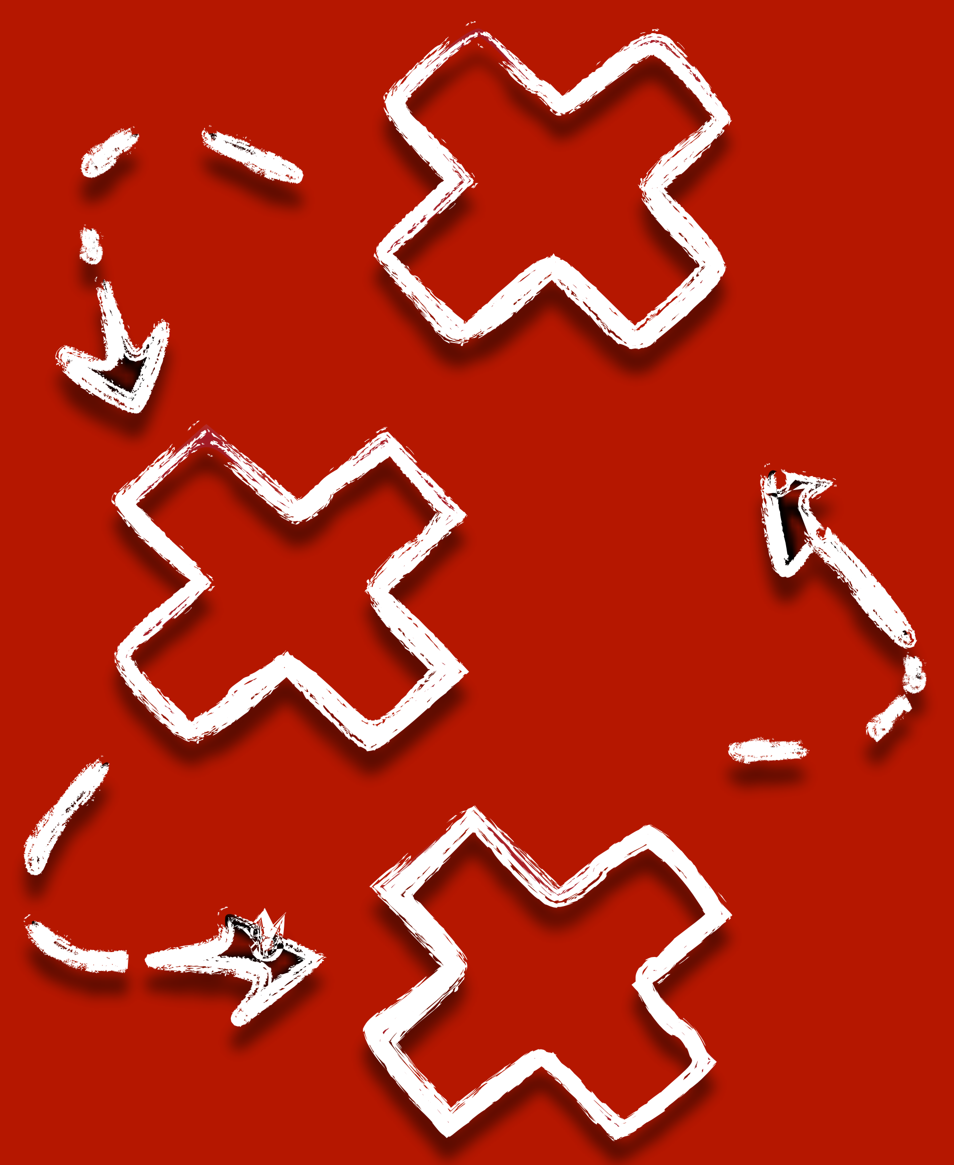

Causal Data Science

Lecture 6:1 Adjustment criterion

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UvA - Spring 2023

Last class: Identification strategies for causal effects

- Given a causal graph G, an **identification strategy** is a formula to estimate an interventional distribution from a combination of observational ones
- **Backdoor criterion (last class), Adjustment criterion (this class)**

$$p(x_j \mid \mathrm{do}(x_i)) = \int_{x_{\mathbf{Z}}} p(x_j \mid x_i, x_{\mathbf{Z}}) p(x_{\mathbf{Z}}) dx_{\mathbf{Z}}$$

- **Frontdoor criterion (this class)**

$$p(x_j \mid \mathrm{do}(x_i')) = \int_{x_{\mathbf{M}}} p(x_{\mathbf{M}} \mid x_i') \int_{x_i} p(x_j \mid x_{\mathbf{M}}, x_i') p(x_i) dx_i$$

- **Instrumental variables (this class)**

Last class: Identification strategies for causal effects

- Given a causal graph G, an **identification strategy** is a formula to estimate an interventional distribution from a combination of observational ones

- **Backdoor criterion (last class), Adjustment criterion (this class)**

$$P(X_j | \mathrm{do}(X_i)) = \sum_{x_{\mathbf{Z}}} P(X_j | X_i, X_{\mathbf{Z}} = x_{\mathbf{Z}}) P(X_{\mathbf{Z}} = x_{\mathbf{Z}})$$

- **Frontdoor criterion (this class)**

$$P(X_j | \mathrm{do}(X_i = x_i)) = \sum_{x_{\mathbf{M}}} P(X_{\mathbf{M}} = x_{\mathbf{M}} | X_i = x_i) \sum_{x_i'} P(X_j | X_{\mathbf{M}} = x_{\mathbf{M}}, X_i = x_i') P(X_i = x_i')$$

- **Instrumental variables (this class)**

Last class: Backdoor criterion [Pearl 2009]

- Given a CBN (G, p) with $G = (\mathbf{V}, \mathbf{E})$, a set $\mathbf{Z} \subseteq \mathbf{V} \backslash \{i, j\}$ satisfies the **backdoor criterion** for estimating the causal effect of X_i on X_j with $i \neq j$:
  - $\mathbf{Z}$ does **not contain any descendant of i,** $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**
  - $\mathbf{Z}$ blocks all **backdoor paths** from i to j (all paths that start with an arrow into $i \leftarrow \dots\ j$)

The backdoor criterion finds **some (not necessarily all)** valid adjustment sets

Complete: Adjustment criterion [Shpitser et al, Perković et al]

- Given a CBN (G, p), a set $\mathbf{Z} \subseteq \mathbf{V} \backslash \{i, j\}$ satisfies the **adjustment criterion** for estimating the causal effect of X_i on X_j with $i \neq j$:
  1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**
  2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**

