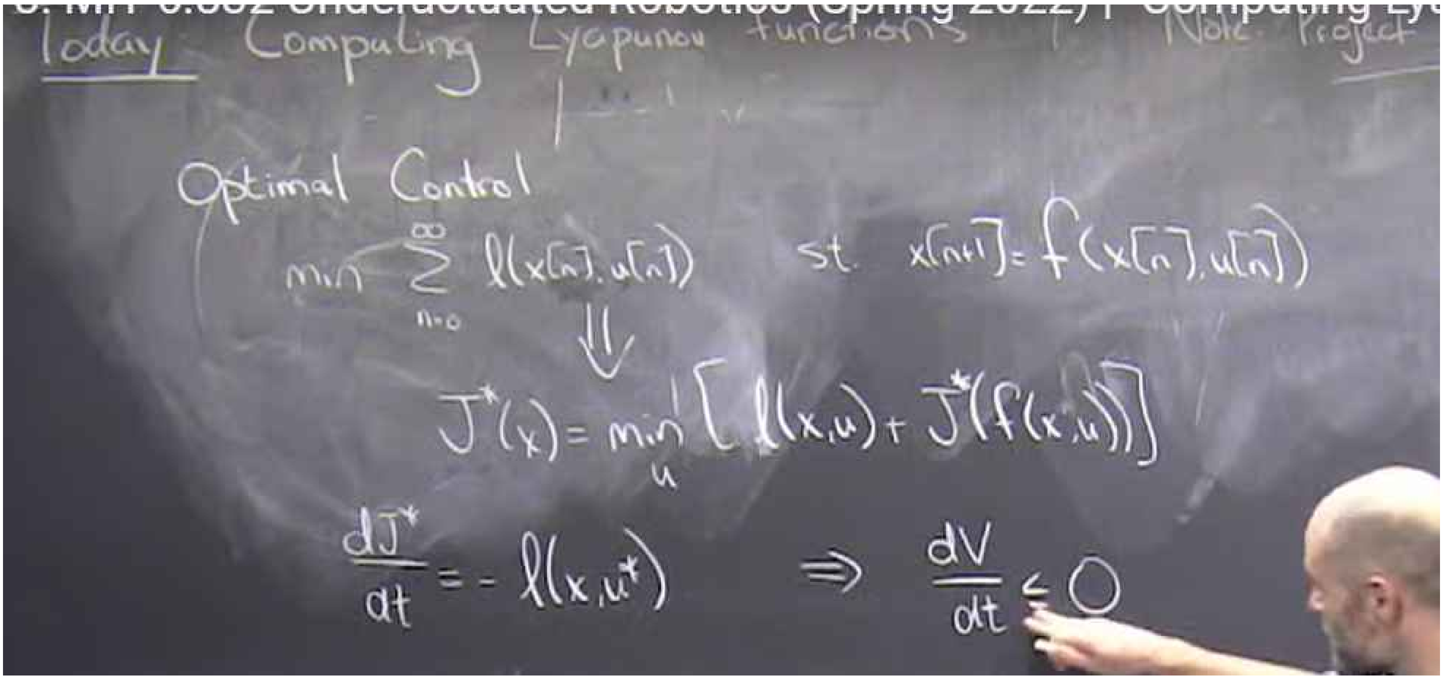
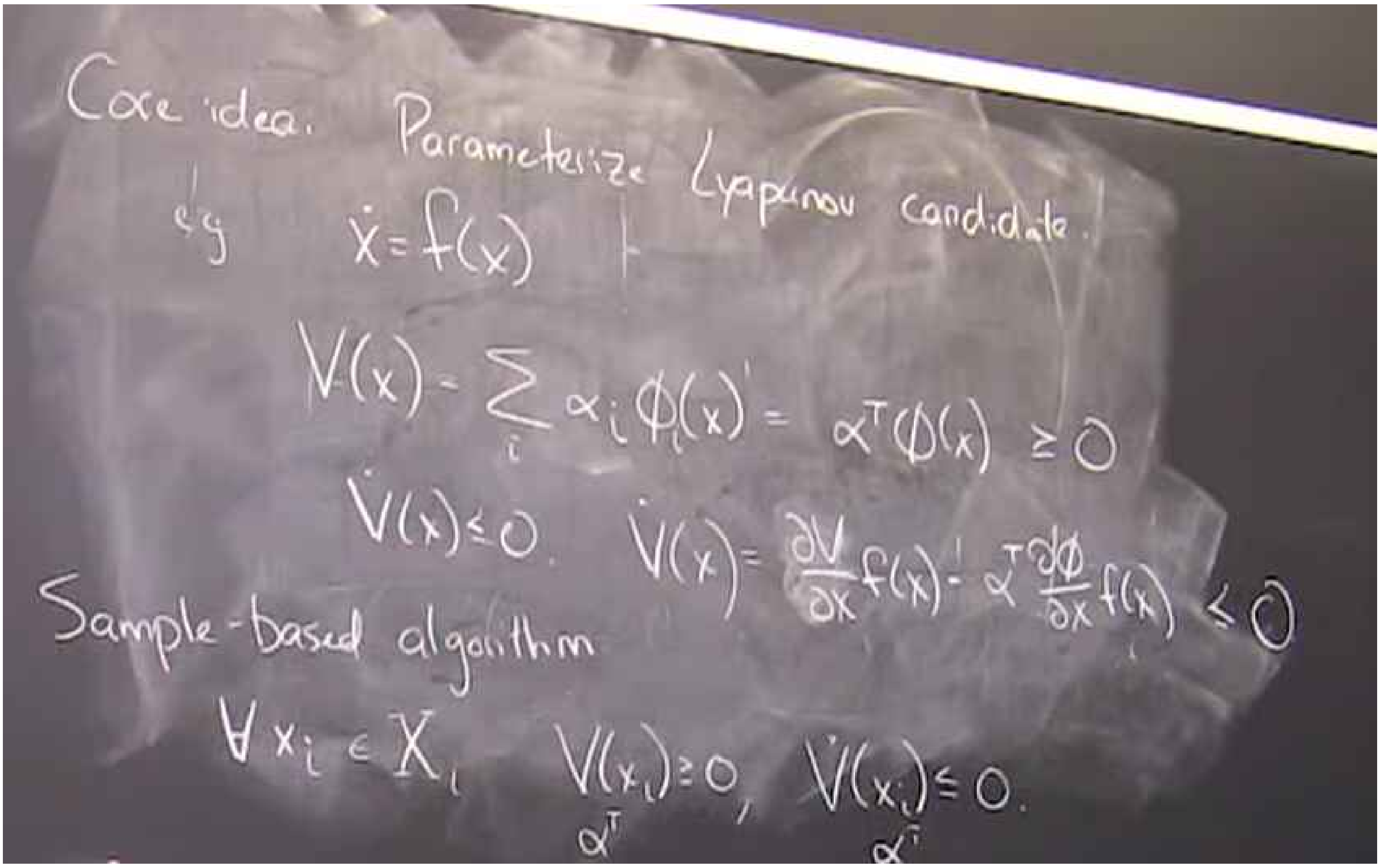
8. Computing Lyapunov Functions I

