Our whole built environment, from modern bridges to skyscrapers, has required architects and engineers to determine the forces and stresses within these structures. The object is to keep these structures standing, or "static"—that is, not in motion, especially not falling down.

The study of statics applies equally well to the human body, including balance, the forces in muscles, joints, and bones, and ultimately the possibility of fracture.

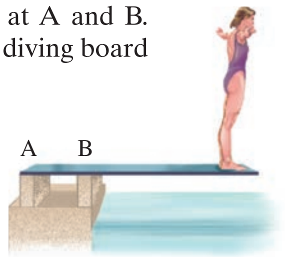
CHAPTER 9

Static Equilibrium; Elasticity and Fracture

CONTENTS

CHAPTER-OPENING QUESTION—Guess now!

The diving board shown here is held by two supports at A and B. Which statement is true about the forces exerted *on* the diving board at A and B?



(a) $\vec{\mathbf{F}}_A$ is down, $\vec{\mathbf{F}}_B$ is up, and F_B is larger than F_A.
(b) Both forces are up and F_B is larger than F_A.
(c) $\vec{\mathbf{F}}_A$ is down, $\vec{\mathbf{F}}_B$ is up, and F_A is larger than F_B.
(d) Both forces are down and approximately equal.
(e) $\vec{\mathbf{F}}_B$ is down, $\vec{\mathbf{F}}_A$ is up, and they are equal.

In this Chapter, we will study a special case in mechanics—when the net force and the net torque on an object, or system of objects, are both zero. In this case both the linear acceleration and the angular acceleration of the object or system are zero. The object is either at rest, or its center of mass is moving at constant velocity. We will be concerned mainly with the first situation, in which the object or objects are all at rest, or *static* (= not moving).

The net force and the net torque can be zero, but this does not imply that no forces at all act on the objects. In fact it is virtually impossible to find an object on which no forces act. Just how and where these forces act can be very important, both for buildings and other structures, and in the human body.

Sometimes, as we shall see in this Chapter, the forces may be so great that the object is seriously *deformed*, or it may even *fracture* (break)—and avoiding such problems gives this field of *statics* even greater importance.

Statics is concerned with the calculation of the forces acting on and within structures that are in *equilibrium*. Determination of these forces, which occupies us in the first part of this Chapter, then allows a determination of whether the structures can sustain the forces without significant deformation or fracture, subjects we discuss later in this Chapter. These techniques can be applied in a wide range of fields. Architects and engineers must be able to calculate the forces on the structural components of buildings, bridges, machines, vehicles, and other structures, since any material will buckle or break if too much force is applied (Fig. 9–1). In the human body a knowledge of the forces in muscles and joints is of great value for doctors, physical therapists, and athletes.

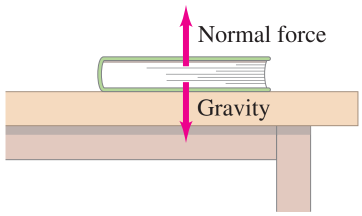
FIGURE 9–1 Elevated walkway collapse in a Kansas City hotel in 1981. How a simple physics calculation could have prevented the tragic loss of over 100 lives is considered in Example 9–12.

9–1 The Conditions for Equilibrium

Objects in daily life have at least one force acting on them (gravity). If they are at rest, then there must be other forces acting on them as well so that the net force is zero. A book at rest on a table, for example, has two forces acting on it, the downward force of gravity and the normal force the table exerts upward on it (Fig. 9–2). Because the book is at rest, Newton's second law tells us that the net force on it is zero. Thus the upward force exerted by the table on the book must be equal in magnitude to the force of gravity acting downward on the book. Such an object is said to be in **equilibrium** (Latin for "equal forces" or "balance") under the action of these two forces.

Do not confuse the two forces in Fig. 9–2 with the equal and opposite forces of Newton's third law, which act on different objects. In Fig. 9–2 , both forces act on the same object; and they happen to add up to zero.

FIGURE 9–2 The book is in equilibrium; the net force on it is zero.



EXAMPLE 9–1 **Straightening teeth.** The wire band shown in Fig. 9–3a has a tension F_T of 2.0 N along it. It therefore exerts forces of 2.0 N on the highlighted tooth (to which it is attached) in the two directions shown. Calculate the resultant force on the tooth due to the wire, F_R.

