AI Lab Assignment 2

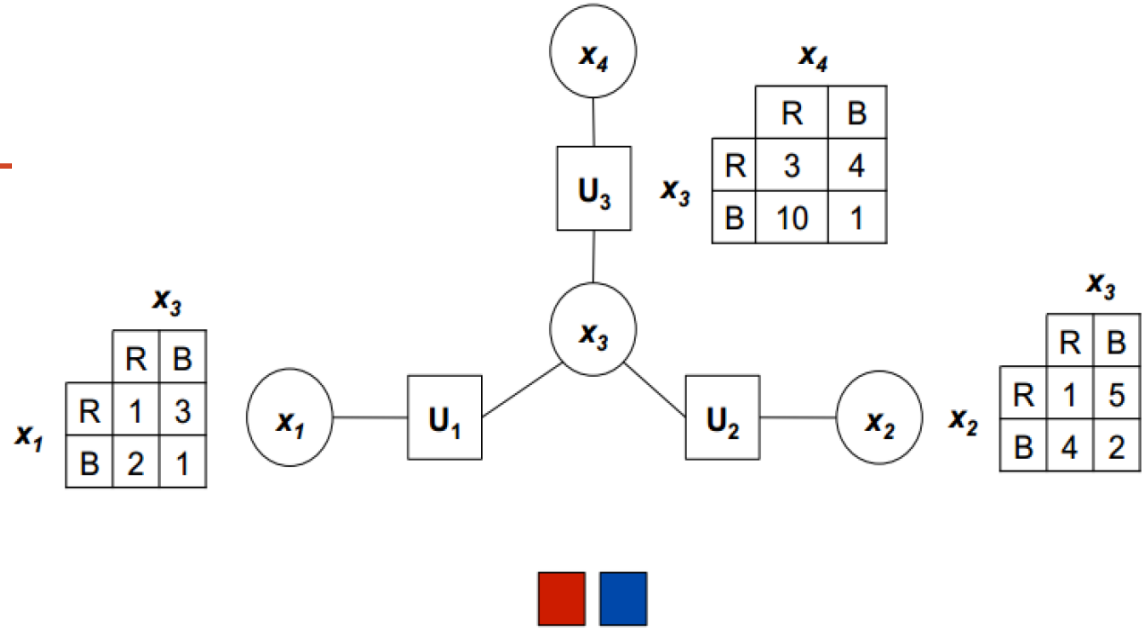
Domian Pruning while passing messages in CSPs/COPs

Max-Sum Message Passing to solve COPs

x_1	x_3	U_1
R	R	1
R	B	3
B	R	2
B	B	1

x_3	x_4	U_3
R	R	3
R	B	4
B	R	10
B	B	1

x_2	x_3	U_2
R	R	1
R	B	5
B	R	4
B	B	2



$$Q_{i\to j}(x_i) = \sum_{k \epsilon M_i - j} R_{k\to i}(x_i)$$

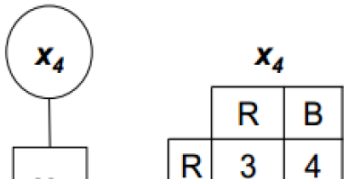
$$R_{j\to i}(x_i) = max_{x_j - i}[U_j(x_j) + \sum_{k \epsilon N_j - i} Q_{k\to j}(x_k)]$$

Once the variable node has received a message from each of its connected function nodes, the optimal variable assignment is found by locally calculating the function, $z_i(x_i)$,

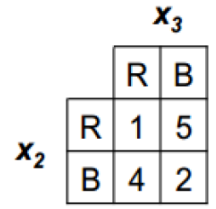
$$Z_i(x_i) = \sum_{j \epsilon M_i} R_{j\to i}(x_i)$$

where M_i is a set of function indexes, indicating which function nodes are connected to variable node x_i.

