Q1

(d)

x_2 is part of basis

$C_B = \begin{pmatrix} 2 & 1-\Delta & 0 \end{pmatrix}$

$y = \begin{pmatrix} 1 & -2 & 6 \\ -1 & 3 & -11 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1-\Delta \\ 0 \end{pmatrix} = \begin{pmatrix} \Delta \\ 1-3\Delta \\ 0 \end{pmatrix}$

Δ has negative impact on x_2:

$$1 - 3\Delta < 0$$

$$\frac{1}{3} < \Delta$$

$\therefore, C_2 = 1 - \frac{1}{3} = \frac{2}{3}, x_2 = 0$ in the optimal solution.

Q2

(a)

Sets

- P - players
- C - cities
- S - positions

Data

- C_p - cost of a player $p \in P$
- R_p - rating of a player $p \in P$
- $H_{p,c} \in \{0, 1\}$ - player $p \in P$ is from city $c \in C$
- L_p - sqares player $p \in P$ can play
- B - budget of the team
- M - minimum number of cities which must be represented in the team
- O - maximum number of players that can be selected from any one city

Variables

$x_{p,s} \in \{0, 1\}$ - player $p \in P$ play in position $s \in S$

Objective

$\max\left(\sum_{p \in P} \sum_{s \in S} x_{p,s} \times R_p\right)$

Constraints

- within budget

$$\sum_{p \in P} \sum_{s \in S} x_{p,s} \times C_p \leq B$$

- all position filled

$$\sum_{p \in P} \sum_{s \in S} x_{p,s} = 9$$

- player plays at most one position

$$\sum_{\{s \in S\}} x_{p,s} \leq 1, \forall p \in P$$

- city reqs

$$\sum_{p \in P} \sum_{\{s \in S\}} x_{p,s} \times H_{p,c} \leq O, \forall c \in C$$

$$\sum_{p \in P} \sum_{c \in C} H_{p,c} \times \sum_{\{s \in S\}} x_{p,s} \geq M$$

(b)

Data

- $A = \{\{0, 1\}, \{0.3\}, ...\}$ - a collection of neighbours

Variables

- $y_a \in \{0, 1\}$ - binary variable indicating whether adjacent squares $a \in A$ have players from the same city

Objective

$$\max\left(\left(\sum_{p \in P} \sum_{s \in S} x_{p,s} \times R_p\right) + \left(\sum_{a \in A} y_a\right)\right)$$

Constraints

- $y_a = 1 \Leftrightarrow$ players on both posistions $s \in a$ are from the same city

$$\sum_{p \in P} \sum_{s \in a} x_{p,s} = 2 \times y_a, \forall a \in A$$

Q3

(a)

Data

- $k_0 = k$ - currently owns k sheep
- p_i - profit per sheep when sell in year i

Stages

- $0 \leq i \leq T$ - years

State

- k_i - number of sheep in year i

Action

- $0 \leq S_i \leq 2 \times k_i$ - number of sheep sold in year i

Value Function

- $V(i, k_i) :=$ expected profit if we have k_i sheep at the start of year i
- Base case:

$$\forall i, k_i < 0 \longrightarrow V(i, k_i) = 0$$

$i = T : V(T, k_T) = p_t \times 2 \times k_T$

- General case:

$$V(i, k_i) = p_i \times S_i + V(i + 1, 2 \times k_i - S_i)$$

(b)

1.

Data

- PFG $= 0.8$ - the chance of soil being good with fertiliser the following year given it's good one year
- PFB $= 0.6$ - the chance of soil being good with fertiliser the following year given it's bad one year
- PG $= 0.4$ - the chance of soil being good without fertiliser the following year given it's good one year
- PB $= 0.1$ the chance of soil being good without fertiliser the following year given it's bad one year
- $E_S = \begin{cases} 700 \text{ , if } S{=}1 \\ 400 \text{ , if } S{=}0 \end{cases}$ - expectation of growth given soil condition S
- $C = 150$ - Applying fertiliser in a year costs 150

Stages

- $0 \leq t \leq T$ - years

State

- $S_t \in \{0, 1\}$ - the condition of the soil in year t, where
  - 0 - bad
  - 1 - good

Action

- $F_t \in \{0, 1\}$ - fertilise in year t, where
  - 0 - don't fertilise
  - 1 - fertilise

Value Function

- $V_{t(S_t)}$:= expected profit if we start year t with soil condition S_t
- Base case: The gardener will retire after T years, with the plot then having no value, regardless of the soil condition.

$$t = T, V_{T(S_T)} = 0$$

- General case:

$$V_t(S_t) = \max\left\{E_{S_t} - C + V_{t+1}^{\text{fertilise}}, E_{S_t} + V_{t+1}^{\text{not fertilise}}\right\}$$

  where
  - $V_{t+1}^{\text{fertilise}} = \begin{cases} \text{PFG } \times V_{t+1}(1) + (1 - \text{PFG}) \times V_{t+1}(0) \text{ , if } S_t{=}1 \\ \text{PFB } \times V_{t+1}(1) + (1 - \text{PFB}) \times V_{t+1}(0) \text{ , if } S_t{=}0 \end{cases}$
  - $V_{t+1}^{\text{not fertilise}} = \begin{cases} \text{PG } \times V_{t+1}(1) + (1 - \text{PG}) \times V_{t+1}(0) \text{ , if } S_t{=}1 \\ \text{PB } \times V_{t+1}(1) + (1 - \text{PB}) \times V_{t+1}(0) \text{ , if } S_t{=}0 \end{cases}$