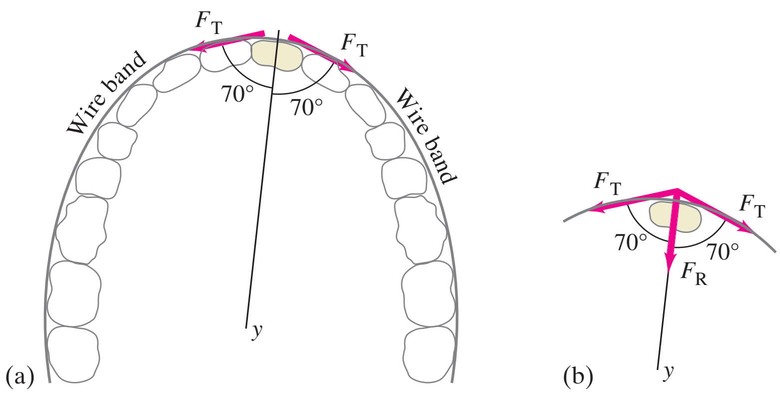


FIGURE 9–3 Forces on a tooth. Example 9–1.

APPROACH Since the two forces F_T are equal, their sum will be directed along the line that bisects the angle between them, which we have chosen to be the y axis. The x components of the two forces add up to zero.

SOLUTION The y component of each force is $(2.0\text{ N})(\cos 70°) = 0.68\text{ N}$: adding the two together, we get a resultant force $F_R = 1.4\text{ N}$ as shown in Fig. 9–3b. We assume that the tooth is in equilibrium because the gums exert a nearly equal magnitude force in the opposite direction. Actually that is not quite so since the objective is to move the tooth ever so slowly.

NOTE If the wire is firmly attached to the tooth, the tension to the right, say, can be made larger than that to the left, and the resultant force would correspondingly be directed more toward the right.

PHYSICS APPLIED
Braces for teeth

The First Condition for Equilibrium

For an object to be at rest, Newton's second law tells us that the sum of the forces acting on it must add up to zero. Since force is a vector, the components of the net force must each be zero. Hence, a condition for equilibrium is that

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \Sigma F_z = 0. \tag{9–1}$$

We will mainly be dealing with forces that act in a plane, so we usually need only the x and y components. We must remember that if a particular force component points along the negative x or y axis, it must have a negative sign. Equations 9–1 represent the **first condition for equilibrium.**

We saw in Chapter 4 that to solve Problems involving forces, we need to draw a *free-body diagram*, indicating *all* the forces on a given object (see Section 4–7).

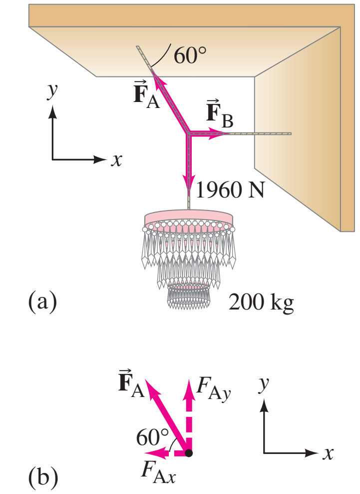


FIGURE 9–4 Example 9–2.

EXAMPLE 9–2 Chandelier cord tension. Calculate the tensions $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ in the two cords that are connected to the vertical cord supporting the 200-kg chandelier in Fig. 9–4. Ignore the mass of the cords.

APPROACH We need a free-body diagram, but for which object? If we choose the chandelier, the cord supporting it must exert a force equal to the chandelier's weight $mg = (200\,\text{kg})(9.80\,\text{m/s}^2) = 1960\,\text{N}$. But the forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ don't get involved. Instead, let us choose as our object the point where the three cords join (it could be a knot). The free-body diagram is then as shown in Fig. 9–4a. The three forces—$\vec{\mathbf{F}}_A$, $\vec{\mathbf{F}}_B$, and the tension in the vertical cord equal to the weight of the 200-kg chandelier—act at this point where the three cords join. For this junction point we write $\Sigma F_x = 0$ and $\Sigma F_y = 0$, since the problem is laid out in two dimensions. The directions of $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ are known, since tension in a cord can only be along the cord—any other direction would cause the cord to bend, as already pointed out in Chapter 4. Thus, our unknowns are the magnitudes F_A and F_B.

