THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________
Seat Number __________
Student Number |__|__|__|__|__|__|__|__|
Family Name ____________________
First Name ____________________

School of Mathematics & Physics
Semester One Examinations, 2023
MATH3202 Operations Research and Mathematical Planning

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Planning Time: 10 minutes

Exam Conditions:

•This is an Open Book examination
•Casio FX82 series or UQ approved and labelled calculator only
•During Planning Time - Students are encouraged to review and plan responses to the exam questions
•This examination paper will be released to the Library

Materials Permitted in the Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)

* Open-book: Any additional written or printed material is permitted; material may also be annotated.

Materials to be supplied to Students:
Additional exam materials (e.g. answer booklets, rough paper) will be provided upon request.

1 x 14-Page Answer Booklet

Instructions to Students:
If you believe there is missing or incorrect information impacting your ability to answer any question, please state this when writing your answer.

For Examiner Use Only

Question	Mark

Total _________

Question 1 – Revised Simplex Algorithm

10 marks total

Suppose we are solving the following linear programming problem:

$$\text{maximise } z = 3x_1 + 2x_2$$

subject to

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 10 \\ 3x_1 + 2x_2 + x_4 &= 16 \\ x_1 + 4x_2 + x_5 &= 15 \end{aligned}$$

Assume we have a current basis of x_1, x_4, x_5. Demonstrate your understanding of the Revised Simplex Algorithm and Sensitivity Analysis by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective? *[2 marks]*

b) What is the entering variable for the next step of the revised simplex algorithm, and what is the leaving variable? *[2 marks]*

c) What is the new objective value? Verify that the new solution is optimal. *[2 marks]*

d) Assuming no other changes, what value does the objective coefficient of x_3 have to increase by so that x_3 is nonzero in the optimal solution? *[2 marks]*

e) If the right-hand side of the second constraint is changed to $16 + \delta$ for some small value of $\delta > 0$, what will happen to the value of z? *[2 marks]*

Hint: The following information may be useful

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 \\ 3 & 2 & 0 \\ 1 & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ -3 & 2 & 0 \\ 10 & -7 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2 – Linear and Integer Programming

16 marks total

Big Manufacturing Company (BMC) want to completely redesign their warehouse network. In order to do this, BMC have prepared the following data:

- A list of the stores that BMC supplies, together with the tonnes per week of products ordered by each store.
- The location of potential warehouse sites. For each site BMC knows the cost over the planning horizon of establishing and running a warehouse at that site and the capacity of the warehouse that would be established.
- For each potential warehouse site and each store, BMC knows the total cost of supplying the store from the warehouse, over the planning horizon.

Part A

Formulate the warehouse location problem to minimise the cost of establishing warehouses and supplying stores. Clearly define all sets, data, and variables, and the objective function and constraints, and state any additional assumptions you make about the problem. *[8 marks]*

Part B

Some of the stores are part of a chain of stores. For reliability reasons, the owners of the chain requires that no more than two stores are supplied from the same warehouse. They also require that at least K different warehouses are used to supply their stores overall. Extend your model to include these requirements. Clearly define any new sets, data, variables and constraints. *[8 marks]*

Question 3 – Dynamic Programming

14 marks total

Part A

An elite runner is considering her strategy for the next running season. There are T races in the season and she wants to maximise the expected number of races she will win.

If she chooses to enter a race, her probability of winning the race is $\frac{1}{2+f}$ where f is her fatigue score going into the race. After the race, her fatigue score increases by 3. If she chooses not to enter a race, her fatigue score decreases by 4, to a minimum of 0.

a) Provide a general dynamic programming formulation to maximise the runner's expected number of races won over the season. You should use Bellman's equation and identify the data, stages, state, actions and value function.
[5 marks]

b) Suppose she starts a season of four races with 0 fatigue score. Which races should she enter and what is her expected number of wins? *[3 marks]*

Part B

Having used the strategy for a season, the runner realises that after any race she runs, there is 50% chance her fatigue score will increase by 3, and a 50% chance it will only increase by 2. Additionally, for the next season she needs to win 3 races to qualify for the national team. Her goal now is to maximise her chances of winning 3 or more races.

Provide a general dynamic programming formulation to maximise the runner's chances of winning 3 or more races. You should use Bellman's equation and identify the data, stages, state, actions and value function. *[6 marks]*

END OF EXAMINATION