Millikan Oil Drop

___Meansurement of the electron charge

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In [244]:

```
import numpy as np
import pandas as pd
import matplotlib.pylab as plt
import os
```

In [245]:

```
rho=886 # kg/m^3
g=9.8 # m/s^2
b=8.20*10**(-3) # Pa*m
p=101204 #Pa
e=1.6*10**(-19)
V_std=1 #V

def get_eta(vid):
    # determine viscosity for each measurement
    vid=int(vid)
    eta= 1.8330*10**(-5) # N*s/m^2
    if 1<=vid<=5:
        return 1.8310*10**(-5)
    elif 6<=vid<=13:
        return 1.8340*10**(-5)
    else:
        return 1.8280*10**(-5)


def get_V(vid):
     # determine voltage for each measurement
    V=505 #V
    vid=int(vid)
    if 1<=vid<=5:
        return 504
    elif 6<=vid<=13:
        return 501
    else:
        return 505
```

In [246]:

```
d_array=10**(-3)*np.array([7.55,7.59,7.60,7.60,7.60,7.61]) # unit: m
d_mean=d_array.mean()
d_std=d_array.std()
print("d_mean: ",d_mean)
print("d_std: ",d_std)
```

```
d_mean:  0.00759166666667
d_std:  1.95078331845e-05
```

In [247]:

```
def reject_outliers(data, m=3):
    '''
    remove anomalous data points that outside 2 standard deviation in the array
    '''
#     data=data[abs(data - np.mean(data)) < m * np.std(data)]
    return data[abs(data - np.mean(data)) < m * np.std(data)]
```

Load data from files

In [248]:

```
data_path = "./data/"
statistics=[]
for file_name in os.listdir(data_path):
    name=file_name[:3]
    obj_drop=pd.read_csv(data_path+file_name).dropna()
    # seperate rising and falling velocities, remove anomalous velocities at switching field direction
    v_y=obj_drop["v_{y}"].values
    n_points=len(v_y)
    v_r=reject_outliers(v_y[v_y>0])
    v_f=reject_outliers(v_y[v_y<0])
    # calculate mean and deviation
    (v_r_mean,v_r_std)=(v_r.mean(),v_r.std())
    (v_f_mean,v_f_std)=(np.abs(v_f.mean()),v_f.std())
    # calculate other properties
    vid=file_name[:2]
    eta=get_eta(vid)
    V=get_V(vid)
    a=np.sqrt((b/2/p)**2+9*eta*v_f_mean/2/rho/g)-b/(2*p) #droplet radius
    m=4*np.pi/3*a**3*rho # droplet mass
    q=m*g*d_mean*(v_f_mean+v_r_mean)/V/v_f_mean #droplet charge
    # long formula for error propagation
    q_std=q*np.sqrt((((b**2*g*rho+18*p**2*eta*v_f_mean+3*b*g*rho*p*np.sqrt(b**2/p**2+18*eta*v_f_mean/g/rh
o))/(2*v_f_mean*(b**2*g*rho+18*p**2*eta*v_f_mean))+1/(v_f_mean+v_r_mean))*v_f_std)**2+(1/(v_f_mean+v_r_me
an)*v_r_std)**2+(1/d_mean*d_std)**2)
    # stack to dataframe
    statistics.append(np.array((name,n_points,v_r_mean,v_r_std,v_f_mean,v_f_std,a,m,q,q_std)))
```

Calculation of the attached charge

In [249]:

```
labels=["name","n_points","v_r_mean","v_r_std","v_f_mean","v_f_std","a","m","q","q_std"]
overall=pd.DataFrame(statistics,columns=labels,dtype="float32")
```