SOLUTION We first resolve $\vec{\mathbf{F}}_A$ into its horizontal (x) and vertical (y) components. Although we don't know the value of F_A, we can write (see Fig. 9–4b) $F_{Ax} = -F_A \cos 60°$ and $F_{Ay} = F_A \sin 60°$. $\vec{\mathbf{F}}_B$ has only an x component. In the vertical direction, we have the downward force exerted by the vertical cord equal to the weight of the chandelier $mg = (200\,\text{kg})(g)$, and the vertical component of $\vec{\mathbf{F}}_A$ upward:

$$\begin{aligned} \Sigma F_y &= 0 \\ F_A \sin 60° - (200\,\text{kg})(g) &= 0 \end{aligned}$$

so

$$F_A = \frac{(200\,\text{kg})g}{\sin 60°} = (231\,\text{kg})g = (231\,\text{kg})(9.80\,\text{m/s}^2) = 2260\,\text{N}.$$

In the horizontal direction, with $\Sigma F_x = 0$,

$$\Sigma F_x = F_B - F_A \cos 60° = 0.$$

Thus

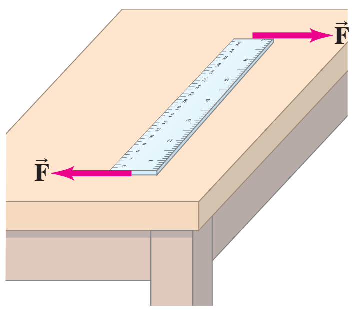
$$F_B = F_A \cos 60° = (231\,\text{kg})(g)(0.500) = (115\,\text{kg})g = 1130\,\text{N}.$$

The magnitudes of $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ determine the strength of cord or wire that must be used. In this case, the cord must be able to support a mass of more than 230 kg.

NOTE We didn't insert the value of g, the acceleration due to gravity, until the end. In this way we found the magnitude of the force in terms of g times the number of kilograms (which may be a more familiar quantity than newtons).

EXERCISE A In Example 9–2, F_A has to be greater than the chandelier's weight, mg. Why?

FIGURE 9–5 Although the net force on it is zero, the ruler will move (rotate). A pair of equal forces acting in opposite directions but at different points on an object (as shown here) is referred to as a *couple*.



The Second Condition for Equilibrium

Although Eqs. 9–1 are a necessary condition for an object to be in equilibrium, they are not always a sufficient condition. Figure 9–5 shows an object on which the net force is zero. Although the two forces labeled $\vec{\mathbf{F}}$ add up to give zero net force on the object, they do give rise to a net torque that will rotate the object.

Referring to Eq. 8–14, $\Sigma\tau = I\alpha$, we see that if an object is to remain at rest, the net torque applied to it (calculated about *any* axis) must be zero. Thus we have the **second condition for equilibrium**: that the sum of the torques acting on an object, as calculated about any axis, must be zero:

$$\Sigma\tau = 0. \tag{9–2}$$

This condition will ensure that the angular acceleration, α, about any axis will be zero. If the object is not rotating initially ($\omega = 0$), it will not start rotating. Equations 9–1 and 9–2 are the only requirements for an object to be in equilibrium.

We will mainly consider cases in which the forces all act in a plane (we call it the xy plane). In such cases the torque is calculated about an axis that is perpendicular to the xy plane. *The choice of this axis is arbitrary*. If the object is at rest, then $\Sigma\tau = 0$ is valid about any axis. Therefore we can choose any axis that makes our calculation easier. Once the axis is chosen, all torques must be calculated about that axis.

CAUTION

Axis choice for $\Sigma\tau = 0$ is arbitrary. All torques must be calculated about the same axis.

CONCEPTUAL EXAMPLE 9–3 **A lever.** The bar in Fig. 9–6 is being used as a lever to pry up a large rock. The small rock acts as a fulcrum (pivot point). The force F_P required at the long end of the bar can be quite a bit smaller than the rock's weight mg, since it is the *torques* that balance in the rotation about the fulcrum. If, however, the leverage isn't sufficient, and the large rock isn't budged, what are two ways to increase the lever arm?

