Communication 11

Sets
- C - cows
- T - time (week)

Data
- P - price of the milk from per unit of grass (\$) $= 4.2$
- R_t - units of grass required to feed the herd in week t
- $G(S_t)$ - units of grass available next week given S_t the amount at the start of week t
- S_0 - units of grass on the field at time initially $= 100$
- MF - maximum units of feed that can be converted into milk across the herd $= 40$

Stages
- Weeks - $0 \leq t \leq 51$

State
- S_t - pasture at the start of week t

Action
- $A_t = [0, \min(S_t, \mathrm{MF})]$ - extra feed to the herd on week t

Value Function

$$V_t(S_t) = \text{maximum expected income if we start week } t \text{ with } S_t \text{ pasture}$$

Base Case
- Insufficient units of pasture to meet the feeding requirement $\rightarrow$ Infeasible

$$\forall 0 \leq t \leq 51,\ S_t \leq R_t \rightarrow V_t(S_t) = -\infty$$

- End of the season, compute feed amount for week 51 maximising the profit

$$V_{51} = \max(a \times P,\ \forall a \in A_{51})$$

General Case
- explore the action space A_t to find the optimal feeding strategy that maximises the profit

$$V_t(S_t) = \max(a \times P + V_{t+1}(G(S_t) - a - R_t),\ \forall a \in A_t)$$

Communication 12

Sets

- C - cows
- T - time (week)

Data

- P - price of the milk from per unit of grass (\$) $= 4.2$
- R_t - units of grass required to feed the herd in week t
- $G(S_t)$ - units of grass available next week given S_t the amount at the start of week t
- S_0 - units of grass on the field at time initially $= 100$
- MF - maximum units of feed that can be converted into milk across the herd $= 40$
- MG - minimum units of grass before penalty is applied $= 150$
- L - penalty cost per unit under 150 (\$) $= 5$

Stages

- Weeks - $0 \leq t \leq 51$

State

- S_t - pasture at the start of week t

Action

- $A_t = [0, \min(S_t, \mathrm{MF})]$ - extra feed to the herd on week t

Value Function

$$V_t(S_t) = \text{maximum expected income if we start week } t \text{ with } S_t \text{ pasture}$$

Base Case

- Insufficient units of pasture to meet the feeding requirement $\rightarrow$ Infeasible

$$\forall 0 \leq t \leq 51,\ S_t \leq R_t \rightarrow V_t(S_t) = -\infty$$

- End of the season, compute feed amount for week 51 maximising the profit taking penalty for each unit under 150 into consideration

$$V_{51} = \max(a \times P - L \times (G(S_{51}) - a - R_{51}),\ \forall a \in A_{51}))$$

General Case

- explore the action space A_t to find the optimal feeding strategy that maximises the profit

$$V_t(S_t) = \max(a \times P + V_{t+1}(G(S_t) - a - R_t),\ \forall a \in A_t)$$

Communication 13

Sets

- C - cows
- T - time (week)

Data

- P - price of the milk from per unit of grass ($) $= 4.2$
- R_t - units of grass required to feed the herd in week t
- $G(S_t, \text{good})$ - units of grass available next week if the weather is good, given S_t pasture at the start of week t
- $G(S_t, \text{bad})$ - units of grass available next week if the weather is bad, given S_t pasture at the start of week t
- S_0 - units of grass on the field at time initially $= 100$
- MF - maximum units of feed that can be converted into milk across the herd $= 40$
- MG - minimum units of grass before penalty is applied $= 150$
- L - penalty cost per unit under 150 ($) $= 5$
- P_{good} - probability of having good weather in the region $= 0.5$

Stages

- Weeks - $0 \leq t \leq 51$

State

- S_t - pasture at the start of week t

Action

- $A_t = [0, \min(S_t, \text{MF})]$ - extra feed to the herd on week t

Value Function

$$V_t(S_t) = \text{maximum expected income if we start week } t \text{ with } S_t \text{ pasture}$$

