Robust fingerprint verification using *m-triplets*

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***Abstract*—Fingerprint verification has become one of the most active research areas nowadays. A key component of an accurate fingerprint verification system is the fingerprint matching algorithm. An accurate matching algorithm uses a robust fingerprint representation. In this paper, we introduce *m-triplets*, a new minutiae triplet representation and similarity for fingerprint verification. The proposed similarity shifts the triplets to find the best minutiae correspondence and it uses rules that discard not matching minutiae triplets without comparing the whole representation; hence, achieving high matching speed. To test the quality of the introduced representation and similarity, we modify a popular fingerprint verification algorithm. The modified algorithm achieves the best accuracy and speed in all the databases of FVC2004 compared with four accurate verification algorithms.**

I. Introduction

Fingerprint recognition [1] plays an important role in legal and civilian applications. Developing a good fingerprint matching algorithm capable of dealing with low fingerprint quality, nonlinear distortions, limited memory and low matching time is an active research area in these applications.

Most authors identify two main types of fingerprint matching algorithms: correlation-based matching and minutiae-based matching. The Fingerprint Verification Competitions (FVC) [2] shows that the minutiae-based matching is the most popular approach. The minutiae-based approach consists on finding the maximum number of matching minutiae pairs in two given fingerprints.

Forensic experts usually establish the minutia pairing by searching similar *local minutiae structures*. Local minutiae structures [3] (or *minutiae descriptors*) are typically represented by one or more minutiae and additional information that makes the structures invariant to rotation and translation.

Minutiae descriptors can be mainly classified in: *ridge-based descriptors* [4]–[7], *texture-based descriptors* [8]–[11], and *neighboring minutiae-based descriptors* [12], [13]. Ridge-based descriptors associate each minutiae with the information related to the ridge it resides. Texture-based descriptors enrich each minutia with the ridge orientation, the frequency information or both, at selected sampling points around it. Neighboring minutiae-based descriptors combine each minutia with information of certain neighboring minutiae.

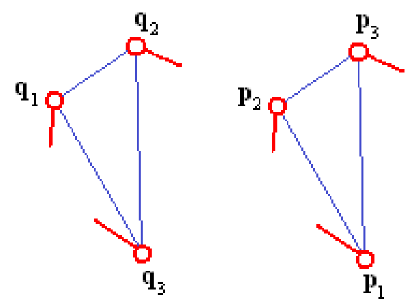

Fig. 1. A true matching pair of minutiae triplets

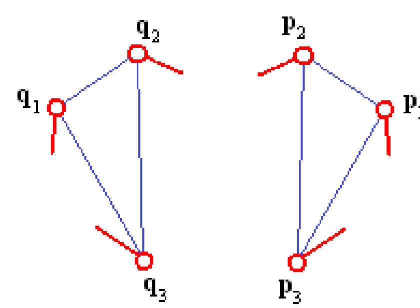

Fig. 2. A minutiae triplet $(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3})$ and its reflection $(\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3})$

Minutiae triplets [14]–[26] are special cases of the neighboring minutiae-based descriptors and they represent the relationships among three minutiae. The analysis of the different algorithms based on minutiae triplets reveals that they have the following problems:

- **Minutiae order dependency**. Some representations fail matching triplets due to the order imposed in its minutiae. For example, the representation proposed by Jiang and Yau [14] and its variants fail comparing the triplets in Figure 1 if $\mathbf{q_1}$ is the central minutia of the first triplet and $\mathbf{p_2}$ is not the central minutia of the second triplet
- **Mirror effect** We say that a fingerprint matching algorithm suffers from the mirror effect when it finds minutiae triplets that match their reflected versions. Figure 2 shows

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an example of the mirror effect, containing the triplet $(\mathbf{q_1}, \mathbf{q_2}, \mathbf{q_3})$ and the reflected version $(\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3})$

In this paper, we propose *m-triplets*, a new minutiae triplet representation and similarity measure to overcome the minutiae order dependency and the mirror effect. To achieve these goals, our minutiae triplet representation contains clockwise-arranged minutiae without a central minutia and the similarity measure tests three possible shifts of the triplets to find the best minutiae correspondence. Moreover, it applies rules for fast discarding candidate tests, consequently saving computations. In order to increase the minutiae discriminating power, the proposed similarity takes into account the small image rotation that is present in the fingerprint verification problems.

