TP2 MachineLearning BIM 1part FEI

December 3, 2020

Emotion Recognition based on facial landmarks

This part of the practical session is about **emotion recognition** based on facial landmarks. We will use the FEI dataset (https://fei.edu.br/~cet/facedatabase.html) to recognize the emotion of a person by analyzing 68 facial landmarks (already estimated and placed). Below, you will find a picture with an example. We will focus on two emotions neutral and happy.

Deadline: Upload this notebook and the one about Toy Examples as a single .zip file to the Moodle. You have one week.

Otherwise, you can also load them from your local machine using the following code

Let's load the Python packages containing the functions needed for the practical session.

```
[3]: import numpy as np
     from time import time
     import itertools
     from sklearn.model_selection import train_test_split
     from sklearn.metrics.pairwise import paired_distances
     from sklearn.model_selection import cross_val_score, cross_validate,_
     →GridSearchCV, KFold, StratifiedKFold
     from sklearn.metrics import classification_report
     from sklearn.utils.multiclass import unique_labels
     from sklearn.metrics import confusion_matrix
     from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
     from sklearn.discriminant_analysis import QuadraticDiscriminantAnalysis
     from sklearn.naive_bayes import GaussianNB
     from sklearn.linear_model import LogisticRegression
     from sklearn.neighbors import KNeighborsClassifier
     from sklearn import decomposition
     from sklearn.preprocessing import StandardScaler, MinMaxScaler
     import matplotlib.pyplot as plt
     # this is needed to plot figures within the notebook
     %matplotlib inline
     np.random.seed(seed=666)
     import warnings
     warnings.filterwarnings("ignore", category=DeprecationWarning)
     warnings.simplefilter(action='ignore', category=FutureWarning)
     from sklearn.exceptions import ConvergenceWarning
     warnings.filterwarnings(action='ignore', category=ConvergenceWarning)
```

We also load a user-defined function useful for plotting the confusion matrix

```
print(cm)
fig, ax = plt.subplots()
im = ax.imshow(cm, interpolation='nearest', cmap=cmap)
ax.figure.colorbar(im, ax=ax)
ax.set(xticks=np.arange(cm.shape[1]),
       yticks=np.arange(cm.shape[0]),
       xticklabels=classes, yticklabels=classes,
       title=title.
       ylabel='True label',
       xlabel='Predicted label')
# Rotate the tick labels and set their alignment.
plt.setp(ax.get_xticklabels(), rotation=45, ha="right",
        rotation_mode="anchor")
# Loop over data dimensions and create text annotations.
fmt = '.2f' if normalize else 'd'
thresh = cm.max() / 2.
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, format(cm[i, j], fmt),
                ha="center", va="center",
                color="white" if cm[i, j] > thresh else "black")
fig.tight_layout()
return ax
```

Now, let's load the data

e have a list of images, the position of the original landmarks (aligned to the images), the position of the landmarks after a normalization process called Generalized Procrustes Analysis (please refer to https://en.wikipedia.org/wiki/Generalized_Procrustes_analysis), the outputs with the class labels and the names of the images.

Generalized Procrustes Analysis (GPA) is used to keep only shape differences between the configurations of landmarks. That is to say, we align all configurations to an average one using only rigid transformations (uniform scaling, rotation and translation). This means that if I take a facial picture of subject A, then step back, translate and rotate a bit the camera and retake a facial picture of the same subject (who has not moved) the two picture will be different with therefore different landmark position. However, after a GPA, the two landmark positions should be perfectly aligned removing the "nuisance" differences related to rotation, translation and uniform scaling.

```
[5]: # Parameters
dim=2 # dimension
# Loading data
```

```
with np.load(Working_directory + 'Data_FEI.npz') as data:
    Images=data['Images_FEI'] # list of images
    X = data['Landmarks_FEI'] # original landmarks
    XGPA = data['Landmarks_FEI_GPA'] # landmarks after GPA (Generalized_
    →Prcrustes Analysis, https://en.wikipedia.org/wiki/
    →Generalized_Procrustes_analysis)
    Y = data['Emotions_FEI'] # class, O for neutral and 1 for happy
    Names = data['Names_FEI']
N,M = X.shape # number subjects
M = int(M/2) # Number of landmarks (they are in 2D)
print('Number of subjects:', N, '; Number of landmarks:',M)
class_names = ["neutral", "happy"]
```

Number of subjects: 400; Number of landmarks: 68

Here, we show an example of facial landmarks

```
[6]: # Plot the facial landmarks

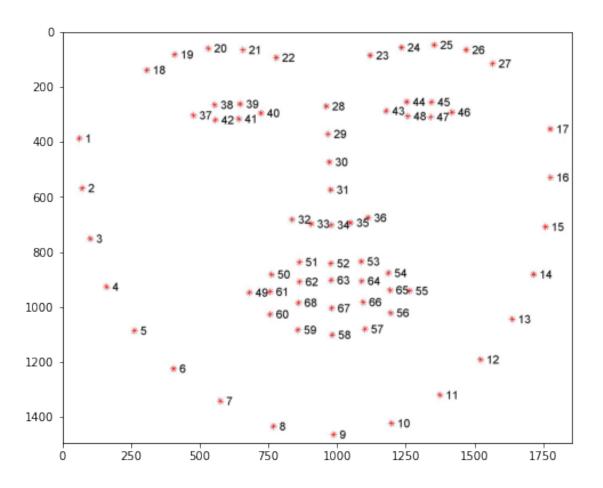
Example=plt.imread(Working_directory + 'facial_landmarks_68markup.jpg') #

Junction to read a jpg image

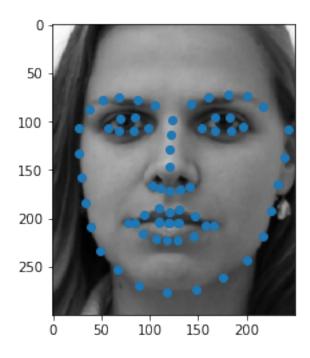
plt.figure(figsize = (8,8)) # Size of the plot

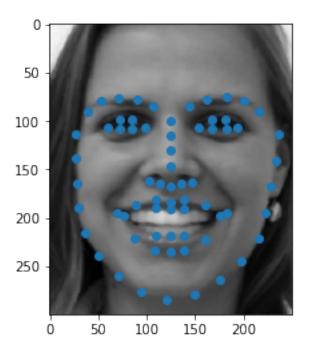
plt.imshow(Example)

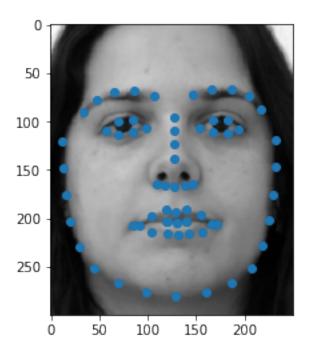
plt.show()
```

