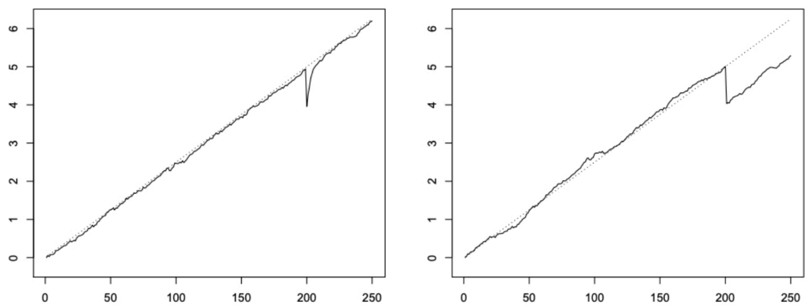
M2 Time Series: Take-Home Exam (Total 50 points, Due date: April 11, 3pm)

Instruction: Hand-writing is not accepted. You don't have to put your code, but please describe briefly how you proceeded your simulation and empirical studies. Your report does not look like an academic paper, but please make it as if I were your boss at your internship.

1. (Multiple Choice, 4 points) Consider the following two sample paths.



One is generated from a trend stationary process, whereas the other one is generated from a unit root with drift.

a. Which one is generated from a trend stationary process?
   (1) Left
   (2) Right
   (3) Both

b. Consider the time series on the left panel. You want to test the null hypothesis of unit root using ADF test. Which test equation is a proper one?
   (1) $X_t = \alpha X_{t-1} + u_t$
   (2) $X_t = c_0 + \alpha X_{t-1} + u_t$
   (3) $X_t = c_0 + c_1 t + \alpha X_{t-1} + u_t$

2. (Multiple Choice, 4 points) Let $X_t, Y_t \sim I(1)$, and consider the following nonstationary regression model

$$Y_t = \beta X_t + u_t.$$

For the regression error process u_t, we consider the following two conditions

$$\text{Condition 1: } u_t = 0.5u_{t-1} + 0.5u_{t-2} + \varepsilon_t,$$
$$\text{Condition 2: } u_t = 0.5u_{t-1} - 0.5u_{t-2} + \varepsilon_t,$$

where $\varepsilon_t \sim iid(0, \sigma^2)$. Let $\hat{\beta}$ be the OLS estimator for β. Find ALL correct statements (choose up to 4 answers).

(1) Under Condition 1, Y_t and X_t are cointegrated.

(2) Under Condition 2, Y_t and X_t are cointegrated.

(3) Under Condition 1, the regression is spurious.

(4) Under Condition 2, the regression is spurious.

(5) Under Condition 1, $\hat{\beta}$ is super consistent, i.e., $\hat{\beta} - \beta = O_p(1/T)$.

(6) Under Condition 2, $\hat{\beta}$ is super consistent, i.e., $\hat{\beta} - \beta = O_p(1/T)$.

(7) Under Condition 1, $\hat{\beta}$ is $\sqrt{T}$-consistent, i.e., $\hat{\beta} - \beta = O_p(1/\sqrt{T})$, not super consistent.

(8) Under Condition 2, $\hat{\beta}$ is $\sqrt{T}$-consistent, i.e., $\hat{\beta} - \beta = O_p(1/\sqrt{T})$, not super consistent.

(9) Under Condition 1, $\hat{\beta}$ is inconsistent, i.e., $\hat{\beta} \not\to_p \beta$.

(10) Under Condition 2, $\hat{\beta}$ is inconsistent, i.e., $\hat{\beta} \not\to_p \beta$.

3. (Short Answer; Multiple Choice, 6 points) Consider bivariate SF-VAR(1) models

$$\text{Model 1:} \begin{bmatrix} 1 & 0 \\ 0.2 & 1 \end{bmatrix} \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.5 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \quad \Sigma_\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\text{Model 2:} \begin{bmatrix} 1 & 0 \\ -0.2 & 1 \end{bmatrix} \begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.5 \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}, \quad \Sigma_\varepsilon = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

For the above VARs, (y_i) does not Granger-cause (y_j) for $i, j = 1, 2$ and $i \neq j$ if

$$\mathbb{E}[y_{j,t+1}|y_{1,t}, y_{2,t}] = \mathbb{E}[y_{j,t+1}|y_{j,t}].$$

a. Let $e_t = (e_{1,t}, e_{2,t})'$ be reduced form errors of the RF-VAR(1) corresponding to the SF-VAR(1) above. Find $cov(e_{1,t}, e_{2,t})$ for each model.

b. Find ALL correct statements about Model 1 (choose up to 2 answers).

(1) (y_1) does not Granger-cause (y_2).

(2) (y_2) does not Granger-cause (y_1).

(3) (y_1) Granger-causes (y_2).

(4) (y_2) Granger-causes (y_1).

c. Find ALL correct statements about Model 2 (choose up to 2 answers).

