

## Communication 3

### Sets

- $F$  - set of supplying farms

### Data

- $O_w$  - price of organic whole milk (\$/L)
- $O_l$  - price of organic low fat milk (\$/L)
- $C_w$  - price of whole milk (\$/L)
- $C_l$  - price of low fat milk (\$/L)
- $F_w$  - fat content of whole milk (%)
- $F_l$  - fat content of low fat milk (%)
- $S_f$  - supply from each farm  $f \in F$  (L)
- $F_f$  - fat content of milk from each farm  $f \in F$  (%)
- $Org_f$  - boolean value representing whether a farm is organic
- MaxPercentLow - maximum percentage of normal and organic milk that can be low fat
- MaxPercentOrg - maximum percentage of total supply that can be organic

### Variables

- $w_f$  - volume of normal whole milk produced from farm  $f \in F$  (L)
- $x_f$  - volume of normal low fat milk produced from farm  $f \in F$  (L)
- $y_f$  - volume of organic whole milk produced from farm  $f \in F$  (L)
- $z_f$  - volume of organic low fat milk produced from farm  $f \in F$  (L)

### Objective function

The goal is to determine the required volumes of milk to be processed such that the overall income is maximised.

$$\max \left( \sum_{f \in F} O_w \times y_f + O_l \times z_f + C_w \times w_f + C_l \times x_f \right)$$

### Constraints

- For each farm, the sum of all processed milk is equal to the supply. This ensures supply is not wasted.

$$S_f = w_f + x_f + y_f + z_f, \quad \forall f \in F$$

- For non-organic farms, the volume of organic whole and low fat milk produced from its supply must be zero.

$$w_f = 0, \quad \forall f \in F \text{ if not } Org_f$$

$$x_f = 0, \quad \forall f \in F \text{ if not } Org_f$$

- The supplied milk is processed into whole milk and low fat milk so that the total milk fat content of production is at most the total milk fat of supply - excess milk fat can be potentially used for other purposes.

$$\sum_{f \in F} F_w \times (w_f + y_f) + F_l \times (x_f + z_f) \leq \sum_{f \in F} S_f \times F_f$$

- The percentage milk fat in organic products must be less than or equal to the percentage milk fat of their supply (since organic products can only be produced from organic supply).

$$\sum_{f \in F} F_w \times y_f + F_l \times z_f \leq \sum_{f \in F \text{ if } Org_f} S_f \times F_f$$

- Low fat milk can make up at most 25% of the total of low fat and whole milk, for each of organic and normal products.

$$\text{MaxPercentLow} \times \left( \sum_{f \in F} w_f + x_f \right) \geq \sum_{f \in F} x_f$$

$$\text{MaxPercentOrg} \times \left( \sum_{f \in F} y_f + z_f \right) \geq \sum_{f \in F} z_f$$

- Organic products can make up at most 15% of all milk sold.

$$\text{MaxPercentOrg} \times \left( \sum_{f \in F} S_f \right) - \left( \sum_{f \in F} w_f + x_f \right) \geq 0$$