Complete: Adjustment criterion [Shpitser et al, Perković et al]

- Given a CBN (G, p), a set $\mathbf{Z} \subseteq \mathbf{V} \setminus \{i, j\}$ satisfies the **adjustment criterion** for estimating the causal effect of X_i on X_j with $i \neq j$:
  1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**
  2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**

The adjustment criteria finds **all valid adjustment sets** (but there are other sets that allow identification of total causal effects - e.g. frontdoor criterion)

Complete: Adjustment criterion [Shpitser et al, Perković et al]

- Given a CBN (G, p), a set $\mathbf{Z} \subseteq \mathbf{V} \setminus \{i, j\}$ satisfies the **adjustment criterion** for estimating the causal effect of X_i on X_j with $i \neq j$:

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**

   Backdoor criterion: $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$

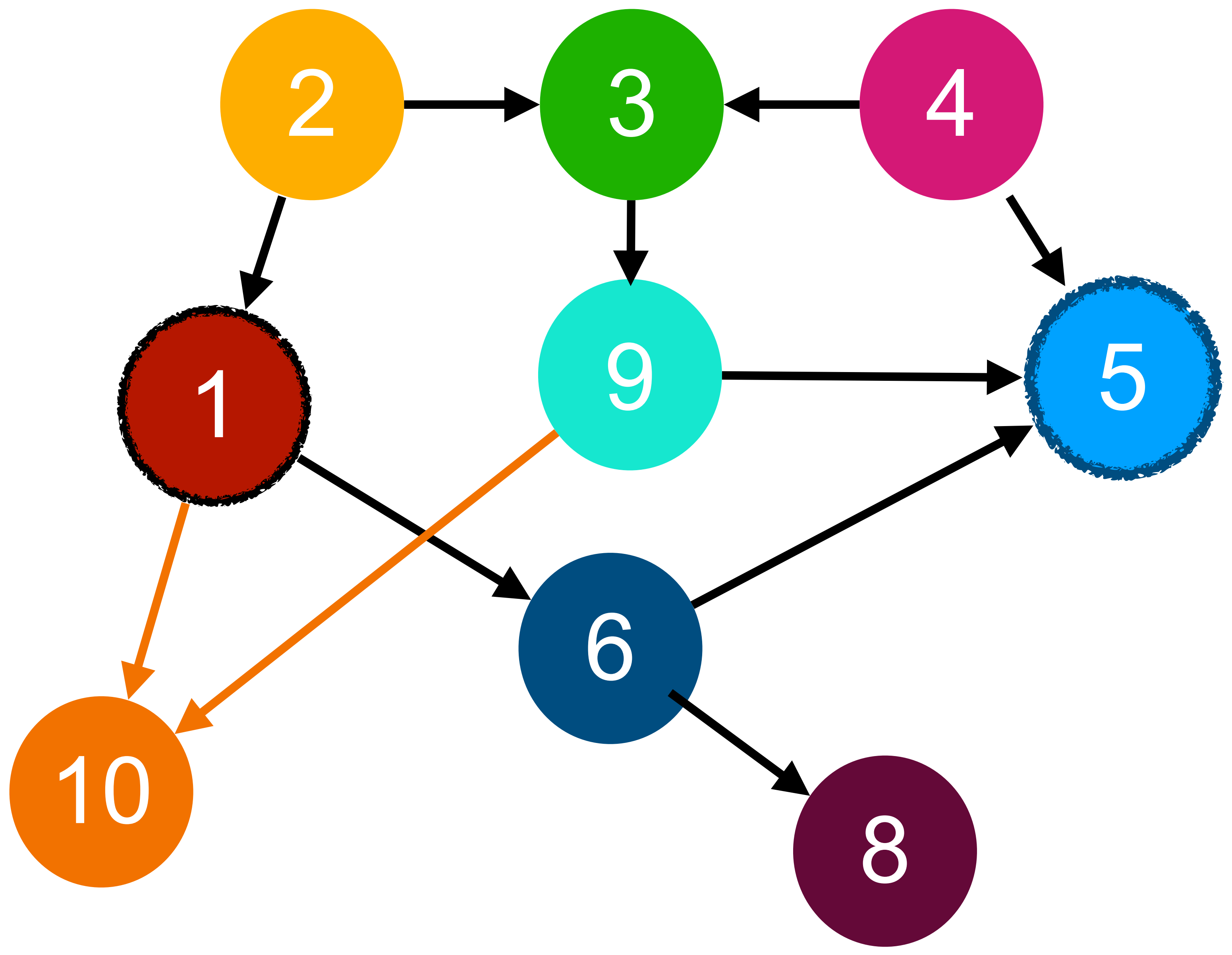
2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**

   Backdoor criterion: all backdoor paths

Adjustment criterion $X_1 \to X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j** **Backdoor:** $\mathrm{Desc}(1)$?

