Approximate quantum cloning

The making of a quantum spy

Quantum Information Project report

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1
Introduction

The no-cloning theorem is a fundamental result of quantum mechanics, which states that a generic quantum state can not be copied exactly [?]. However, imperfect cloning is not forbidden, as was observed by Bužek and Hillery in 1996, who first proposed an approximate quantum cloning machine (QCM) for qubits [?]. Such QCM produces two identical imperfect copies of an arbitrary input state, making use of an extra ancillary qubit. The quality of the copies is independent of the input state, therefore it is known as universal quantum cloning machine (UQCM). Other QCMs were then proposed, for example the phase-covariant quantum cloning machine (PCQCM), which optimally clones the states on an equator of the Bloch sphere [?]. Another category of quantum cloning machines is economical quantum cloning machines (EQCM), which do not use the extra ancillary qubit [?].

Quantum cloning is of particular interest in the context of quantum key distribution (QKD) [?]. The security of QKD protocols relies on the no-cloning theorem, therefore a QCM is a suitable means of attack. An eavesdropper could intercept the state that Alice is sending to Bob, clone it approximately and send the imperfect copy to Bob. Since, the copy is not exact, Alice and Bob would find some missing correlations after their measurements. Nonetheless, if the errors introduced by the QCM could be confused with generic noise and the eavesdropper could go unnoticed.

In this project we have implemented an experimental realization of the universal, the phase-covariant and the economical phase-covariant QCMs on the quantum processors available via Quantum Inspire [?] and IBM Quantum Experience [?]. The purpose of the present work is to compare the performances of different backends with each other and with the theoretical expectations, addressing the question of whether current quantum computers could be used as QCMs, focusing in particular on their use for eavesdropping. We considered three different sets of input states in order to evaluate the QCMs: the whole Bloch sphere, the equator of the Bloch sphere in the xz plane and the BB84 states (i.e. the computational and the Hadamard basis). In each of these cases we have calculated the average fidelity of the two copies. Moreover, we also studied the results calibrating the readout.

The report is structured as follows: in Section 2 we introduce the afore-mentioned QCMs, in Section 3 we discuss the experimental setup with which we have implemented and tested them on real hardware, in Section 4 we examine the obtained results, in Section 5 we draw some conclusions and discuss possible further developments.

2

Approximate quantum cloning

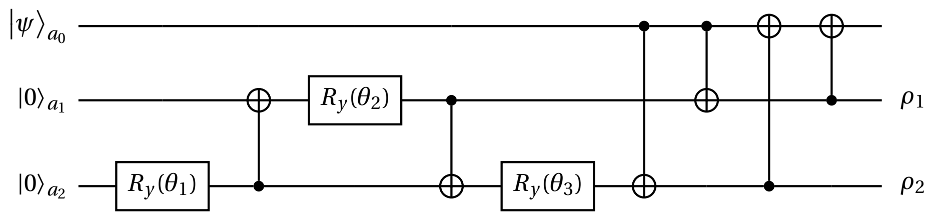
2.1. No-cloning theorem

It is a fundamental result of quantum mechanics that a generic quantum states can not be copied exactly.

2.2. Universal quantum cloning machine

2.3. Phase covariant quantum cloning machine

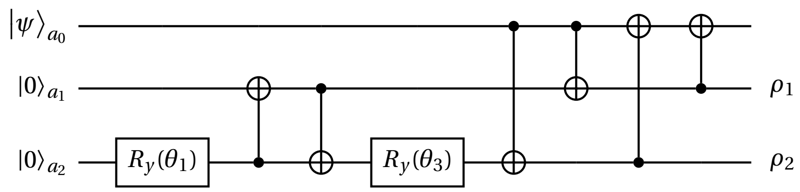
The circuit for the phase covariant quantum cloning machine (PCQCM) proposed in [?] is similar to the one for the universal quantum cloning machine (UQCM), but the output copies are on the register of the ancillae:



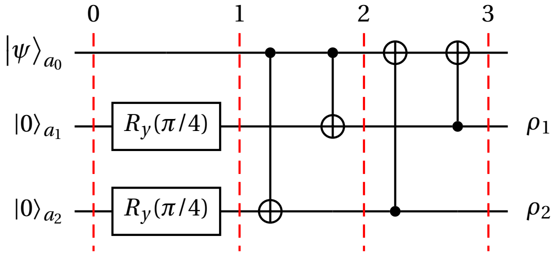
The only significant difference is the preparation of the two ancillae. In this case, different angles of rotation are used. In order to optimally clone the states on the xz equator, the following angles have to be chosen:

$$\theta_1 = \theta_3 = \frac{\pi}{4}, \quad \theta_2 = 0.$$

Hence, the circuit above can be rewritten removing the rotation $R_y(\theta_2)$:



