THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

This exam paper must not be removed from the venue

Venue ____________________

Seat Number ________

Student Number |__|__|__|__|__|__|__|__|

Family Name ____________________

First Name ____________________

School of Mathematics & Physics

EXAMINATION

Semester One Final Examinations, 2017

MATH3202-1 Operations Research and Mathematical Planning (Theory Exam)

This paper is for St Lucia Campus students.

Examination Duration: 120 minutes

Reading Time: 10 minutes

Exam Conditions:

This is a Central Examination

This is an Open Book Examination

During reading time - write only on the rough paper provided

This examination paper will be released to the Library

Materials Permitted In The Exam Venue:

(No electronic aids are permitted e.g. laptops, phones)

Calculators - Casio FX82 series or UQ approved (labelled)

Materials To Be Supplied To Students:

1 x 14 Page Answer Booklet

Instructions To Students:

There are **40** marks available on this exam from **3** questions.

Provide your answers in the booklet provided.

Additional exam materials (eg. answer booklets, rough paper) will be provided upon request.

For Examiner Use Only

Question	Mark

Total ______

Question 1 – Revised Simplex Algorithm

10 marks

Suppose we are solving the following linear programming problem:

$$maximise\ z = 20x_1 + 10x_2 + 15x_3$$

Subject to:

$$\begin{aligned} 3x_1 + 2x_2 + 5x_3 + x_4 &= 55 \\ 2x_1 + x_2 + x_3 + x_5 &= 26 \\ x_1 + x_2 + 3x_3 + x_6 &= 30 \\ 5x_1 + 2x_2 + 4x_3 + x_7 &= 57 \end{aligned}$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0.$$

Assume we have a current basis of x_1, x_2, x_4, x_6. Demonstrate your understanding of the steps of the Revised Simplex Algorithm by answering the following:

a) What is the basic feasible solution at this stage? What is the value of the objective?

b) What is the entering variable for the next step of the Revised Simplex Algorithm?

c) What is the leaving variable?

d) What is the new value of the objective? Verify that the new solution is optimal.

e) If the right-hand side of the third constraint is changed to $30 + \delta$ for some value of $\delta > 0$, what will happen to the value of z?

f) Assuming no other data changes, what value does the objective function coefficient of x_3 have to reduce to so that x_3 is zero in the optimal solution?

Hint The following information may be useful:

$$\begin{bmatrix} 3 & 2 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 5 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -2 & 0 & 1 \\ 0 & 5 & 0 & -2 \\ 1 & -4 & 0 & 1 \\ 0 & -3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 5 & 0 \\ 2 & 1 & 1 & 0 \\ 1 & 1 & 3 & 1 \\ 5 & 2 & 4 & 0 \end{bmatrix} \begin{bmatrix} -\frac{2}{5} & -\frac{2}{5} & 0 & \frac{3}{5} \\ \frac{3}{5} & \frac{13}{5} & 0 & -\frac{7}{5} \\ \frac{1}{5} & -\frac{4}{5} & 0 & \frac{1}{5} \\ -\frac{4}{5} & \frac{1}{5} & 1 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Question 2 – Linear and Integer Programming

16 marks total

A company facing rapid growth in the demand for its product needs to plan the process of building new factories over the next few years. They supply you with the following information:

- For each factory location, they specify:
    - Whether or not a factory has currently been built.
    - The cost to build a factory there and have it ready for production by the start of a certain year – this cost varies by factory and by year.
    - The production capacity, in units per year, for the factory.
    - The production cost per unit for the factory in each year.
- The demand, in number of units, of the company's product for each year of the planning horizon.
- The cost of unmet demand, supplied as a cost per unit for each year in the planning horizon.
- An annual capital budget that represents the total amount that can be spent on building. If money is unspent, it rolls over to the next year so that total capital expenditure up to any year must not exceed the total capital budget up to that year.

The company wishes to minimise the total cost of production and unmet demand over the planning horizon, subject to the various capacities, without breaking the capital budget.

a) Develop an integer programming model to determine the optimal plan for building and operating factories. Clearly define all sets, data, variables, objective function and constraints. *[12 marks]*

b) Suppose the company now has the option of upgrading an existing factory (including any new factories that have been built). In addition to the above information, they now provide the capital cost to upgrade each factory as well as the production capacity and production cost for the upgraded factory. Revise your formulation to determine the optimal plan for building, upgrading and operating factories over the planning horizon. *[4 marks]*

Question 3 – Dynamic Programming

14 marks total

A farmer grows three horticultural crops in successive seasons on 100 hectares (ha). Each crop takes one season to reach maturity from the time of planting. The yield of each crop (in hundreds of tonnes per 100 ha) is given by

$$y_i = w_i - 0.1w_i^2, \quad i = 1,2,3$$

where w_i is the depth of water in centimetres received by the crop grown in the ith season. The depth of water depends on the height of water released from storage at the beginning of each season (u_i metres) and rainfall received during each season (q_i centimetres). The area of the dam is 1 ha so

$$w_i = u_i + q_i, \quad i = 1,2,3$$

The dam is full at the beginning of the first season with a water height of 3 m. The amount of water which can be released at the beginning of any season is limited to integer values of metres of water, and by the amount in storage. Rainfall augments the water in storage. The catchment area is 100 ha, so 1 cm of rainfall raises the level of the dam (x_i) by 1 m, provided the dam is not full.

It is assumed that the expected rainfall q_i is known for the next three seasons, as is the price b_i (in dollars per tonne) received for the ith season crop. The farmer would like to know the water release schedule that will maximize the revenue from the crops.

a) Provide a dynamic programming formulation to solve this problem. You should use Bellman's equations and identify the data, stages, states, actions and the transition and value functions. *[8 marks]*

b) Suppose that the expected rainfall for the next three seasons is $q = [2,1,1]$ and the price received for the ith season crop is $b = [50,100,150]$. Apply your formulation to find the optimal water release schedule. *[6 marks]*

END OF EXAMINATION