THE UNIVERSITY OF QUEENSLAND
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This exam paper must not be removed from the venue

Venue	____________________
Seat Number	__________
Student Number	|__|__|__|__|__|__|__|__|
Family Name	____________________
First Name	____________________

School of Mathematics & Physics
Semester One Examinations, 2022
MATH3202 Operations Research and Mathematical Planning (Theory)

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Planning Time: 10 minutes

Exam Conditions:

- This is an Open Book examination
- Casio FX82 series or UQ approved or labelled calculator only
- During Planning Time - Students are encouraged to review and plan responses to the exam questions
- This examination paper will be released to the Library

Materials Permitted in the Exam Venue:
(No electronic aids are permitted e.g. laptops, phones)

Open-book: Any additional written or printed material is permitted; material may also be annotated.

Materials to be supplied to Students:
Additional exam materials (e.g. answer booklets, rough paper) will be provided upon request.

1 x 14-Page Answer Booklet

Instructions to Students:
If you believe there is missing or incorrect information impacting your ability to answer any question, please state this when writing your answer.

For Examiner Use Only

Question	Mark
1	
2a	
2b	
3a	
3b	

Total _________

Question 1 – Revised Simplex Algorithm

10 marks total

Suppose we are solving the following linear programming problem.

$$\text{Maximise } z = 2x_1 + x_2$$

Subject to:

$$\begin{aligned} 3x_1 + x_2 + x_3 &= 18 \\ 2x_1 + x_2 + x_4 &= 13 \\ 4x_1 + 5x_2 + x_5 &= 39 \end{aligned}$$

Assume we have a current basis of x_1, x_4, x_5. Demonstrate your understanding of the Revised Simplex Algorithm and Sensitivity Analysis by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective? *[2 marks]*

b) What is the entering variable for the next step of the revised simplex algorithm, and what is the leaving variable? *[2 marks]*

c) What is the new objective value? Verify that the new solution is optimal. *[2 marks]*

d) Assuming no other changes, what value does the objective coefficient of x_2 have to reduce to so that x_2 is zero in the optimal solution? *[2 marks]*

e) If the right-hand side of the second constraint is changed to $13 + \delta$ for some small value of $\delta > 0$, what will happen to the value of z? *[2 marks]*

Hint: The following information may be useful

$$\begin{bmatrix} 3 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ -\frac{4}{3} & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & 1 & 0 \\ 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 6 & -11 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 2 – Linear and Integer Programming

16 marks total

Grid Ball is a new sports game that is played on a grid with numbered squares, as follows:

0	1	2
3	4	5
6	7	8

A Grid Ball team has nine players who must be selected from a large pool, with one player selected for each square.

For each player in the pool, we know:

- Their cost (a fee based on many factors);
- Their rating (a number from 1 to 100, with higher numbers better);
- The city they are from; and
- The squares they can play.

When selecting a team, you need to consider:

- A given total budget for players;
- The minimum number of cities which must be represented in the team; and
- The maximum number of players that can be selected from any one city.

a) Assuming a selection panel wishes to maximise the total rating of the team, formulate the Grid Ball team selection problem as an MIP. Clearly define all sets, data, and variables, and the objective function and constraints, and state any additional assumptions you make about the problem. You may wish to use variables x_{is} to indicate that player i is selected to play in position s, with other variables as required. *[10 marks]*

b) There are 12 pairs of adjacent squares (0,1), (0,3), (1,2), (1,4) … (7,8). If the two players in adjacent squares are from the same city, then due to their familiarity with each other the team is considered to have extra "chemistry". Assume we score one additional rating point for every pair of adjacent squares which have players from the same city. Extend your model to maximise the total rating from player ratings and chemistry points. Specify the new objective, variables, and constraints. *[6 marks]*

Question 3 – Dynamic Programming

14 marks total

a) A farmer currently owns k sheep. Each year she decides how many sheep to sell and how many to keep. For each sheep she sells in year i, her profit per sheep is p_i. The number of sheep in year $i + 1$ will be double the number of sheep kept in year i. She will sell all her sheep at the end of T years.

Provide a general dynamic programming formulation to maximise the farmer's total profit over the T years. You should use Bellman's equation and identify the data, stages, state, actions and value function. *[5 marks]*

b) The farmer's husband is an avid gardener who attends a small vegetable plot in their yard. Every year, at the start of the growing season, he tests the soil condition and rates it as 'good' or 'poor'. If it is good then he expects to grow \$700 worth of vegetables for the season, while if it is poor then he expects only \$400.

Each season, the gardener can choose to fertilise the soil. If the soil is good one year, then there is an 80% chance it will be good the following year with fertiliser but only a 40% chance without fertiliser. If the soil is poor one year, then there is a 60% chance it will be good the following year with fertiliser but only a 10% chance without fertiliser. Applying fertiliser in a year costs \$150.

The gardener will retire after T years, with the plot then having no value, regardless of the soil condition.

1. Provide a dynamic programming formulation to maximise the gardener's profit over the T years. You should use Bellman's equation and identify the data, stages, states, actions, and the value function. *[6 marks]*

2. Suppose there are two years before his retirement. Should he fertilise his garden? *[3 marks]*

END OF EXAMINATION