The first two CNOT gates in this circuit act as a SWAP gate (because $|0\rangle_{a_1}$ is the input state of the central qubit). Hence, these two CNOT gates are not necessary. It is possible to ignore them, provided that $R_y(\theta_3)$ is moved to the second qubit. Since $\theta_1 = \theta_3 = \theta = \frac{\pi}{4}$, the final circuit is obtained



We have managed to greatly simplify the circuit. We have removed 5 CNOT gates from the circuit that we were using on real quantum processors with linear connectivity: the 2 CNOT gates mentioned here and the 3 CNOT gates that were necessary in order to implement the SWAP gate that was necessary to respect the connectivity constraints. With this new circuit, only the top qubit has to be able to connect to the other two qubits. As a final observation, since this circuit is much more simple than the one for the universal QCM, the average fidelity over the whole Bloch sphere could be greater in this case than with the universal QCM.

We will now show, step by step, that this circuit does clone states on the xz-equator. We consider a generic input state

$$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle\,,$$

therefore the initial state of the full circuit is

$$|\Psi_0\rangle = |\psi\rangle_{a_0}|0\rangle_{a_1}|0\rangle_{a_2}\,.$$

Each ancilla, after the rotation, is

$$R_y(\pi/4)\,|0\rangle = \cos(\pi/8)\,|0\rangle - \sin(\pi/8)\,|1\rangle\,.$$

Hence, the two ancilla are prepared in the following state:

$$\begin{aligned}|\phi\rangle &= \big(\cos(\pi/8)\,|0\rangle + \sin(\pi/8)\,|1\rangle\big)\big(\cos(\pi/8)\,|0\rangle + \sin(\pi/8)\,|1\rangle\big) = \\ &= \cos^2(\pi/8)\,|00\rangle + \cos(\pi/8)\sin(\pi/8)(|01\rangle + |10\rangle) + \sin^2(\theta)\,|11\rangle\,.\end{aligned}$$

Right before the CNOT gates, the state of the three qubits is:

$$\begin{aligned}|\Psi_1\rangle = \big(&\cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle\big)\big(\cos(\pi/8)\,|0\rangle + \sin(\pi/8)\,|1\rangle\big)\big(\cos(\pi/8)\,|0\rangle + \sin(\pi/8)\,|1\rangle\big) = \\ &= \cos(\theta/2)\cos^2(\pi/8)\,|000\rangle + e^{i\phi}\sin(\theta/2)\cos^2(\pi/8)\,|100\rangle \\ &+ \cos(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|010\rangle + e^{i\phi}\sin(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|110\rangle \\ &+ \cos(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|001\rangle + e^{i\phi}\sin(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|101\rangle \\ &+ \cos(\theta/2)\sin^2(\pi/8)\,|011\rangle + e^{i\phi}\sin(\theta/2)\sin^2(\pi/8)\,|111\rangle\,.\end{aligned}$$

After applying the first two CNOT gates:

$$\begin{aligned}|\Psi_2\rangle = &\cos(\theta/2)\cos^2(\pi/8)\,|000\rangle + e^{i\phi}\sin(\theta/2)\cos^2(\pi/8)\,|111\rangle \\ &+ \cos(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|010\rangle + e^{i\phi}\sin(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|101\rangle \\ &+ \cos(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|001\rangle + e^{i\phi}\sin(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|110\rangle \\ &+ \cos(\theta/2)\sin^2(\pi/8)\,|011\rangle + e^{i\phi}\sin(\theta/2)\sin^2(\pi/8)\,|100\rangle\,.\end{aligned}$$

After applying the last two CNOT gates:

$$\begin{aligned}|\Psi_3\rangle = &\cos(\theta/2)\cos^2(\pi/8)\,|000\rangle + e^{i\phi}\sin(\theta/2)\cos^2(\pi/8)\,|111\rangle \\ &+ \cos(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|110\rangle + e^{i\phi}\sin(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|001\rangle \\ &+ \cos(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|101\rangle + e^{i\phi}\sin(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|010\rangle \\ &+ \cos(\theta/2)\sin^2(\pi/8)\,|011\rangle + e^{i\phi}\sin(\theta/2)\sin^2(\pi/8)\,|100\rangle\,.\end{aligned}$$

