# Milikan Oil Drop

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## Abstract

The quantized nature of the electric charge is observed and measured via suspension of minuscule non-volatile oil droplets in a constant electric field. Successive measurements of oil droplets' falling and rising velocity in the absense and presense of a constant electric field are measured respectively to determine the charge embedded in suspect oil droplets. The charge of an electron is determined to be $1.57 \times 10^{-19} \pm 7 \times 10^{-20}$ C, laying in one sigma range and 1.56% from the accepted value.

## INTRODUCTION

In 1909, Robert A. Millikan and Harvey Fletcher first measured the charge of an electron by exploiting very low mass oil droplets in a viscous medium to greatly resolve the electric force imparted on them by a constant electric field. The dynamics of such particles can be exactly calculated in the simple case where only the electric and gravitational force act on a sole oil droplet. The experimental setup suspends a small number of oil droplets in a chamber isolated from atmospheric noise. A constant electric field is supplied by a parallel plate capacitor through which the oil droplets may fall. In the atmosphere of the chamber, once the oil droplet has reached terminal velocity its dynamics are governed by (also illustrated in Fig.1 on the left),

$$mg = kv_f \tag{1}$$

with $m$ being the mass of the droplet, $g$ the acceleration near the surface of the earth, $k$ the coefficient of friction between the air and the droplet, and $v_f$ the velocity of the fall. The drops are then subjected to an electric field (Fig. 1, right), and are then governed by,

$$Eq = mg + kv_r \tag{2}$$

at terminal velocity where $E$ is the electric density, $q$ is the charge, and $v_r$ is the droplet's rising velocity. Together, the two equations allow for the elimination of $k$ and produce a clear equation for the charge $q$:

$$q = \frac{mg(v_f + v_r)}{Ev_f} \tag{3}$$

This is an exact value for the charge of a given droplet, however, it remains to measure the mass which cannot be easily done by direct methods. Instead, we may leverage the known density of the oil in use and Stokes' law for the frictional force exerted on spherical objects to determine the mass as a function of terminal velocity:

$$m = \frac{4\pi}{3}(\frac{9\eta v_f}{2\rho g})^3 \rho g \tag{4}$$

An additional correction factor is needed, however, as Stokes' law breaks down for the length and velocity scales of interest in this measurement. The final calculation of q takes the form

$$q = \frac{4\pi}{3}\left[\sqrt{\left(\frac{b}{2p}\right)^2 + \frac{9\eta v_f}{2\rho g}} - \frac{b}{2p}\right]^3 \frac{\rho g d(v_f + v_r)}{V v_f} \tag{5}$$

where $d$ is the distance between the plates of the capacitor, $\rho$ is the density of the oil, $\eta$ is the viscosity of air, $b$ is a known constant, and $V$ is the potential difference across the plates.

**METHODS**

In preparation for the experiment the thickness of the spacer plate, equivalent to the plate separation distance, was measured as well as the resistance of a thermisistor located inside the chamber; the thermisistor's resistance gives a direct measure of the temperature in the chamber and thus gives a direct measure of the viscosity of air in the chamber. Consistent temperature is important for collecting data so that the conditions remain constant and velocities remain relatively unaffected. The choice of non-volatile oil droplets is natural so as to insure that a great number of measurements can be made. The oil droplets are sprayed into a tightly sealed chamber that minimizes atmospheric noise in the droplets' dynamics. Droplets are illuminated by an LED light source. The chamber is equipped with a parallel plate capacitor through which the droplets can fall. A switch allows for the application of a specified voltage on either of the plates generating a uniform electric field. The oil is inserted through a small opening at the top plate using an atomizer that serves to break the oil into small droplets that can be viewed using a magnifying lens. The chamber is attached to a level and stable apparatus to ensure that the electric field is always parallel or anti-parallel to the direction of the gravitational field. Successive measurements of the droplets falling and rising velocity are taken via video recording of the view through the magnifying lens in order to determine the charge on a given droplet. The pixel length traveled by oil droplets can be calibrated according to a grid of known spacing that is located in the far field of the viewing scope's vision. Once enough measurements have been made at a given ionization, the droplets are subjected to our ionizing source Th-232 to induce a new ionization. In this way measurements of different charges can be taken while avoiding statistical uncertainty that comes from the different masses and behaviors of different droplets. The velocities are determined via an image analysis software called Tracker that maps the trajectories of oil droplets and calculates a velocity for each frame given the particle's position in the frame before and after. Once all velocities are acquired, a python program can be written to calculate and tabulate all measured values of $q$.