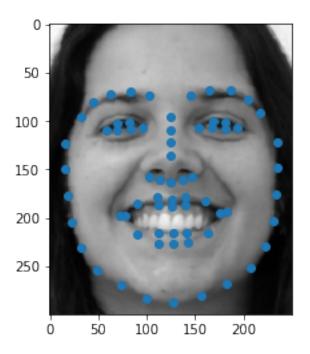


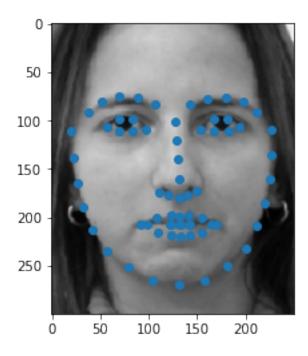
```
[7]: # plot the first 6 images of the data-set
for i in range(0,6):
    image = Images[i,:,:]
    plt.figure()
    plt.imshow(image, cmap='gray', origin='upper')
    landmark=X[i,:]
    x=landmark[::2]
    y=landmark[1::2]
    plt.plot(x,y,'o')
    plt.show()
```

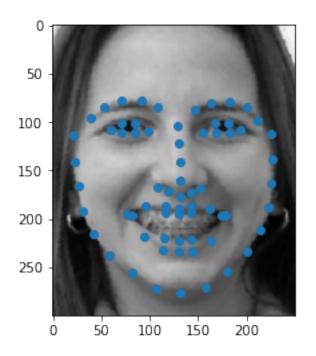












Question (IMP+IMH): after plotting the first 6 images of the data-set, what do you notice? Do you notice a regular pattern? Do you think that it would be worth it to randomly shuffle the data?

Answer:

Yes we do need to shuffle since we see its alternate between with smile and without a smile and we might get a biased algorithm.

```
[8]: # Shuffle data randomly
indeces=np.arange(N) # Integers from 0 to N-1
#print(indeces)

#Hint: Use np.random.shuffle
np.random.shuffle(indeces)
#print(indeces)

XpGPA=XGPA[indeces]
Xp=X[indeces]
Yp=Y[indeces]
Imagesp=Images[indeces]
Xmean = np.mean(XpGPA,axis=0) # Compute average

Namesp=[''] * N
for i in range(0,N):
    Namesp[i]=Names[indeces[i]]
```

Among the loaded data, we also have aligned landmarks after a Generalized Procrustes Analysis. Let's check them and compare them with the landmarks before alignement.

QUESTION (IMP+IMH): Please comment the results. What can you notice?

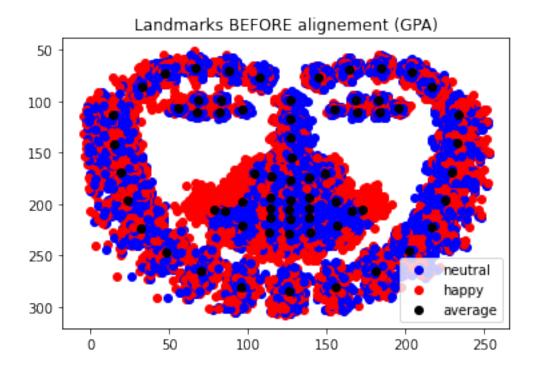
Answer:

We can see that after the GPA we get much clearer "face" with smile.

```
[9]: # Plot all landmarks BEFORE GPA
plt.figure()
for i in range(0,N):
    landmark=Xp[i]
    x=landmark[::2]
    y=landmark[1::2]
    if Yp[i].astype(int)==0:
        neutral=plt.scatter(x, y, c='b')
    else:
        happy=plt.scatter(x, y, c='r')
Xaverage = np.mean(Xp,axis=0) # Compute average
average=plt.scatter(Xaverage[::2],Xaverage[1::2],color='k')
plt.legend((neutral,happy,average),('neutral','happy','average'))
```

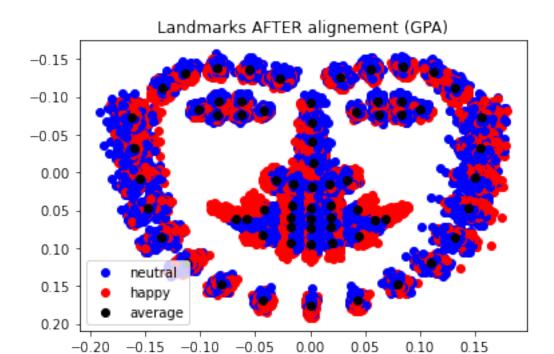
```
plt.gca().invert_yaxis()
plt.title('Landmarks BEFORE alignement (GPA)')
```

[9]: Text(0.5, 1.0, 'Landmarks BEFORE alignement (GPA)')



```
[10]: # Plot all landmarks AFTER GPA
plt.figure()
for i in range(0,N):
    landmark=XpGPA[i]
    x=landmark[::2]
    y=landmark[1::2]
    if Yp[i].astype(int)==0:
        neutral=plt.scatter(x, y, c='b')
    else:
        happy=plt.scatter(x, y, c='r')
    average=plt.scatter(Xmean[::2],Xmean[1::2],color='k')
    plt.legend((neutral,happy,average),('neutral','happy','average'))
    plt.gca().invert_yaxis()
    plt.title('Landmarks AFTER alignement (GPA)')
```

[10]: Text(0.5, 1.0, 'Landmarks AFTER alignement (GPA)')



We need now to compute some features for the classification algorithms. As first idea, we could use the paired Euclidean distances between the landmarks of every subject and the landmarks of the average configuration.

```
[11]: # Compute distances from the average configuration (features)

dist_average=np.zeros((N,M))
  average=np.reshape(Xmean,(M,2)) # Reshape average as matrix
#print(Xmean.shape)
#print(average.shape)

for i in range(N):
    landmark=np.reshape(XpGPA[i],(M,2)) # Reshape all landmarks as matrices
    dist_average[i]=paired_distances(landmark, average)

print('Number of subjects N is: ', dist_average.shape[0], '; number of_
    →features is: ', dist_average.shape[1] )
```

Number of subjects N is: 400 ; number of features is:

Question (IMP+IMH): One usual question in Machine Learning is, do we need to

scale/normalize the features? What do you think? Should we do it in this case? Compute both scaled and normalized data.