In [250]:

```
overall#he droplet charge calculated
```

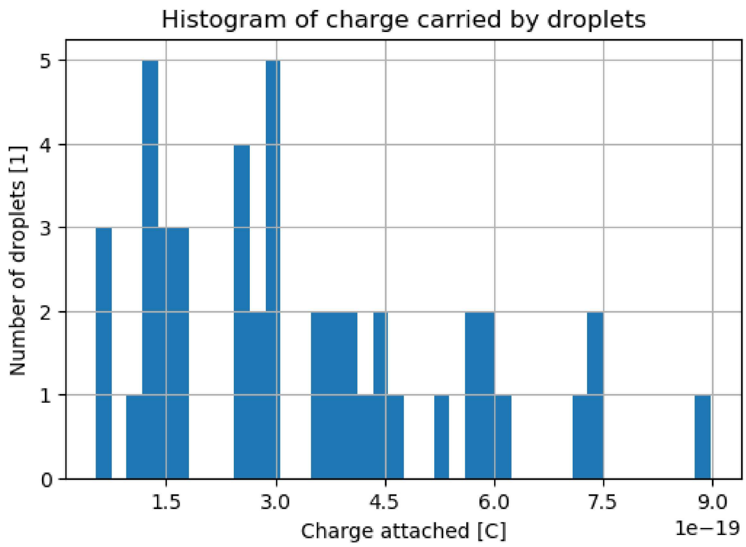
	name	n_points	v_r_mean	v_r_std	v_f_mean	v_f_std	a	m	q	q_std
0	01A	960.0	0.000097	0.000011	0.000066	0.000014	7.541285e-07	1.591690e-15	5.771566e-19	1.261804e-19
1	01B	1140.0	0.000054	0.000011	0.000079	0.000042	8.265568e-07	2.095755e-15	5.222924e-19	3.264994e-19
2	01C	835.0	0.000137	0.000017	0.000067	0.000013	7.569128e-07	1.609384e-15	7.241972e-19	1.445641e-19
3	01D	571.0	0.000044	0.000011	0.000049	0.000012	6.452343e-07	9.969526e-16	2.782697e-19	8.106526e-20
4	02A	1249.0	0.000083	0.000024	0.000037	0.000013	5.572819e-07	6.423136e-16	3.039631e-19	1.163181e-19
5	02B	704.0	0.000020	0.000011	0.000041	0.000019	5.825815e-07	7.338251e-16	1.607750e-19	9.995117e-20
6	04A	350.0	0.000062	0.000011	0.000064	0.000015	7.403968e-07	1.506316e-15	4.361534e-19	1.158103e-19
7	04B	154.0	0.000142	0.000083	0.000028	0.000037	4.748169e-07	3.972838e-16	3.573406e-19	4.076589e-19
8	04C	109.0	0.000039	0.000011	0.000051	0.000021	6.554863e-07	1.045233e-15	2.717123e-19	1.341717e-19
9	04D	48.0	0.000036	0.000012	0.000051	0.000009	6.539939e-07	1.038110e-15	2.628353e-19	6.389324e-20
10	04E	432.0	0.000018	0.000010	0.000060	0.000027	7.163554e-07	1.364294e-15	2.599770e-19	1.618532e-19
11	05A	112.0	0.000234	0.000080	0.000054	0.000035	6.732974e-07	1.132774e-15	8.973500e-19	5.176288e-19
12	06A	124.0	0.000059	0.000015	0.000020	0.000008	3.993551e-07	2.363742e-16	1.369184e-19	5.247263e-20
13	06B	356.0	0.000041	0.000038	0.000023	0.000013	4.255928e-07	2.860917e-16	1.200775e-19	9.921591e-20
14	06D	396.0	0.000132	0.000099	0.000033	0.000014	5.232803e-07	5.317720e-16	3.920115e-19	2.710639e-19
15	06E	206.0	0.000100	0.000063	0.000025	0.000014	4.472741e-07	3.320806e-16	2.476113e-19	1.724092e-19
16	06F	121.0	0.000075	0.000026	0.000054	0.000006	6.764853e-07	1.148940e-15	4.075351e-19	9.300076e-20
17	06H	385.0	0.000049	0.000031	0.000021	0.000013	4.050480e-07	2.466276e-16	1.238745e-19	8.883027e-20
18	07A	336.0	0.000014	0.000008	0.000023	0.000024	4.302464e-07	2.955794e-16	6.979906e-20	9.213031e-20
19	08A	872.0	0.000033	0.000038	0.000014	0.000010	3.236547e-07	1.258253e-16	6.339593e-20	6.636466e-20
20	08B	124.0	0.000106	0.000125	0.000023	0.000036	4.323034e-07	2.998390e-16	2.464733e-19	3.887668e-19
21	09A	409.0	0.000052	0.000022	0.000024	0.000014	4.384432e-07	3.127969e-16	1.477152e-19	8.911137e-20