Today: Computing Lyapunov functions



Condition relaxed from bellman equation to lyapunov.



Core Idea: Parameterize Lyapunov Candidates

Formulate a search on possible lyapunov function

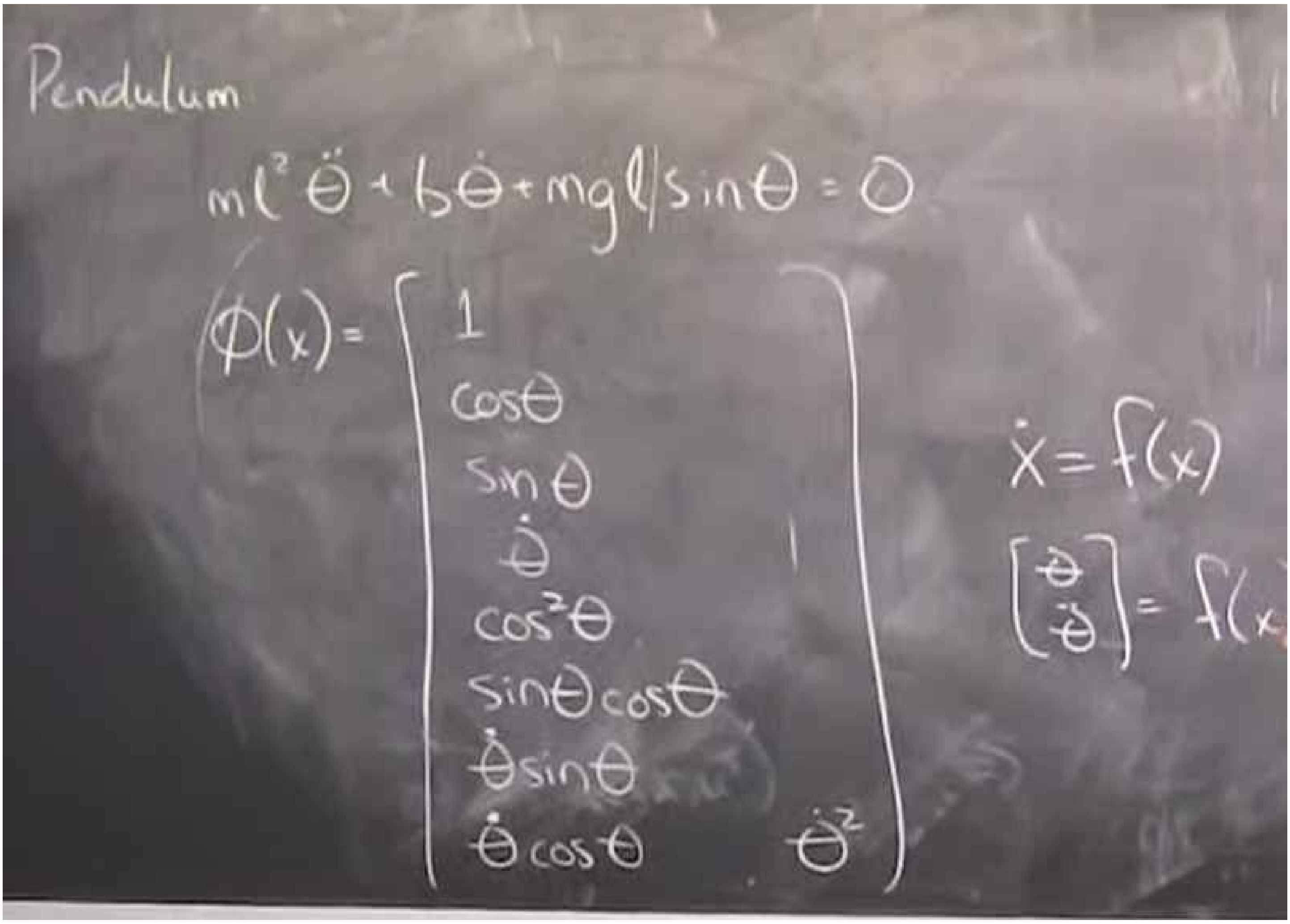
e.g.