Answer:

I wouldent scale or normalize the features since all of them are in the same scale already and we wont normalize it since we wont want to "loss" the meaning of the diffrence between the values.

```
[12]: # Scale data (each feature will have average equal to 0 and unit variance)
      # https://scikit-learn.org/stable/modules/generated/sklearn.preprocessing.
      \rightarrow StandardScaler.html
      scaler = StandardScaler()
      scaler.fit(dist_average)
      dist_average_scale=scaler.transform(dist_average)
      print('Scaler')
      print('Number of subjects N is: ', dist_average_scale.shape[0], '; number of_
       →features is: ', dist_average_scale.shape[1] )
      # Normalize data (each feature will be scaled into the range 0,1)
      # https://scikit-learn.org/stable/modules/generated/sklearn.preprocessing.
       \hookrightarrow MinMaxScaler.html
      normalizer = MinMaxScaler()
      normalizer.fit(dist average)
      dist_average_normalize=normalizer.transform(dist_average)
      print('Normalizer')
      print('Number of subjects N is: ', dist_average_normalize.shape[0], '; number_u
       →of features is: ', dist_average_normalize.shape[1] )
```

Scaler

```
Number of subjects N is: 400 ; number of features is: 68 Normalizer
Number of subjects N is: 400 ; number of features is: 68
```

Let's divide the data-set into Training and Test sets using original, scaled and normalized data.

Let's try to fit LDA to all training sets and predict the error on their respective test sets.

Question (IMP+IMH): Compare the performnces between original, scaled and normalized data. Comment the results.

Answer:

We can see that indeed there is no sense to normalize and scale the data because the result stays the same with the normalize and scale or without it.

```
[14]: # Fitting LDA to original data
print("Fitting LDA to training set")
t0 = time()
lda = LinearDiscriminantAnalysis()
lda.fit(X_train, y_train)
y_pred = lda.predict(X_test)
print("done in %0.3fs" % (time() - t0))
print(classification_report(y_test, y_pred))

# Compute confusion matrix
cnf_matrix = confusion_matrix(y_test, y_pred)

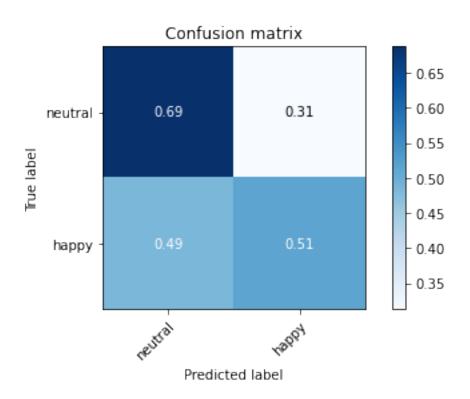
# Plot normalized confusion matrix
plt.figure()
plot_confusion_matrix(cnf_matrix, classes=class_names, normalize=True)
plt.show()
```

Fitting LDA to training set done in 0.006s

precision recall f1	l-score support
0 0.57 0.69	0.62 64
1 0.64 0.51	0.57 68
.cy	0.60 132
vg 0.60 0.60	0.60 132
vg 0.60 0.60	0.60 132

```
Normalized confusion matrix [[0.6875 0.3125 ] [0.48529412 0.51470588]]
```

<Figure size 432x288 with 0 Axes>



```
[15]: # Fitting LDA to scaled data
print("Fitting LDA to scaled dataset")
t0 = time()
lda = LinearDiscriminantAnalysis()
lda.fit(X_train_scale, y_train_scale)
y_pred_scale = lda.predict(X_test_scale)
print("done in %0.3fs" % (time() - t0))
print(classification_report(y_test_scale, y_pred_scale))

# Compute confusion matrix
cnf_matrix_scale = confusion_matrix(y_test_scale, y_pred_scale)

# Plot normalized confusion matrix
plt.figure()
plot_confusion_matrix(cnf_matrix_scale, classes=class_names, normalize=True)
plt.show()
```

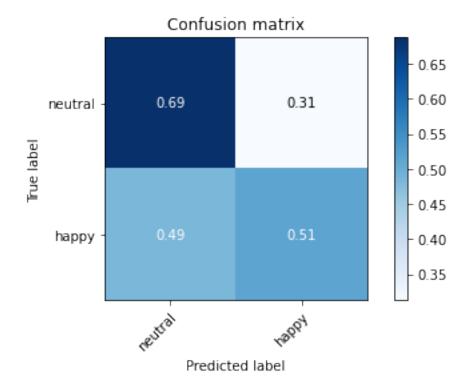
Fitting LDA to scaled dataset done in 0.007s

p:	recision	recall	f1-score	support
0	0.57	0.69	0.62	64
1	0.64	0.51	0.57	68

accuracy			0.60	132
macro avg	0.60	0.60	0.60	132
weighted avg	0.60	0.60	0.60	132

Normalized confusion matrix [[0.6875 0.3125] [0.48529412 0.51470588]]

<Figure size 432x288 with 0 Axes>



```
[16]: # Fitting LDA to normalized data
print("Fitting LDA to normalized dataset")
t0 = time()
lda_normalize = LinearDiscriminantAnalysis()
lda_normalize.fit(X_train_normalize, y_train_normalize)
y_pred_normalize = lda_normalize.predict(X_test_normalize)
print("done in %0.3fs" % (time() - t0))
print(classification_report(y_test_normalize, y_pred_normalize))

# Compute confusion matrix
cnf_matrix_normalize = confusion_matrix(y_test_normalize, y_pred_normalize)

# Plot normalized confusion matrix
plt.figure()
```

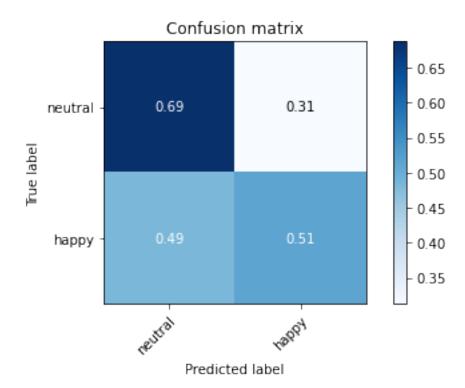
plot_confusion_matrix(cnf_matrix_normalize, classes=class_names, normalize=True)
plt.show()

Fitting LDA to normalized dataset done in 0.006s

	precision	recall	f1-score	support
0	0.57	0.69	0.62	64
1	0.64	0.51	0.57	68
			0.00	400
accuracy			0.60	132
macro avg	0.60	0.60	0.60	132
weighted avg	0.60	0.60	0.60	132

Normalized confusion matrix

<Figure size 432x288 with 0 Axes>



We can then use the function 'cross_val_score' to compute the CV score. Let's use all methods seen today.

