Cavendish Torsion Balance

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Abstract

We reproduce the famous experiment performed by Henry Cavendish in 1798 - the measurement of gravitational constant G. The tiny gravitation is converted to two measurable quantities, the oscillation period T and location shift ΔS, using a well designed torsion balance. We collected data using a programmed camera and image processing software. The values of T and ΔS are given by data fitting and we find $(6.51 \pm 0.2) \times 10^{-11}$ $\mathrm{m}^3/(\mathrm{kg}\cdot \mathrm{s}^2)$, which is 2.4% lower than expectation. Possible sources of error are discussed.

INTRODUCTION

The torsion balance consists of two small-mass balls suspended from a highly sensitive copper torsion ribbon (the pendulum) and a pair of large-mass tungsten spheres that can be positioned as required. In the idealized approximation, the balance is correctly aligned on both vertical and horizontal directions so that the gravitation between the two small spheres and earth is canceled. After moving the tungsten spheres, the pendulum will oscillate under the gravitational torque between them. An optical lever, composed of a laser source and a mirror fixed to the pendulum, is used to measure the corresponding twist accurately. Once the oscillation period T and the distance ΔS between the two oscillation equilibrium positions being measured, the gravitational torque applied on the pendulum could be derived and thus gives the value of the constant G.

$$G = \pi^2 \Delta S b^2 \left(\frac{d^2 + 2r^2/5}{T^2 m L d} \right) \tag{1}$$

METHOD

The apparatus of Cavendish Torsion Balance is shown in Fig. 1. At first, with proper adjustment, we make sure that the apparatus is correctly aligned and the swing of laser projection on the screen is symmetric. Then the tungsten spheres are placed at positionI and II respectively and the data collecting procedure is initiated after the oscillation going stable.

Construction Details

Variables on the right side of Eqn. 1 are given by Lab Manual: radius of the large spheres r=9.55mm, arm length of pendulum d=50mm, central distance between the paired balls b=42.2mm, mass of one large tungsten m=1.5kg. The errors of these given values can be neglected. Distance L between the screen and torsion balance is measured four times by Tongxie and Yuning respectively, as shown in the Table IV. We take L=135.1 $\pm$ 0.8 cm, which includes the error of measurements and the precision of the measuring tape we use.

Measurement Details

The laser dot on the screen is captured at a set frequency by a programmed camera and its trajectory is given by an image processing software, in the form of "position-time" data array attached in the appendix.

With hundreds of data points, values of the oscillation period and equilibrium position can be well estimated using a nonlinear fitting. The model is supposed to be a damped oscillation, as shown below.

$$S(t) = Ae^{-\lambda t} cos(\omega t + \varphi) + B \tag{2}$$

The fitting curves of positionI and positionII are shown in Fig. 2 and the estimated values of the fitting parameters is given in Table I II. Given the estimated fitting parameters, we have $\Delta S = |B_1 - B_2|$ and $T = \frac{4\pi}{\omega_1 + \omega_2}$.

Since the Eqn. 1 ignored the gravitation of the further tungsten ball, the G constant derived will be lower than the expected value. This systematic error can be corrected with the equation below:

$$G_0 = G/(1 - \beta) \tag{3}$$

in which

$$\beta = \frac{b^3}{(b^2 + 4d^2)^{\frac{3}{2}}} \tag{4}$$

Finally, G_0 is the corrected result of our measurement.

RESULTS

The final values and errors of ΔS, T and L is given in Table III. Here the errors consist of two part, deviation in data fitting and instrumental error.

The gravitational constant G_0 derived from Eqn. 1 and Eqn. 3 is $(6.51 \pm 0.2) \times 10^{-11}$ $m^3/(kg \cdot s^2)$, 2.4% lower than the recognized value of G, which is 6.67×10^{-11} $m^3/(kg \cdot s^2)$. Since the estimated G deviates from the expectation by about one standard deviation, it's reasonable to attribute the deviation to the random measurement fluctuation.

Further, we analyze some possible perturbation in the experiment that may lead to a deviation of the final result.

1) Height difference

If the small spheres and large spheres not strictly leveled, calculation shows that the height deviation of one millimeter will eventually bring a 0.08% reduction in the estimated G_0 value, which is small enough to be neglected. Supposing the height perturbation to be δh, the corresponding perturbation is:

$$\left(\frac{b}{\sqrt{b^2 + \delta h^2}}\right)^3$$

2) Air Resistance

Rough estimates show that the air resistance(10^{-11}N) during the oscillation is three orders of magnitude smaller than that of gravity(10^{-8}N), so its effect on measurement can be neglected.

