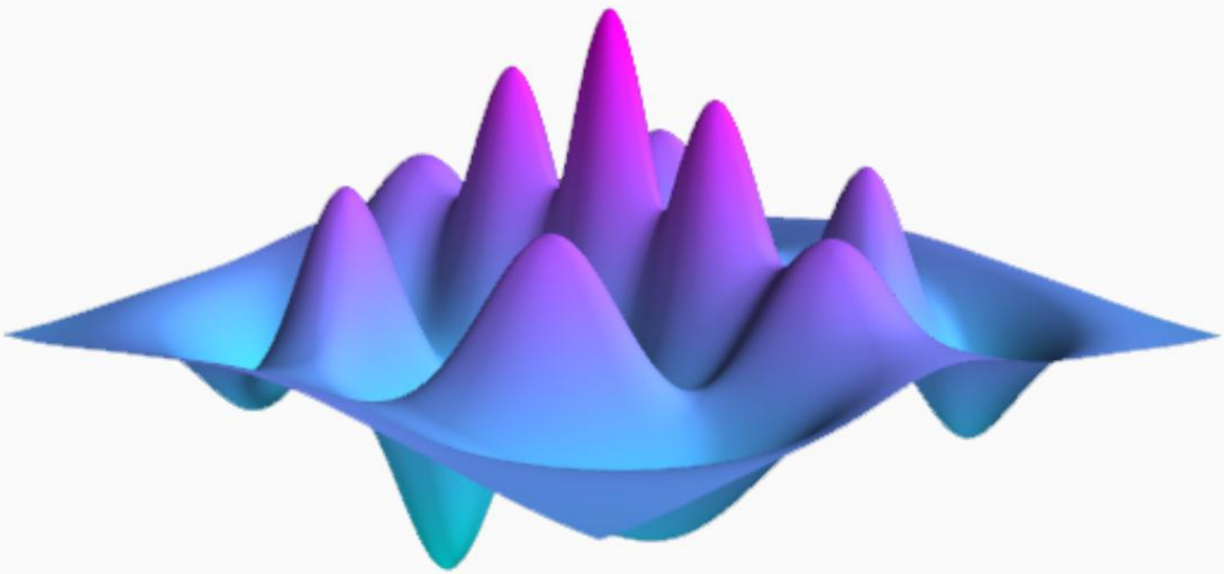
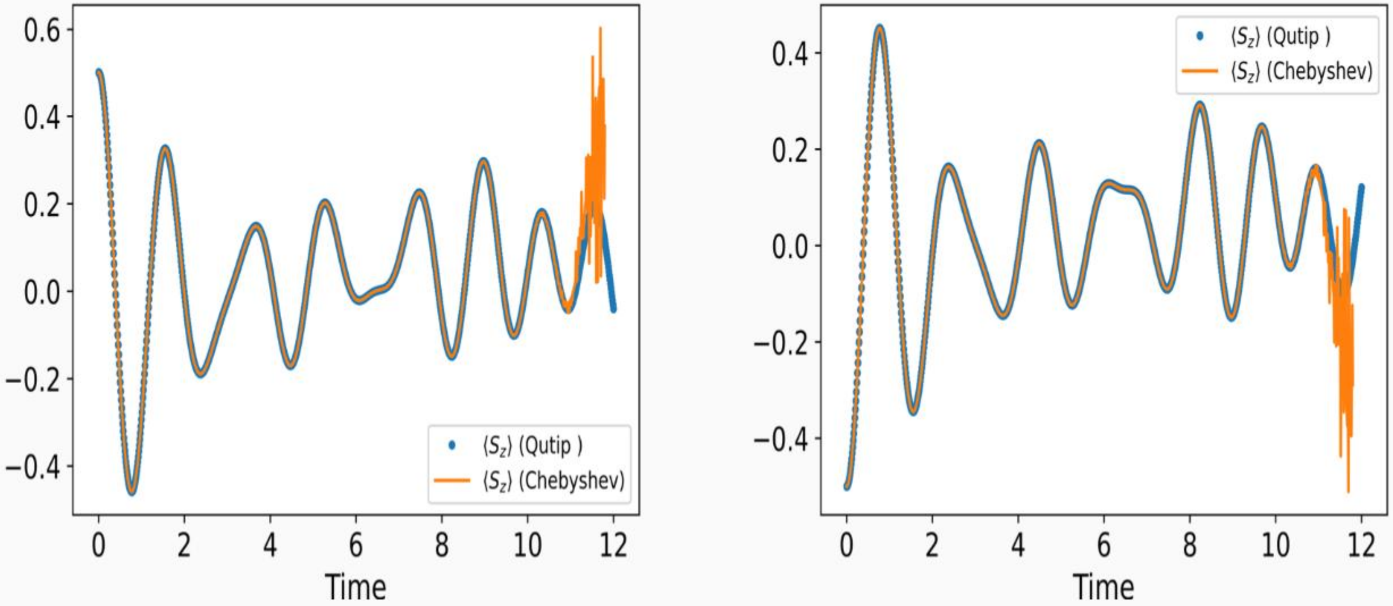