Question (IMP+IMH):

- 1. Analyze the code and explain what it does.
- 2. compare the performances between original, scaled and normalized data

Answers:

- 1. the code takes different subsets of training and test (cv=5 so that means 5 different subsets) and gives the accuracy of each subset using the model that was defined for him. we get 5 different models for each method and therefor 5 accuracies.
- 2. we can see again and reasure that we do not need to sacling and normlize this data

```
[17]: # Cross-validation for Model Assessment
      # raw data
      print('raw data')
      # Fitting LDA
      print("Fitting LDA")
      t0 = time()
      lda = LinearDiscriminantAnalysis()
      lda_score = cross_val_score(lda,X=dist_average, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(lda_score.mean(),__
      →lda score.std() ))
      # Fitting QDA
      print("Fitting QDA")
      t0 = time()
      qda = QuadraticDiscriminantAnalysis()
      qda_score = cross_val_score(qda, X=dist_average, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(qda_score.mean(),__
      →qda_score.std() ))
      # Fitting Logistic-regression
      print("Fitting Logistic Regression")
      t0 = time()
      logit = LogisticRegression(solver='lbfgs')
      logit_score = cross_val_score(logit,X=dist_average, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(),
       →logit_score.std() ))
      # Fitting Naive-Bayes
      print("Fitting Naive-Bayes")
      t0 = time()
      GNB = GaussianNB()
      GNB_score = cross_val_score(GNB,X=dist_average, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
```

```
print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(),
      →GNB score.std() ))
     # Fitting K-nearest neighbour
     print("Fitting K-nearest neighbour")
     t0 = time()
     neigh = KNeighborsClassifier(n_neighbors=3)
     neigh_score = cross_val_score(neigh,X=dist_average, y=np.ravel(Yp),cv=5)
     print("done in %0.3fs" % (time() - t0))
     print(" Average and std CV score : {0} +- {1}".format(neigh_score.mean(), __
      →neigh_score.std() ))
     raw data
     Fitting LDA
     done in 0.033s
     Average and std CV score: 0.55749999999999 +- 0.045138675213169485
     Fitting QDA
     done in 0.017s
      Average and std CV score: 0.5625 +- 0.044721359549995794
     Fitting Logistic Regression
     done in 0.023s
      Average and std CV score: 0.53499999999999999999 +- 0.0483476990145343
     Fitting Naive-Bayes
     done in 0.011s
      Average and std CV score: 0.5725 +- 0.058843011479699094
     Fitting K-nearest neighbour
     done in 0.043s
     [18]: # Cross-validation for Model Assessment
     # normalized data
     print('normalized data')
     # Fitting LDA
     print("Fitting LDA")
     t0 = time()
     lda = LinearDiscriminantAnalysis()
     lda score = cross val score(lda,X=dist average normalize, y=np.ravel(Yp),cv=5)
     print("done in %0.3fs" % (time() - t0))
     print(" Average and std CV score : {0} +- {1}".format(lda_score.mean(), __
      →lda_score.std() ))
     # Fitting QDA
     print("Fitting QDA")
     t0 = time()
     qda = QuadraticDiscriminantAnalysis()
     qda_score = cross_val_score(qda,X=dist_average_normalize, y=np.ravel(Yp),cv=5)
     print("done in %0.3fs" % (time() - t0))
```

```
print(" Average and std CV score : {0} +- {1}".format(qda_score.mean(),_
 →qda_score.std() ))
# Fitting Logistic-regression
print("Fitting Logistic Regression")
t0 = time()
logit = LogisticRegression(solver='lbfgs')
logit_score = cross_val_score(logit,X=dist_average_normalize, y=np.
 →ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(), __
 →logit score.std() ))
# Fitting Naive-Bayes
print("Fitting Naive-Bayes")
t0 = time()
GNB = GaussianNB()
GNB_score = cross_val_score(GNB,X=dist_average_normalize, y=np.ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(),
 →GNB_score.std() ))
# Fitting K-nearest neighbour
print("Fitting K-nearest neighbour")
t0 = time()
neigh = KNeighborsClassifier(n_neighbors=3)
neigh_score = cross_val_score(neigh, X=dist_average_normalize, y=np.
 \rightarrowravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(neigh_score.mean(), __
 →neigh score.std() ))
normalized data
Fitting LDA
done in 0.037s
Average and std CV score: 0.557499999999999999999 +- 0.045138675213169485
Fitting QDA
done in 0.024s
Average and std CV score: 0.5625 +- 0.044721359549995794
Fitting Logistic Regression
done in 0.059s
Average and std CV score: 0.567500000000001 +- 0.03674234614174766
Fitting Naive-Bayes
done in 0.007s
Average and std CV score : 0.5725 +- 0.058843011479699094
Fitting K-nearest neighbour
done in 0.053s
```

```
[19]: # Cross-validation for Model Assessment
      # scaled data
      print('scaled data')
      # Fitting LDA
      print("Fitting LDA")
      t0 = time()
      lda = LinearDiscriminantAnalysis()
      lda_score = cross_val_score(lda,X=dist_average_scale, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(lda_score.mean(), __
      →lda score.std() ))
      # Fitting QDA
      print("Fitting QDA")
      t0 = time()
      qda = QuadraticDiscriminantAnalysis()
      qda_score = cross_val_score(qda,X=dist_average_scale, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(qda_score.mean(),_
      →qda_score.std() ))
      # Fitting Logistic-regression
      print("Fitting Logistic Regression")
      t0 = time()
      logit = LogisticRegression(solver='lbfgs')
      logit_score = cross_val_score(logit,X=dist_average_scale, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(),
      →logit_score.std() ))
      # Fitting Naive-Bayes
      print("Fitting Naive-Bayes")
      t0 = time()
      GNB = GaussianNB()
      GNB_score = cross_val_score(GNB,X=dist_average_scale, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(),
      →GNB score.std() ))
      # Fitting K-nearest neighbour
      print("Fitting K-nearest neighbour")
      t0 = time()
      neigh = KNeighborsClassifier(n_neighbors=3)
      neigh_score = cross_val_score(neigh,X=dist_average_scale, y=np.ravel(Yp),cv=5)
      print("done in %0.3fs" % (time() - t0))
```

```
print(" Average and std CV score : {0} +- {1}".format(neigh_score.mean(),⊔
→neigh_score.std() ))
```

```
scaled data
Fitting LDA
done in 0.030s
Average and std CV score: 0.55749999999999 +- 0.045138675213169485
Fitting QDA
done in 0.027s
 Average and std CV score: 0.5625 +- 0.044721359549995794
Fitting Logistic Regression
done in 0.122s
Average and std CV score: 0.55 +- 0.01767766952966367
Fitting Naive-Bayes
done in 0.009s
Average and std CV score: 0.5725 +- 0.058843011479699094
Fitting K-nearest neighbour
done in 0.053s
 Average and std CV score: 0.5625 +- 0.017677669529663688
```

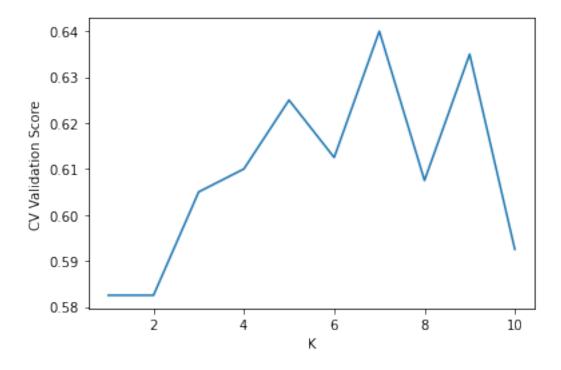
In the previous example we have fixed the hyper-parameter K to 3. We could use CV to find the best value.