**Error Analysis**

We attempted to calculate the velocities of droplets using a stopwatch and our knowledge of the distances on the grid, however this presents great systematic error in time measurements which are subject to human reflex. Video analysis provided greater reliability and accuracy. To handle the size of the data collected, video files were compressed to a 720 by 1280 pixel resolution and a frame rate of 5fps. This introduces uncertainty in distance and time measurements of roughly $2 \times 10^{-6}m$ and $0.2s$ respectively. Of further consideration is error in distance measurements due to an oil droplet's spacing between it and the grid located in the image's background. If we assume the grid is located a distance $D$ from the lens, the particle is situated a distance $\Delta$ from the grid, and that the distance traveled by the particle can be roughly approximate by an arclength the difference in perceived distance traveled and the actual distance traveled is given by, $D\theta - (D - \Delta)\theta = \Delta\theta$, and the percent error in measurement is given by $\frac{\Delta}{D}$ For the length scales of interest this amounts to less than a one percent error when $\Delta$ is less than 0.6mm. Thus, we can expect this error to contribute an order of magnitude or two less than other measured values to the overall error. Velocity of droplets perpendicular to the viewing plane should also be considered as a factor contributing to the overall error. It is expected that this error also contributes at about the same order as that of error due to the droplets distance from the grid plane since the perpendicular velocity is expected to be relatively small compared to velocities in the plane. Smaller uncertainties exist in measurements of the voltage, pressure, and temperature. Error propagation is handled through python, with the following formula,

$$\sigma_q = q\sqrt{(\frac{\partial \log q}{\partial v_r}\sigma_{v_r})^2 + (\frac{\partial \log q}{\partial v_f}\sigma_{v_f})^2 + (\frac{\sigma_V}{V})^2 + (\frac{\sigma_d}{d})^2} \tag{6}$$

The calculation of errors is presented in the Jupyter notebook and measured values of all quantities are presented in an attached table as appendix.

## RESULTS AND DISCUSSION

The charges of various particles are tabulated in a histogram in Fig. 3. The distribution displays a discreteness in the charges of the oil droplets observed, with distinct peaks forming at particular values of $q$. The centers of these peaks are evaluated and the differences between each peak and their neighbor are taken. We obtain five differences, the average of these is calculated to be $1.7 \times 10^{-19} \pm 7 \times 10^{-20}C$. We can arrive at a different value of electron's charge if we utilize optimization algorithms. Assuming the quantized nature of the electron, the measured values of q can be binned into separate peaks based on some variable value for the fundamental charge. A python script can be run to evaluate the charge based on some assumed value of $e$ and the difference between the estimated $e$ and assumed can be minimized to yield a value of $e$. This procedure is actually an optimization based on properly designed loss function. Here we adopt the error function as the square difference between averaged estimated $e$ value and assumed $e$. The optimization curve is displayed in Fig. 4. Optimization yields a value of $1.57 \times 10^{-19}C$ for the electric charge. The Standard deviation in the measured values of q is calculated to be of the order $10^{-20}$. Observations are consistent with a quantized theory of the electric charge in which case one would expect, given enough resolution, to see particles coming in discrete charges spaced apart at integer multiples of a common unit. Ideally one would see discrete spikes spaced apart by a value $e$, which is the actual charge of an electron, however stochastic processes lead to a widening of the distribution. Many of our measurements for $q$ came from different particles rather than a singular particle. Better experimental handling in the future can lead to a greater number of charge measurements for a single particle, eliminating some uncertainty due to the use of different particles. We do not have a sufficient quantitative understanding of error due to velocities perpendicular to the image plane and error due to the particle's distance from the screen, so it is possible that these effects are more considerable than we have assumed.

