

Quantum project: Report 1

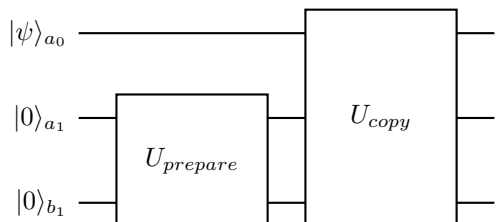
November 23, 2020

1 Introduction

Copying arbitrary quantum information is forbidden by the laws of quantum mechanics. Quantum communication protocols, e.g. QKD, rely on the fact that an eavesdropper will be noticed when intercepting and reading the information, which ensures that communication remains private. In this work we explore an imperfect quantum copy machine and its implementation on quantum computing platforms such as Quantum Inspire (QI). First, we will analyze the circuit that implements this copy machine and obtain a clear idea of what the output state is. Secondly, we implement this circuit for all single qubit states and show the results of the devices Starmon5 and the QXsimulator.

2 Universal quantum copy machine

The quantum circuit corresponding to the UQCM contains two stages, as can be seen in the following circuit,

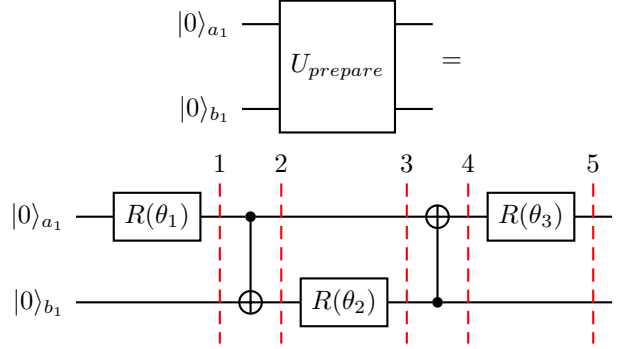


2.1 Preparation of the state

The first stage of the UQCM starts before interacting with the input qubit. The two bottom qubits are required to be in an arbitrary state,

$$|\psi\rangle = C_1|00\rangle + C_2|01\rangle + C_3|10\rangle + C_4|11\rangle. \quad (1)$$

For such purpose we will use the following circuit with an input state $|00\rangle_{a_1 b_1}$.



We analyse each stage of the preparation process. Simply denote $|00\rangle_{a_1 b_1}$ as $|00\rangle$, where the qubit from left to right side is always $a_1 b_1$.

The rotation gate here is defined by

$$R(\theta) = R_y(2\theta) = \cos(2\theta/2)\hat{I} - i\sin(2\theta/2)\hat{\sigma}_y = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$$

And the formula can be given by,

$$\begin{aligned} |\psi_0\rangle &= |00\rangle \\ |\psi_1\rangle &= (\cos \theta_1 |0\rangle + \sin \theta_1 |1\rangle)|0\rangle \\ |\psi_2\rangle &= \cos \theta_1 |00\rangle + \sin \theta_1 |11\rangle \\ |\psi_3\rangle &= (\cos \theta_1 \cos \theta_2 |0\rangle - \sin \theta_2 \sin \theta_1 |1\rangle)|0\rangle + (\cos \theta_1 \sin \theta_2 |0\rangle + \cos \theta_2 \sin \theta_1 |1\rangle)|1\rangle \\ |\psi_4\rangle &= (\cos \theta_1 \cos \theta_2 |0\rangle - \sin \theta_2 \sin \theta_1 |1\rangle)|0\rangle + (\cos \theta_1 \sin \theta_2 |1\rangle + \cos \theta_2 \sin \theta_1 |0\rangle)|1\rangle \\ &= \cos \theta_1 \cos \theta_2 |00\rangle + \cos \theta_1 \sin \theta_2 |11\rangle + \cos \theta_2 \sin \theta_1 |01\rangle - \sin \theta_2 \sin \theta_1 |10\rangle \\ |\psi_5\rangle &= \cos \theta_2 (\cos \theta_1 |0\rangle + \sin \theta_1 |1\rangle)(\cos \theta_3 |0\rangle + \sin \theta_3 |1\rangle) \\ &\quad + \sin \theta_2 (\cos \theta_1 |0\rangle - \sin \theta_1 |1\rangle)(-\sin \theta_3 |0\rangle + \cos \theta_3 |1\rangle) \end{aligned}$$