Figure 6: Qutip Logo

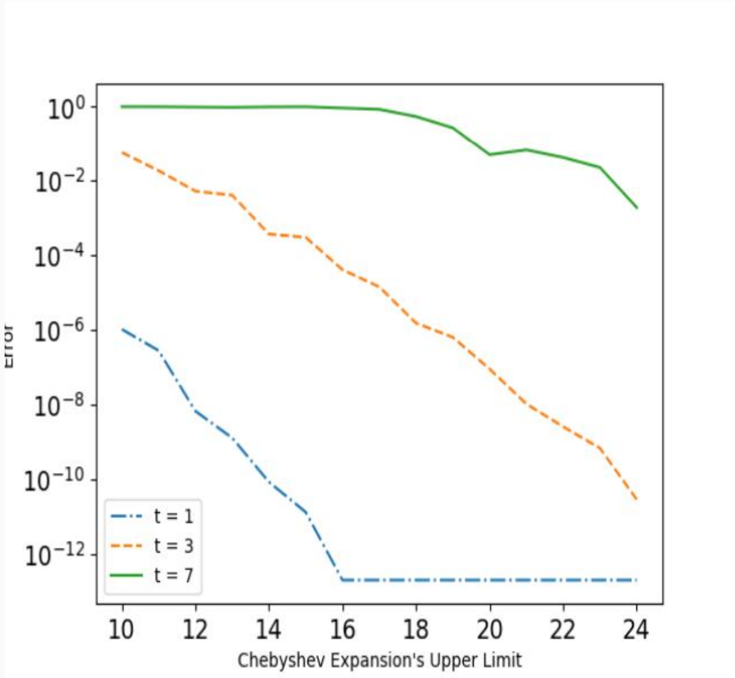
Qutip



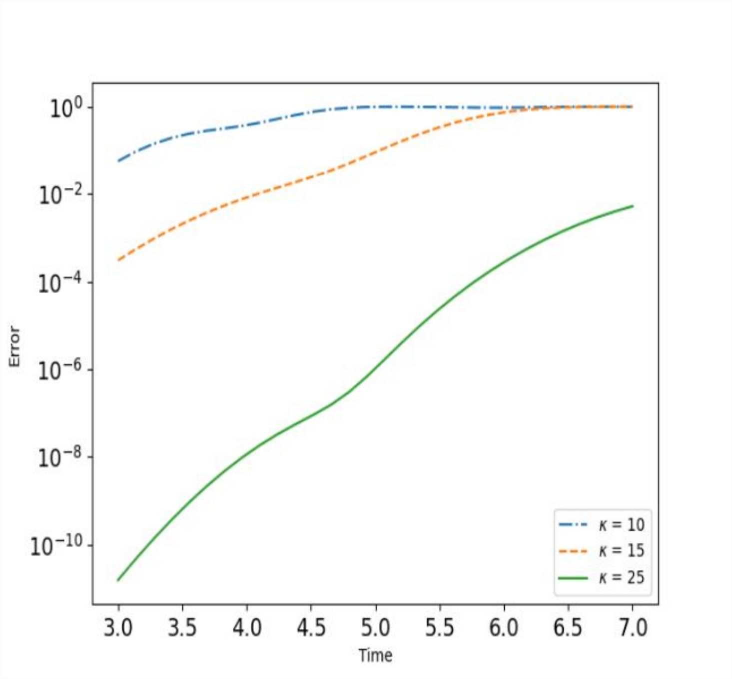
(a) $S_1^z(t)$ (b) $S_2^z(t)$

Figure 7

Error



(a) Evolution operator error rate for a constant time



(b) Evolution operator error rate for a constant chebyshev expansion upper limit

Figure 8

Thanks For Your Attention :)

References i

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