$$\dot{x} = f(x)$$

$$V(x) = \sum_i \alpha_i \phi_i(x) = \alpha^T \phi(x) \geq 0$$, using function approxiations

$$\dot{V}(x) \leq 0, \ \ \dot{V}(x) = \frac{\partial V}{\partial x} f(x) = \alpha^T \frac{\partial \phi}{\partial x} f(x) \leq 0$$

Write a linear program, to generate a $\phi(x)$ using optimization



How do I prove to all x?

Idea: generate lyapunov like in this form:

$$V(x) = \sum_i \alpha_i^T \phi_i^2(x) \geq 0, \ \alpha \geq 0$$

$$= \phi^T(x) \begin{bmatrix} \alpha_{11} & \alpha_{12} & .. \\ .. & .. & .. \end{bmatrix} \phi(x)$$

We will use the form of optimization, e.g.

$$min_\alpha \alpha$$

$$s.t. \begin{bmatrix} \alpha & 0 \\ 0 & 1 \end{bmatrix} \geq 0$$, semi-definite program

But how do I parameterize $\dot{V}(x) = \frac{\partial V}{\partial x} f(x)$ to make it less than zero?

Question:

In stead of using $x^T P x$ to parameterize Lyapunov function $V(x)$

We now using $\phi^T(x) P \phi(x)$ for parameterization

So $\dot{V} = \frac{\partial(V)}{\partial(x)} f(x) = \phi^T(x) P \frac{\partial \phi}{\partial x} f(x) + f^T(x) \frac{\partial \phi^T}{\partial x} P \phi(x)$ is it equal to $\phi^T(x) P_2 \phi(x)$

Basis set changed, the dynamics stays in basis set 1, and stays in basis 2.

Assume dynamics is polynomial

Polynomial systems

$$\dot{x} = p(x) \ is \ polynomial$$

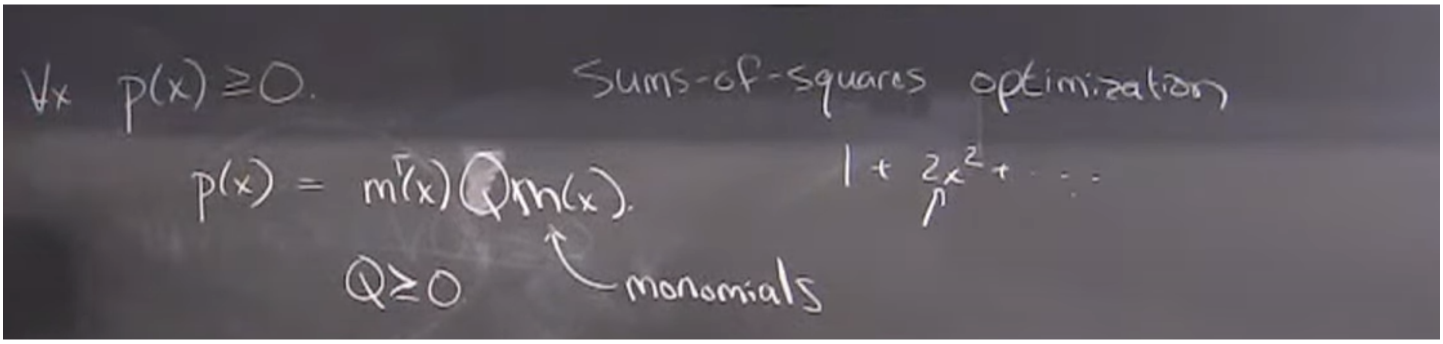
$$\phi(x) \ is \ polynomial$$

$$\frac{\partial \phi}{\partial x} \ is \ polynomial$$

$$\frac{\partial \phi}{\partial x} f(x) \ is \ polynomial$$

$$\phi_2(x) \ is \ second \ basis \ polynomial \ with \ matching \ degrees \ to \ \frac{\partial \phi}{\partial x} f(x)$$

Sum of squares Optimization:



Searching for Lyapunov function vis SOS