

Pt11

A new farmer is starting out in our area and needs support in managing his herd of four cows: Emma, Betty, Clarabelle and Phoebe. He has a one-hectare field for the cows to graze on and needs to plan how he should feed the cows each week for a 52-week season.

We measure the amount of grass available in the field in 10 kg units. This field currently has 100 units of grass (i.e. 1000 kg). Each week grass will grow and our modelling suggests the growth is described well by the following function:

```
def pasture(p):  
    return round(p + 0.61*p*(1-p/300))
```

For example, if there are 100 units of grass at the start of a week then there will be 141 units of grass at the start of the following week.

However, each week the farmer needs to feed some of the grass to the herd. The herd requires a minimum amount of feed units to remain healthy and this depends on the time in the season. If the weeks are numbered from 0 to 51 then the required feed is given by the following function:

```
def required(t):  
    if t < 18:  
        y = 10.088 + 0.1445*t  
    else:  
        y = 10.088 + 0.1445*18 + 0.01151*(t-18)**2  
    return round(y)
```

At the start of the season the cows have all recently calved and started lactating. The feed required is low because they are using up reserves put on during pregnancy. At the end of the season they will be pregnant again and so their energy demands increase.

The farmer can choose to feed his herd more than this required amount of feed. The cows turn this extra energy into milk. One unit of feed is converted by a cow into milk that sells for \$4.20. Each cow can turn up to 10 units of feed into milk each week.

The farmer will only consider whole numbers of units of feed. By convention, we calculate the new amount of pasture for the following week before subtracting the amount used for feed in the current week (since in reality these are happening simultaneously).

How much should the farmer feed his herd each week during the season? Please provide us with the total revenue from milk sold.

Sets

- C - cows
- T - time (week)

Data

- P - price of the milk from per unit of grass (\$)
- R_t - required grass at time t per cow (10kg)
- G_t - growth of the grass at time t (10kg)
- F_0 - grass on the field at time initially (10kg)

Variables

- X_{ct} - amount of grass feed to cow c at time t
- F_t - grass on the field at time t (10kg)

Objective function

$$\max \left(P \times \sum_{t \in T} \sum_{c \in C} (X_{ct} - R_t) \right)$$

Constraints

- Cows can't eat more than the amount of existing grass at any week

$$\forall t \in T, \sum_{c \in C} X_{ct} \leq F_t$$

- Each cow needs to eat the minimum requirement every week

$$\forall t \in T, \forall c \in C, X_{ct} \geq R_t$$

- Grass balance & non neg

$$\forall t \in T, F_{t+1} = F_t + G_t - \sum_{c \in C} X_{ct}$$

$$\forall t \in T, F_t \geq 0$$