2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**

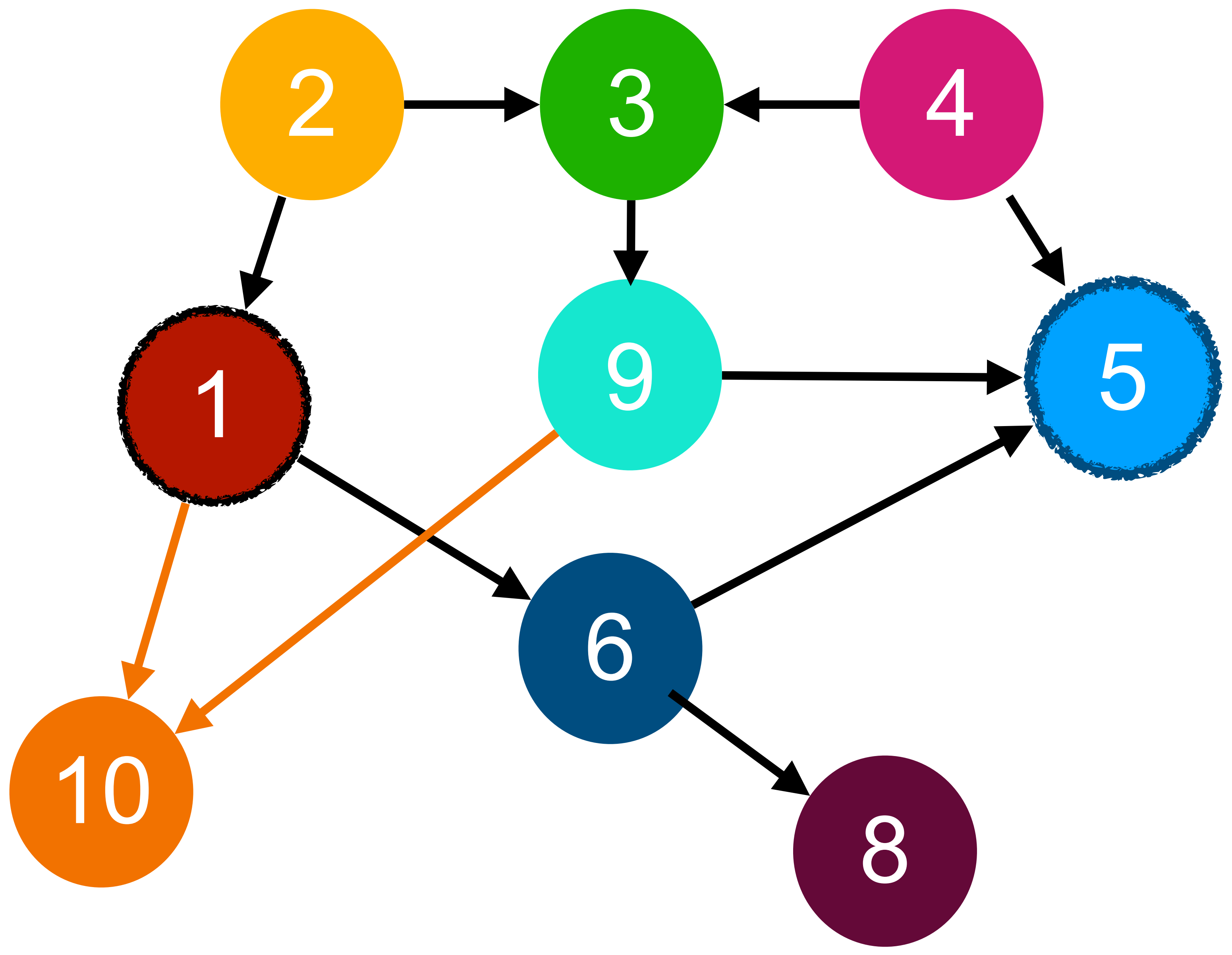


Adjustment criterion $X_1 \to X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**

   Backdoor: 6,8,10 ∉ Z

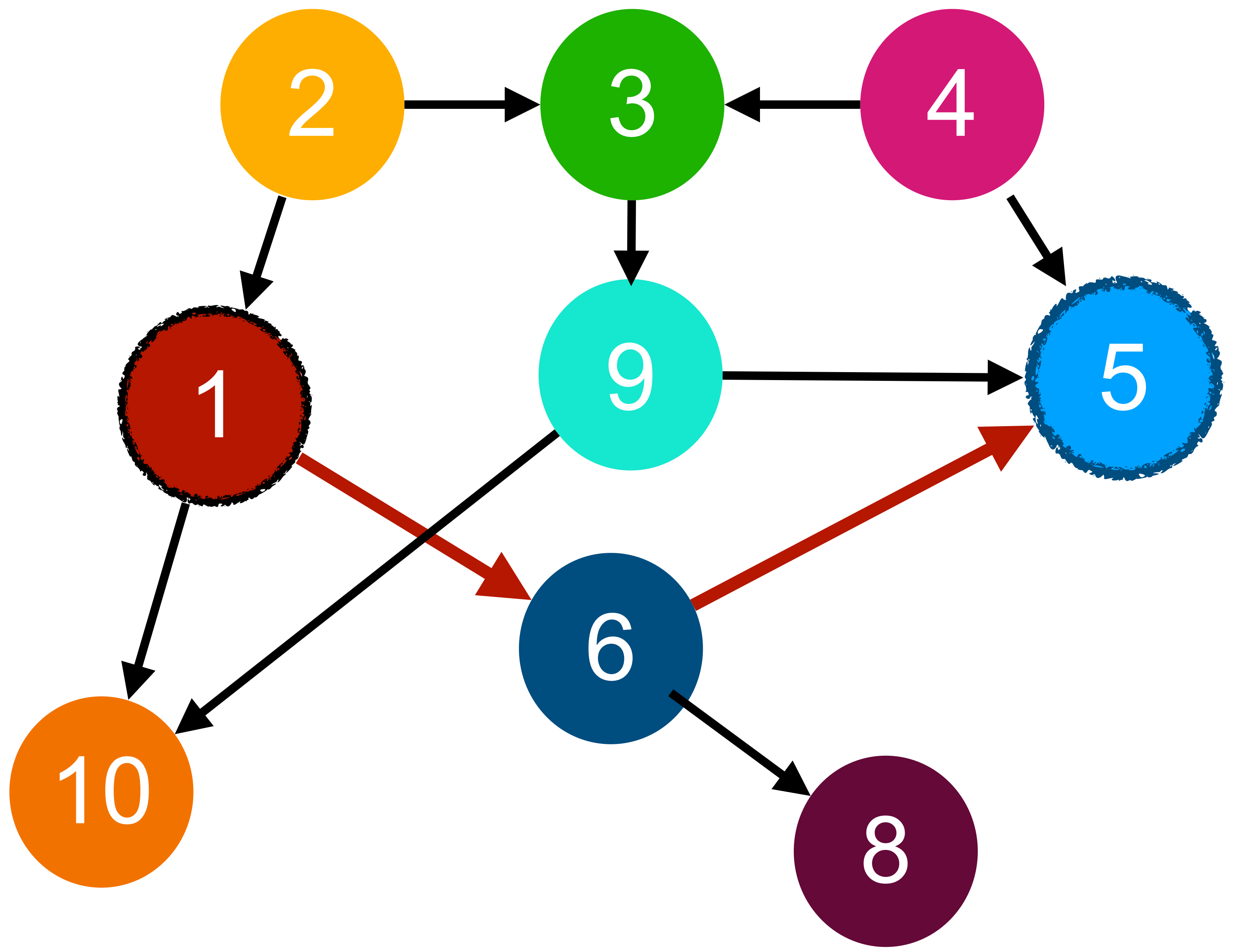
2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**