Question(IMP+IMH): Comment the code and explain what it does.

Answer:

The code search for the best K for the K-nearest neighbour between thr values 1 to 10 these values used in GridSearchCV to find the optimal one for the whole dataset, then plot the score of the CV for every number of K

[20]: Text(0, 0.5, 'CV Validation Score')



We could also use CV to assess the prediction error (generalization error) in a left-out test set.

Question(IMP+IMH): Comment the code and explain what it does.

Answer:

The code search for the best K for the K-nearest neighbor between the values 1 to 10 these values used in GridSearchCV to find the optimal one for the X_train and y_train dataset therefor we comput it for 5 (cv=5) subsets of train and validation sets, then plots the score of the CV for every number. then takes the best model from the CV and computes only its score for the test set.

```
[21]: # We only use the training set for finding the best hyper-parameter

parameters = {'n_neighbors': [1,2,3,4,5,6,7,8,9,10]}

neighCV = KNeighborsClassifier()

grid = GridSearchCV(neighCV, parameters, cv=5, n_jobs=-1)

grid.fit(X_train, y_train)

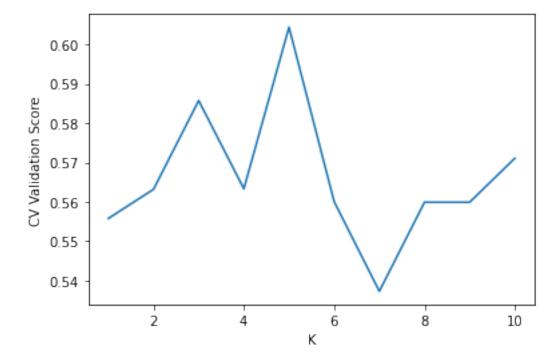
print('The best K is', grid.best_params_.get('n_neighbors'), ' with an average_
→validation score equal to ', grid.best_score_)

# plot the CV validation score for each K value
```

```
plt.plot([1,2,3,4,5,6,7,8,9,10], grid.cv_results_.get('mean_test_score'))
plt.xlabel('K')
plt.ylabel('CV Validation Score')

# Let's now use the best model to assess the test score
BestModel=grid.best_estimator_
print('The test score is', BestModel.score(X_test, y_test))
```

The best K is 5 with an average validation score equal to 0.6044025157232704 The test score is 0.6590909090909091



Question(IMP+IMH): Comment the results of the two previous experiments. What about the best K and validation/test error? Are the results the same? Why in your opinion?

Answer: The results are not the same because the second model has fewer data to train on, but it is the more correct way to train the model. to have a train and validation sets to find the hyperparameters and then try the best model on the test set.

It seems that these features do not work very well... let's try to change them. We can use the distances between all combinations of landmarks. Each subject has $M^*(M-1)/2$ features.

```
[22]: # Use distances between all combinations of landmarks. Each subject has M*(M-1)/
      \rightarrow2 features
      dist_combination=np.zeros((N,int((M*(M-1)/2))))
      for s in range(N):
          temp=[]
          landmarks=np.reshape(XpGPA[s],(M,2))
          for i in range(M-1):
              a=landmarks[i,:]
              for j in range(i+1,M):
                  b=landmarks[j,:]
                  dist_2=np.sqrt(np.dot(a, a) - 2 * np.dot(a, b) + np.dot(b, b))
                  temp.append(dist_2)
          dist_combination[s]=np.array(temp)
      # Scale data (each feature will have average equal to 0 and unit variance)
      scaler.fit(dist_combination)
      dist combination scale=scaler.transform(dist combination)
      print('Number of subjects N is: ', dist_combination_scale.shape[0], '; number_u
       →of features is: ', dist_combination_scale.shape[1] )
```

Number of subjects N is: 400 ; number of features is: 2278

Question (IMP+IMH): Should we scale/normalize the new features?

Answer:

Yes we should since we no longer in a known dimension of the data If not scale, the feature with a higher value range starts dominating in our model and we might have a biased model

Use the classification algorithms seen before to test the discriminative power of the new features.

```
# Plot normalized confusion matrix
plt.figure()
plot_confusion_matrix(cnf_matrix_scale, classes=class_names, normalize=True)
plt.show()
# Cross-validation for Model Assessment
# Fitting LDA
print("Fitting LDA")
t0 = time()
lda = LinearDiscriminantAnalysis()
lda_score = cross_val_score(lda,X=dist_combination_scale, y=np.ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(lda_score.mean(),__
→lda_score.std() ))
# Fitting QDA
print("Fitting QDA")
t0 = time()
qda = QuadraticDiscriminantAnalysis()
qda score = cross val score(qda, X=dist combination scale, y=np.ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(qda_score.mean(),_
→qda_score.std() ))
# Fitting Logistic-regression
print("Fitting Logistic Regression")
t0 = time()
logit = LogisticRegression(solver='lbfgs')
logit_score = cross_val_score(logit,X=dist_combination_scale, y=np.
\rightarrowravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(),
→logit_score.std() ))
# Fitting Naive-Bayes
print("Fitting Naive-Bayes")
t0 = time()
GNB = GaussianNB()
GNB_score = cross_val_score(GNB,X=dist_combination_scale, y=np.ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(), __

GNB_score.std() ))
# Fitting K-nearest neighbour
print("Fitting K-nearest neighbour")
t0 = time()
```

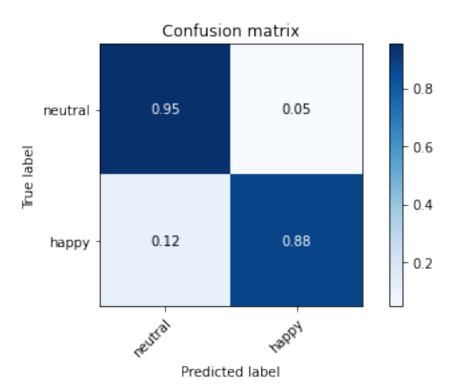
Fitting LDA to scaled dataset done in 0.203s

	precision	recall	f1-score	support
	-			
0	0.88	0.95	0.92	64
1	0.95	0.88	0.92	68
accuracy			0.92	132
macro avg	0.92	0.92	0.92	132
weighted avg	0.92	0.92	0.92	132

Normalized confusion matrix [[0.953125 0.046875]

[0.11764706 0.88235294]]

