Comp Physics HW 2

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1. Newman 6.11:

Solution:
In general, when evaluating the relaxation method we have equations of the form $x = f(x)$. Let us define a parameter Δx that is described by

$$\Delta x = x' - x = f(x) - x. \tag{1}$$

According to Newman, the overrelaxation method involves the iteration of the following equation

$$x' = x + (1+\omega)\Delta x = (1+\omega)f(x) - \omega x \tag{2}$$

Let us define ϵ to be the error on our current solution estimate. This implies that the true solution x^* differs by said error by $x^* = x + \epsilon$. On the next iteration, let's define ϵ' to be the error on next estimate, so that $x^* = x' + \epsilon'$. Performing a Taylor expansion, the value x' after an iteration is given in terms of the previous values by

$$\begin{aligned} x' = f(x^*) = f(x^*) + f'(x^*)(x - x^*) = x^* + f'(x^*)(x - x^*) \\ \Rightarrow x' - x^* = f'(x^*)(x - x^*) = ((1+\omega)f'(x) - \omega)(x - x^*) \end{aligned} \tag{3}$$

Now, finally applying values near the solution x^*, we get

$$\epsilon' = \epsilon\left[(1+\omega)f'(x^*) - \omega\right]$$

Now, we revisit the solutions that deviate slightly by error ϵ

$$x^* = x + \epsilon = x + \frac{\epsilon'}{(1+\omega) - \omega} = x' + \epsilon'$$

Finally, we get

$$\epsilon' \approx \frac{x - x'}{1 - 1/[(1+\omega)f'(x) - \omega]} \tag{4}$$

Results:

Relaxation results...
Answer:0.7968126311118457, Iterations:14

Overrelaxation Results...
Answer:0.7968123729832619, Iterations:4

One may need to use a negative value of ω to avoid cancellation error if the function $f(x)$ is complex polynomial.

2. **Newman 6.13**: Planck's radiaition law says that the intensity of radiation per unit area per unit wavelength λ a black body emits at a temperature T is

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{hc/k_BT\lambda} - 1}$$

   (a) At maximum intensity, the derivative of the radiation law must go to zero implying

$$\frac{dI}{d\lambda} = \frac{-10\pi hc^2\lambda^{-6}(e^{hc/k_BT\lambda} - 1) - 2\pi hc^2\lambda^{-5}(-hc/k_BT\lambda^{-2}e^{hc/k_BT\lambda}}{(e^{hc/k_BT\lambda} - 1)^2} = 0$$

$$= -5(e^{hc/k_BT\lambda} - 1) + \frac{hc}{k_BT\lambda}e^{hc/k_BT\lambda} = 0$$

$$= -5 + 5e^{-hc/k_BT\lambda} + \frac{hc}{k_BT\lambda} = 0$$

   If we define a parameter $x = hc/k_BT\lambda$, we can rewrite the above equation as

$$-5 + 5e^{-x} + x = 0,$$

   which can be solved numerically. From the numeric value of x, we arrive at

$$\lambda = \frac{b}{T};\ b := \frac{hc}{k_Bx} \tag{5}$$

   with b being a constant of proportionality.
   Results:
   The value of x is: 4.965113639831543
   The value of the displacement constant is: 0.0028977732599822937 K m

   At a wavelength of 502nm, the Sun's temperature is roughly: 5772.45669319182 K

3. Could not get the Schecter function to fit at all.