Adjustment criterion $X_1 \to X_5$

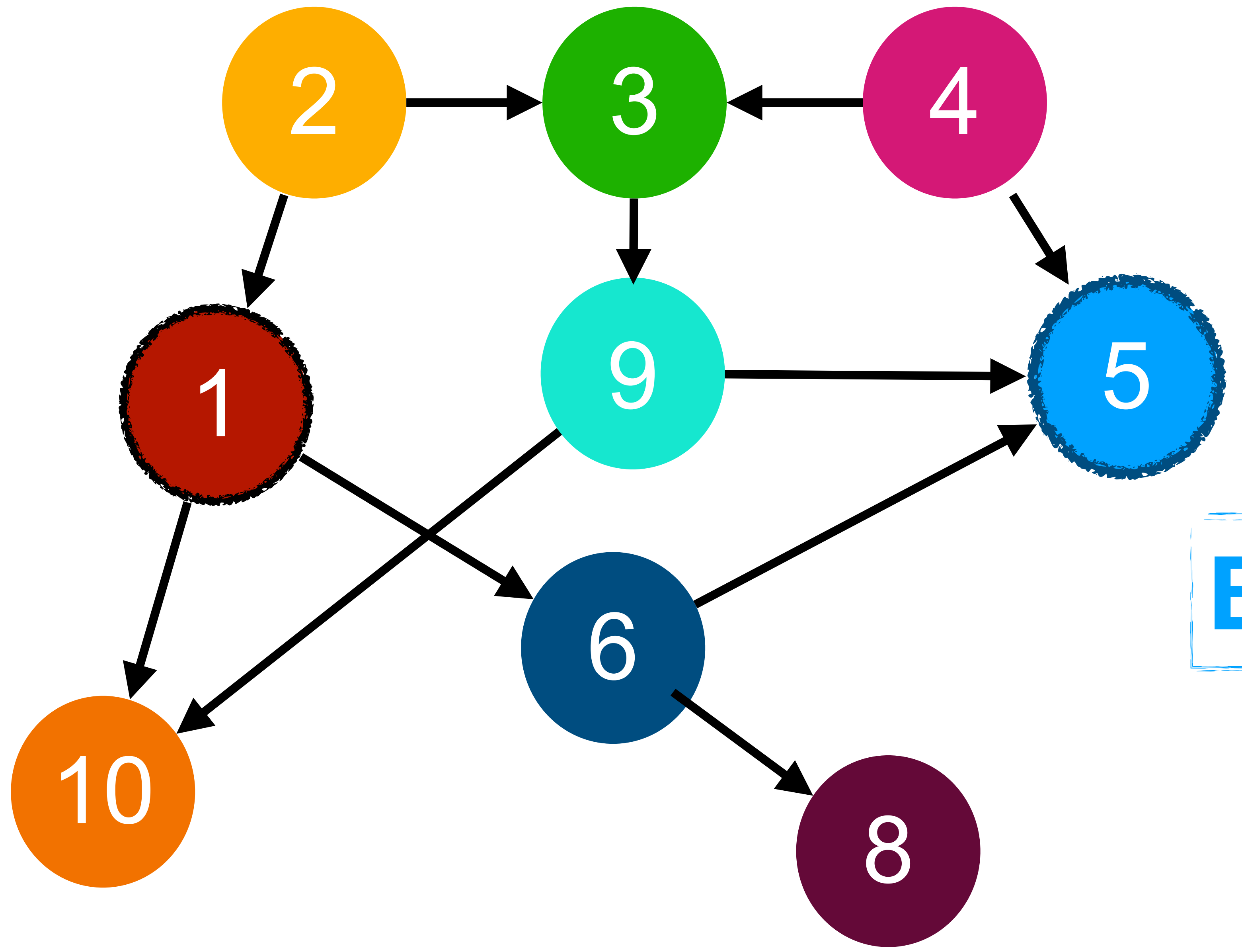
1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**   Backdoor: $6,8,10 \notin Z$ Adj: $6,8 \notin Z$

2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**



Adjustment criterion $X_1 \to X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes** $r \neq i$ **on a directed path from** i **to** j
2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from** i **to** j



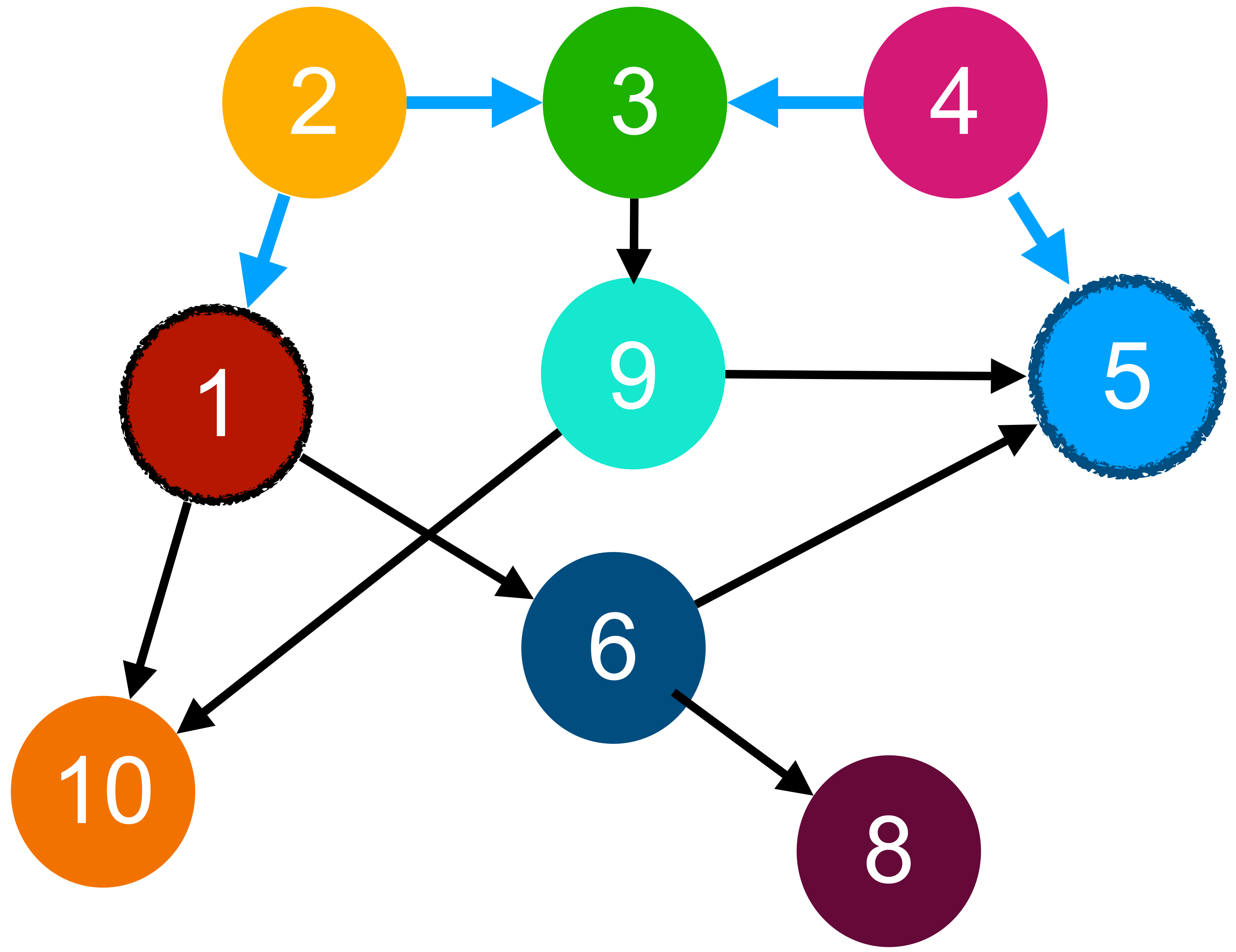
Backdoor criterion: all backdoor paths?