<Figure size 432x288 with 0 Axes>



Fitting LDA

```
done in 1.542s
Average and std CV score: 0.91749999999999999999 +- 0.023184046238739257
Fitting QDA
C:\Users\eidan\anaconda3\envs\env_full\lib\site-
packages\sklearn\discriminant_analysis.py:715: UserWarning: Variables are
collinear
  warnings.warn("Variables are collinear")
C:\Users\eidan\anaconda3\envs\env_full\lib\site-
packages\sklearn\discriminant_analysis.py:715: UserWarning: Variables are
collinear
  warnings.warn("Variables are collinear")
C:\Users\eidan\anaconda3\envs\env_full\lib\site-
packages\sklearn\discriminant analysis.py:715: UserWarning: Variables are
collinear
  warnings.warn("Variables are collinear")
C:\Users\eidan\anaconda3\envs\env_full\lib\site-
packages\sklearn\discriminant analysis.py:715: UserWarning: Variables are
collinear
  warnings.warn("Variables are collinear")
C:\Users\eidan\anaconda3\envs\env_full\lib\site-
packages\sklearn\discriminant_analysis.py:715: UserWarning: Variables are
collinear
  warnings.warn("Variables are collinear")
done in 0.502s
 Average and std CV score: 0.69249999999999 +- 0.07441438033068606
Fitting Logistic Regression
done in 0.632s
 Average and std CV score: 0.9625 +- 0.0262202212042538
Fitting Naive-Bayes
done in 0.096s
Average and std CV score: 0.9475 +- 0.031024184114977156
Fitting K-nearest neighbour
done in 0.952s
Average and std CV score: 0.9225 +- 0.03482097069296032
```

mmmm it seems that some variables are collinear. Collinearity means that one variable can be linearly predicted by the other, basically it means that there is redundancy...

Question (IMP+IMH): Which technique could you use to reduce the collinearity/redundancy? Use it and test the new features.

Answer:

DONE!

Number of subjects N is: 400 ; number of features is: 42

```
[25]: # Test the predictive power of the new features using LDA, QDA, etc.
      # Create training and test set
      X_train_scale, X_test_scale, y_train_scale, y_test_scale =_
      →train test split(dist combination scale pca, np.ravel(Yp), test size=0.33,
      →random state=42)
      # Fitting LDA to scaled data
      print("Fitting LDA to scaled dataset")
      t0 = time()
      lda = LinearDiscriminantAnalysis()
      lda.fit(X_train_scale, y_train_scale)
      y_pred_scale = lda.predict(X_test_scale)
      print("done in %0.3fs" % (time() - t0))
      print(classification_report(y_test_scale, y_pred_scale))
      # Compute confusion matrix
      cnf_matrix_scale = confusion_matrix(y_test_scale, y_pred_scale)
      # Plot normalized confusion matrix
      plt.figure()
      plot_confusion_matrix(cnf_matrix scale, classes=class names, normalize=True)
      plt.show()
      # Cross-validation for Model Assessment
      # Fitting LDA
      print("Fitting LDA")
      t0 = time()
      lda = LinearDiscriminantAnalysis()
```

```
lda_score = cross_val_score(lda,X=dist_combination_scale_pca, y=np.
→ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(lda_score.mean(), __
→lda_score.std() ))
# Fitting QDA
print("Fitting QDA")
t0 = time()
qda = QuadraticDiscriminantAnalysis()
qda_score = cross_val_score(qda,X=dist_combination_scale_pca, y=np.
\rightarrowravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(qda_score.mean(), __
→qda_score.std() ))
# Fitting Logistic-regression
print("Fitting Logistic Regression")
t0 = time()
logit = LogisticRegression(solver='lbfgs')
logit_score = cross_val_score(logit,X=dist_combination_scale_pca, y=np.
\rightarrowravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(),
→logit_score.std() ))
# Fitting Naive-Bayes
print("Fitting Naive-Bayes")
t0 = time()
GNB = GaussianNB()
GNB_score = cross_val_score(GNB,X=dist_combination_scale_pca, y=np.
→ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(),
→GNB_score.std() ))
# Fitting K-nearest neighbour
print("Fitting K-nearest neighbour")
t0 = time()
neigh = KNeighborsClassifier(n_neighbors=3)
neigh_score = cross_val_score(neigh,X=dist_combination_scale_pca, y=np.
→ravel(Yp),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(neigh_score.mean(),
 →neigh_score.std() ))
```

Fitting LDA to scaled dataset

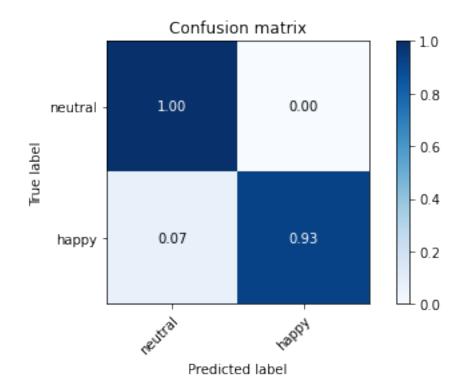
done in 0.005s

	precision	recall	f1-score	support
0	0.93	1.00	0.96	64
1	1.00	0.93	0.96	68
accuracy			0.96	132
macro avg	0.96	0.96	0.96	132
weighted avg	0.96	0.96	0.96	132

 ${\tt Normalized \ confusion \ matrix}$

[[1. 0.] [0.07352941 0.92647059]]

<Figure size 432x288 with 0 Axes>



Fitting LDA done in 0.027s

Fitting QDA done in 0.020s

Average and std CV score : 0.9125 +- 0.023717082451262854

Fitting Logistic Regression

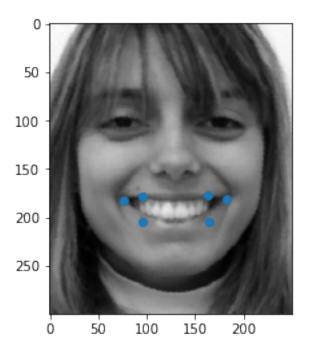
done in 0.163s

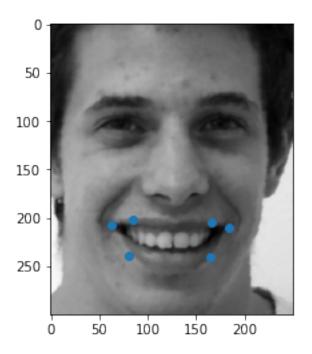
A second solution, would be to manually select few landmarks

```
[26]: # Select lateral landmarks mouth
      select_land=[49,50,60,55,54,56]
      indeces_central=[]
      for k in range(0,len(select_land)):
          indeces_central.append(select_land[k]*2-2) # Remember that landmarks are_
       \rightarrowM*2 vectors (odds values are the x and even values are the y)
          indeces central.append(select land[k]*2-1)
      indeces_central=np.array(indeces_central,dtype=int)
      Ms=int(len(indeces_central)/2)
      Xps=np.zeros((N,Ms*dim))
      XpsGPA=np.zeros((N,Ms*dim))
      for i in range(0,N):
          XpsGPA[i,:]=XpGPA[i,indeces_central]
          Xps[i,:]=Xp[i,indeces_central]
      Yps=Yp
      print('Number of subjects N is: ', XpsGPA.shape[0], '; number of features is: u
       →', XpsGPA.shape[1] )
```