(1) (y_1) does not Granger-cause (y_2).

(2) (y_2) does not Granger-cause (y_1).

(3) (y_1) Granger-causes (y_2).

(4) (y_2) Granger-causes (y_1).

4. (Computer Exercise, 10 points) Obtain quarterly Real Personal Consumption Expenditures for the 1955Q1-2018Q4 sample, available on FRED under code `PCECC96`.

Construct the log changes in the Real Personal Consumption Expenditures $\Delta \log c_t = \log c_t - \log c_{t-1}$ where c_t is the original quarterly Real Personal Consumption Expenditures.

(a) Split the sample into two parts: first one up to 2008Q4, second one from 2009Q1 onward. Use `auto.arima` with `ic = aic` and `stationary = TRUE`, `stepwise = FALSE`, `approximation = FALSE` to find the best model. Check the estimated model for adequacy, diagnose residuals using `ggtsdiag`.

(b) Use the estimated model with `forecast` to generate 1 to 36 step ahead forecast for the prediction subsample, 2009Q1-2018Q4.

(c) Use `slide` from the `tsibble` package to generate a rolling scheme forecast, in particular a sequence of 1 period ahead forecasts for the prediction subsample, 2009Q1-2018Q4.

(d) Plot the multistep forecast and the 1 step ahead rolling forecasts, with their confidence intervals.

(e) Use `accuracy` to evaluate the out of sample accuracy of the two sets of forecasts.

5. (Computer Exercise, 20 points) Obtain monthly data for Total Nonfarm Payroll Employment for the 1975M1-2018M12 sample (Not Seasonally Adjusted) available on FRED under code `PAYNSA`.

(a) Construct the following transformed time series

- change in Total Nonfarm Payroll Employment $\Delta E_t = E_t - E_{t-1}$
- log of Total Nonfarm Payroll Employment $\log E_t$
- log change in Total Nonfarm Payroll Employment $\Delta \log E_t = \log E_t - \log E_{t-1}$
- 12 month log change in Total Nonfarm Payroll Employment $\Delta_{12} \log E_t = \log E_t - \log E_{t-12}$
- twice differenced Total Nonfarm Payroll Employment $\Delta\Delta_{12} \log E_t = \Delta_{12} \log E_t - \Delta_{12} \log E_{t-1}$

Plot the original and the transformed time series. Comment on their trends, volatility, seasonal patterns.

(b) Use `ggseasonplot` to create seasonal plots for ΔE_t and $\Delta \log E_t$. Comment on the seasonal patterns.

(c) Plot ACF and PACF for $\log E_t, \Delta \log E_t, \Delta_{12} \log E_t, \Delta\Delta_{12} \log E_t$. Comment on their shape.

(d) Perform the ADF and KPSS tests on $\log E_t, \Delta_{12} \log E_t, \Delta\Delta_{12} \log E_t$. Summarize the results.

(e) Split the sample into two parts: estimation sample from 1975M1 to 2014M12, and prediction sample from 2015M1 to 2018M12. Use ACF and PACF from (c) to identify and estimate a suitable model for $\Delta\Delta_{12} \log E_t$ using `arima`. Check the estimated model for adequacy - diagnose residuals using `ggtsdiag`.

(f) Use `auto.arima` to find the best model for $\log E_t$. Check the estimated model for adequacy - diagnose residuals using `ggtsdiag`.

(g) Use `slide` from `tsibble` package to create a rolling scheme sequence of 1 period ahead forecasts for the prediction subsample 2015M1-2018M12 using the same model specification as in (f).

(h) Plot the forecast for E_t from (g) together with its confidence intervals and the actual data for the period 2008M1-2018M12.

(i) Use the forecast for E_t from (g) to construct the forecast for ΔE_t, plot it together with the actual data.

(j) Construct and plot the forecast errors for E_t and for ΔE_t.

6. (Computer Exercise, 6 points) The response of hours worked to different shocks has been studied extensively since Gali (1999), who argued that hours worked show a decline in response to a positive technology shock. In this problem, you will replicate some of his results.

   Obtain the following two quarterly time series for the period 1947Q1-2017Q4 from FRED: labor productivity, measured as Nonfarm Business Sector: Real Output Per Hour of All Persons OPHNFB and for total hours worked, measured as Nonfarm Business Sector: Hours of All Persons HOANBS.

   (a) Test the log of real output per hour $y_{1,t} = \log OPHNFB_t$ and the log of hours $y_{2,t} = \log HOANBS_t$ for the presence of unit root using ADF test. Afterwards apply the ADF unit root test also to the first differences, $\Delta y_{1,t}$ and $\Delta y_{2,t}$. Comment on results.

   (b) Estimate a bivariate reduced form VAR for $y_t = (\Delta y_{1,t}, \Delta y_{2,t})'$, using AIC information criteria to select number of lags.