The above state is invariant under the exchange of the second and third qubits, where the copies are made. This means that the single qubit states of the two copies are the same, i.e.:

$$\rho_{a_1} = \rho_{a_2} \quad \text{where} \quad \rho_{a_1} = \mathrm{Tr}_{a_0,a_2}[|\Psi_3\rangle\langle\Psi_3|], \quad \rho_{a_2} = \mathrm{Tr}_{a_0,a_1}[|\Psi_3\rangle\langle\Psi_3|]$$

In order to calculate the expected fidelity of the copies (we will consider the copy ρ_{a_1}), we first isolate the qubits a_0 and a_2:

$$\begin{aligned}|\Psi_3\rangle = |0\rangle &\left(\cos(\theta/2)\cos^2(\pi/8)\,|0\rangle + e^{i\phi}\sin(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|1\rangle\right)|0\rangle\\ &+|0\rangle\left(e^{i\phi}\sin(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|0\rangle + \cos(\theta/2)\sin^2(\pi/8)\,|1\rangle\right)|1\rangle\\ &+|1\rangle\left(e^{i\phi}\sin(\theta/2)\sin^2(\pi/8)\,|0\rangle + \cos(\theta/2)\sin(\pi/8)\cos(\pi/8)\,|1\rangle\right)|0\rangle\\ &+|1\rangle\left(\cos(\theta/2)\cos(\pi/8)\sin(\pi/8)\,|0\rangle + e^{i\phi}\sin(\theta/2)\cos^2(\pi/8)\,|1\rangle\right)|1\rangle\,.\end{aligned}$$

The single copy fidelity is then (θ and ϕ are the angles on the Bloch sphere of the input state):

$$\begin{aligned}F(\theta,\phi) &= \langle\psi|\,\rho_{a_1}\,|\psi\rangle = \langle\psi|\,\mathrm{Tr}_{a_0,a_2}[|\Psi_3\rangle\langle\Psi_3|]\,|\psi\rangle =\\ &= \left(\cos^2(\theta/2)\cos^2(\pi/8) + \sin^2(\theta/2)\cos(\pi/8)\sin(\pi/8)\right)^2\\ &+ \left|e^{i\phi}\cos(\theta/2)\sin(\theta/2)\sin(\pi/8)\cos(\pi/8) + e^{-i\phi}\sin(\theta/2)\cos(\theta/2)\sin^2(\pi/8)\right|^2\\ &+ \left|e^{i\phi}\cos(\theta/2)\sin(\theta/2)\sin^2(\pi/8) + e^{-i\phi}\sin(\theta/2)\cos(\theta/2)\sin(\pi/8)\cos(\pi/8)\right|^2\\ &+ \left(\cos^2(\theta/2)\cos(\pi/8)\sin(\pi/8) + \sin^2(\theta/2)\cos^2(\pi/8)\right)^2.\end{aligned}$$

The fidelity can be computed analytically. The final result is:

$$F(\theta,\phi) = \frac{3\left(4-\sqrt{2}\right) + \left(3\sqrt{2}-4\right)\left[\cos(2\phi) + \cos(2\theta)\left(1-\cos(2\phi)\right)\right]}{16(2-\sqrt{2})}.$$

It is possible to observe that if $\phi = 0$ is set, the fidelity becomes a constant:

$$F(\theta,0) = \frac{3\left(4-\sqrt{2}\right) + \left(3\sqrt{2}-4\right)}{16(2-\sqrt{2})} = \frac{8}{16(2-\sqrt{2})} = \frac{1}{2}\frac{2+\sqrt{2}}{2} = \frac{1}{2} + \frac{1}{\sqrt{8}}.$$

Therefore, for input states on the xz-equator, the fidelity of the PCQCM is:

$$F^{ideal}_{equator} = \frac{1}{2} + \frac{1}{\sqrt{8}} \approx 0.854 \tag{2.1}$$

As discussed in [**?**] this is the optimal fidelity for a PCQCM. Moreover, this is the optimal result attainable also when trying to maximize the fidelity for BB84 input states. Indeed, the optimization over the two different sets of output is equivalent and leads therefore to the same result [**?**].

