1 Correlation Function

We begin by writing the angular space observable, X, in terms of the harmonic counterpart

$$X(\Omega) = \sum_{lm} \tilde{X}_{lm} Y_{lm}(\Omega) \tag{1}$$

where Ω refers to the angular coordinates on the sky. The angular cross correlation function of two (scalar) tracers, X, Z of large scale structure can be written in terms of their harmonic space counter parts, $\tilde{X}, \tilde{Z}$ as

$$\begin{align}
\langle XZ\rangle(\theta) &= \left\langle \sum_{\ell,m}\sum_{\ell',m'} \tilde{X}_{\ell m}\tilde{Z}_{\ell' m'} Y_{\ell m}(\Omega) Y_{\ell' m'}(\Omega+\theta) \right\rangle \tag{2}\\
&= \sum_{\ell,m} C_\ell Y_{\ell m}(\Omega) Y_{\ell m}(\Omega+\theta) \tag{3}\\
\langle XZ\rangle(\theta) &= \frac{1}{4\pi}\sum_\ell (2\ell+1) C_\ell P_\ell(\cos\theta) \tag{4}
\end{align}$$

We used the identities

$$\begin{align}
\langle \tilde{X}_{\ell m}\tilde{Z}_{\ell' m'}\rangle &= C_\ell \delta_D(m,m')\delta_D(\ell,\ell') \tag{5}\\
\sum_{m=-\ell}^{m=\ell} Y_{\ell m}(\Omega) Y_{\ell m}(\Omega+\theta) &= \frac{2\ell+1}{4\pi} \tag{6}
\end{align}$$

For the case of shear, since it is a spin-2 object, eq.1 is written in terms of spin harmonics (see for ex. Castro et al., 2005; Kilbinger et al., 2017). Rest of the analysis proceeds similarly, using the relation for spin harmonics, analogous to eq. 6 Hu & White (1997).

Expression of ξ_+ is same as eq. 4. Expressions for galaxy lensing cross correlation de Putter & Takada (2010) and ξ_- is given by

$$\begin{align}
\langle g\gamma_T\rangle(\theta) &= \frac{1}{4\pi}\sum_\ell \frac{(2\ell+1)}{\ell(\ell+1)} C_\ell^{g\kappa} P_\ell^2(\cos\theta) \tag{7}\\
\xi_+(\theta) &= \frac{1}{4\pi}\sum_\ell (2\ell+1) C_\ell^{\kappa\kappa} P_\ell(\cos\theta) \tag{8}\\
\xi_-(\theta) &= \frac{1}{4\pi}\sum_\ell \frac{(\ell-4)!}{(\ell+4)!}\ell^4 (2\ell+1) C_\ell^{\kappa\kappa} P_\ell^4(\cos\theta) \tag{9}
\end{align}$$

Sukhdeep: ξ_- is a bit of cheating. I'm not familiar with spin harmonics, so I simply took the relation between P_ℓ^m and $J_m(\ell\theta)$ to get this expression from the Hankel transform for ξ_-. It may not be very accurate at large scale (low ℓ) as is evident from $g\gamma_T$ expression. I can sort this out later. If somebody knows correct expression already, please feel free to put it in.

The ccl_tracer_corr_legendre routine computes these transform to convert C_ℓ to correlation functions. We use the associated Legendre function implementation from gsl library. ccl_tracer_corr_legendre routine evaluations can be slow, especially for P_ℓ^m with $m > 0$. Note that P_ℓ^m evaluations need to be done only once and can then be saved as long as ℓ, θ values do not change. This is not yet implemented, but will be done soon.

1.1 Hankel Transform

Expression in eq. 7–9 can be written as Hankel transforms using the relation between P_ℓ^m and bessel functions J_m

$$P_\ell^m(\cos\theta) = (-1)^m \frac{(\ell+m)!}{(\ell-m)!} \ell^{-m} J_m(\ell\theta) \tag{10}$$

We get the following analogous expressions (flat-sky limit)

$$\langle g\gamma_T \rangle(\theta) = \frac{1}{2\pi} \int d\ell \ell C_\ell J_2(\ell\theta) \tag{11}$$

$$\xi_+(\theta) = \frac{1}{2\pi} \int d\ell \ell C_\ell J_0(\ell\theta) \tag{12}$$

$$\xi_-(\theta) = \frac{1}{2\pi} \int d\ell \ell C_\ell J_4(\ell\theta) \tag{13}$$

To evaluate Hankel transform, we use the fast FFTlog routine(Hamilton, 2000; Talman, 2009). In brief, FFTlog works on functions periodic in log space, by writing the Hankel Transform as a convolution between bessel function and the function of interest (in this case C_ℓ). The convolution can then be evaluated using Fourier transforms, with Fourier transform of bessel function evaluated using analytical functions while Fourier transform of C_ℓ and the inverse Fourier transform of the product evaluated using fast fourier transform routines.

References

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