Master 2 Econometrics and Empirical Economics
Financial Econometrics:
Homework # 1 Part I

You can do the work in a team of two persons.

1. **Descriptive statistics of financial data.** Consider the attached file. It contains the daily returns of Spyder and realized volatility (third column). In the sequel, study the two series.

   (a) Provide the following sample statistics: mean, variance, skewness, kurtosis, ACF at several lags, and the ACF of the squared returns. Comment the results.

   (b) Provide graphs of the rolling means, variances, skewness, kurtosis. Consider several windows and provide the best graphs (the window should be the same for a given asset). Comment the results.

2. **GARCH models**. Consider the model

$$R_t = \mu + \sigma_t z_t,\ z_t\ i.i.d.\ D(0,1),\ \sigma_t^2 = \omega + \alpha(R_{t-1} - \theta\sigma_{t-1})^2 + \beta\sigma_{t-1}^2.$$

   We denote $\psi = (\mu, \omega, \alpha, \beta, \theta)$. Simulate the following designs: 1) $\psi = (0; 0.01; 0.05; 0.90; 0)$, $z_t \sim \mathcal{N}(0,1)$, 2) $\psi = (0; 0.01; 0.05; 0.6; 2)$, $z_t \sim \mathcal{N}(0,1)$, with $T = 250$ and $T = 1,000$. (It is always useful to remove the first observations of the simulated model; here remove the 250 observations, meaning that when you want a sample size T, simulate T+250 observations and remove the first 250 observations). For each design, simulate at least 100 replications (ideally 1,000 replications).

   (a) For each design and each sample size, estimate ψ by the ML method. Provide the simulation mean of ψ and their standard deviation (for a given sample, you will estimate a unique ψ; then compute the mean of the estimator across the replications). Comment the results.

   (b) Consider again the same designs but assume that z_t is a standardized $T(10)$ (standardized means that the variance is one). I remind you that the definition of a T(n) is $T(n) = X/\sqrt{Y/n}$, $X \sim \mathcal{N}(0,1)$, $Y \sim \chi^2(n)$, X and Y are independent. Do the same work as in the previous question a). Comment the results and compare with the results in a).

(c) Estimate the GARCH model (with and without leverage effect) by using the Spyder daily returns. Test the presence of leverage effect. Provide the diagnostics (in sample and out-of-sample ones).

(d) Compare the GARCH variance with the riskmetriks variance and realized volatility.

3. **Forecast comparison of GARCH and RV.** Compare the forecasts of GARCH and realized variance. You should find the best AR process that describes well the realized variance data.

4. **Value-at-Risk.** Consider again the same Spyder daily return. You have two models (GARCH and GARCH with leverage). Consider also the i.i.d. normal model. If you are able to compute the MLE with student error, do it. So, at the end you have three (or six models if you are able to compute the Student MLEs) for each sample. For each model, compute the 5% conditional VaR. Derive the sample of violations of the VaR. Do the three tests studied in Lecture 3. Comment the results.