Number of subjects N is: 400 ; number of features is: 12

```
[27]: # plot two test images
for i in range(0,2):
    image = Imagesp[i,:,:]
    plt.figure()
    plt.imshow(image, cmap='gray', origin='upper')
    landmark=Xps[i,:]
    x=landmark[::2]
    y=landmark[1::2]
    plt.plot(x,y,'o')
    plt.show()
```

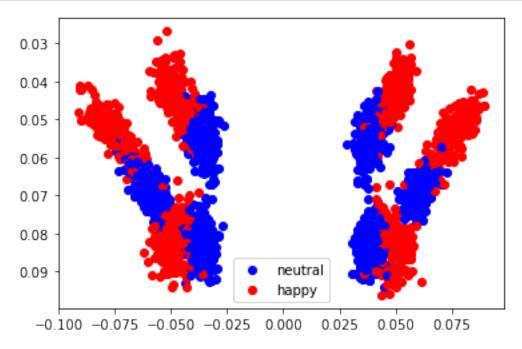




```
[28]: # Plot only selected landmarks
plt.figure()
for i in range(0,N):
    landmark=XpsGPA[i]
```

```
x=landmark[::2]
y=landmark[1::2]
if Yps[i].astype(int)==0:
    neutral=plt.scatter(x, y, c='b')
else:
    happy=plt.scatter(x, y, c='r')

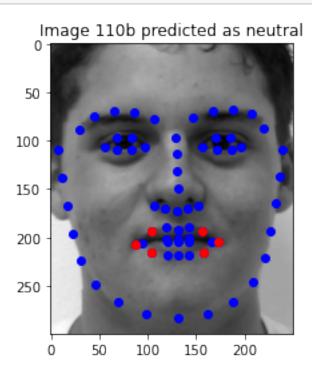
plt.legend((neutral,happy),('neutral','happy'))
plt.gca().invert_yaxis()
```

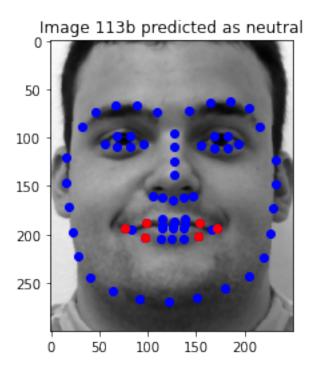


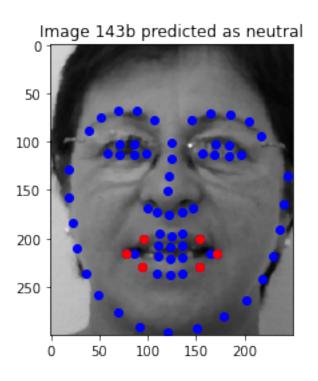
```
print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(),
       →logit_score.std() ))
      # Fitting Naive-Bayes
      print("Fitting Naive-Bayes")
      t0 = time()
      GNB = GaussianNB()
      GNB_score = cross_val_score(GNB,X=XpsGPA, y=np.ravel(Yps),cv=5)
      print("done in %0.3fs" % (time() - t0))
      print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(),
       →GNB_score.std() ))
     Fitting QDA
     done in 0.021s
      Average and std CV score: 0.96 +- 0.031024184114977135
     Fitting Logistic Regression
     done in 0.039s
      Average and std CV score: 0.945 +- 0.016955824957813174
     Fitting Naive-Bayes
     done in 0.010s
      Average and std CV score: 0.9375 +- 0.01936491673103706
[30]: # Fitting LDA
      print("Fitting LDA")
      lda = LinearDiscriminantAnalysis()
      lda_validate = cross_validate(lda, X=XpsGPA, y=np.ravel(Yps), cv=5, n_jobs=-1,__
      →return_train_score=True, return_estimator=True )
      print(" Average and std train score : {0} +- {1}".
       →format(lda_validate['train_score'].mean(), lda_validate['train_score'].std()_⊔
      →))
      print(" Average and std test score : {0} +- {1}".
       -format(lda_validate['test_score'].mean(), lda_validate['test_score'].std() ))
      # Let's look for the best CV model (the one with the best test score)
      best_estimator=lda_validate['estimator'][np.argmax(lda_validate['test_score'])]
      C=best_estimator.predict(XpsGPA)
      # Let's find the images where it did a mistake
      error=np.ravel(np.array(np.where(np.abs(C-np.ravel(Yps)))))
      if len(error)>5:
          kk=5
      else:
          kk=len(error)
     Fitting LDA
```

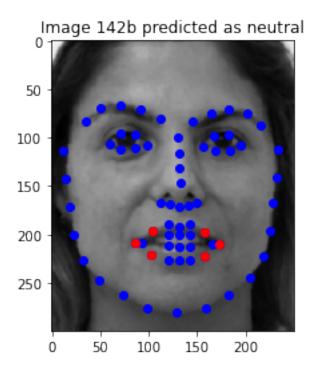
Average and std train score: 0.95625 +- 0.006846531968814562 Average and std test score: 0.94000000000001 +- 0.021505813167606556 Let's plot some images where the best model was wrong.

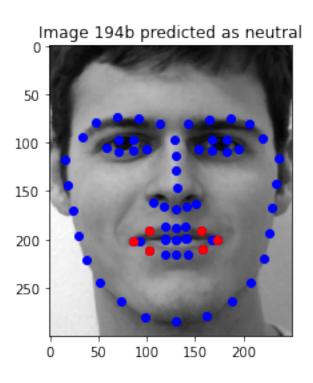
```
[31]: # plot error images
      for i in range(0,kk):
          image = Imagesp[error[i],:,:]
          plt.figure()
          plt.imshow(image, cmap='gray', origin='upper')
          landmarkALL=Xp[error[i],:]
          landmark=Xps[error[i],:]
          xALL=landmarkALL[::2]
          yALL=landmarkALL[1::2]
          x=landmark[::2]
          y=landmark[1::2]
          plt.plot(xALL,yALL,'ob')
          plt.plot(x,y,'or')
          if C[error[i]]==0:
              plt.title('Image ' + Namesp[error[i]] + ' predicted as neutral')
          elif C[error[i]]==1:
              plt.title('Image ' + Namesp[error[i]] + ' predicted as happy')
          plt.show()
```











Question (IMP+IMH): Comment the results. Why did the algorithm make a mistake? Would you choose other landmarks? Try at least another combination of landmarks

Answer:

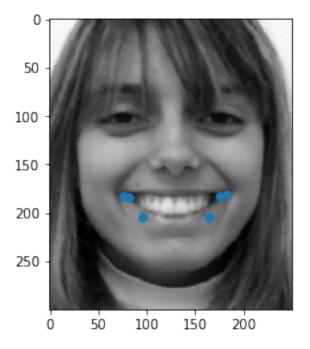
I would choose a bit different landmarks since it makes sense to choose the edge of the smile has the landmarks as I did.

