Communication 5

Sets

- F - set of supplying farms
- D - set of days

Data

- W_w - wholesale price of whole milk (\$/L)

- W_l - wholesale price of low fat milk (\$/L)

- F_w - fat content of whole milk (%)

- F_l - fat content of low fat milk (%)

- C_s - cost of storage (\$/L/day)

- D_t - demand for each milk variety each day (L)

- S_f - supply from each farm $f \in F$ (L)

- F_f - fat content of milk from each farm $f \in F$ (%)

Variables

- x_{tf} - volume of whole milk produced from farm $f \in F$ on day $t \in D$ (L)

- y_{tf} - volume of low fat milk produced from farm $f \in F$ on day $t \in D$ (L)

- z_t - volume of whole milk stored on day $t \in D$ (L)

- w_t - volume of low fat milk stored on day $t \in D$ (L)

- a_t - total volume of whole milk sold on day $t \in D$ (L)

- b_t - total volume of low fat milk sold on day $t \in D$ (L)

Objective function

The goal of the program is to determine the volume of each variety of milk that needs to be processed to achieve a maximal profit and hence establish what this optimal profit is.

$$\max\left(\sum_{t \in D} W_w \times a_t + W_l \times b_t - (z_t + w_t) \times C_s \right)$$

Constraints

- The total milk processed each day from each farm is less than or equal to that farm's daily supply

$$x_{tf} + y_{tf} \leq S_f, \quad \forall\, f \in F,\, t \in D$$

- The cumulatively fat content of processed milk is less than or equal to the fat content of supply

$$\sum_{f \in F} \left(F_w \times x_{tf} + F_l \times y_{tf} \right) \leq \sum_{f \in F} S_f \times Ff$$

Monday: - On mondays, for each milk variety, the volume of stored milk must equal processed milk minus sold milk

$$z_t = \left(\sum_{f \in F} x_{tf} \right) - a_t, \quad \forall\, t \in D, \quad \text{and} \quad w_t = \left(\sum_{f \in F} y_{tf} \right) - b_t, \quad \forall\, t \in D$$

Other Days: - On days other than monday, for each milk variety, the volume of stored milk must equal the sum of the processed milk from that day and stored milk from the previous day (this makes up all available milk to be sold) minus sold milk

$$z_t = \left(\sum_{f \in F} x_{tf} \right) + z_{t-1} - a_t, \quad \forall \, t \in D, \quad \text{and} \quad w_t = \left(\sum_{f \in F} y_{tf} \right) + w_{t-1} - b_t, \quad \forall \, t \in D$$

- For each milk variety, the total sold milk must be greater than or equal to the stored milk from the previous day (as this milk has to be sold and cannot remain in storage)

$$a_t = z_{t-1}, \quad \forall \, t \in D, \quad \text{and} \quad b_t = w_{t-1}, \quad \forall \, t \in D$$