Section B: Report to the client

Communication 1

Based on your initial communication, we understand you desire to determine the volume of whole and low fat milk which should be produced from your total supply to maximise income, all subject to constraints on milk fat.

Your pricing for each of the milk types - whole and low fat - indicates that we would expect low fat milk production should be maximised as it shall reap the greatest profit. However, due to the requirement that all fat of your supply is to be used, a large proportion of your supply must still be processed into whole milk. Low fat milk only contains 1% fat so limiting production only to low fat would not utilise all fat from supply.

Through mathematical modelling, it was determined that to optimise profit, 35316.67L of whole milk and 5183.33L of low fat milk should be processed from the supply of the five farms. This split of whole and low fat milk will result in an optimal income of \$44653.67.

Communication 2

In response to Haven and Silo Springs converting to organic production and the inclusion of your new organic products, the model we provided previously has been revised to include additional constraints.

In this revised model, organic milks will now achieve the greatest profit and so, it is inferred that these products will be maximised by the model.

In our model, have ensured that all organic products are processed only from the supply of the organic suppliers - Haven and Silo Springs - to guarantee the quality of the product. Based on your communication with us, it was also specified that the fat content of organic product is equal to the fat content of organic supply, and similarly for non-organic product and supply.

With the addition of these new constraints, our mathematical formulations suggest you process 14363.33L of whole organic milk, 1936.67L of low fat organic milk, 20953.33L of whole milk and 3246.67L of low fat milk from your total supply. This results in an overall income of \$47913.67. With the inclusion of organic productions, your projected income has increased by \$3260.

Communication 3

Thank you for bringing these marketing restrictions to our attention. In response to these restrictions, we have once again revised our model and hope it will suit your needs.

We have added constraints on the percentage of production which can be low fat for each of organic and non-organic products. Further, it is constrained that no more that 15% of production is organic so that we are following market trends as required.

Based on our model, the optimal income from the given supply under current regulation is \$45967.50. The following table is a plan that would achieve such income.

Table1.1.1

	Barnwood	Thistlebrook	Rustic Ranch	Haven	Silo Springs	*Total*
Organic Whole Milk	0	0	0	0	4556.25	4556.25
Organic Low Fat Milk	0	0	0	1518.75	0	1518.75
Standard Whole Milk	6293.75	0	9300	7681.25	2543.75	25818.75
Standard Low Fat Milk	3306.25	5300	0	0	0	8606.25

The revised model suggests that from the total supply, 4556.25L of whole organic milk, 1518.75L of low fat organic milk, 25818.75L of whole milk and 8606.25L of low fat milk is processed.

Sensitivity Analysis

Due to stringent regulations, a significant portion of organic milk supply is diverted for the production of non-organic milk, thereby compromising overall income for producers. Total income would increase \$81 per 1% up until 40.25% (23.37%) to reach the maximal value of \$48012.50 (\$46645.47)

To mitigate this issue, actions can be taken to loosen the regulations, such as advocating for an increase in the percentage of organic milk allowed and collaborating with decision-makers. Additionally, promoting organic farms could bolster support for organic milk production.

We do not have information regarding the cost of producing organic VS non organic milk. If the cost is higher for organic milk, it is advisable to reduce the organic supply under current regulation - organic products can make up at most 15% of all milk sold.

Communication 4

We are pleased to hear of your increased revenue and have devised a new model to optimise your operations based on your new suppliers and the changed requirements of your operation.

In this new model, as requested, we have removed all constraints regarding organic products and focus only on standard whole milk and low fat milk. In line with the new information regarding demand for the products, we have implemented a constraint to ensure that the volume of sold milk of each type does not exceed demand on any given day.

Similarly to out previous responses, the percentage fat in the product is less than or equal to the percentage fat of supply to reflect what is realistically possible in such a scenario.

The wholesale price of low fat milk is greater than the wholesale price of whole milk, so it is intuitive that the solution maximises low fat milk production and this is reflected in the results.

As there is no limit on production apart from the limit on supply from each farm, the final solution reflects that demand is met on each day.

Out devised plan using mathematical modelling is as follows:

Day	Category	Sold(L)	Demand(L)	Day	Catogory	Amount(L)
Mon	Whole	13778	13778	Mon	Whole	18774.33
	Low Fat	3485	3485		Low Fat	10462.6
Tue	Whole	27488	27488	Tue	Whole	31039.67
	Low Fat	6896	6896		Low Fat	10313.33
Wed	Whole	68427	68427	Wed	Whole	2366
	Low Fat	17060	17060		Low Fat	0
Thu	Whole	13740	13740	Thu	Whole	25826
	Low Fat	3543	3543		Low Fat	5757
Fri	Whole	27428	27428	Fri	Whole	37218.67
	Low Fat	6756	6756		Low Fat	678.3
Sat	Whole	27519	27519	Sat	Whole	49453
	Low Fat	6794	6794		Low Fat	6633
Sun	Whole	81853	82193	Sun	Whole	0
	Low Fat	20733	20733		Low Fat	0

Communication 5

We recognise the importance of this constraint on duration of milk storage and have once again updated out model to reflect this requirement.

To address the constraint, we have enforced that each day the cumulative sold milk must include any milk stored from the previous day. This ensures that there will never be milk in storage for more than one day, and no stored milk goes to waste.

Using this new model, the optimised plan for your milk processing over the next seven days, split into milk to sell and to store, is as follows:

Day	Category	Sold(L)	Demand(L)	Day	Catogory	Amount(L)
Mon	Whole	13778	13778	*Mon*	Whole	19975
	Low Fat	3485	3485		Low Fat	6896
Tue	Whole	27488	27488	*Tue*	Whole	32240.33
	Low Fat	6896	6896		Low Fat	6746.67
Wed	Whole	68427	68427	*Wed*	Whole	0
	Low Fat	17060	17060		Low Fat	0
Thu	Whole	13740	13740	*Thu*	Whole	15241
	Low Fat	3543	3543		Low Fat	6756
Fri	Whole	27428	27428	*Fri*	Whole	27519
	Low Fat	6756	6756		Low Fat	6794
Sat	Whole	27519	27519	*Sat*	Whole	39753.33
	Low Fat	6794	6794		Low Fat	6746.67
Sun	Whole	72267	82193	*Sun*	Whole	0
	Low Fat	20733	20733		Low Fat	0

The total weekly income you can expect using this plan is $277194.54. This income is based on 250647L of whole milk and 65267L of low fat milk being sold in total over the week. As shown above, with this plan, milk demands for each variety are met every day except Sunday which should be a source of satisfaction for customers.

In addition to creating this model, we have perform some analysis on the data to provide insight into potential opportunities for growth. Our analysis has shown that on each day of the week except sunday, demand for both whole milk and low fat milk is being met. This is restricting profit as it is assumed in the model that demand cannot be exceeded. Thus, an oppotunity for growth exists here. Through implementing marketing strategies it may be possible to increase demand for the products, thus allowing more sales.

An opportunity for growth exists in increasing the supply from each of the farms. For example, the daily supply from Fresh Pail is currently 9300L. For every litre of milk added to this supply, an extra $0.90 can be earned on a Sunday, $0.85 on a Saturday, $0.10 on a Wednesday and $0.05 on either a Tuesday or Friday. However, it should be noted that this is not an unbounded constraint and after a certain volume of supply, this increase in profit is no longer quaranteed. The bound changes depending on the day.

In consideration of the constraint on fat percentage, each day there is left over fat from supply which is not utilised - displayed in the slack variables of the model. It is recommended that this excess fat is stored and put to use in other capacities to eliminate waste.