# Code Documentation for HiQ Competition

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## 1 Problem 1

The Hamiltonian is given by,

$$H(s) = (1-s)H_0 + sH_I \,, \tag{1.1}$$

where

$$H_0 = -B\sum_{k=1}^{N} X_k \tag{1.2}$$

$$H_I = -\alpha\sum_{k=1}^{N} X_k - \beta\sum_{k=1}^{N} Z_k - J\sum_{k=1}^{N-1} X_k X_{k+1} - J\sum_{k=1}^{N-1} Z_k Z_{k+1} \,, \tag{1.3}$$

and $X_k$ and $Z_k$ are Pauli-$X$ and Pauli-$Z$ acting on the $k$-th qubit. Here, $N = 13$, $J = 2$, $\alpha = 0.5$, $\beta = 1.0$, $B = 0.5$ and $T = 3$.

Let $s := t/T$ and $H(t) := (1 - t/T)H_0 + (t/T)H_1$. Then the evolution that we want to achieve is,

$$\exp\left(-i\int_0^T H(t)\,\mathrm{d}t\right) |+\rangle^N \;. \tag{1.4}$$

Such an evolution can be accomplished by Trotterization. First,

$$\int_0^T H(t)\,\mathrm{d}t \approx \sum_{j=0}^{M-2} \frac{H(j\Delta t) + H((j+1)\Delta t)}{2}\Delta t = \frac{H(0)}{2}\Delta t + \sum_{j=1}^{M-2} H(j\Delta t)\Delta t + \frac{H(T)}{2}\Delta t \,, \tag{1.5}$$

where $M = 21$ and $(M-1)\Delta t = T$, which implies $\Delta t = 3/20$. So we can implement the evolution operator by,

$$\exp\left(-i\int_0^T H(t)\,\mathrm{d}t\right) \approx e^{-iH(0)\Delta t/2}\prod_{j=1}^{M-2} e^{-iH(j\Delta t)\Delta t} e^{-iH(T)\Delta t/2} \tag{1.6}$$

$$\approx e^{-iH_0\Delta t/2}\prod_{j=1}^{M-2} e^{-i(j\Delta t^2/T)H_I} e^{-i\Delta t(1-j\Delta t/T)H_0} e^{-iH_I\Delta t/2} \tag{1.7}$$

These operators can be decomposed into single- or two-qubit gates.

- $j = 0$:

$$e^{-iH_0\Delta t/2} = \prod_{k=1}^{N} e^{iBX_k\Delta t/2} \tag{1.8}$$

- $1 \leq j \leq M-2$: For the $H_0$ part, we have,

$$e^{-i\Delta t(1-j\Delta t/T)H_0} = \prod_{k=1}^{N} e^{iB\Delta t(1-j\Delta t/T)X_k} . \tag{1.9}$$

  For the $H_I$ part, we have,

$$\begin{aligned} e^{-i(j\Delta t^2/T)H_I} = &\prod_{k=1}^{N} e^{i\beta(j\Delta t^2/T)Z_k} \prod_{k=1}^{N-1} e^{iJ(j\Delta t^2/T)Z_k Z_{k+1}} &(1.10)\\ &\prod_{k=1}^{N} e^{i\alpha(j\Delta t^2/T)X_k} \prod_{k=1}^{N-1} e^{iJ(j\Delta t^2/T)X_k X_{k+1}} . &(1.11)\end{aligned}$$

  Combining two parts, the evolution operator for the $j$-th time step is given by,

$$\begin{aligned} &\prod_{k=1}^{N} e^{i\beta(j\Delta t^2/T)Z_k} \prod_{k=1}^{N-1} e^{iJ(j\Delta t^2/T)Z_k Z_{k+1}} &(1.12)\\ &\prod_{k=1}^{N} e^{i\left(\alpha(j\Delta t^2/T)+B\Delta t(1-j\Delta t/T)\right)X_k} \prod_{k=1}^{N-1} e^{iJ(j\Delta t^2/T)X_k X_{k+1}} . &(1.13)\end{aligned}$$

- $j = M-1$:

$$\begin{aligned} e^{-i(\Delta t/2)H_I} = &\prod_{k=1}^{N} e^{i\beta(\Delta t/2)Z_k} \prod_{k=1}^{N-1} e^{iJ(\Delta t/2)Z_k Z_{k+1}} &(1.14)\\ &\prod_{k=1}^{N} e^{i\alpha(\Delta t/2)X_k} \prod_{k=1}^{N-1} e^{iJ(\Delta t/2)X_k X_{k+1}} . &(1.15)\end{aligned}$$

We apply these gates to the initial state $|+\rangle^N$ sequentially for $j = 0, 1, \cdots, 20$. Let the prepared state be $|\psi\rangle$. Then $E_D = \langle\psi|H_I|\psi\rangle$.

# 2 Problem 2

The target of this task is to perform state preparation on restricted quantum chips. Noting that the expressive power of a highly entangled parametrized quantum circuit (PQC) is very strong. A sufficient number of ansatzs should estimate any quantum states properly. To build up an highly entangled PQC, we only need strong entangling gates between parameterized ansatzs.

In this case, the ansatz contains only single qubit quantum gates, which doesn't depends on the connectivity of the quantum chips. Therefore, all we need is to construct an entangling gate based on the topology of the chips. For a fully connected quantum chip, a simplest idea is to connect each neighboring qubits with a CNOT gate. But any Multilayer Parameterized Quantum Circuits (MPQC) (see Fig. 1) should work similarly. By the definition of MPQC, we need $n-1$ CNOT gates that connects all qubits.

With restricted connectivity, a good idea is to centralize the CNOT gates on qubits with high connectivity (so the entanglement spread fast). Therefore, we can choose the qubit with highest connectivity as the control bit (root) and performs CNOT with all its neighbors. Then we should choose the child qubit connected to most untouched qubits and perform CNOT onto them.

## 2.1 Cost function

Suppose $\mathbf{a}$ is the target state vector. We separate its real and imaginary parts $\mathbf{a} = \mathbf{b} + i\mathbf{c}$, where $\mathbf{b}$ and $\mathbf{c}$ are real. Let $\mathbf{a}' = \mathbf{b}' + i\mathbf{c}'$ be our prepared state. Then we need to minimize $|\mathbf{b} - \mathbf{b}'|$ and $|\mathbf{c} - \mathbf{c}'|$. The distance $|\mathbf{b} - \mathbf{b}'|$ is defined as $\sqrt{\sum_i (b_i - b_i')^2}$.

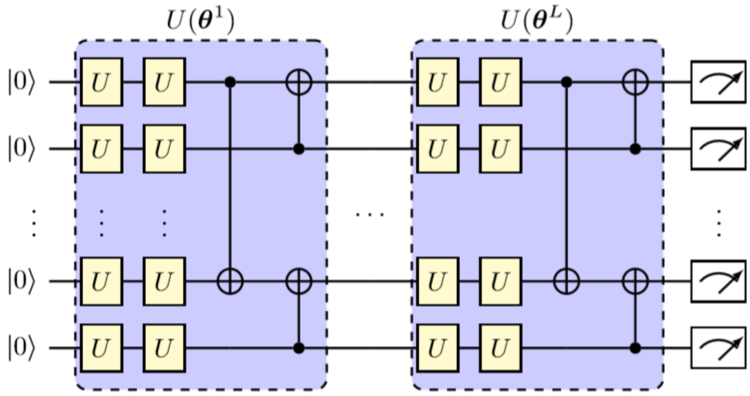


Figure 1: MPQC. arXiv:1810.11922