Cavendish

Load and pre-process data

In[486]:=
```
position1data = Import[NotebookDirectory[] <> "day4_position1_data.csv"];
position2data = Import[NotebookDirectory[] <> "day4_position2_data.csv"];
```

In[488]:=
```
(*offset vals*)
```

In[489]:=
```
{offval1, offval2} = {9.1517, 9.1216};

start = 2; end = 400;
```

In[491]:=
```
{curve1, curve2} =
  {#[[start ;; end, -1]], #[[start ;; end, -2]]}ᵀ & /@ {position1data, position2data};
```

Method-I Fitting with dumped oscillation model

In[492]:=
```
{model, params} = {A Exp[-λ t] Cos[ω t + ϕ] + B, {{A, 15}, {λ, 0.001}, {ω, 0.012}, ϕ, {B, 30}}};
```

In[493]:=
```
fit1 = NonlinearModelFit[curve1, model, params, t, MaxIterations → 100];
```

In[494]:=
```
fit2 = NonlinearModelFit[curve2, model, params, t, MaxIterations → 100];
```

In[495]:=
```
param1 = Evaluate@fit1["BestFitParameters"];
param2 = Evaluate@fit2["BestFitParameters"];
```

In[497]:= $\{\sigma_{\omega 1}, \sigma_{B1}\}$ = Part[First[fit1[{"ParameterErrors"}]], {3, 5}]

Out[497]= $\{8.35197 \times 10^{-7}, 0.00299906\}$

In[498]:= $\{\sigma_{\omega 2}, \sigma_{B2}\}$ = Part[First[fit2[{"ParameterErrors"}]], {3, 5}]

Out[498]= $\{2.53344 \times 10^{-6}, 0.00511381\}$

In[499]:=
```
fit1[{"ParameterTable"}] // First
```

Out[499]=

	Estimate	Standard Error	t-Statistic	P-Value
A	−16.5432	0.00976245	−1694.58	0.
λ	0.000299712	9.27725×10^{-7}	323.061	0.
ω	0.0125738	8.35197×10^{-7}	15054.9	0.
ϕ	0.537226	0.000519459	1034.2	0.
B	31.3377	0.00299906	10449.2	0.

In[521]:=
```
fit2[{"ParameterTable"}] // First
```

Out[521]=

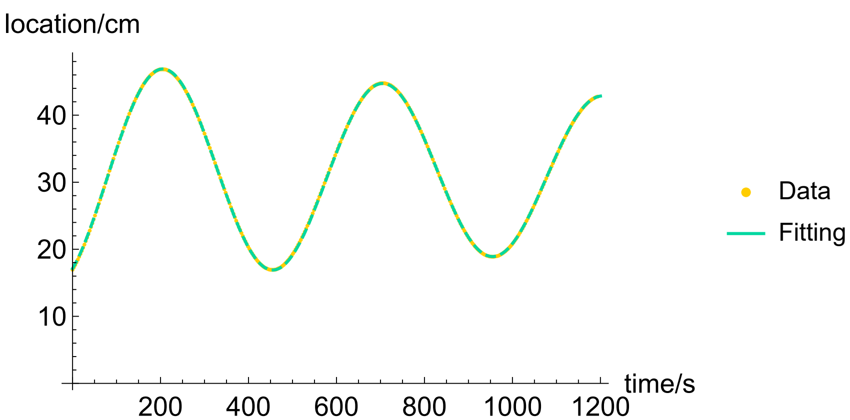
	Estimate	Standard Error	t-Statistic	P-Value
A	9.98778	0.015527	643.25	0.
λ	0.000269804	2.36035×10^{-6}	114.307	3.16603×10^{-304}
ω	0.0125537	2.53344×10^{-6}	4955.21	0.
ϕ	1.60345	0.00155875	1028.68	0.
B	27.8822	0.00511381	5452.33	0.

In[501]:=
```
{c1, c2} = {Interpreter["ComputedColor"]["RGB 255 205 0"],
  Interpreter["ComputedColor"]["RGB 0 215 160"]}
```

Out[501]= { , }

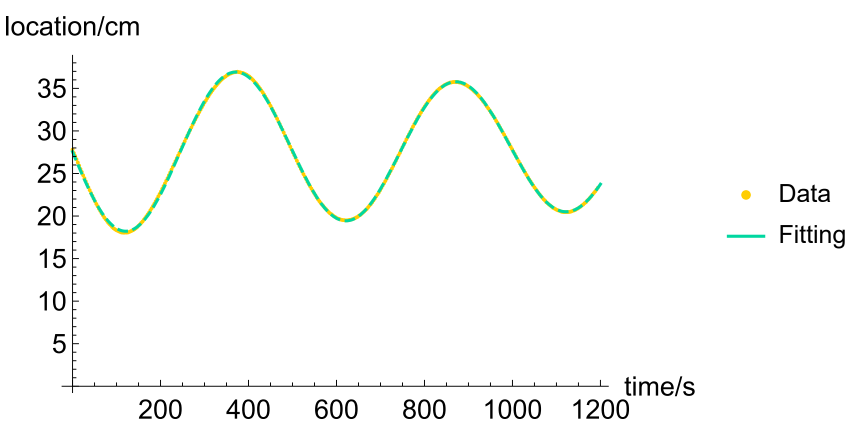
In[502]:=
```
Legended[Show[ListPlot[curve1, PlotStyle → {c1, PointSize[Small]}],
  Plot[model /. param1, {t, 0, 3 end}, PlotStyle → {Dashed, c2}],
  AxesLabel → {"time/s", "location/cm"}, AxesStyle → Black, BaseStyle → {FontSize → 14}],
 LineLegend[{c1, c2}, {"Data", "Fitting"}, Joined → {False, True}]]
```