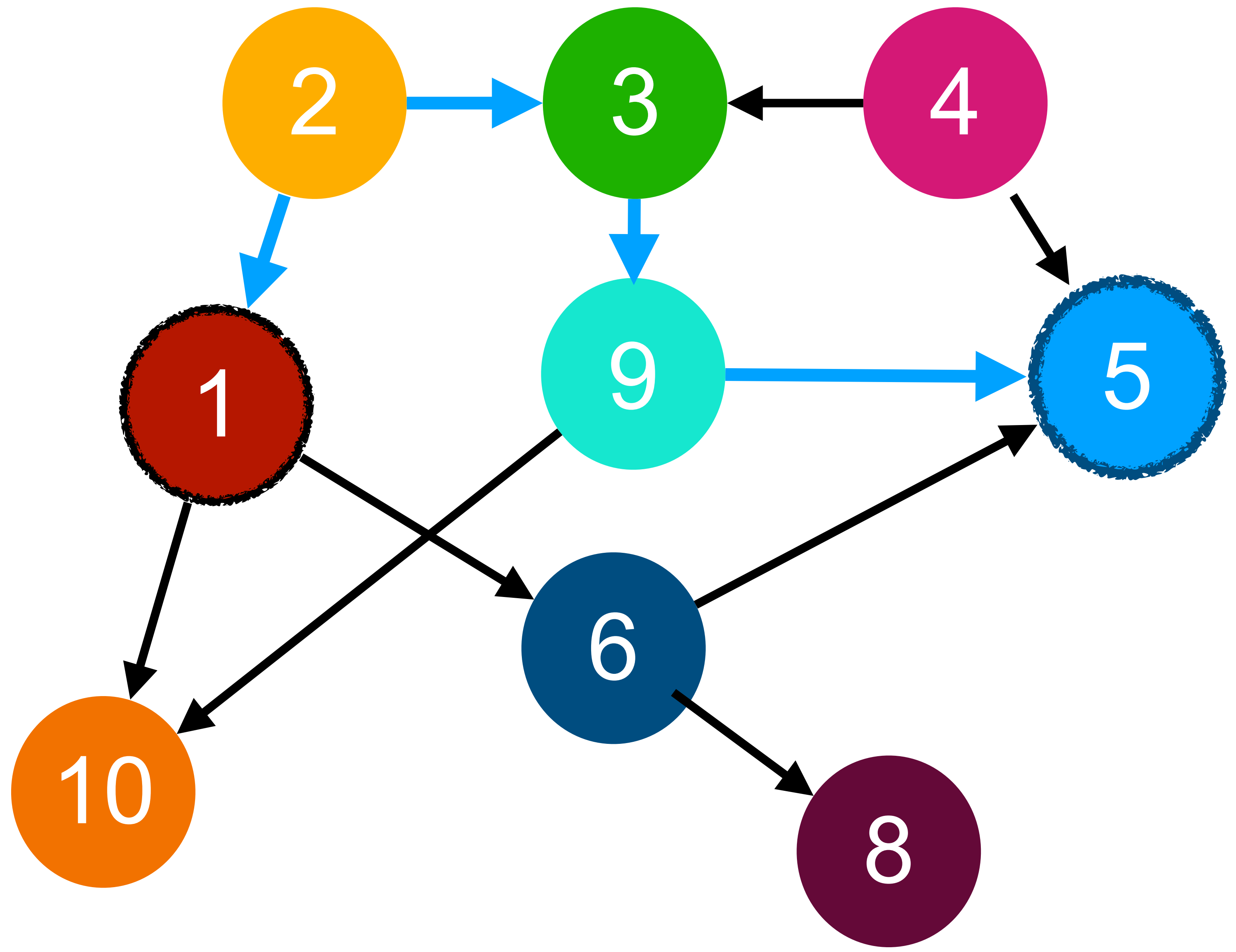
Adjustment criterion $X_1 \to X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**

Backdoor criterion: all backdoor paths?