22	09B	364.0	0.000265	0.000189	0.000027	0.000018	4.649105e-07	3.729327e-16	6.043287e-19	4.903621e-19
23	10A	243.0	0.000061	0.000059	0.000016	0.000011	3.526907e-07	1.628188e-16	1.160608e-19	1.134375e-19
24	11A	354.0	0.000074	0.000132	0.000018	0.000012	3.737323e-07	1.937332e-16	1.484244e-19	2.289573e-19
25	11B	708.0	0.000023	0.000038	0.000015	0.000013	3.367016e-07	1.416635e-16	5.406359e-20	7.306212e-20
26	13A	150.0	0.000096	0.000025	0.000043	0.000013	6.023454e-07	8.110714e-16	3.868082e-19	1.296810e-19
27	13B	285.0	0.000175	0.000073	0.000022	0.000013	4.190767e-07	2.731510e-16	3.630988e-19	2.040628e-19
28	13C	154.0	0.000127	0.000021	0.000033	0.000013	5.208464e-07	5.243863e-16	3.784747e-19	1.335429e-19
29	13D	478.0	0.000118	0.000015	0.000041	0.000013	5.847114e-07	7.419029e-16	4.276518e-19	1.221074e-19
30	13E	1089.0	0.000078	0.000021	0.000039	0.000019	5.660419e-07	6.730821e-16	3.010972e-19	1.458919e-19
31	13F	181.0	0.000172	0.000019	0.000041	0.000011	5.882166e-07	7.553256e-16	5.767505e-19	1.348694e-19
32	13G	161.0	0.000064	0.000014	0.000042	0.000013	5.909288e-07	7.658219e-16	2.890824e-19	9.779566e-20
33	13H	155.0	0.000087	0.000029	0.000018	0.000012	3.789482e-07	2.019583e-16	1.716367e-19	1.039117e-19
34	13I	157.0	0.000042	0.000020	0.000024	0.000015	4.390456e-07	3.140878e-16	1.278192e-19	8.687053e-20
35	13J	100.0	0.000076	0.000008	0.000038	0.000007	5.609695e-07	6.551489e-16	2.921510e-19	5.208002e-20
36	13K	92.0	0.000098	0.000035	0.000017	0.000009	3.612755e-07	1.749999e-16	1.777900e-19	9.502843e-20
37	13L	110.0	0.000043	0.000008	0.000026	0.000008	4.614205e-07	3.645968e-16	1.418015e-19	4.567891e-20
38	14A	624.0	0.000041	0.000012	0.000096	0.000023	9.119827e-07	2.815024e-15	5.939706e-19	1.914406e-19
39	14B	482.0	0.000040	0.000015	0.000096	0.000017	9.128012e-07	2.822610e-15	5.904479e-19	1.499494e-19
40	14C	286.0	0.000051	0.000026	0.000020	0.000012	3.920644e-07	2.236632e-16	1.193663e-19	7.944851e-20
41	16A	587.0	0.000100	0.000013	0.000051	0.000014	6.569278e-07	1.052144e-15	4.582708e-19	1.200143e-19
42	17A	821.0	0.000111	0.000023	0.000079	0.000028	8.230424e-07	2.069136e-15	7.365568e-19	2.759127e-19
43	17B	308.0	0.000114	0.000012	0.000077	0.000016	8.158515e-07	2.015374e-15	7.355359e-19	1.582469e-19
44	17C	389.0	0.000036	0.000012	0.000078	0.000012	8.191622e-07	2.040009e-15	4.409025e-19	9.907497e-20
45	18A	603.0	0.000097	0.000017	0.000033	0.000012	5.205169e-07	5.233916e-16	3.026955e-19	1.057990e-19