We modify a popular fingerprint verification algorithm [16] to work with our new minutiae triplet representation and similarity. We test the behavior of the modified algorithm in the FVC2004 fingerprint databases, comparing its performance with four popular algorithms. Experimental results show that our proposal achieves the highest accuracy and lowest matching time for all the databases of FVC2004, according to the evaluation protocols of FVC [2].

II. New Representation and Comparison Function for Minutiae Triplets

A. Definitions

This section defines some basic functions that we use throughout the paper. Given two minutiae $\mathbf{p_i} = (x_i, y_i, \theta_i)$ and $\mathbf{p_j} = (x_j, y_j, \theta_j)$, $\mathrm{ed}(\mathbf{p_i}, \mathbf{p_j})$ represents the Euclidean distance between the coordinates of $\mathbf{p_i}$ and $\mathbf{p_j}$.

$$\mathrm{ed}(\mathbf{p_i}, \mathbf{p_j}) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{1}$$

For two given angles α and β, $\mathrm{ad}_{2\pi}(\alpha, \beta)$ computes the angle required to rotate a vector with angle β in clockwise sense to superpose it to another vector with the same origin and angle α.

$$\mathrm{ad}_{2\pi}(\alpha, \beta) = \begin{cases} \beta - \alpha & \text{if } \beta > \alpha \\ \beta - \alpha + 2\pi & \text{otherwise} \end{cases} \tag{2}$$

Finally, for a given pair of minutiae $\mathbf{p_i}$ and $\mathbf{p_j}$, $\mathrm{ang}(\mathbf{p_i}, \mathbf{p_j})$ computes the angle of the vector with initial point at $\mathbf{p_i}$ and terminal point at $\mathbf{p_j}$

$$\mathrm{ang}(\mathbf{p_i}, \mathbf{p_j}) = \begin{cases} \arctan(\Delta y/\Delta x) & \text{if } \Delta x > 0 \wedge \Delta y \geq 0 \\ \arctan(\Delta y/\Delta x) + 2\pi & \text{if } \Delta x > 0 \wedge \Delta y < 0 \\ \arctan(\Delta y/\Delta x) + \pi & \text{if } \Delta x < 0 \\ \pi/2 & \Delta x = 0 \wedge \Delta y > 0 \\ 3\pi/2 & \Delta x = 0 \wedge \Delta y < 0 \end{cases} \tag{3}$$

where $\Delta y = y_i - y_j$ and $\Delta x = x_i - x_j$.

B. Feature Representation

In this section, we introduce *m-triplets*, a robust feature representation based on minutiae triplets. Provided that a fingerprint is described by the minutia set P, our representation is a t-uple with the following components (see Figure 3):

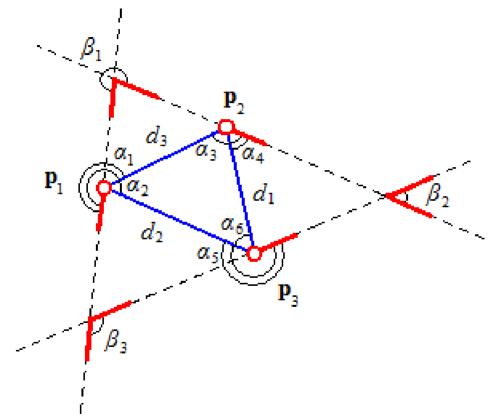

Fig. 3. The components of the new feature representation proposed in this paper

- minutiae $\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3} \in P$, clockwise arranged starting on $\mathbf{p_1}$.
- $d_{i \in 1 \dots 3}$, where d_i is the euclidean distance between the minutiae different than $\mathbf{p_i}$.
- d_{max}, d_{mid} and d_{min} are the maximum, middle and minimum distance in the triplet, respectively.
- $\alpha_{i \in 1 \dots 6}$ are the angles $\mathrm{ad}_{2\pi}\left(\mathrm{ang}\left(\mathbf{p}, \mathbf{q}\right), \theta\right)$ required to rotate the direction θ of a minutia to superpose it to the vectors associated with the other two minutiae in the triplet.
- $\beta_i = \mathrm{ad}_{2\pi}\left(\theta_j, \theta_k\right)$ is the angle required to rotate the direction of the minutia $\mathbf{p_k}$ in order to superpose it to the direction of the minutia $\mathbf{p_j}$.