**FIGURES**

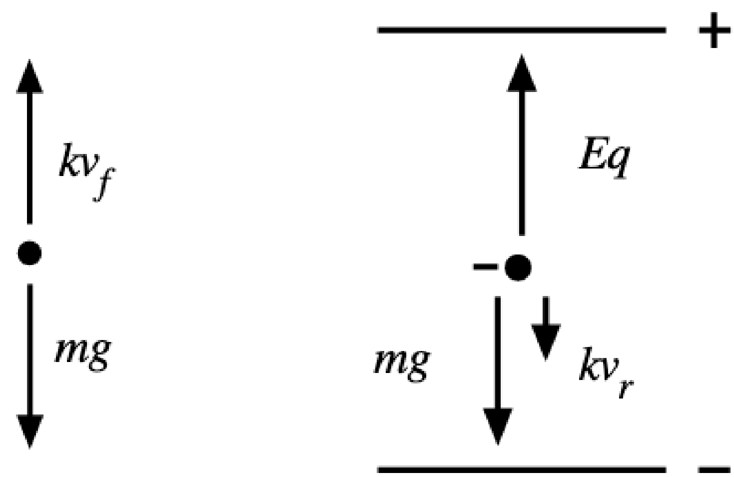


FIG. 1. The left free body diagram demonstrates the forces acting on a droplet subjected to only gravity. The free body diagram to the right represents the forces acting on the oil drop subjected to the electric field.

| Mass $(10^{-16} kg)$ | Velocity Rise $(mm/s)$ | Velocity Fall $(mm/s)$ | Charge $(10^{-19} C)$ | Percent Error for Charge (%) |
|---|---|---|---|---|
| 6.793227 | 0.018 | 0.039 | 1.478002 | 7.75 |
| 2.367041 | 0.062 | 0.020 | 1.425056 | 11.06 |
| 1.602079 | 0.106 | 0.016 | 1.818649 | 13.51 |
| 2.792898 | 0.056 | 0.022 | 1.456100 | 9.12 |
| 1.610011 | 0.095 | 0.016 | 1.665219 | 3.93 |
| 1.677208 | 0.111 | 0.016 | 1.929991 | 20.46 |
| 3.777965 | 0.043 | 0.027 | 1.454119 | 9.24 |

FIG. 2. This is a table representing some of our best results for charges from the experiment with percent errors from the actual charge of an electron. Including their averaged velocities and mass.