It might be of some interest to consider the performances of the PCQCM over the whole Bloch sphere. The average fidelity over the Bloch sphere S is (integrating over solid angle):

$$\overline{F} = \frac{1}{\int_S \mathrm{d}\Omega}\int_S F(\Omega)\,\mathrm{d}\Omega = \frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi)\sin(\theta)\,\mathrm{d}\theta\,\mathrm{d}\phi = \frac{7+2\sqrt{2}}{12} \approx 0.819.$$

The standard deviation can be calculated similarly:

$$\sigma_F = \sqrt{\overline{F^2} - \overline{F}^2} = \frac{1}{6}\sqrt{\frac{3-2\sqrt{2}}{5}} \approx 0.031.$$

We conclude that when considering the PCQCM over the whole Bloch sphere, it is not too far from the optimal bound of the UQCM (which was $F = 5/6 \approx 0.833$). This could have some interesting consequences: since we managed to greatly simplify its circuit, the PCQCM could turn to be better than the UQCM when using real hardware, in contrast with what is expected theoretically. A similar analysis was carried out in [**?**] in the context of economical QCM.

2.4. Economical phase covariant quantum cloning machine

Given 2 qubits, with the first B (Bob) to be the target and the second E (Eve) to be the copy, an economical quantum cloning machnie (EQCM) on can be defined by [**?**]

$$U_c\,|00\rangle_{BE} = |00\rangle_{BE} \tag{2.2}$$

$$U_c\,|10\rangle_{BE} = \cos\alpha\,|10\rangle_{BE} + \sin\alpha\,|01\rangle_{BE} \tag{2.3}$$

With target to be an equatorial state in the XY plane of Bloch sphere

$$\left|\psi\right\rangle_B = \frac{1}{\sqrt{2}}(|0\rangle + e^{i\phi}\,|1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi} \end{pmatrix}$$

and Eve's qubit initialized at $|0\rangle$ state. Apply the unitary to the two-qubit input state and we get

$$U_c\left|\psi\right\rangle_B|0\rangle_E = \frac{1}{\sqrt{2}}|00\rangle_{BE} + \frac{e^{i\phi}}{\sqrt{2}}(\cos\alpha\,|10\rangle_{BE} + \sin\alpha\,|01\rangle_{BE}) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\phi}\cos(\alpha) \\ e^{i\phi}\sin(\alpha) \\ 0 \end{pmatrix}$$

And the density matrix of the system after applying the unitary is

$$\rho_{BE} = \frac{1}{2}\begin{pmatrix} 1 & e^{-i\phi}\cos(\alpha) & e^{-i\phi}\sin(\alpha) & 0 \\ e^{i\phi}\cos(\alpha) & \cos^2(\alpha) & \frac{1}{2}\sin(2\alpha) & 0 \\ e^{i\phi}\sin(\alpha) & \frac{1}{2}\sin(2\alpha) & \sin^2(\alpha) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.4}$$

Use partial trace to get the density matrix of subsystem from the mixed state

$$\mathrm{Tr}_E(\rho_{BE}) = \begin{pmatrix} \frac{\sin^2(\alpha)}{2} + \frac{1}{2} & \frac{1}{2}e^{-i\phi}\cos(\alpha) \\ \frac{1}{2}e^{i\phi}\cos(\alpha) & \frac{\cos^2(\alpha)}{2} \end{pmatrix}, \quad \mathrm{Tr}_B(\rho_{BE}) = \begin{pmatrix} \frac{\cos^2(\alpha)}{2} + \frac{1}{2} & \frac{1}{2}e^{-i\phi}\sin(\alpha) \\ \frac{1}{2}e^{i\phi}\sin(\alpha) & \frac{\sin^2(\alpha)}{2} \end{pmatrix}$$

We can calculate the fidelity

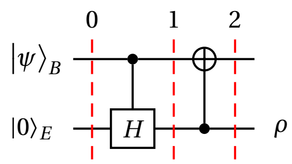
$$F_B = \left\langle\psi_B\right|\mathrm{Tr}_E(\rho_{BE})\left|\psi_B\right\rangle = \frac{1+\cos\alpha}{2}$$

$$F_E = \left\langle\psi_B\right|\mathrm{Tr}_B(\rho_{BE})\left|\psi_B\right\rangle = \frac{1+\sin\alpha}{2}$$