Base Case

- Insufficient units of pasture to meet the feeding requirement $\rightarrow$ Infeasible

$$\forall 0 \leq t \leq 51,\ S_t \leq R_t \rightarrow V_t(S_t) = -\infty$$

- End of the season, compute feed amount for week 51 maximising the profit, and apply penalty for each unit under 150 taking both good and bad weather into consideration

$$V_{51}(S_{51}) = \max\big(a \times P - L \times \big(P_{\text{good}} \times (G(S_{51}, \text{good}) - a - R_{51}) + \big(1 - P_{\text{good}}\big) \times (G(S_{51}, \text{bad}) - a - R_{51})\big),\ \forall a \in A_{51}\big)$$

General Case

- explore the action space A_t to find the optimal feeding strategy that maximises the profit

$$V_t(S_t) = \max\left(a \times P + P_{\text{good}} \times V_{t+1}(G(S_t, \text{good}) - a - R_t) + \left(1 - P_{\text{good}}\right) \times V_{t+1}(G(S_t, \text{bad}) - a - R_t),\ \forall a \in A_t\right)$$

Communication 14

Sets

- C - cows
- T - time (week)

Data

- P - price of the milk from per unit of grass (\$) $= 4.2$
- R_t - units of grass required to feed the herd in week t
- $G(S_t, \text{good})$ - units of grass available next week if the weather is good, given S_t pasture at the start of week t
- $G(S_t, \text{bad})$ - units of grass available next week if the weather is bad, given S_t pasture at the start of week t
- S_0 - units of grass on the field at time initially $= 100$
- MF - maximum units of feed that can be converted into milk across the herd $= 10 \times (4 - d)$
- MG - minimum units of grass before penalty is applied $= 150$
- L - penalty cost per unit under 150 (\$) $= 5$
- P_{good} - probability of having good weather in the region $= 0.5$
- DRF - dry reduced feed in units of grass $= 3$

Stages

- Weeks - $0 \leq t \leq 51$

State

- S_t - pasture at the start of week t
- d - number of dried cows

Action

- $A_t = [0, \min(S_t, \text{MF})]$ - extra feed to the herd on week t
- $D = \{d, d + 1\}$ - dry a cow or not

Value Function

$$V_t(S_t, d) = \text{maximum expected income if we start week } t \text{ with } S_t \text{ pasture and } d \text{ cows dried}$$

Base Case

- Insufficient units of pasture to meet the feeding requirement $\rightarrow$ Infeasible
  $\forall 0 \leq t \leq 51,\ S_t \leq R_t - d \times \text{DRF} \rightarrow V_t(S_t, d) = -\infty$
- End of the season, apply penalty for each unit under 150
  - All cows dried $\rightarrow$ deterministic
    $V_{51}(S_{51}, 4) = -L \times \big(P_{\text{good}} \times (G(S_{51}, \text{good}) - R_{51} + 4 \times \text{DRF}) + \big(1 - P_{\text{good}}\big) \times (G(S_{51}, \text{bad})) - R_{51} + 4 \times \text{DRF}\big)$

- otherwise $\rightarrow$ compute feed amount for week 51 taking penalty into consideration to maximise the profit
  $V_{51}(S_{51}, d) = \max\big(a \times P - L \times \big(P_{\text{good}} \times (G(S_{51}, \text{good}) - a - R_{51} + d \times \text{DRF}) + \big(1 - P_{\text{good}}\big) \times (G(S_{51}, \text{bad}) - a - R_{51} + d \times \text{DRF})\big),\ \forall a \in A_{51})\big)$

General Case

- All cows dried $\rightarrow$ deterministic, compute to end of the season
  $V_t(S_t, 4) = P_{\text{good}} \times V_{t+1}(G(S_t, \text{good}) - a - R_t - 4 \times \text{DRF}) + \big(1 - P_{\text{good}}\big) \times V_{t+1}(G(S_t, \text{bad}) - a - R_t - 4 \times \text{DRF})$