Max-Sum Message Passing to solve COPs



Complete Message Passing				
Steps	Messages			Message Type
1	$Q_{1\to1}(x_1)=\{0,0\}$	$Q_{2\to2}(x_2)=\{0,0\}$	$Q_{4\to3}(x_4)=\{0,0\}$	Variable to Function
2	$R_{1\to3}(x_3)=\{2,3\}$	$R_{2\to3}(x_3)=\{4,5\}$	$R_{3\to3}(x_3)=\{4,10\}$	Function to Variable
3	$Q_{3\to1}(x_3)=\{8,15\}$	$Q_{3\to2}(x_3)=\{6,13\}$	$Q_{3\to3}(x_3)=\{6,8\}$	Variable to Function
4	$R_{1\to1}(x_1)=\{18,16\}$	$R_{2\to2}(x_2)=\{18,15\}$	$R_{3\to4}(x_4)=\{18,10\}$	Function to Variable
5	$Z_1(x_1)=\{18,1$ $Q_{i\to j}(x_i)=\sum_{k\epsilon M_i-j} R_{k\to i}(x_i)\{10,18\}$; $Z_4(x_4)=\{18,10\}$			At Completion (When each Variable has received messages from all of its neighbours)
6	$=R$;			Final Color Assignment to Variables after finding $argmax_{x_i} z_i(x_i)$



$$R_{j\to i}(x_i) = max_{x_j-i}[U_j(x_j) + \sum_{k\epsilon N_j-i} Q_{k\to j}(x_k)]$$

$$Z_i(x_i) = \sum_{j\epsilon M_i} R_{j\to i}(x_i)$$

$$Q_{3\to3}(x_3) = R_{1\to3}(x_3) + R_{2\to3}(x_3)$$
$$Q_{3\to3}(x_3) = \{2,3\} + \{4,5\} = \{6,8\}$$

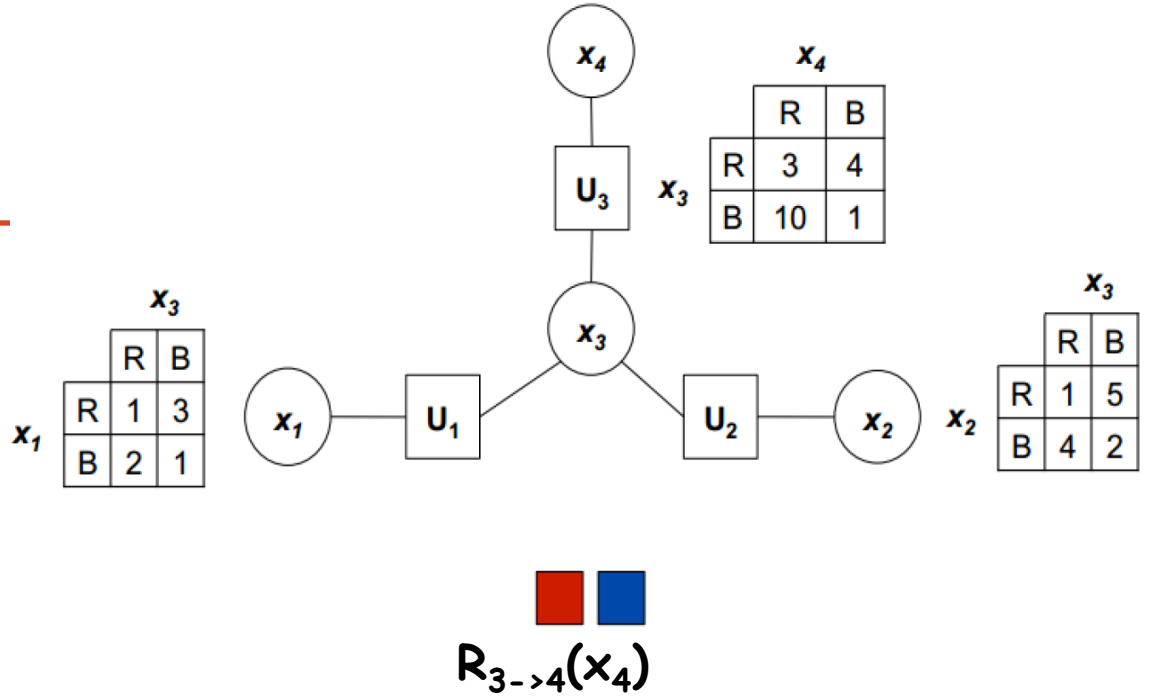
$$Z_3(x_3) = R_{1\to3}(x_3) + R_{2\to3}(x_3) + R_{3\to3}(x_3)$$
$$Z_3(x_3) = \{2,3\} + \{4,5\} + \{4,10\} = \{10,18\}$$