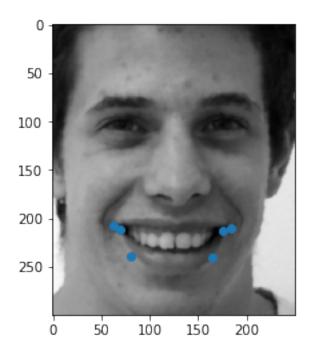
```
[32]: # Select lateral landmarks mouth
      select_land=[49,61,60,56,65,55]
      indeces central=[]
      for k in range(0,len(select land)):
          indeces_central.append(select_land[k]*2-2) # Remember that landmarks are_
       \rightarrowM*2 vectors (odds values are the x and even values are the y)
          indeces_central.append(select_land[k]*2-1)
      indeces_central=np.array(indeces_central,dtype=int)
      Ms=int(len(indeces_central)/2)
      Xps=np.zeros((N,Ms*dim))
      XpsGPA=np.zeros((N,Ms*dim))
      for i in range(0,N):
          XpsGPA[i,:]=XpGPA[i,indeces_central]
          Xps[i,:]=Xp[i,indeces_central]
      Yps=Yp
      print('Number of subjects N is: ', XpsGPA.shape[0], '; number of features is: u
      # plot two test images
      for i in range (0,2):
          image = Imagesp[i,:,:]
          plt.figure()
          plt.imshow(image, cmap='gray', origin='upper')
          landmark=Xps[i,:]
          x=landmark[::2]
          y=landmark[1::2]
          plt.plot(x,y,'o')
          plt.show()
      # Plot only selected landmarks
      plt.figure()
      for i in range(0,N):
          landmark=XpsGPA[i]
          x=landmark[::2]
          y=landmark[1::2]
          if Yps[i].astype(int)==0:
              neutral=plt.scatter(x, y, c='b')
          else:
              happy=plt.scatter(x, y, c='r')
      plt.legend((neutral, happy), ('neutral', 'happy'))
      plt.gca().invert_yaxis()
```

```
# Fitting QDA
print("Fitting QDA")
t0 = time()
qda = QuadraticDiscriminantAnalysis()
qda_score = cross_val_score(qda,X=XpsGPA, y=np.ravel(Yps),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(qda_score.mean(),__
→qda_score.std() ))
# Fitting Logistic-regression
print("Fitting Logistic Regression")
t0 = time()
logit = LogisticRegression(solver='lbfgs')
logit_score = cross_val_score(logit, X=XpsGPA, y=np.ravel(Yps),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(logit_score.mean(),
→logit_score.std() ))
# Fitting Naive-Bayes
print("Fitting Naive-Bayes")
t0 = time()
GNB = GaussianNB()
GNB_score = cross_val_score(GNB,X=XpsGPA, y=np.ravel(Yps),cv=5)
print("done in %0.3fs" % (time() - t0))
print(" Average and std CV score : {0} +- {1}".format(GNB_score.mean(),
→GNB score.std() ))
# Fitting LDA
print("Fitting LDA")
lda = LinearDiscriminantAnalysis()
lda_validate = cross_validate(lda,X=XpsGPA, y=np.ravel(Yps), cv=5, n_jobs=-1,__
→return_train_score=True, return_estimator=True )
print(" Average and std train score : {0} +- {1}".
→format(lda_validate['train_score'].mean(), lda_validate['train_score'].std()_⊔
→))
print(" Average and std test score : {0} +- {1}".
-format(lda_validate['test_score'].mean(), lda_validate['test_score'].std() ))
# Let's look for the best CV model (the one with the best test score)
best_estimator=lda_validate['estimator'][np.argmax(lda_validate['test_score'])]
C=best_estimator.predict(XpsGPA)
# Let's find the images where it did a mistake
error=np.ravel(np.array(np.where(np.abs(C-np.ravel(Yps)))))
if len(error)>5:
   kk=5
```

```
else:
    kk=len(error)
```

Number of subjects N is: 400; number of features is: 12





```
Fitting QDA done in 0.007s
```

Average and std CV score : 0.9625 +- 0.02091650066335188

Fitting Logistic Regression

done in 0.018s

Average and std CV score : 0.942500000000001 +- 0.016955824957813174

Fitting Naive-Bayes

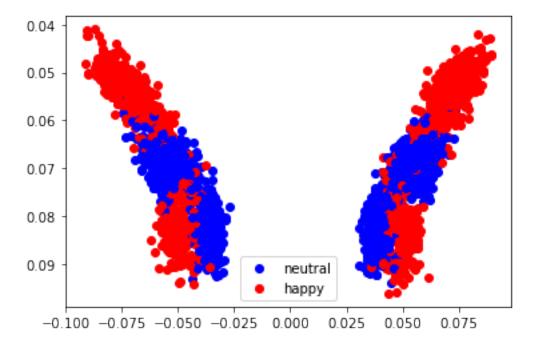
done in 0.010s

Average and std CV score : 0.94000000000001 +- 0.02893959225697557

Fitting LDA

Average and std train score : 0.956874999999999 +- 0.008244316223920583

Average and std test score : 0.9525 +- 0.026692695630078273



Here, we use Nested Cross-Validation for finding the generalization error and the best K value

```
[33]: # Fitting K-nearest neighbour with Nested Cross-Validation

print("Fitting K-nearest neighbour with Nested CV")

t0 = time()

neigh = KNeighborsClassifier()

parameters = {'n_neighbors': [1,2,3,4,5,6,7,8,9,10,11,12,13,14,15]}

inner_cv = StratifiedKFold(n_splits=5, shuffle=True, random_state=666) # we fix_u

the random state to always have the same results if we relaunch the code
```

Fitting K-nearest neighbour with Nested CV done in 0.870s

Average and std Nested Cv train score: 0.95625 +- 0.00624999999999998

Average and std Nested Cv test score: 0.9550000000000001 +- 0.020310096011589906

Question (IMP+IMH): Are Training and Test scores similar? What does it mean?

Answer:

yes they are similar that's means we have a very good moodel with no overfitting. The perfect model.

Question (IMP+IMH) (OPTIONAL): Please propose at least another set of features using landmarks and/or pixel intensities of the images and test its discriminative power

[]: