Q1

(d)

x_2 is part of basis

$C_B = \begin{pmatrix} 2 & 1-\Delta & 0 \end{pmatrix}$

$y = \begin{pmatrix} 1 & -2 & 6 \\ -1 & 3 & -11 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 1-\Delta \\ 0 \end{pmatrix} = \begin{pmatrix} \Delta \\ 1-3\Delta \\ 0 \end{pmatrix}$

Δ has negative impact on x_2:

$$1 - 3\Delta < 0$$

$$\frac{1}{3} < \Delta$$

$\therefore, C_2 = 1 - \frac{1}{3} = \frac{2}{3}, x_2 = 0$ in the optimal solution.

Q2

Sets

- P - players
- C - cities
- S - positions

Data

- C_p - cost of a player $p \in P$
- R_p - rating of a player $p \in P$
- $H_{p,c} \in \{0, 1\}$ - player $p \in P$ is from city $c \in C$
- L_p - sqares player $p \in P$ can play
- B - budget of the team
- M - minimum number of cities which must be represented in the team
- O - maximum number of players that can be selected from any one city

Variables

$x_{p,s} \in \{0, 1\}$ - player $p \in P$ play in position $s \in S$

Objective

$\sum_{p \in P} \sum_{s \in S} x_{p,s} \times R_p$

Constraints

- within budget

$$\sum_{p \in P} \sum_{s \in S} x_{p,s} \times C_p \leq B$$

- all position filled

$$\sum_{p \in P} \sum_{s \in S} x_{p,s} = 9$$

- player plays at most one position

$$\sum_{\{s \in S\}} x_{p,s} \leq 1, \forall p \in P$$

- city reqs

$$\sum_{p \in P} \sum_{\{s \in S\}} x_{p,s} \times H_{p,c} \leq O, \forall c \in C$$

$$\sum_{p \in P} \sum_{c \in C} H_{p,c} \times \sum_{\{s \in S\}} x_{p,s} \geq M$$