RESPONSE One way is to increase the lever arm of the force F_P by slipping a pipe over the end of the bar and thereby pushing with a longer lever arm. A second way is to move the fulcrum closer to the large rock. This may change the long lever arm R only a little, but it changes the short lever arm r by a substantial fraction and therefore changes the ratio of R/r dramatically. In order to pry the rock, the torque due to F_P must at least balance the torque due to mg; that is, $mgr = F_P R$ and

$$\frac{r}{R} = \frac{F_P}{mg}.$$

With r smaller, the weight mg can be balanced with less force F_P. The ratio R/r is the **mechanical advantage** of the system. A lever is a "simple machine." We discussed another simple machine, the pulley, in Chapter 4, Example 4–14.

PHYSICS APPLIED

The lever

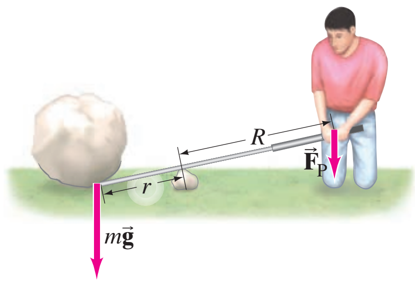


FIGURE 9–6 Example 9–3. A lever can "multiply" your force.

EXERCISE B For simplicity, we wrote the equation in Example 9–3 as if the lever were perpendicular to the forces. Would the equation be valid even for a lever at an angle as shown in Fig. 9–6?

9–2 Solving Statics Problems

The subject of statics is important because it allows us to calculate certain forces on (or within) a structure when some of the forces on it are already known. We will mainly consider situations in which all the forces act in a plane, so we can have two force equations (x and y components) and one torque equation, for a total of three equations. Of course, you do not have to use all three equations if they are not needed. When using a torque equation, a torque that tends to rotate the object counterclockwise is usually considered positive, whereas a torque that tends to rotate it clockwise is considered negative. (But the opposite convention would be OK too.)

PROBLEM SOLVING

$\tau > 0$ counterclockwise
$\tau < 0$ clockwise

One of the forces that acts on objects is the force of gravity. As we discussed in Section 7–8, we can consider the force of gravity on an object as acting at its center of gravity (CG) or center of mass (CM), which for practical purposes are the same point. For uniform symmetrically shaped objects, the CG is at the geometric center. For more complicated objects, the CG can be determined as discussed in Section 7–8.

There is no single technique for attacking statics problems, but the following procedure may be helpful.

PROBLEM SOLVING

Statics

1. Choose one object at a time for consideration. Make a careful **free-body diagram** by showing all the forces acting on that object, including gravity, and the points at which these forces act. If you aren't sure of the direction of a force, choose a direction; if the actual direction of the force (or component of a force) is opposite, your eventual calculation will give a result with a minus sign.
2. Choose a convenient **coordinate system**, and resolve the forces into their components.
3. Using letters to represent unknowns, write down the **equilibrium equations** for the **forces**:

$$\Sigma F_x = 0$$

and

$$\Sigma F_y = 0,$$

assuming all the forces act in a plane.

4. For the **torque equation**,

$$\Sigma \tau = 0,$$

choose any axis perpendicular to the xy plane that might make the calculation easier. (For example, you can reduce the number of unknowns in the resulting equation by choosing the axis so that one of the unknown forces acts through that axis; then this force will have zero lever arm and produce zero torque, and so won't appear in the torque equation.) Pay careful attention to determining the lever arm for each force correctly. Give each torque a + or − sign to indicate torque direction. For example, if torques tending to rotate the object counterclockwise are positive, then those tending to rotate it clockwise are negative.

5. **Solve** these equations for the unknowns. Three equations allow a maximum of three unknowns to be solved for. They can be forces, distances, or even angles.

PHYSICS APPLIED

Balancing a seesaw

EXAMPLE 9–4 **Balancing a seesaw.** A board of mass $M = 4.0\,\text{kg}$ serves as a seesaw for two children, as shown in Fig. 9–7a. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.