Minutiae in m-triplets are arranged in clockwise direction without central minutia. Therefore, in order to avoid the minutiae order dependency, the similarity function considers the three minutiae rotations in clockwise sense. Using clockwise rotations also avoids the mirror effect.

C. M-triplets similarity

In this section, we introduce our m-triplets similarity. It is designed to highly distinguish between similar and non-similar m-triplets. Given two m-triplets $\mathbf{t}$ and $\mathbf{r}$, we propose $\mathrm{s}_2(\mathbf{t}, \mathbf{r})$ (4) to compare m-triplets.

$$\mathrm{s}_2(\mathbf{t}, \mathbf{r}) = \max\left\{\mathrm{s}_1(\mathbf{t}, \mathbf{r}), \mathrm{s}_1\left(\mathbf{t}, \mathrm{shift}(\mathbf{r})\right), \mathrm{s}_1\left(\mathbf{t}, \mathrm{shift}\left(\mathrm{shift}(\mathbf{r})\right)\right)\right\} \tag{4}$$

where: $\mathrm{shift}(\mathbf{r})$ is the clockwise-shifted m-triplet $\mathbf{r}$ and $\mathrm{s}_1(\mathbf{t}, \mathbf{r})$ is the base similarity function (5).

$$\mathrm{s}_1(\mathbf{t}, \mathbf{r}) = \begin{cases} 0 \text{ if } \mathrm{s}_\theta(\mathbf{t}, \mathbf{r}){=}0 \vee \mathrm{s}_d(\mathbf{t}, \mathbf{r}){=}0 \vee \mathrm{s}_\alpha(\mathbf{t}, \mathbf{r}){=}0 \vee \mathrm{s}_\beta(\mathbf{t}, \mathbf{r}){=}0 \\ 1\text{-}\left(1\text{-}\mathrm{s}_d(\mathbf{t}, \mathbf{r})\right)\left(1\text{-}\mathrm{s}_\alpha(\mathbf{t}, \mathbf{r})\right)\left(1\text{-}\mathrm{s}_\beta(\mathbf{t}, \mathbf{r})\right), \text{ otherwise} \end{cases} \tag{5}$$

The base similarity function s_1 is defined using functions s_θ, s_d, s_α, and s_β, which consider different components of the m-triplets. According to (5), two m-triplets are totally dissimilar if they have at least one component totally dissimilar. If all component similarities are above zero, the produce rule makes the similarity hight if at least one component is close to 1.

The function s_θ (6) takes advantage of the small image rotation on the fingerprint verification problems. We incorporate this information into the m-triplets similarity function to increase minutia discrimination for such problems. Consequently, two m-triplets are dissimilar if their angles differ more than $\pi/4$.

$$\mathrm{s}_\theta(\mathbf{t},\mathbf{r}) = \begin{cases} 0 & \text{if } \bigvee_{i=1}^{3}\left(\mathrm{ad}_\pi(\theta_i^{\mathbf{t}},\theta_i^{\mathbf{r}}) > \pi/4\right) \\ 1 & \text{otherwise} \end{cases} \tag{6}$$

The function s_d (7) compares m-triplets in terms of the lengths of the sides of the triangle formed by the minutiae.

$$\mathrm{s}_d(\mathbf{t},\mathbf{r}) = \begin{cases} 0 \text{ if } \bigvee_{i=1}^{3}\left(|d_i^{\mathbf{t}} - d_i^{\mathbf{r}}| > t_l\right) \\ 1 - \max_{i=1\ldots3}\{|d_i^{\mathbf{t}} - d_i^{\mathbf{r}}|\}/t_l, \text{ otherwise} \end{cases} \tag{7}$$

Equation 7 returns values in the interval $[0,1]$. It returns 0 if at least one length difference is greater than threshold t_l. It returns 1 if all length differences are 0; that is, the triangles formed by both m-triplets are equal.