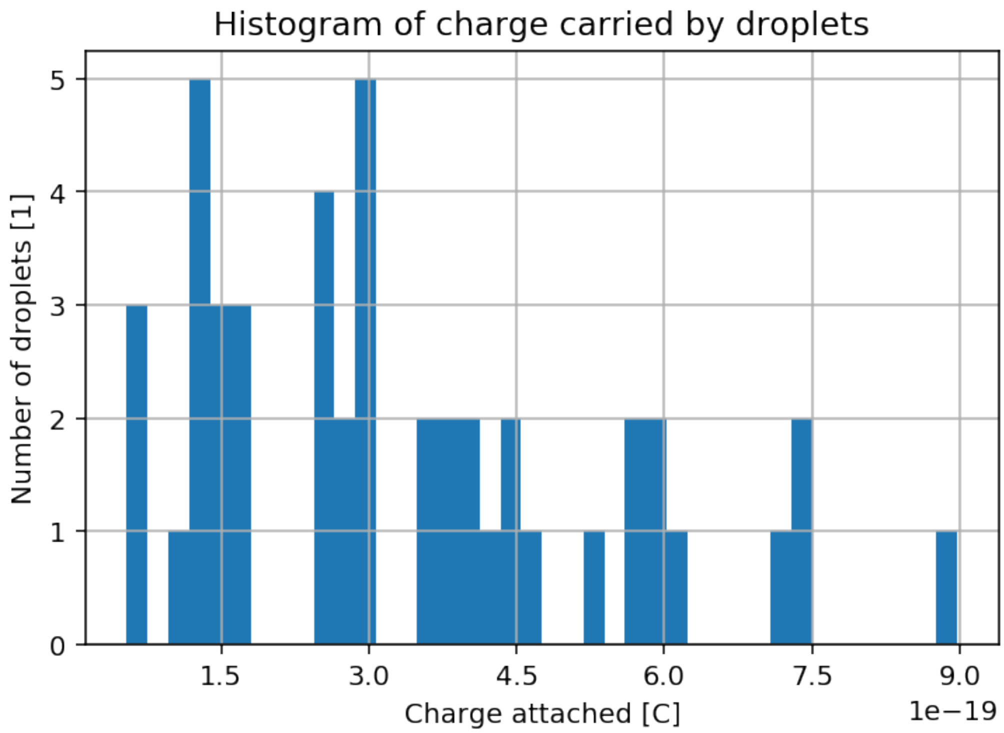


FIG. 3. The histogram shows the quantized distribution of charge carried by droplets.

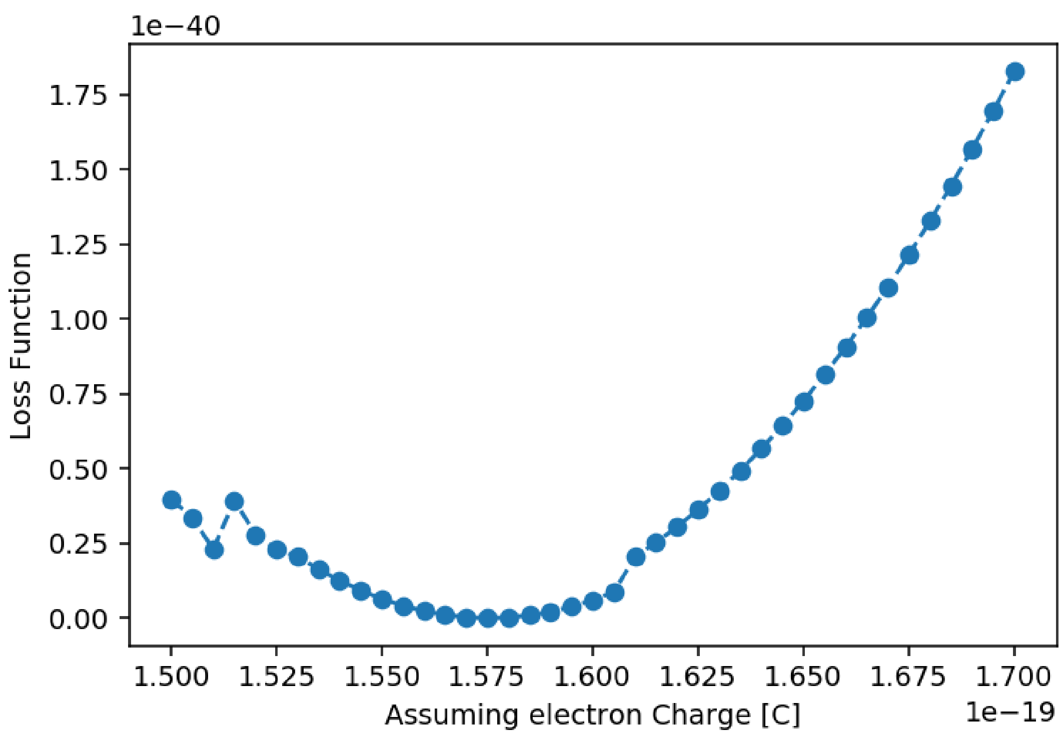


FIG. 4. The plot presents the loss function with varying electron charge.