APPROACH We follow the steps of the Problem Solving Strategy above.

SOLUTION

1. **Free-body diagram.** We choose the board as our object, and assume it is horizontal. Its free-body diagram is shown in Fig. 9–7b. The forces acting on the board are the forces exerted downward on it by each child, $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$, the upward force exerted by the pivot $\vec{\mathbf{F}}_N$, and the force of gravity on the board $(= M\vec{\mathbf{g}})$ which acts at the center of the uniform board.
2. **Coordinate system.** We choose y to be vertical, with positive upward, and x horizontal to the right, with origin at the pivot.
3. **Force equation.** All the forces are in the y (vertical) direction, so

$$\Sigma F_y = 0$$
$$F_N - m_A g - m_B g - Mg = 0,$$

where $F_A = m_A g$ and $F_B = m_B g$.

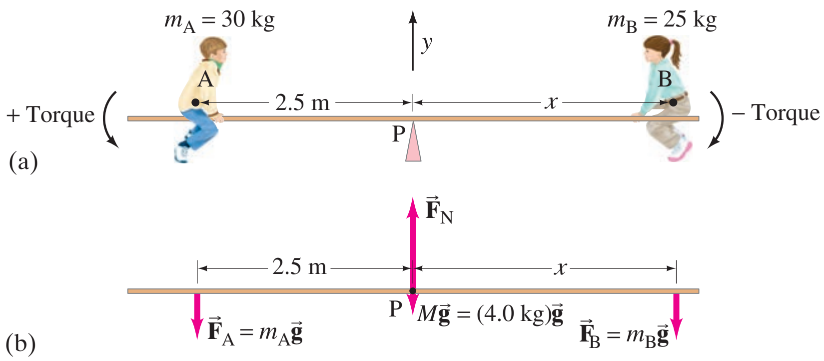


FIGURE 9–7 (a) Two children on a seesaw, Example 9–4. (b) Free-body diagram of the board.

4. **Torque equation.** Let us calculate the torque about an axis through the board at the pivot point, P. Then the lever arms for F_N and for the weight of the board are zero, and they will contribute zero torque about point P. Thus the torque equation will involve only the forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$, which are equal to the weights of the children. The torque exerted by each child will be mg times the appropriate lever arm, which here is the distance of each child from the pivot point. $\vec{\mathbf{F}}_A$ tends to rotate the board counterclockwise $(+)$ and $\vec{\mathbf{F}}_B$ clockwise $(-)$, so the torque equation is

$$\begin{aligned} \Sigma\tau &= 0 \\ m_A g(2.5\,\mathrm{m}) - m_B gx + Mg(0\,\mathrm{m}) + F_N(0\,\mathrm{m}) &= 0 \end{aligned}$$

or

$$m_A g(2.5\,\mathrm{m}) - m_B gx = 0,$$

where two terms were dropped because their lever arms were zero.

5. **Solve.** We solve the torque equation for x and find

$$x = \frac{m_A}{m_B}(2.5\,\mathrm{m}) = \frac{30\,\mathrm{kg}}{25\,\mathrm{kg}}(2.5\,\mathrm{m}) = 3.0\,\mathrm{m}.$$

To balance the seesaw, child B must sit so that her CG is 3.0 m from the pivot point. This makes sense: since she is lighter, she must sit farther from the pivot than the heavier child in order to provide torques of equal magnitude.

EXERCISE C We did not need to use the force equation to solve Example 9–4 because of our choice of the axis. Use the force equation to find the force exerted by the pivot.

EXAMPLE 9–5 **Forces on a beam and supports.** A uniform 1500-kg beam, 20.0 m long, supports a 15,000-kg printing press 5.0 m from the right support column (Fig. 9–8). Calculate the force on each of the vertical support columns.

APPROACH We analyze the forces on the beam (the force the beam exerts on each column is equal and opposite to the force exerted by the column on the beam). We label these forces $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$ in Fig. 9–8. The weight of the beam itself acts at its center of gravity, 10.0 m from either end. We choose a convenient axis for writing the torque equation: the point of application of $\vec{\mathbf{F}}_A$ (labeled P), so $\vec{\mathbf{F}}_A$ will not enter the equation (its lever arm will be zero) and we will have an equation in only one unknown, F_B.

