

Sets

- T - time (month)

Data

- S - goal
- n - interest (month)
- I_t - income in month t
- E_t - essential spending in month t
- m - number of month to achieve the goal

Stages

- Months - $0 \leq t \leq m$

State

- S_t - amount left to save at the start of month t

Action

- $X_t = [0, \min(S_t, I_t - E_t)]$ - amount put into saving month t

Value Function

$V_t(S_t)$ = minimum amount need to save in month t to achieve the goal on time

Base Case

- No longer possible

$$\forall t, S_t > \sum_t^m (I_t - E_t) \times (1 + n)^{m-t} < \text{goal} \rightarrow V_t(S_t) = \infty$$

- Due date of the goal

$$V_m = S_m$$

General Case

- explore the action space X_t to find the optimal saving strategy that minimises the total saving amount

$$V_t(S_t) = \min(x + V_{t+1}(S_t - x \times (1 + n)^{m-t}), \forall x \in X_t)$$