2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**

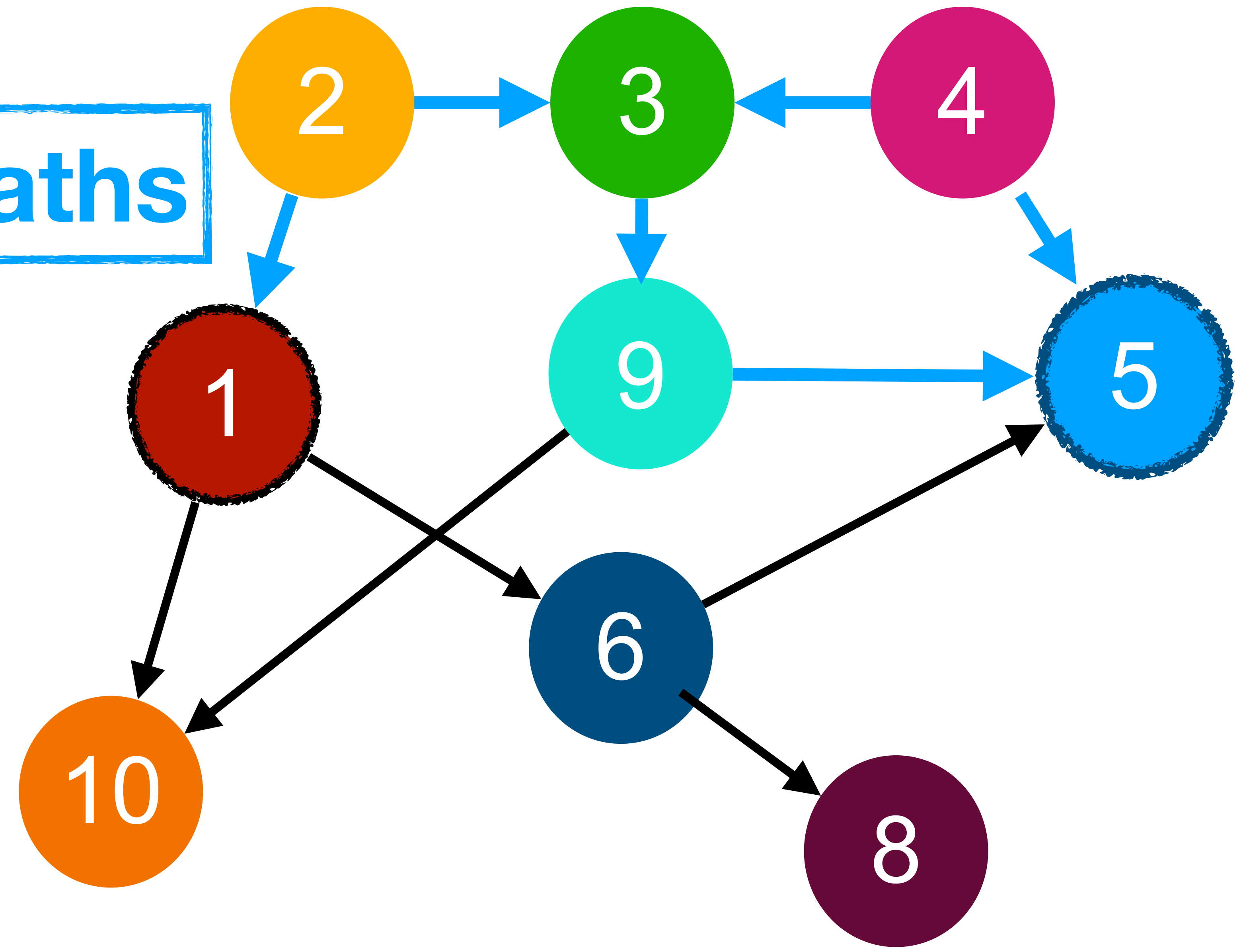




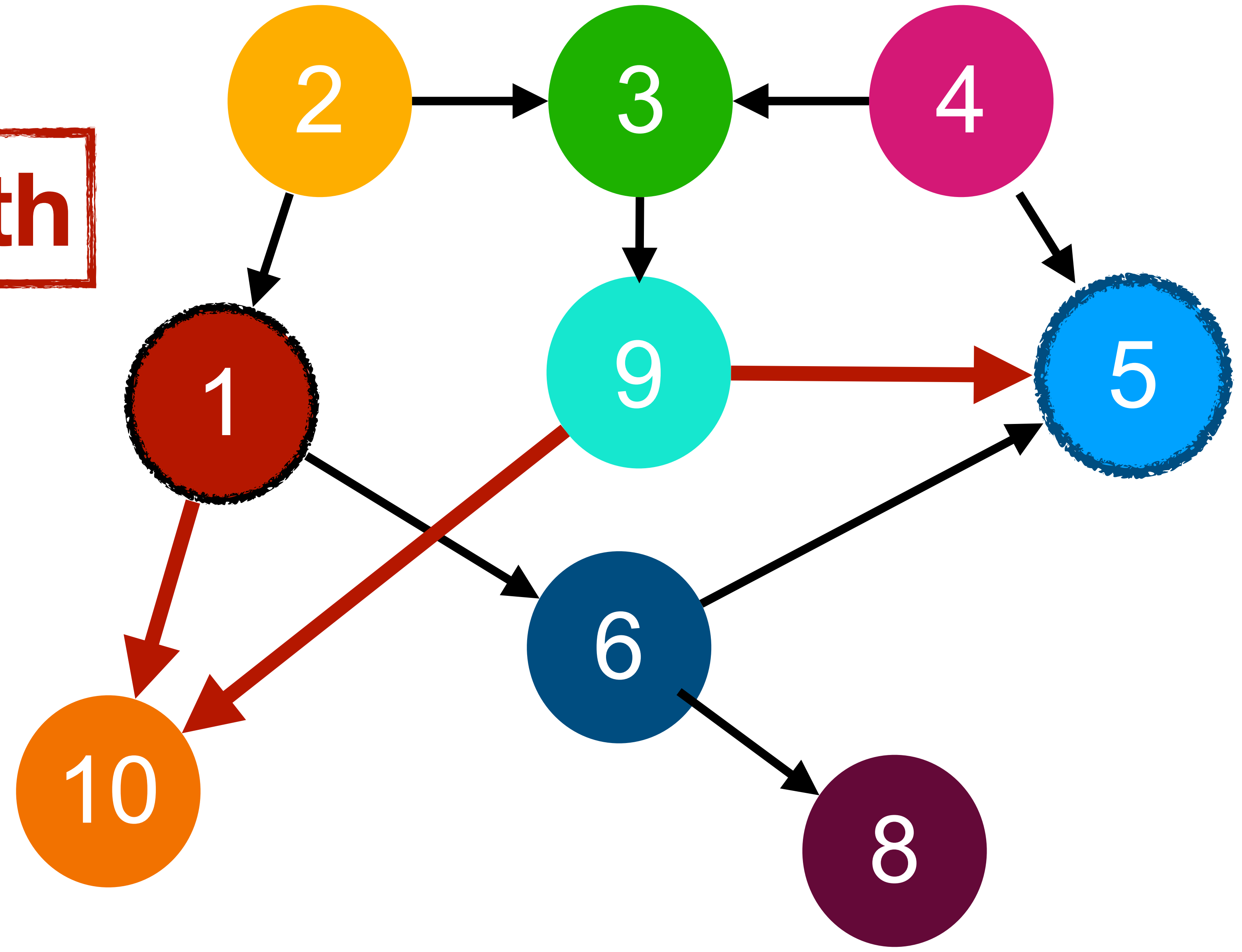
Adjustment criterion $X_1 \to X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**
2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**