SOLUTION The torque equation, $\Sigma\tau = 0$, with the counterclockwise direction as positive, gives

$$\Sigma\tau = -(10.0\,\mathrm{m})(1500\,\mathrm{kg})g - (15.0\,\mathrm{m})(15{,}000\,\mathrm{kg})g + (20.0\,\mathrm{m})F_B = 0.$$

Solving for F_B, we find $F_B = (12{,}000\,\mathrm{kg})g = 118{,}000\,\mathrm{N}$. To find F_A, we use $\Sigma F_y = 0$, with $+y$ upward:

$$\Sigma F_y = F_A - (1500\,\mathrm{kg})g - (15{,}000\,\mathrm{kg})g + F_B = 0.$$

Putting in $F_B = (12{,}000\,\mathrm{kg})g$, we find that $F_A = (4500\,\mathrm{kg})g = 44{,}100\,\mathrm{N}$.

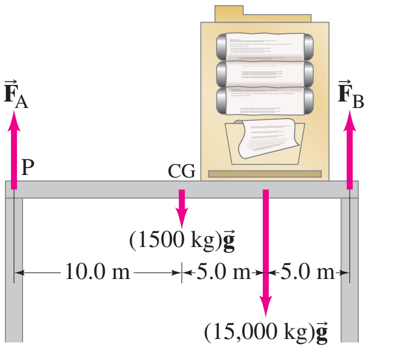


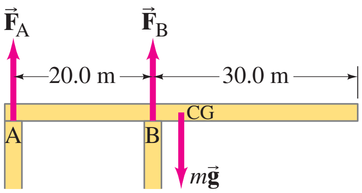
FIGURE 9–8 A 1500-kg beam supports a 15,000-kg machine. Example 9–5.

Figure 9–9 shows a uniform beam that extends beyond its support like a diving board. Such a beam is called a **cantilever**. The forces acting on the beam in Fig. 9–9 are those due to the supports, $\vec{\mathbf{F}}_A$ and $\vec{\mathbf{F}}_B$, and the force of gravity which acts at the CG, 5.0 m to the right of the right-hand support. If you follow the procedure of the last Example and calculate F_A and F_B, assuming they point upward as shown in Fig. 9–9, you will find that F_A comes out negative. If the beam has a mass of 1200 kg and a weight $mg = 12{,}000\,\mathrm{N}$, then $F_B = 15{,}000\,\mathrm{N}$ and $F_A = -3000\,\mathrm{N}$ (see Problem 10). Whenever an unknown force comes out negative, it merely means that the force actually points in the opposite direction from what you assumed. Thus in Fig. 9–9, $\vec{\mathbf{F}}_A$ actually must pull downward (by means of bolts, screws, fasteners, and/or glue). To see why $\vec{\mathbf{F}}_A$ has to act downward, note that the board's weight acting at the CG would otherwise rotate the board clockwise about support B.

PHYSICS APPLIED
Cantilever

PROBLEM SOLVING
If a force comes out negative

FIGURE 9–9 A cantilever.



EXERCISE D Return to the Chapter-Opening Question, page 230, and answer it again now. Try to explain why you may have answered differently the first time.

Our next Example involves a beam that is attached to a wall by a hinge and is supported by a cable or cord (Fig. 9–10). It is important to remember that a flexible cable can support a force only along its length. (If there were a component of force perpendicular to the cable, it would bend because it is flexible.) But for a rigid device, such as the hinge in Fig. 9–10, the force can be in any direction and we can know the direction only after solving the equations. (The hinge is assumed small and smooth, so it can exert no internal torque on the beam.)

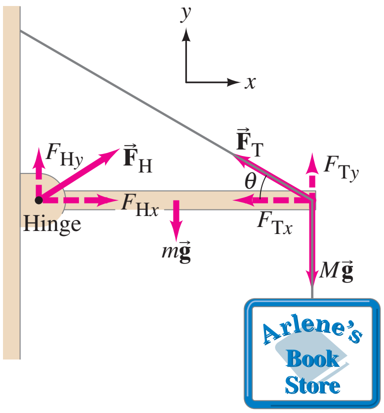


FIGURE 9–10 Example 9–6.

