Communication 5

Sets

- F - set of supplying farms
- T - set of days

Data

- W_w - wholesale price of whole milk (\$/L)
- W_l - wholesale price of low fat milk (\$/L)
- F_w - fat content of whole milk (%)
- F_l - fat content of low fat milk (%)
- C_s - cost of storage (\$/L/day)
- Dw_t - demand for whole milk each day (L)
- Dl_t - demand for low fat milk each day (L)
- S_f - supply from each farm $f \in F$ (L)
- F_f - fat content of milk from each farm $f \in F$ (%)

Variables

- x_{tf} - volume of whole milk produced from farm $f \in F$ on day $t \in T$ (L)
- y_{tf} - volume of low fat milk produced from farm $f \in F$ on day $t \in T$ (L)
- z_t - volume of whole milk stored on day $t \in T$ (L)
- w_t - volume of low fat milk stored on day $t \in T$ (L)
- a_t - total volume of whole milk sold on day $t \in T$ (L)
- b_t - total volume of low fat milk sold on day $t \in T$ (L)

Objective function

The goal of the program is to determine the volume of each variety of milk that needs to be processed from the daily supply to achieve a maximal profit, and hence to establish what this optimal profit is.

$$\max\left(\sum_{t \in D} W_w \times a_t + W_l \times b_t - (z_t + w_t) \times C_s\right)$$

Constraints

- Each day, for each of the milk varieties, the volume of sold milk cannot exceed the demand for that variety

$$a_t \leq Dw_t, \quad \forall\, t \in D$$

$$b_t \leq Dl_t, \quad \forall\, t \in D$$

- The total milk processed each day from each farm must be less than or equal to that farm's daily supply

$$x_{tf} + y_{tf} \leq S_f, \quad \forall\, f \in F,\, t \in D$$

- Each day, the cumulative fat content of processed milk must be less than or equal to the fat content of its input. It is assumed in this constraint that milk fat left over from previous days cannot be used in processing on subsequent days.

$$\sum_{f \in F} \left(F_w \times x_{tf} + F_l \times y_{tf}\right) \leq \sum_{f \in F} S_f \times Ff$$

Monday: - On mondays, for each milk variety, the volume of stored milk must equal processed milk minus sold milk

$$z_t = \left(\sum_{f \in F} x_{tf} \right) - a_t, \quad \forall\, t \in D, \quad \text{and} \quad w_t = \left(\sum_{f \in F} y_{tf} \right) - b_t, \quad \forall\, t \in D$$

Other Days: - On days other than monday, for each milk variety, the volume of stored milk must equal the sum of the processed milk from that day and stored milk from the previous day (this makes up all available milk to be sold) minus sold milk

$$z_t = \left(\sum_{f \in F} x_{tf} \right) + z_{t-1} - a_t, \quad \forall\, t \in D, \quad \text{and} \quad w_t = \left(\sum_{f \in F} y_{tf} \right) + w_{t-1} - b_t, \quad \forall\, t \in D$$

- For each milk variety, the total sold milk must be greater than or equal to the stored milk from the previous day (as this milk has to be sold and cannot remain in storage)

$$a_t = z_{t-1}, \quad \forall\, t \in D, \quad \text{and} \quad b_t = w_{t-1}, \quad \forall\, t \in D$$