The function s_α (8) compares m-triplets based on the angles formed by minutiae directions and the sides of the triangles (angles α on Figure 3).

$$\mathrm{s}_\alpha(\mathbf{t},\mathbf{r}) = \begin{cases} 0 \text{ if } \bigvee_{i=1}^{6}\left(\mathrm{ad}_\pi(\alpha_i^{\mathbf{t}},\alpha_i^{\mathbf{r}}) > t_a\right) \\ 1 - \max_{i=1\ldots6}\{\mathrm{ad}_\pi(\alpha_i^{\mathbf{t}},\alpha_i^{\mathbf{r}})\}/t_a, \text{ otherwise} \end{cases} \tag{8}$$

The function s_β (Equation 9) compares m-triplets based on relative minutiae directions (see angles β on Figure 3)

$$\mathrm{s}_\beta(\mathbf{t},\mathbf{r}) = \begin{cases} 0 \text{ if } \bigvee_{i=1}^{3}\left(\mathrm{ad}_\pi(\beta_i^{\mathbf{t}},\beta_i^{\mathbf{r}}) > t_a\right) \\ 1 - \max_{i=1\ldots3}\{\mathrm{ad}_\pi(\beta_i^{\mathbf{t}},\beta_i^{\mathbf{r}})\}/t_a, \text{ otherwise} \end{cases} \tag{9}$$

Equations 8 and 9 return values in the interval $[0,1]$. They return 0 if at least two compared angles differ more than threshold t_a. The less the angles differ, the higher is the value returned by the equations; therefore, they return 1 if the compared angles are identical.

In order to reduce the similarity computation time, we propose the following three theorems:

Theorem 1. *Given two m-triplets* $\mathbf{t}$ *and* $\mathbf{r}$*, if* $|d_{max}^{\mathbf{t}} - d_{max}^{\mathbf{r}}| > t_l$ *then* $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$

Theorem 2. *Given two m-triplets* $\mathbf{t}$ *and* $\mathbf{r}$*, if* $|d_{mid}^{\mathbf{t}} - d_{mid}^{\mathbf{r}}| > t_l$ *then* $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$

Theorem 3. *Given two m-triplets* $\mathbf{t}$ *and* $\mathbf{r}$*, if* $|d_{min}^{\mathbf{t}} - d_{min}^{\mathbf{r}}| > t_l$ *then* $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$

The proofs of theorems 1 and 2 appear in section II-C1 and II-C2 respectivelly. We obviate the proof of theorem 3 because it is similar to the proof of theorem 1.

Based on this theorem, we can modify s_2 (4) to detect if $\mathbf{t}$ and $\mathbf{r}$ are not similar without performing any costly shifts, hence reducing the similarity computation time. This way, our m-triplets similarity can be re-written as (10).

$$\mathrm{s}_2(\mathbf{t},\mathbf{r}) = \begin{cases} 0 \text{ if } (|d_{max}^{\mathbf{t}} - d_{max}^{\mathbf{r}}| > t_l) \vee (|d_{mid}^{\mathbf{t}} - d_{mid}^{\mathbf{r}}| > t_l) \vee \\ \vee(|d_{min}^{\mathbf{t}} - d_{min}^{\mathbf{r}}| > t_l) \\ \max\{\mathrm{s}_1(\mathbf{t},\mathbf{r}), \mathrm{s}_1(\mathbf{t},\mathrm{shift}(\mathbf{r})), \mathrm{s}_1(\mathbf{t},\mathrm{shift}(\mathrm{shift}(\mathbf{r})))\}, \\ \text{otherwise} \end{cases} \tag{10}$$

Local distance threshold t_l and angle threshold t_a are parameters of the algorithm.