EXAMPLE 9–6 **Hinged beam and cable.** A uniform beam, 2.20 m long with mass $m = 25.0$ kg, is mounted by a small hinge on a wall as shown in Fig. 9–10. The beam is held in a horizontal position by a cable that makes an angle $\theta = 30.0°$. The beam supports a sign of mass $M = 28.0$ kg suspended from its end. Determine the components of the force $\vec{\mathbf{F}}_H$ that the (smooth) hinge exerts on the beam, and the tension F_T in the supporting cable.

APPROACH Figure 9–10 is the free-body diagram for the beam, showing all the forces acting on the beam. It also shows the components of $\vec{\mathbf{F}}_T$ and a guess for the direction of $\vec{\mathbf{F}}_H$. We have three unknowns, F_{Hx}, F_{Hy}, and F_T (we are given θ), so we will need all three equations, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma\tau = 0$.

SOLUTION The sum of the forces in the vertical (y) direction is

$$\begin{aligned} \Sigma F_y &= 0 \\ F_{Hy} + F_{Ty} - mg - Mg &= 0. \end{aligned} \qquad \textbf{(i)}$$

In the horizontal (x) direction, the sum of the forces is

$$\begin{aligned} \Sigma F_x &= 0 \\ F_{Hx} - F_{Tx} &= 0. \end{aligned} \qquad \textbf{(ii)}$$

For the torque equation, we choose the axis at the point where $\vec{\mathbf{F}}_T$ and $M\vec{\mathbf{g}}$ act. Then our torque equation will contain only one unknown, F_{Hy}, because the lever arms for $\vec{\mathbf{F}}_T$, $M\vec{\mathbf{g}}$, and F_{Hx} are zero. We choose torques that tend to rotate the beam counterclockwise as positive. The weight mg of the (uniform) beam acts at its center, so we have

$$\begin{aligned} \Sigma\tau &= 0 \\ -(F_{Hy})(2.20\text{ m}) + mg(1.10\text{ m}) &= 0. \end{aligned}$$

We solve for F_{Hy}:

$$F_{Hy} = \left(\frac{1.10\text{ m}}{2.20\text{ m}}\right)mg = (0.500)(25.0\text{ kg})(9.80\text{ m/s}^2) = 123\text{ N}. \qquad \textbf{(iii)}$$

Next, since the tension $\vec{\mathbf{F}}_T$ in the cable acts along the cable ($\theta = 30.0°$), we see from Fig. 9–10 that $\tan\theta = F_{Ty}/F_{Tx}$, or

$$F_{Ty} = F_{Tx}\tan\theta = F_{Tx}(\tan 30.0°). \qquad \textbf{(iv)}$$

Equation (i) above gives

$$F_{Ty} = (m + M)g - F_{Hy} = (53.0\text{ kg})(9.80\text{ m/s}^2) - 123\text{ N} = 396\text{ N}.$$

Equations (iv) and (ii) give

$$\begin{aligned} F_{Tx} &= F_{Ty}/\tan 30.0° = 396\text{ N}/\tan 30.0° = 686\text{ N}; \\ F_{Hx} &= F_{Tx} = 686\text{ N}. \end{aligned}$$

The components of $\vec{\mathbf{F}}_H$ are $F_{Hy} = 123$ N and $F_{Hx} = 686$ N. The tension in the wire is $F_T = \sqrt{F_{Tx}^2 + F_{Ty}^2} = \sqrt{(686\text{ N})^2 + (396\text{ N})^2} = 792$ N.†

†Our calculation used numbers rounded off to 3 significant figures. If you keep an extra digit, or leave the numbers in your calculator, you get $F_{Ty} = 396.5$ N, $F_{Tx} = 686.8$ N, and $F_T = 793$ N, all within the expected precision of 3 significant figures (Section 1–4).

Alternate Solution Let us see the effect of choosing a different axis for calculating torques, such as an axis through the hinge. Then the lever arm for F_H is zero, and the torque equation $(\Sigma\tau = 0)$ becomes

$$-mg(1.10\text{ m}) - Mg(2.20\text{ m}) + F_{Ty}(2.20\text{ m}) = 0.$$

We solve this for F_{Ty} and find

$$F_{Ty} = \frac{m}{2}g + Mg = (12.5\text{ kg} + 28.0\text{ kg})(9.80\text{ m/s}^2) = 397\text{ N}.$$

We get the same result, within the precision of our significant figures.