By the definition of a quantum cloning machine, the post-copy states must preserve same fidelities where $F_B = F_E$, equals to $\sin\alpha = \cos\alpha$. Thus we acquire that $\alpha = \pi/4$, and the fidelity of a economical quantum cloning machine on this equator would be

$$F_{equator} = \frac{1+\cos\pi/4}{2} \approx 0.8535$$

The unitary U_c can be implemented with a controlled-Hadamard gate and a controlled-NOT gate

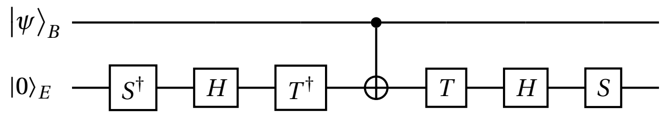


Verify the circuit complys with the definition in 2.2 and 2.3

$$U_c\,|00\rangle = \mathrm{CNOT}_{EB}\mathrm{CH}_{BE}\,|00\rangle_{BE} = |00\rangle_{BE}$$

$$\begin{aligned} U_c\,|10\rangle &= \mathrm{CNOT}_{EB}\mathrm{CH}_{BE}\,|10\rangle_{BE} \\ &= \mathrm{CNOT}_{EB}\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)_{BE} \\ &= \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle)_{BE} \end{aligned}$$

The controlled-Hadamard gate can be compiled into common-used quantum gates



where

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$$

As the name shows, the economical quantum cloning machine is designed to use 2 instead of 3 qubits, which limited its performance on the whole bloch sphere. The controlled-Hadamard gate can be write in matrix form by definition. With the CNOT gate known, we can write the unitary of cloning machine

$$\mathrm{CH}_{BE} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_c = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Consider the generic input state to be copied

$$|\psi\rangle_B = \cos(\theta/2)\,|0\rangle_B + e^{i\phi}\sin(\theta/2)\,|1\rangle_B\,,$$

The post-copy state is

$$|\Psi\rangle_{BE} = U_c(|\psi\rangle_B \otimes |0\rangle_E), \quad \rho_{BE} = |\Psi\rangle\langle\Psi|_{BE} = \begin{pmatrix} \cos^2\left(\frac{\theta}{2}\right) & \frac{e^{-i\phi}\sin(\theta)}{2\sqrt{2}} & \frac{e^{-i\phi}\sin(\theta)}{2\sqrt{2}} & 0 \\ \frac{e^{i\phi}\sin(\theta)}{2\sqrt{2}} & \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right) & \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right) & 0 \\ \frac{e^{i\phi}\sin(\theta)}{2\sqrt{2}} & \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right) & \frac{1}{2}\sin^2\left(\frac{\theta}{2}\right) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

The above density matrix comply with 2.4. This assured the two qubits will share the same state after being copied.

$$\mathrm{Tr}_E[\rho_{BE}] = \mathrm{Tr}_B[\rho_{BE}] = \frac{1}{4}\begin{pmatrix} \cos(\theta)+3 & \sqrt{2}e^{-i\phi}\sin(\theta) \\ \sqrt{2}e^{i\phi}\sin(\theta) & 1-\cos(\theta) \end{pmatrix}$$

Then we have the fidelity

$$F(\theta,\phi) = \langle\psi_B|\mathrm{Tr}_E[\rho_{BE}]|\psi_B\rangle = \frac{1}{8}\left(2\cos(\theta) - \left(\sqrt{2}-1\right)\cos(2\theta) + \sqrt{2}+5\right)$$

The fidelity is not related with ϕ and it can be verified that on XY equator

$$F(\pi/2,\phi) = \frac{1}{4}\left(\sqrt{2}+2\right)$$

which is exactly what we expected. The fidelity is shown below The global fidelity on the whole bloch sphere is

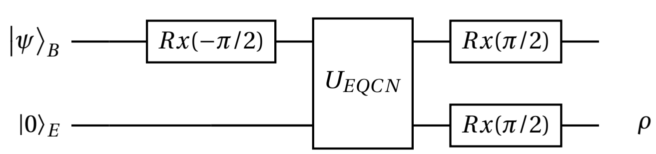
$$\overline{F} = \frac{1}{\int_S \mathrm{d}\Omega}\int_S F(\Omega)\,\mathrm{d}\Omega = \frac{1}{2}\int_0^\pi F(\theta,\phi)\sin(\theta)\,\mathrm{d}\theta = \frac{1}{12}\left(2\sqrt{2}+7\right) \approx 0.819036$$

With the lowest fidelity occurs at $F(\pi) = 1/2$.