- otherwise $\rightarrow$ explore the action space $D \times A_t$ - (dry a cow?) $\times$ (different amount of extra feed) to find the optimal strategy that maximises the profit
  $V_t(S_t, d) = \max\big(a \times P + P_{\text{good}} \times V_{t+1}(G(S_t, \text{good}) - a - R_t - d \times \text{DRF}, d') + \big(1 - P_{\text{good}}\big) \times V_{t+1}(G(S_t, \text{bad}) - a - R_t - d \times \text{DRF}, d'),\ \forall a \in A_t,\ \forall d' \in D\big)$

Communication 15

Sets

- C - cows
- T - time (week)

Data

- P - price of the milk from per unit of grass (\$) $= 4.2$
- $G(S_t, \text{good})$ - units of grass available next week if the weather is good, given S_t pasture at the start of week t
- $G(S_t, \text{bad})$ - units of grass available next week if the weather is bad, given S_t pasture at the start of week t
- S_0 - units of grass on the field at time initially $= 100$
- MF - maximum units of feed that can be converted into milk across the herd $= 10 \times \sum l$
- MG - minimum units of grass before penalty is applied $= 150$
- L - penalty cost per unit under 150 (\$) $= 5$
- P_{good} - probability of having good weather in the region $= 0.5$
- $R_t(l_t)$ - units of grass required to feed each cow in week t given lactating tuple l_t
- $l_0 = (1, 1, 1, 1)$
- $l_{\text{dried}} = (0, 0, 0, 0)$

Stages

- Weeks - $0 \leq t \leq 51$

State

- S_t - pasture at the start of week t
- l_t - 4D tuple, specify which cows are still lactating in week t

Action

- $A_t = [0, \min(S_t, \text{MF})]$ - extra feed to the herd on week t
- D_c - dry cow c if c is still lactating

Value Function

$$V_t(S_t, l_t) = \text{maximum expected income if we start week } t \text{ with } S_t \text{ pasture and lactating pattern } l_t$$

Base Case

- Insufficient units of pasture to meet the feeding requirement $\rightarrow$ Infeasible
  $\forall 0 \leq t \leq 51,\ S_t \leq R_t(l_t) \rightarrow V_t(S_t, l_t) = -\infty$
- End of the season, apply penalty for each unit under 150

- All cows dried → deterministic
  $V_{51}(S_{51}, l_{\text{dried}}) = -L \times \left(P_{\text{good}} \times (G(S_{51}, \text{good}) - R_{51}(l_{\text{dried}})) + \left(1 - P_{\text{good}}\right) \times (G(S_{51}, \text{bad})) - R_{51}(l_{\text{dried}})\right)$

- otherwise → compute feed amount for week 51 taking penalty into consideration to maximise the profit
  $V_{51}(S_{51}, l_{51}) = \max\left(a \times P - L \times \left(P_{\text{good}} \times (G(S_{51}, \text{good}) - a - R_{51}(l_{51})) + \left(1 - P_{\text{good}}\right) \times (G(S_{51}, \text{bad}) - a - R_{51}(l_{51}))\right),\ \forall a \in A_{51}\right)$

General Case

- All cows dried → deterministic, compute to end of the season
  $V_t(S_t, l_{\text{dried}}) = P_{\text{good}} \times V_{t+1}(G(S_t, \text{good}) - a - R_t(l_{\text{dried}}),\ l_{\text{dried}}) + \left(1 - P_{\text{good}}\right) \times V_{t+1}(G(S_t, \text{bad}) - a - R_t(l_{\text{dried}}),\ l_{\text{dried}})$

- otherwise → explore the action space $D_c \times A_t$ - (drying cow c) $\times$ (different amount of extra feed) to find the optimal strategy that maximises the profit
  $V_t(S_t, l_t) = \max\left(a \times P + P_{\text{good}} \times V_{t+1}(G(S_t, \text{good}) - a - R_t(l_t), l_{t+1}) + \left(1 - P_{\text{good}}\right) \times V_{t+1}(G(S_t, \text{bad}) - a - R_t(l_t), l_{t+1}),\ \forall a \in A_t,\ \forall l_{t+1} \in \{l_t, l_t \text{ with one of the 1s changed to a 0}\}\right)$