Out[502]=



In[503]:=
```
Legended[Show[ListPlot[curve2, PlotStyle → {c1, PointSize[Small]}],
  Plot[model /. param2, {t, 0, 3 end}, PlotStyle → {Dashed, c2}],
  AxesLabel → {"time/s", "location/cm"}, AxesStyle → Black, BaseStyle → {FontSize → 14}],
 LineLegend[{c1, c2}, {"Data", "Fitting"}, Joined → {False, True}]]
```

Out[503]=



Calculate gravitational constant

In[504]:= $\Delta S = \text{Quantity}[\text{Abs}[(B\ /.\ \text{param1}) + \text{offval1} - (B\ /.\ \text{param2}) - \text{offval2}], \text{"cm"}]$

Out[504]= 3.48566 cm

In[505]:= $T = \text{Quantity}[4\pi / ((\omega\ /.\ \text{param1}) + (\omega\ /.\ \text{param2})), \text{"s"}]$

Out[505]= 500.104 s

In[506]:=
```
{r, d, b, m} =
  {Quantity[9.55, "mm"], Quantity[50, "mm"], Quantity[42.2, "mm"], Quantity[1.5, "kg"]};
```

In[507]:= $L_{arr} = \{134.5, 134.7, 135.0, 135.5, 135.3, 135.6, 135.0, 135.2\};$

In[508]:= $\{L, \sigma_L\} = \text{Quantity}[\{\text{Mean@}L_{arr}, 2 * \text{StandardDeviation@}L_{arr}\}, \text{"cm"}]$

Out[508]= {135.1 cm , 0.755929 cm}

In[509]:= $G = \pi^2\, \Delta S\, b^2 \dfrac{d^2 + 2/5\, r^2}{T^2\, m\, L\, d}$ // UnitConvert

Out[509]= $6.13203 \times 10^{-11}\ \text{m}^3/(\text{kg}\,\text{s}^2)$

Correction and error

In[510]:= $\beta = \dfrac{b^3}{(b^2 + 4\, d^2)^{3/2}}$

Out[510]= 0.0587723

In[511]:= $G_0 = G / (1 - \beta)$

Out[511]= $6.51493 \times 10^{-11}\ \text{m}^3/(\text{kg}\,\text{s}^2)$

In[512]:= $\sigma_{S\text{-def}} = 0.1$ (*cm max presion of the metric scale*);

$\sigma_{\Delta S} = \text{Quantity}\left[\sqrt{(\sigma_{B1} + \sigma_{B2})^2 + (\sigma_{S\text{-def}})^2}, \text{"cm"}\right]$

Out[513]= 0.100329 cm

In[514]:= $\sigma_{T\text{-def}} = 10^{-6}$; (*the computer is precise enough for timing*)

$\sigma_T = T \sqrt{\text{Max}\left[\dfrac{\sigma_{\omega 1}}{\omega\ /.\ \text{param1}}, \dfrac{\sigma_{\omega 2}}{\omega\ /.\ \text{param2}}\right]^2 + (\sigma_{T\text{-def}})^2}$

Out[514]= 0.100926 s

In[515]:= $\sigma_{L\text{-def}} = \text{Quantity}[0.5, \text{"mm"}];$

$\sigma_L = \sqrt{(\sigma_L)^2 + \sigma_{L\text{-def}}^2}$

Out[516]= 7.57581 mm

In[517]:= δ_m = Quantity[10, "g"]

Out[517]= 10 g

In[518]:= $\sigma_{G0} = G_0 \sqrt{\left(\frac{\sigma_{\Delta S}}{\Delta S}\right)^2 + 4\left(\frac{\sigma_T}{T}\right)^2 + \left(\frac{\sigma_L}{L}\right)^2 + \left(\frac{\delta_m}{m}\right)^2}$

Out[518]= 1.95939×10^{-12} m^3/ (kg s^2)

In[519]:= *G* // UnitConvert

Out[519]= 6.674×10^{-11} m^3/ (kg s^2)

In[520]:= Abs@$(G_0 - G)$

Out[520]= 0.0238465 *G*