THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

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Family Name _____________________
First Name _____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2019

MATH3202 Operations Research and Math Planning (Theory)

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Any calculator permitted - unrestricted

Materials To Be Supplied To Students:

1 x 14-Page Answer Booklet

Instructions To Students:

There are **40** marks available on this exam from 3 questions.

Write your answers in the answer booklet.

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Question	Mark

Total ________

Question 1 – Revised Simplex Algorithm

10 marks total

Suppose we are solving the following linear programming problem.

$$maximise\ z = x_1 + x_2$$

Subject to:

$$\begin{aligned} 5x_1 + 3x_2 + x_3 &= 9 \\ 3x_1 + 5x_2 + x_4 &= 12 \\ x_1 + x_5 &= 1 \end{aligned}$$

Assume we have a current basis of x_1, x_2, x_4. Demonstrate your understanding of the Revised Simplex Algorithm and Sensitivity Analysis by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective? *[2 marks]*
b) What is the entering variable for the next step of the revised simplex algorithm, and what is the leaving variable? *[2 marks]*
c) What is the new objective value? Verify that the new solution is optimal. *[2 marks]*
d) Assuming no other changes, what value does the objective coefficient of x_1 have to reduce to so that x_1 is zero in the optimal solution? *[2 marks]*
e) If the right hand side of the second constraint is changed to $12 + \delta$ for some small value of $\delta > 0$, what will happen to the value of z? *[2 marks]*

Hint: The following information may be useful

$$\begin{bmatrix} 5 & 3 & 0 \\ 3 & 5 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1/3 & 0 & -5/3 \\ -5/3 & 1 & 16/3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 & 0 \\ 3 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5/16 & -3/16 & 0 \\ -3/16 & 5/16 & 0 \\ -5/16 & 3/16 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2 – Linear and Integer Programming

16 marks total

Big Mining Company (BMC) mines coking coal which is used in the production of iron and steel. BMC wish to plan the operation of their mine and their wash plant for the next few months.

BMC's mine is divided into different regions, each of which supplies coal with different characteristics. The most important characteristic of the coal is its **ash** content. The lower the ash content is, the higher the price the coal can be sold for. BMC sells a number of different coal products, which they refer to as **blends**. Each blend has a maximum ash content and a selling price.

To improve the ash content of the coal, BMC process all coal through a **wash plant**. The wash plant can be operated at different settings. For each setting and each mine region, BMC know the ash content of the washed coal and how much washed coal will be produced for each tonne of input coal (yield). An example table is below:

	Wash Plant Setting			
Region	**1**	**2**	**3**	**4**
Region 1	11.2%, 0.95	10.8%, 0.92	10.3%, 0.90	9.7%, 0.88
Region 2	9.3%, 0.95	…	…	…
Region 3	…	…	…	…

In this example, if coal from Region 1 is washed at Setting 3, then the final ash content will be 10.3% and each tonne of coal washed will produce 0.90 tonnes of coal to sell.

Coal from different regions can be mixed together to achieve the desired blend in each month. For example, if a blend has a target ash content of 9.75%, then one tonne of coal from Region 1 could be washed at Setting 3 and mixed with 1 tonne of coal from Region 2 washed at setting 1. This would give a total yield of 0.90 + 0.95 = 1.85 tonnes and an ash content of (10.3% * 0.90 + 9.3% * 0.95) / 1.85 = 9.78%

BMC have the following additional information:

- The maximum amount of coal they can mine from each region in each month
- The minimum and maximum amount of each blend they can sell in each month
- The maximum ash content for each blend and the selling price for each blend
- The maximum throughput of the wash plant in each month.

Your tasks are:

a) Assuming that all coal mined in a month is washed, blended and sold in the same month, and assuming BMC wish to maximise revenue from selling blends, formulate BMC's planning problem as an LP. Clearly define all sets, data, and variables, and the objective function and constraints, and state any additional assumptions you make about the problem. You may wish to use variables x_{ijkt} to indicate the amount of coal from region i processed through the wash plant at setting j to produce blend k in month t. *[12 marks]*

b) For operational purposes BMC wish to limit the number of region/setting/blend combinations used each month to be 10. Furthermore, any combination used must use at least 1000 tonnes of input. Extend your model as an MIP to handle these additional requirements. Specify the new variables and constraints. *[4 marks]*

Question 3 – Dynamic Programming

14 marks total

The DPP (Dodgy Political Party) is faced with a series of four elections over the next year, each three months apart. They have a total of \$10,000,000 to spend on election advertising and they wish to maximise the probability that they will win at least three of the elections.

For each election they know the probability of winning if they spend a specified sum. Example data is given in the following table.

	\$0	**\$1m**	**\$2m**	**\$3m**	**\$4m**	**\$5m**
Election 1	0.5	0.6	0.7	0.75	0.78	0.8
Election 2	0.8	0.85	0.9	0.93	0.95	0.96
Election 3	0.3	0.5	0.65	0.7	0.75	0.78
Election 4	0.7	0.8	0.85	0.88	0.90	0.92

You can assume the DPP does not need to decide how much to spend on an election until they know the outcome of the previous election and that they always spend a whole number of millions.

a) Provide a general dynamic programming formulation to maximise the DPP's chance of winning at least three elections. You should use Bellman's equation and identify the data, state, stages, actions and the transition and value functions. *[8 marks]*

b) Suppose the DPP have won the first two elections and have \$4 million dollars left. How much should they spend on Election 3 and what is their probability of winning at least three elections? *[6 marks]*

END OF EXAMINATION