1) Proof of Theorem 1: **Theorem 1**. *Given two m-triplets* $\mathbf{t}$ *and* $\mathbf{r}$*, if* $|d_{max}^{\mathbf{t}} - d_{max}^{\mathbf{r}}| > t_l$ *then* $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$

Proof: From the definition of m-triplet we have:

$$d_{max}^{\mathbf{r}} \geq d_{mid}^{\mathbf{r}} \tag{11}$$

$$d_{max}^{\mathbf{r}} \geq d_{min}^{\mathbf{r}} \tag{12}$$

$$d_{mid}^{\mathbf{r}} \geq d_{min}^{\mathbf{r}} \tag{13}$$

Assume, without loss of generality, that $d_{max}^{\mathbf{t}} > d_{max}^{\mathbf{r}}$; then, from the hypothesis of the theorem, we infer that:

$$d_{max}^{\mathbf{t}} - d_{max}^{\mathbf{r}} > t_l \tag{14}$$

$$d_{max}^{\mathbf{t}} - t_l > d_{max}^{\mathbf{r}} \tag{15}$$

From (15) and (11) we have $d_{max}^{\mathbf{t}} - t_l > d_{mid}^{\mathbf{r}}$; which is equivalent to

$$d_{max}^{\mathbf{t}} - d_{mid}^{\mathbf{r}} > t_l \tag{16}$$

Similarly, from (15) and (12) we obtain

$$d_{max}^{\mathbf{t}} - d_{min}^{\mathbf{r}} > t_l \tag{17}$$

From (14), (16) and (17) we conclude that $d_{max}^{\mathbf{t}}$ differs more than threshold t_l with respect to $d_{max}^{\mathbf{r}}$, $d_{mid}^{\mathbf{r}}$ and $d_{min}^{\mathbf{r}}$ respectively. Therefore, $\mathrm{s}_d(\mathbf{t},\mathbf{r})$ returns 0 for every clockwise shifting of $\mathbf{r}$ and consequently $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$ ■

2) Proof of Theorem 2: **Theorem 2**. *Given two m-triplets* $\mathbf{t}$ *and* $\mathbf{r}$*, if* $|d_{mid}^{\mathbf{t}} - d_{mid}^{\mathbf{r}}| > t_l$ *then* $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$

Proof: From the definition of m-triplet we have:

$$d_{max}^{\mathbf{t}} \geq d_{mid}^{\mathbf{t}} \tag{18}$$

Assume, without loss of generality, that $d_{mid}^{\mathbf{t}} > d_{mid}^{\mathbf{r}}$; then, from the hypothesis of the theorem, we infer that:

$$d_{mid}^{\mathbf{t}} - d_{mid}^{\mathbf{r}} > t_l \tag{19}$$

$$d_{mid}^{\mathbf{t}} - t_l > d_{mid}^{\mathbf{r}} \tag{20}$$

From (20) and (13) we have $d_{mid}^{\mathbf{t}} - t_l > d_{min}^{\mathbf{r}}$; which is equivalent to

$$d_{mid}^{\mathbf{t}} - d_{min}^{\mathbf{r}} > t_l \tag{21}$$

Similarly, from (18) and (19) we obtain

$$d_{max}^{\mathbf{t}} - d_{mid}^{\mathbf{r}} > t_l \tag{22}$$

Additionally, from (18) and (21) we obtain

$$d_{max}^{\mathbf{t}} - d_{min}^{\mathbf{r}} > t_l \tag{23}$$

From (19), (21), (22) and (23) we conclude that $d_{max}^{\mathbf{t}}$ and $d_{mid}^{\mathbf{t}}$ differ more than threshold t_l with respect to both $d_{mid}^{\mathbf{r}}$, $d_{min}^{\mathbf{r}}$. Therefore, $\mathrm{s}_d(\mathbf{t},\mathbf{r})$ returns 0 for every clockwise shifting of $\mathbf{r}$ and consequently $\mathrm{s}_2(\mathbf{t},\mathbf{r}) = 0$ ■

D. The Parziale and Niel (PN) Fingerprint Verification Algorithm

In order to test the quality of the m-triplets and the proposed similarity function, we modify a known verification algorithm. The algorithm PN [16], works with minutiae triplets found using Delaunay triangulation. This algorithm has the peculiarity that it does not compare the triplets directly, instead, it compares minutiae pairs and the matching triplets are then found from the coincident minutiae pairs. The matching algorithm consists of three major steps:

1) Local minutiae matching: Every minutiae pair related by a segment from the Delaunay triangulation in the query fingerprint is compared with every minutiae pair in the template fingerprint, also obtained by Delaunay triangulation. Then, two triplets are found coincident if they match segment by segment.
2) Global minutiae matching: For each pair of matching triplets the rotation and translation parameters are calculated and the minutiae are aligned. When aligning, two minutiae are found matching if they satisfy certain geometrical restrictions. The alignment that maximizes the amount of matching minutiae is selected as the best.
3) Similarity score computation: The similarity is computed with the formula $100\sqrt{\frac{m^2}{|P||Q|}}$; where m is the number of matching minutiae, P is the set of minutiae in the template fingerprint and Q is the set of minutiae in the query fingerprint.