Max-Sum Message Passing to solve COPs

x_1	x_3	U_1
R	R	1
R	B	3
B	R	2
B	B	1

x_3	x_4	U_3
R	R	3
R	B	4
B	R	10
B	B	1

x_2	x_3	U_2
R	R	1
R	B	5
B	R	4
B	B	2



$R_{3->4}(x_4)$

$$Q_{i\to j}(x_i) = \sum_{k \epsilon M_i - j} R_{k\to i}(x_i)$$

x_3	x_4	U_3	$Q_{3->3}(x_3)$	Sum
R	R	3	6	9
B	R	4	8	18
R	B	10	6	10
B	B	1	8	9

$$R_{j\to i}(x_i) = max_{x_j - i}[U_j(x_j) + \sum_{k \epsilon N_j - i} Q_{k\to j}(x_k)]$$

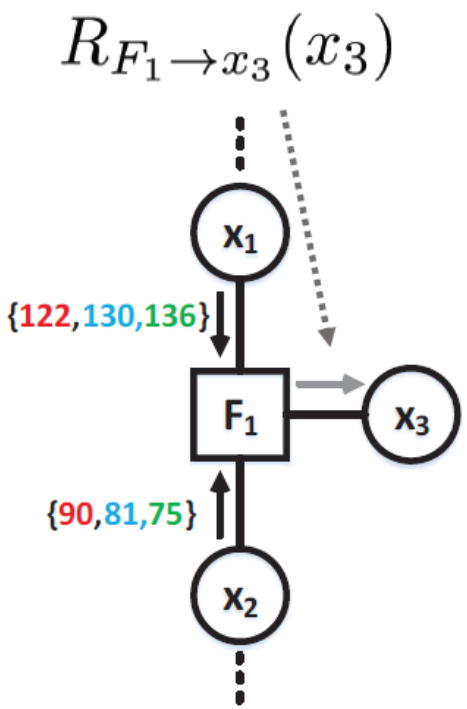
$$Z_i(x_i) = \sum_{j \epsilon M_i} R_{j\to i}(x_i)$$

$$Q_{3\to 3}(x_3) = R_{1\to 3}(x_3) + R_{2\to 3}(x_3)$$
$$Q_{3\to 3}(x_3) = \{2,3\} + \{4,5\} = \{6,8\}$$

$$Z_3(x_3) = R_{1\to 3}(x_3) + R_{2\to 3}(x_3) + R_{3\to 3}(x_3)$$
$$Z_3(x_3) = \{2,3\} + \{4,5\} + \{4,10\} = \{10,18\}$$

Function to Variable Message Computation

Utility (cost) table for the function node, $F_1(x_1, x_2, x_3)$			
x_1	x_2	x_3	F_1
R	R	R	5
R	R	B	10
R	R	G	12
R	B	R	13
R	B	B	12
R	B	G	35
R	G	R	9
R	G	B	38
R	G	G	11
B	R	R	12
B	R	B	14
B	R	G	38
B	B	R	10
B	B	B	3
B	B	G	9
B	G	R	40
B	G	B	14
B	G	G	13
G	R	R	10
G	R	B	37
G	R	G	12
G	B	R	39
G	B	B	13
G	B	G	14
G	G	R	11
G	G	B	12
G	G	G	4



$Q_{x_1 \to F_1}(x_1)$	$Q_{x_2 \to F_1}(x_2)$	F_1		$R_{F_1 \to x_3}(x_3)$
122	90	5	217	
122	90	10	222	
122	90	12	224	
122	81	13	216	
122	81	12	215	
122	81	35	238	
122	75	9	206	
122	75	38	235	
122	75	11	208	
130	90	12	232	
130	90	14	234	
130	90	38	258	{256,263,258}
130	81	10	221	
130	81	3	214	
130	81	9	220	
130	75	40	245	
130	75	14	219	
130	75	13	218	
136	90	10	236	
136	90	37	263	
136	90	12	238	
136	81	39	256	
136	81	13	230	
136	81	14	231	
136	75	11	222	
136	75	12	223	
136	75	4	215	

(a) Computation of a factor-to-variable message (i.e. F_1 to x_3).