Finally, we found that the coefficients of the final state depends on the rotation angles,

$$|\psi_5\rangle = C_1|00\rangle + C_2|01\rangle + C_3|10\rangle + C_4|11\rangle.$$

From which we can observe that,

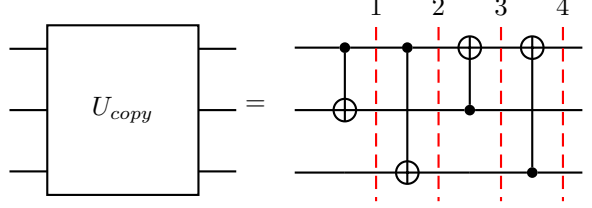
$$\begin{aligned} C_1 &= \sin(\theta_1) \sin(\theta_2) \sin(\theta_3) + \cos(\theta_1) \cos(\theta_2) \cos(\theta_3) \\ C_2 &= \sin(\theta_3) \cos(\theta_1) \cos(\theta_2) - \sin(\theta_1) \sin(\theta_2) \cos(\theta_3) \\ C_3 &= \sin(\theta_1) \cos(\theta_2) \cos(\theta_3) - \sin(\theta_2) \sin(\theta_3) \cos(\theta_1) \\ C_4 &= \sin(\theta_1) \sin(\theta_3) \cos(\theta_2) + \sin(\theta_2) \cos(\theta_1) \cos(\theta_3) \end{aligned}$$

2.2 Copying process

Once we have prepared the state, we do the copy process, which can be described as controlled entanglement between the input and prepared qubits. The circuit

corresponding to the copy process can be observed in the following circuit, where the prepared initial state and the input state that we consider are given respectively by,

$$|\psi\rangle_{a_1, b_1}^{(prep)} = \frac{1}{\sqrt{6}}(2|00\rangle + |01\rangle + |11\rangle), \quad |\psi\rangle_{a_0}^{(in)} = \alpha|0\rangle + \beta|1\rangle. \quad (2)$$



We consider that the input state of the copy machine is $|\psi_0\rangle = |\psi\rangle_{a_0}^{(in)} |\psi\rangle_{a_1, b_1}^{(prep)}$. Then, it will transform in each stage of the copy process as,

$$|\psi_1\rangle = \frac{\alpha}{\sqrt{6}}|0\rangle(2|00\rangle + |01\rangle + |11\rangle) + \frac{\beta}{\sqrt{6}}|1\rangle(2|10\rangle + |11\rangle + |01\rangle) \quad (3)$$

$$|\psi_2\rangle = \frac{\alpha}{\sqrt{6}}|0\rangle(2|00\rangle + |01\rangle + |11\rangle) + \frac{\beta}{\sqrt{6}}|1\rangle(2|11\rangle + |10\rangle + |00\rangle) \quad (4)$$

$$|\psi_3\rangle = \sqrt{\frac{2}{3}}(\alpha|000\rangle + \beta|111\rangle) + \frac{1}{\sqrt{6}}(\alpha|001\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|100\rangle) \quad (5)$$

$$|\psi_4\rangle = \sqrt{\frac{2}{3}}(\alpha|000\rangle + \beta|111\rangle) + \frac{1}{\sqrt{6}}(\alpha|101\rangle + \alpha|011\rangle + \beta|010\rangle + \beta|100\rangle) \quad (6)$$

$$= \left(\sqrt{\frac{2}{3}}\alpha|00\rangle + \beta\frac{1}{\sqrt{6}}(|10\rangle + \beta|01\rangle) \right) |0\rangle + \left(\sqrt{\frac{2}{3}}\beta|11\rangle + \frac{1}{\sqrt{6}}\alpha(|10\rangle + \alpha|01\rangle) \right) |1\rangle \quad (7)$$

$$= |\Phi_0\rangle|0\rangle + |\Phi_1\rangle|1\rangle \quad (8)$$

where we have defined,

$$|\Phi_0\rangle = \left(\sqrt{\frac{2}{3}}\alpha|00\rangle + \beta\frac{1}{\sqrt{3}}|+\rangle \right) \quad (9)$$

$$|\Phi_1\rangle = \left(\sqrt{\frac{2}{3}}\beta|11\rangle + \alpha\frac{1}{\sqrt{3}}|+\rangle \right) \quad (10)$$

3 Implementation of UQCM in QI