the charge per oil droplet calculated from data is

In [251]:

```
plt.figure().dpi=100
plt.xlabel("Charge attached [C]")
plt.ylabel("Number of droplets [1]")
plt.xticks(np.arange(0,9.5E-19,1.5E-19))
plt.title("Histogram of charge carried by droplets")
(overall.q).hist(bins=40)
```

```
<matplotlib.axes._subplots.AxesSubplot at 0x11b4f0f7128>
```



In [252]:

```
def clustering(arr,x):
    arr=list(arr/x)
    num=int(max(arr))
    clusters=[]
    for i in range(num+1):
        clusters.append(x*(np.array(list(filter(lambda x:i<x<=i+1 ,arr)))))
    return clusters
```

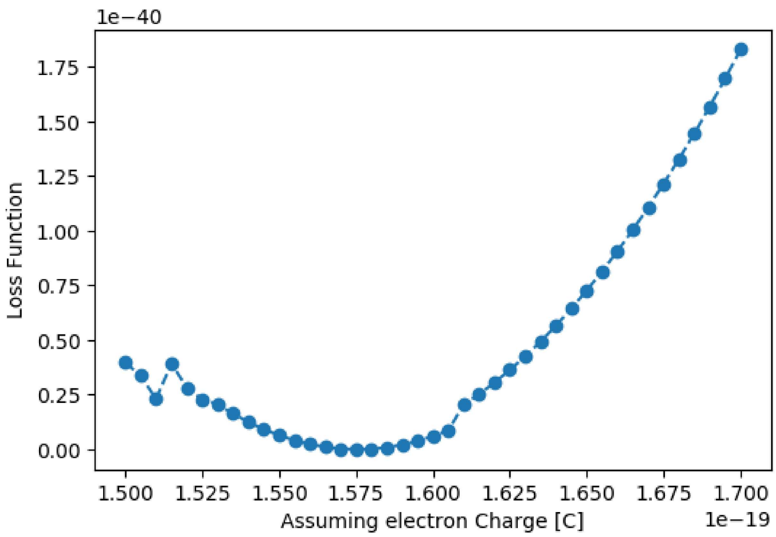
In [275]:

```
def obj_error(x):
    clusters=clustering(overall.q,x)
    test=np.nan_to_num(np.array(list(map(np.mean,clusters))))
    estimate_delta_q=[]
    n_len=len(test)
    for i in range(1,n_len):
        for j in range(i):
            estimate_delta_q.append(abs(test[i]-test[j])/(i-j))
    return  (np.array(estimate_delta_q).mean()-x)**2
#sum((np.array(estimate_delta_q)-e)**2)
#
```

In [287]:

```
steps=0.005E-19
min_bound=1.5E-19
max_bound=1.7E-19
ran=np.arange(min_bound,max_bound,steps)
est=list(map(obj_error,ran))
plt.figure().dpi=100
plt.xlabel("Assuming electron Charge [C]")
plt.ylabel("Loss Function")
plt.plot(ran,est,"--o")
```

```
[<matplotlib.lines.Line2D at 0x11b507a4c88>]
```



In [288]:

```
steps=steps=0.001E-19
ran=np.arange(min_bound,max_bound,steps)
est=list(map(obj_error,ran))
e_estimate,min_loss=min(list(zip(ran,est)),key=lambda x: x[1])
```

In [285]:

```
print("Estimate e value:",e_estimate,"C")
print("Error",(e-e_estimate)/e*100,"%")
```

```
Estimate e value: 1.575e-19 C
Error 1.5625 %
```