When comparing the minutiae triplets, the algorithm does not take into account the spatial position of a segment with respect to the others in the triplet. This leads the algorithm to suffer from the mirror effect. To solve this problem, we use our m-triplets and similarity directly in the matching step. We call the modified algorithm Modified Parziale and Niel algorithm (MPN).

III. Experimental Results

To evaluate MPN algorithm, we use four FVC2004 databases. Although different protocols for performance evaluation exist, we use the FVC protocol [2]. We compare MPN with the original PN algorithm [16], JY [14], TK [11], and QYW [9].

The parameters values of the m-triplets similarity were the same in all databases: local distance threshold $t_l = 12$ and angle threshold $t_a = \pi/6$. We estimate these parameters through experiments using fingerprint databases B of FVC2004.

Table I shows the experimental results. We express in percentage the performance indicators EER, FMR100, FMR1000 and ZeroFMR. The indicator Time refers to the average matching time in milliseconds. We carry out all the experiments on a laptop with an Intel Core 2 Duo processor (1.5 GHz) and 3GB of RAM. The best result per database for each performance indicator appears in bold letter.

Experimental results show that our algorithm achieves the best results in all databases for each performance indicator except for ZeroFMR in DB2_A. MPN is the fastest algorithm because the proposed m-triplets similarity allows discarding false matching m-triplets without comparing the respective whole descriptors. MPN also exhibits superior results compared to the original PN. It is worth to mention that MPN also outperforms JY which is a very popular algorithm based on minutiae triplets.

TABLE I
Experimental results on FVC2004

Database	Algorithm	EER	FMR100	FMR1000	ZeroFMR	Time(ms)
DB1_A	QYW	24.3	60.6	80.3	97.3	47.1
	JY	13.5	28.5	42.8	55.9	14.7
	TK	15.9	29.1	41.8	51.0	26.1
	PN	11.4	17.7	24.4	25.9	17.1
	MPN	**7.7**	**15.0**	**20.9**	**25.3**	**14.2**
DB2_A	QYW	24.8	52.1	58.9	73.7	32.1
	JY	11.0	19.4	28.4	39.2	11.6
	TK	7.8	12.0	18.7	24.9	21.0
	PN	10.0	12.1	15.1	**16.9**	14.1
	MPN	**6.7**	**9.0**	**13.5**	20.9	**10.9**
DB3_A	QYW	19.7	47.7	65.9	87.5	53.4
	JY	12.0	22.1	32.3	41.4	21.6
	TK	9.6	19.7	32.9	37.3	38.4
	PN	7.1	10.7	17.6	24.9	27.0
	MPN	**4.9**	**8.5**	**16.0**	**23.9**	**20.8**
DB4_A	QYW	25.5	55.5	65.8	74.7	27.4
	JY	9.7	16.3	25.3	28.6	19.6
	TK	7.6	11.4	16.6	39.4	20.8
	PN	5.2	6.9	9.3	11.9	18.9
	MPN	**4.1**	**5.3**	**7.7**	**10.4**	**16.0**

IV. Conclusions

In this paper, we propose a new representation and similarity measure for minutiae triplet to overcome the minutiae order dependency and the mirror effect, two common problems appearing in most minutiae triplet based algorithms. We introduce optimizations to avoid comparing the whole descriptor in most cases, hence increasing the matching speed. The new representation and similarity benefit from the small image rotation that is present in fingerprint verification problems to achieve higher minutiae discrimination. For testing the quality of the proposal, we modified a popular fingerprint verification algorithm based on minutiae triplet to work with our new representation and similarity. Experimental results show that the modified algorithm outperforms its predecessor and other three algorithms.

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