Lab Task

Proceedings of the Twenty-Ninth International Joint Conference on Artificial Intelligence (IJCAI-20)

Speeding Up Incomplete GDL-based Algorithms for Multi-agent Optimization with Dense Local Utilities

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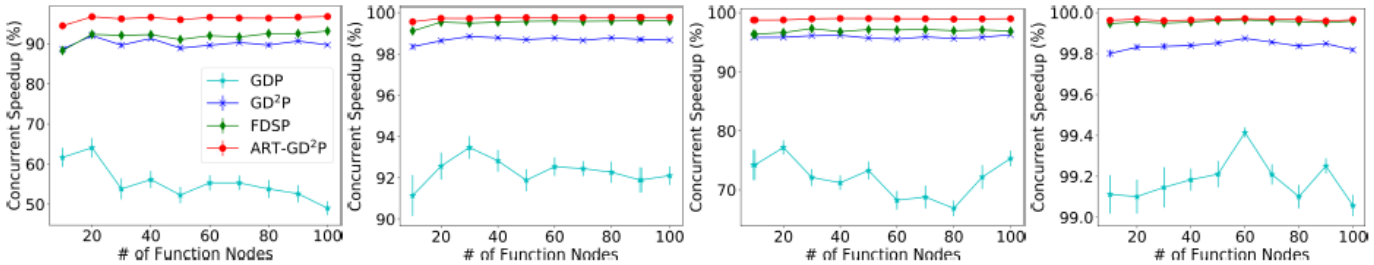
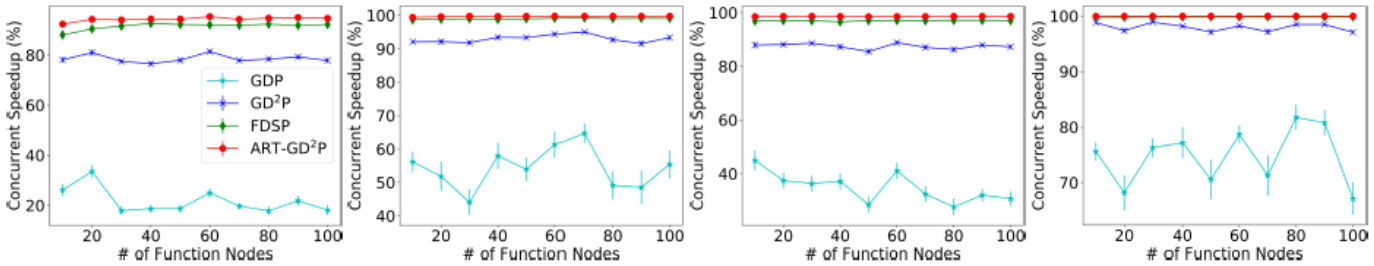
Lab Task



(a) Small domain, low arity (b) Small domain, high arity (c) Large domain, low arity (d) Large domain, high arity

Figure 6: Performance comparison on sparse problems



(a) Small domain, low arity (b) Small domain, high arity (c) Large domain, low arity (d) Large domain, high arity

Figure 7: Performance comparison on dense problems

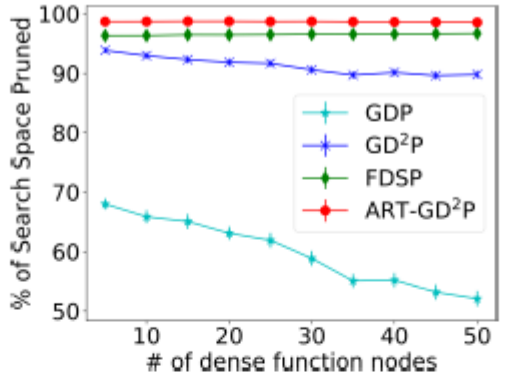
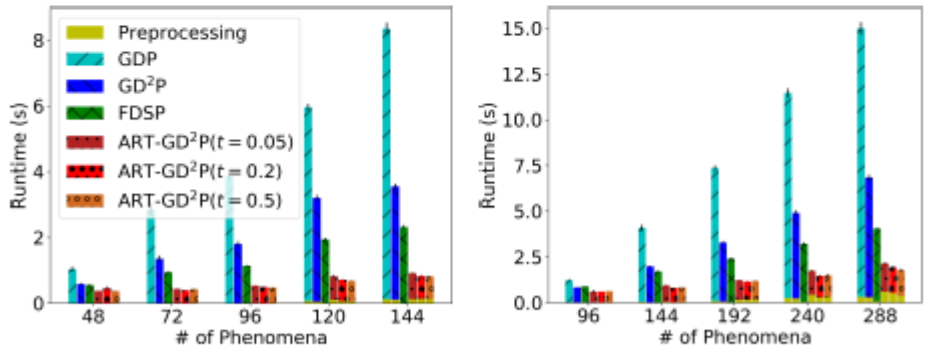


Figure 8: Performance comparison on the problems with dense local utilities



(a) 48 radars (b) 96 radars

Figure 9: Performance comparison on NetRad systems