Backdoor paths



Extra path

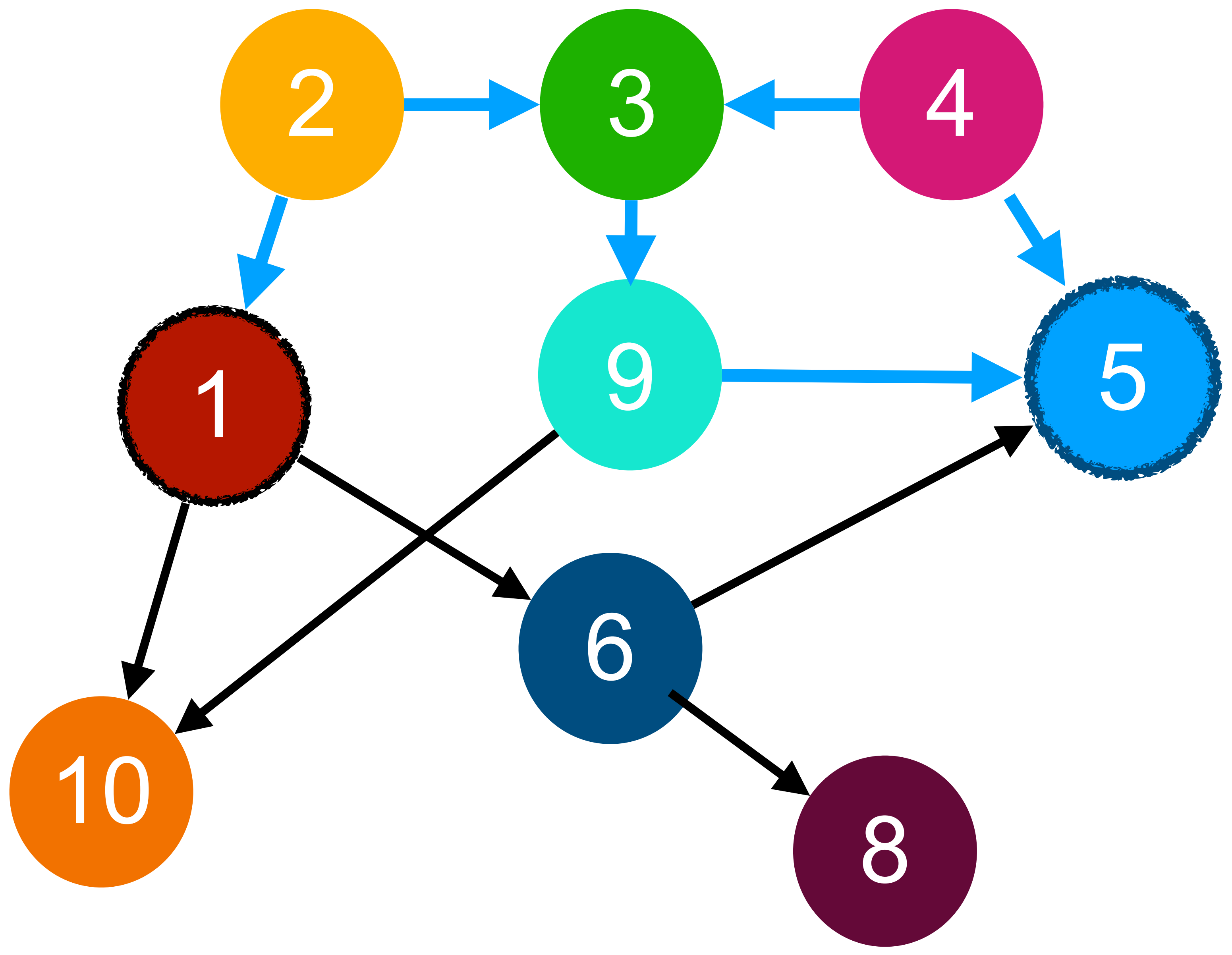


Adjustment criterion $X_1 \to X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**

Backdoor: 6,8,10 $\notin Z$ Adj: 6,8 $\notin Z$

2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**



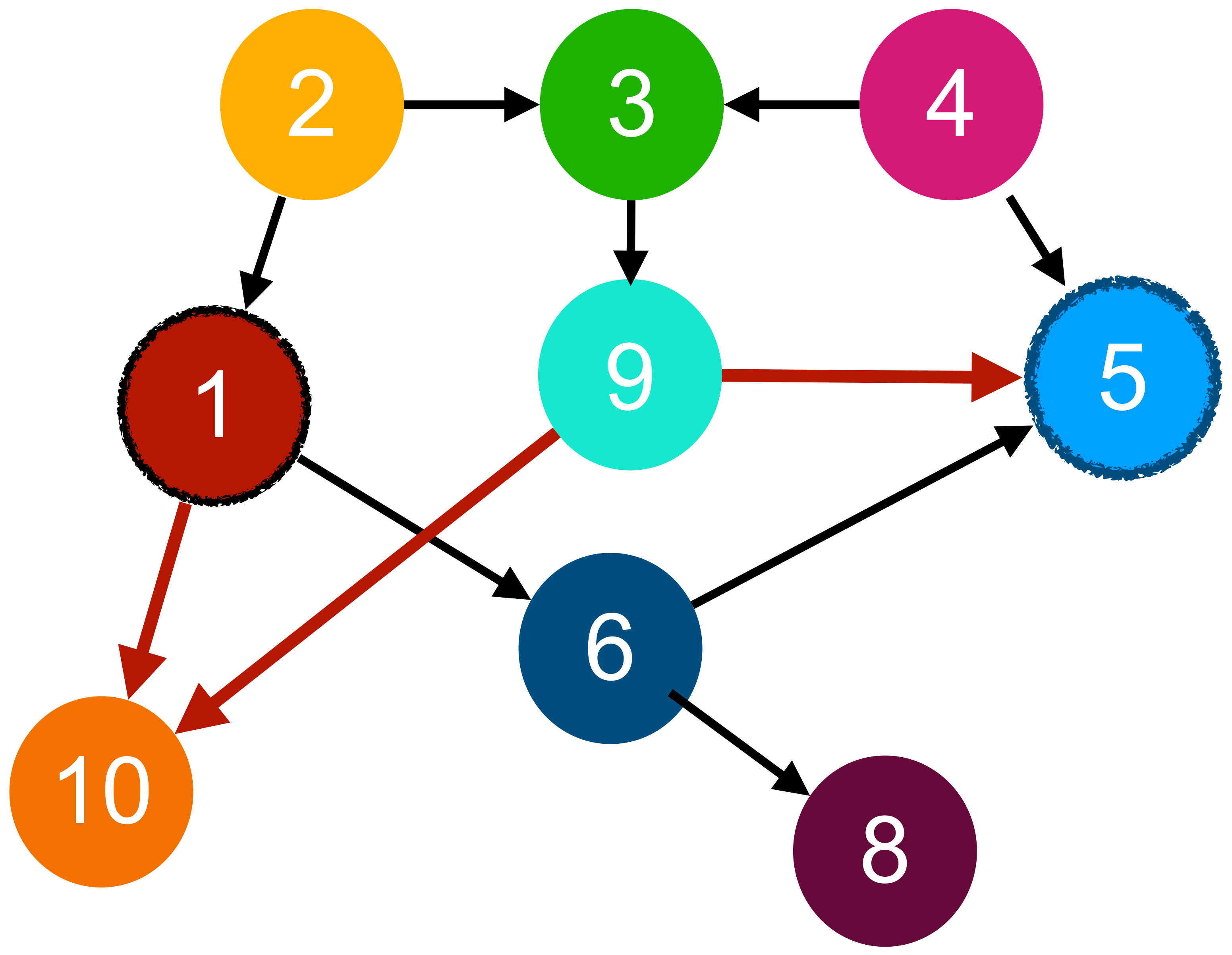
Backdoor: $\{2\}$ + any subset $\{3,4,9\}$; $\{3,4\}$; $\{4,9\}$