3) Deviation of the initial equilibrium position (or the laser is not strictly aligned)

Supposing the equilibrium axis of the pendulum is deviated from the central axis by a small $\delta\theta$, the perturbation effect on the gravitational torque is a second order term which could be neglected. However, the asymmetry of laser reflection will maker the measurement of ΔS larger, which introduces a perturbation term at one order,

$$1 + 2\frac{\delta\theta}{\Delta\theta}$$

in which the $\Delta\theta$ is the twist angle caused by gravitational torque. Since the $\Delta\theta$ is very small, this effect is much more significant than the effect of other possible perturbation.

In future experiments, the direction of laser source and mirror must be properly adjusted, because small deviation in equilibrium position can lead to a significant error at the final value of G_0.

FIGURES

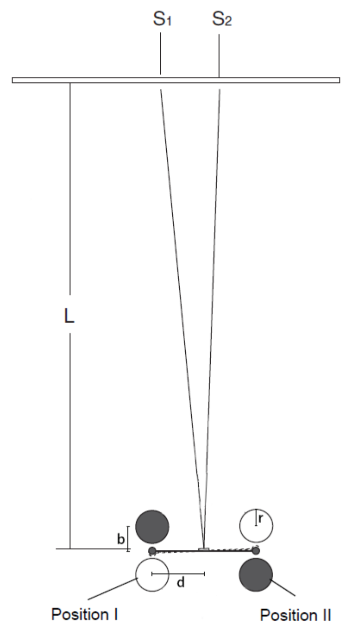
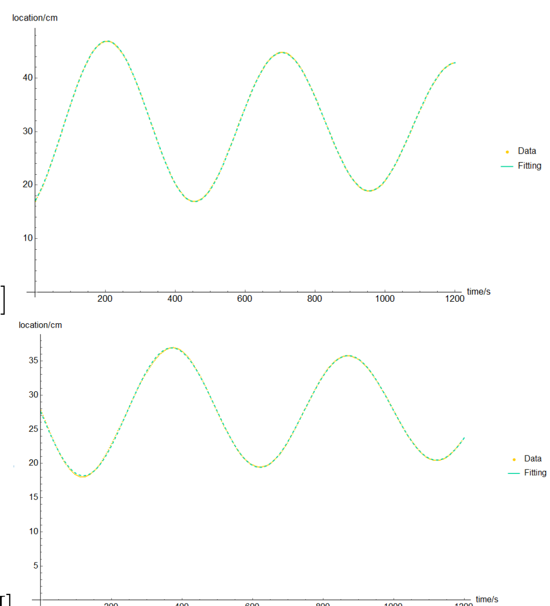


FIG. 1. The Cavendish Torsion in our experiment. The distance L is measured from mirror to the center of screen. b is the distance between the centers of mass of large and small spheres, d is half the length of lever, and r is the radius of the large sphere.



[positionI]

[positionII]

FIG. 2. The fitting curves of positionI and positionII

TABLES

TABLE I. Estimated values of oscillation parameters at Position I.

Parameters	Estimate	Standard Error	t-Statistic	P-Value
A (cm)	-16.543	0.0097	-1694.58	0.
$\lambda(s^{-1})$	2.997×10^{-4}	9.277×10^{-7}	323.061	0.
$\omega(2\pi s^{-1})$	0.0125	$8.352\ \times10^{-7}$	15054.9	0.
$\phi(/)$	0.5372	0.0005	1034.2	0.
B (cm)	31.338	0.003	10449.2	0.

TABLE II. Estimated values of oscillation parameters at Position II.

Parameters	Estimate	Standard Error	t-Statistic	P-Value
A (cm)	9.988	0.015	643.25	0.
$\lambda(s^{-1})$	2.698×10^{-4}	2.360×10^{-6}	114.307	0.
$\omega(2\pi\cdot s^{-1})$	1.255×10^{-2}	2.533×10^{-6}	4955.21	0.
$\phi(/)$	1.6034	0.0015	1028.68	0.
B (cm)	27.882	0.005	5452.33	0.

TABLE III. Values of all measured quantities. "Standard Deviation" is abbreviated as SD.

Quantities	Mean	SD	Relative Error(%)
$\Delta S(cm)$	3.48	0.1	2.9
$T(s)$	500.10	0.10	0.02
L(cm)	135.1	0.8	0.6
$G_0(m^3/(kg\cdot s^2))$	6.51×10^{-11}	0.2×10^{-11}	3.1

TABLE IV. Measurement data of L. "Standard Deviation" is abbreviated as SD. Each one measured for 4 times and the final result is combined.

	Measurement	Mean (cm)	SD (cm)	relative error(%)
Tongxie	4	134.93	0.38	0.3
Yuning	4	135.28	0.22	0.16
Combined	8	135.10	0.35	0.26