NOTE It doesn't matter which axis we choose for $\Sigma\tau = 0$. Using a second axis can serve as a check.

*A More Difficult Example—The Ladder

EXAMPLE 9–7 **Ladder.** A 5.0-m-long ladder leans against a wall at a point 4.0 m above a cement floor as shown in Fig. 9–11. The ladder is uniform and has mass $m = 12.0$ kg. Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

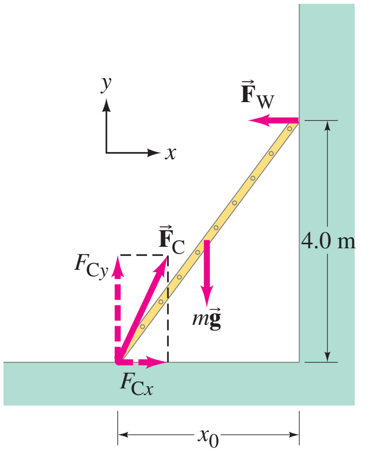


FIGURE 9–11 A ladder leaning against a wall. Example 9–7. The force $\vec{\mathbf{F}}_C$ that the cement floor exerts on the ladder need not be along the ladder which (unlike a cord) is rigid.

APPROACH Figure 9–11 is the free-body diagram for the ladder, showing all the forces acting on the ladder. The wall, since it is frictionless, can exert a force only perpendicular to the wall, and we label that force $\vec{\mathbf{F}}_W$. The cement floor exerts a force $\vec{\mathbf{F}}_C$ which has both horizontal and vertical force components: F_{Cx} is frictional and F_{Cy} is the normal force. Finally, gravity exerts a force $mg = (12.0\text{ kg})(9.80\text{ m/s}^2) = 118\text{ N}$ on the ladder at its midpoint, since the ladder is uniform.

SOLUTION Again we use the equilibrium conditions, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma\tau = 0$. We will need all three since there are three unknowns: F_W, F_{Cx}, and F_{Cy}. The y component of the force equation is

$$\Sigma F_y = F_{Cy} - mg = 0,$$

so immediately we have

$$F_{Cy} = mg = 118\text{ N}.$$

The x component of the force equation is

$$\Sigma F_x = F_{Cx} - F_W = 0.$$

To determine both F_{Cx} and F_W, we need a torque equation. If we choose to calculate torques about an axis through the point where the ladder touches the cement floor, then $\vec{\mathbf{F}}_C$, which acts at this point, will have a lever arm of zero and so won't enter the equation. The ladder touches the floor a distance $x_0 = \sqrt{(5.0\text{ m})^2 - (4.0\text{ m})^2} = 3.0\text{ m}$ from the wall (right triangle, $c^2 = a^2 + b^2$). The lever arm for mg is half this, or 1.5 m, and the lever arm for F_W is 4.0 m, Fig. 9–11. The torque equation about the ladder's contact point on the cement is

$$\Sigma\tau = (4.0\text{ m})F_W - (1.5\text{ m})mg = 0.$$

Thus

$$F_W = \frac{(1.5\text{ m})(12.0\text{ kg})(9.8\text{ m/s}^2)}{4.0\text{ m}} = 44\text{ N}.$$

Then, from the x component of the force equation,

$$F_{Cx} = F_W = 44\text{ N}.$$

Since the components of $\vec{\mathbf{F}}_C$ are $F_{Cx} = 44$ N and $F_{Cy} = 118$ N, then

$$F_C = \sqrt{(44\text{ N})^2 + (118\text{ N})^2} = 126\text{ N} \approx 130\text{ N}$$

(rounded off to two significant figures), and it acts at an angle to the floor of

$$\theta = \tan^{-1}(118\text{ N}/44\text{ N}) = 70°.$$

NOTE The force $\vec{\mathbf{F}}_C$ does *not* have to act along the ladder's direction because the ladder is rigid and not flexible like a cord or cable (see page 89, Chapter 4).