And the deviation

$$\sigma_F = \sqrt{\overline{F^2} - \overline{F}^2} = \frac{1}{12}\sqrt{\frac{1}{5}\left(27-8\sqrt{2}\right)} \approx 0.147603$$

For the BB84 states on the XZ equator, we can use a $Rx(-\pi/2)$ gate to rotate input state onto the XY equator and then rotate it back after copying. This modified EQCM could copy states on the XZ plane and thus can be used to attack a BB84 QKD protocol.



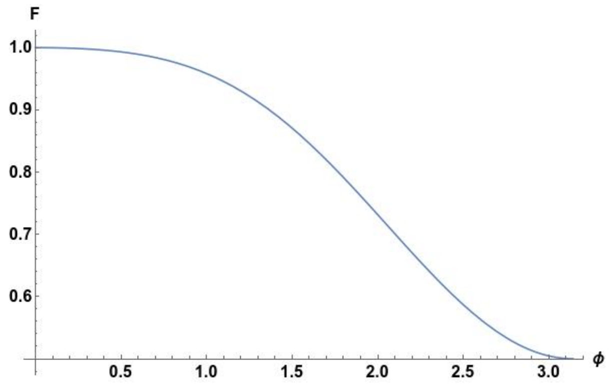


Figure 2.1: Fidelity of Economical Quantum Cloning Machine

2.5. Quantum cloning and quantum key distribution

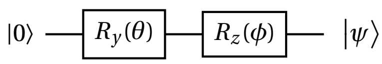
NB: optimal cloning and optimal eavesdropping are different! When using the 3 qubit phase covariant QCM Eve also has the ancilla, which carries some additional information. See [?] "In summary: without ancilla, Eve can make the best possible guess on the bit sent by Alice (because the machine realizes the optimal phase-covariant cloning) but has very poor information about the result obtained by Bob. Adding the ancilla does not modify the estimation of Alice's bit but allows Eve to deterministically symmetrize her information on Alice and Bob's symbols. However, the two machines are equally good from the point of view of cloning."

3
Implementation

3.1. Preparation of the input states

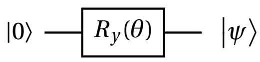
We have prepared a generic state on the Bloch sphere using the following two rotations:

$$\left|\psi\right\rangle = R_z(\phi)R_y(\theta)\left|0\right\rangle = \cos(\theta/2)\left|0\right\rangle + e^{i\phi}\sin(\theta/2)\left|1\right\rangle,$$



where θ is the polar angle and ϕ is the azimuthal angle. For the preparation of an input state on the xz-equator, only the rotation about the y axis is needed:

$$\left|\psi\right\rangle = R_y(\theta)\left|0\right\rangle = \cos(\theta/2)\left|0\right\rangle + \sin(\theta/2)\left|1\right\rangle.$$

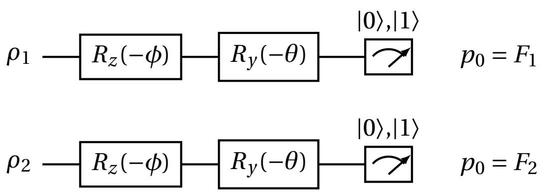


3.2. Measurement of the fidelity of the copies

The fidelity of the two copies can be measured directly. If the input state was $\left|\psi\right\rangle$, it is sufficient to perform a measurement in the $\{\left|\psi\right\rangle, \left|\psi_\perp\right\rangle\}$ basis. The fidelity corresponds to the probability of measuring $\left|\psi\right\rangle$. Indeed, if the copies are $\rho_{1,2}$ and their fidelities with respect to the input state are $F_{1,2}$:

$$p_{1,2}^{\psi} = \mathrm{Tr}\left[\left|\psi\right\rangle\left\langle\psi\right|\rho_{1,2}\right] = \left\langle\psi\right|\rho_{1,2}\left|\psi\right\rangle = F_{1,2}.$$

Since $\left|\psi\right\rangle$ was prepared with the rotation $R_z(\phi)R_y(\theta)$, it follows that a measurement in the $\{\left|\psi\right\rangle, \left|\psi_\perp\right\rangle\}$ basis can be performed implementing the opposite rotation and then measuring in the computational basis.



It is worth mentioning that both the preparation and the fidelity measurement are not required in the normal use of QCMs, they are just a means of testing their performances. It should therefore be taken into account that the additional gates could decrease the quality of the copies because of the greater depth of the run circuits.

3.3. Universal quantum cloning machine

3.4. Phase covariant quantum cloning machine

3.5. Economical phase covariant quantum cloning machine

3.6. Readout calibration

4 Results

NB: COMPARISON WITH OTHER EXPERIMENTAL RESULTS?
https://arxiv.org/pdf/1909.03170.pdf UQCM with gates (not peer-reviewed, but it has some review of modern experimental results)
https://link.aps.org/doi/10.1103/PhysRevLett.88.187901 UQCM with NMR (bad results)
https://science-sciencemag-org.tudelft.idm.oclc.org/content/296/5568/712 (photons)
https://arxiv.org/pdf/quant-ph/0311010.pdf economical phase covariant with NMR

4.1. Universal quantum cloning machine

4.1.1. Sphere

4.1.2. Equator

4.1.3. BB84 states

4.2. Phase covariant quantum cloning machine

THIS RESULTS ARE TEMPORARY AND ARE GOING TO BE MODIFIED WITH THE UPDATED VERSION OF THE CODE.

4.2.1. Sphere

Without readout correction:

	Starmon 5	**Athens**	**Ourense**	**Santiago**	**Valencia**	**Vigo**	**Yorktown**
F_1	0.759	0.779	0.812	0.770	0.805	0.782	0.779
σ_{F_1}	0.033	0.033	0.031	0.043	0.032	0.030	0.028
F_2	0.760	0.824	0.786	0.791	0.794	0.812	0.765
σ_{F_2}	0.038	0.031	0.034	0.043	0.034	0.036	0.009

With readout correction:

	Starmon 5	**Athens**	**Ourense**	**Santiago**	**Valencia**	**Vigo**	**Yorktown**
F_1	0.786	0.779	0.812	0.774	0.799	0.798	0.775
σ_{F_1}	0.034	0.034	0.033	0.045	0.035	0.036	0.030
F_2	0.767	0.823	0.788	0.802	0.787	0.812	0.763
σ_{F_2}	0.040	0.032	0.035	0.057	0.037	0.038	0.043

4.2.2. Equator

	Starmon 5	Athens	Ourense	Santiago	Valencia	Vigo	Yorktown
F_1	0.780	0.807	0.837	0.819	0.842	0.792	0.778
σ_{F_1}	0.029	0.017	0.007	0.008	0.014	0.019	0.045
F_2	0.785	0.850	0.815	0.817	0.823	0.838	0.758
σ_{F_2}	0.038	0.015	0.010	0.009	0.016	0.019	0.005

	Starmon 5	Athens	Ourense	Santiago	Valencia	Vigo	Yorktown
F_1	0.801	0.808	0.840	0.822	0.840	0.814	0.775
σ_{F_1}	0.032	0.017	0.008	0.009	0.015	0.023	0.047
F_2	0.791	0.848	0.818	0.815	0.822	0.841	0.778
σ_{F_2}	0.040	0.015	0.010	0.010	0.019	0.020	0.024

4.2.3. BB84 states

	Starmon 5	Athens	Ourense	Santiago	Valencia	Vigo	Yorktown
F_1	0.787	0.808	0.834	0.789	0.840	0.76	0.76
σ_{F_1}	0.037	0.013	0.009	0.010	0.007	0.03	0.02
F_2	0.790	0.848	0.820	0.835	0.824	0.75	0.76
σ_{F_2}	0.029	0.015	0.011	0.004	0.012	0.03	0.02

	Starmon 5	Athens	Ourense	Santiago	Valencia	Vigo	Yorktown
F_1	0.809	0.807	0.836	0.793	0.6	0.76	0.76
σ_{F_1}	0.041	0.013	0.009	0.011	0.2	0.03	0.02
F_2	0.796	0.847	0.822	0.834	0.6	0.75	0.76
σ_{F_2}	0.031	0.015	0.012	0.004	0.2	0.03	0.02

4.3. Economical phase covariant quantum cloning machine

4.3.1. Sphere

4.3.2. Equator

4.3.3. BB84 states

5

Conclusions

More backends with different technologies.