Adjustment criterion $X_1 \rightarrow X_5$

1. $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**

   Backdoor: $6, 8, 10 \notin Z$ Adj: $6, 8 \notin Z$

2. $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**



Backdoor: $\{2\}$ + anysubset $\{3,4,9\}$; $\{3,4\}$; $\{4,9\}$

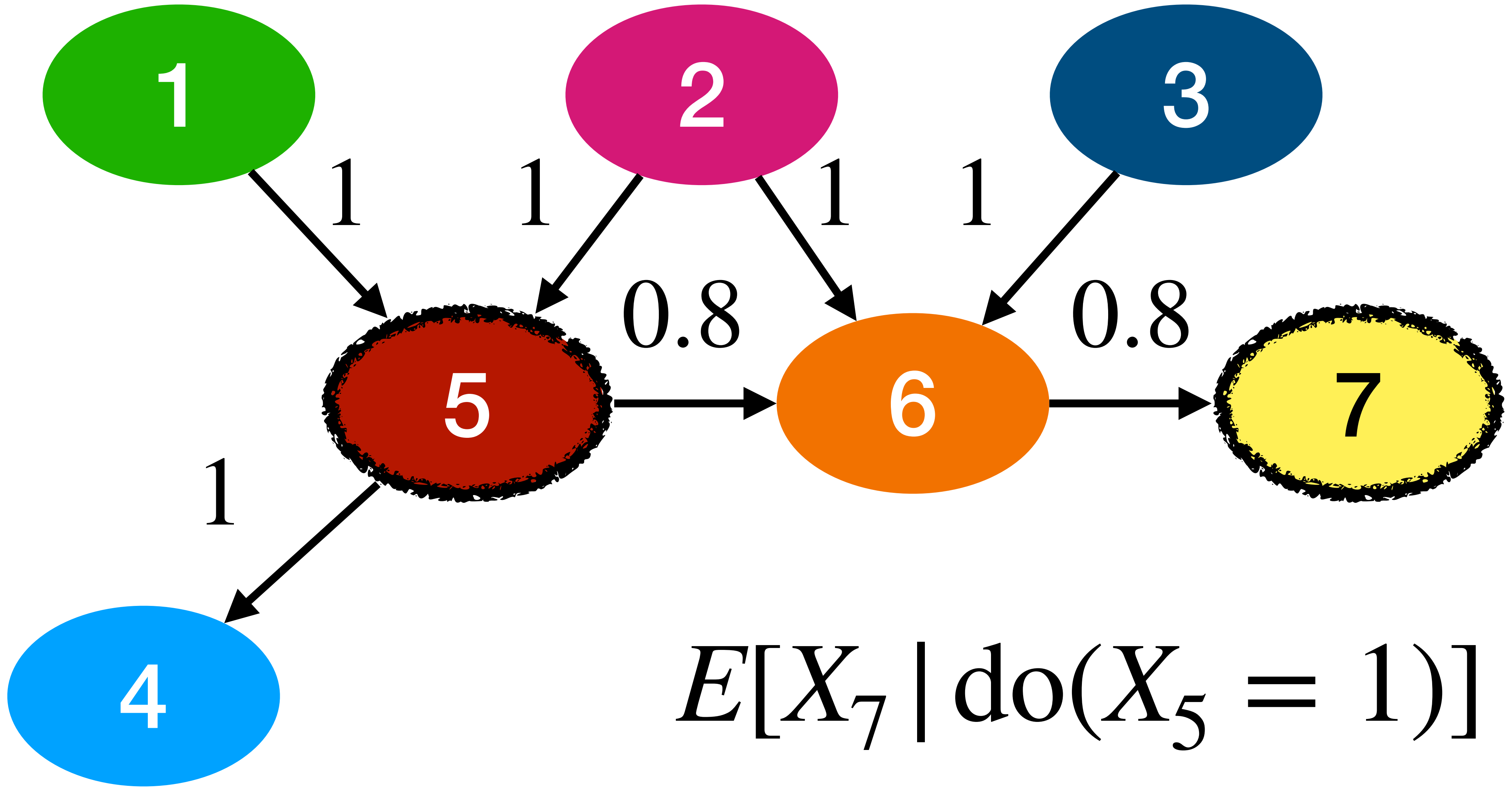
Adj: + $\{10,9,4\}$; $\{10,9,2\}$

$\{10,9,4,3\}$; $\{10,9,2,3\}$

$\{10,9,4,3,2\}$

Optimal adjustment sets in terms of asymptotic variance

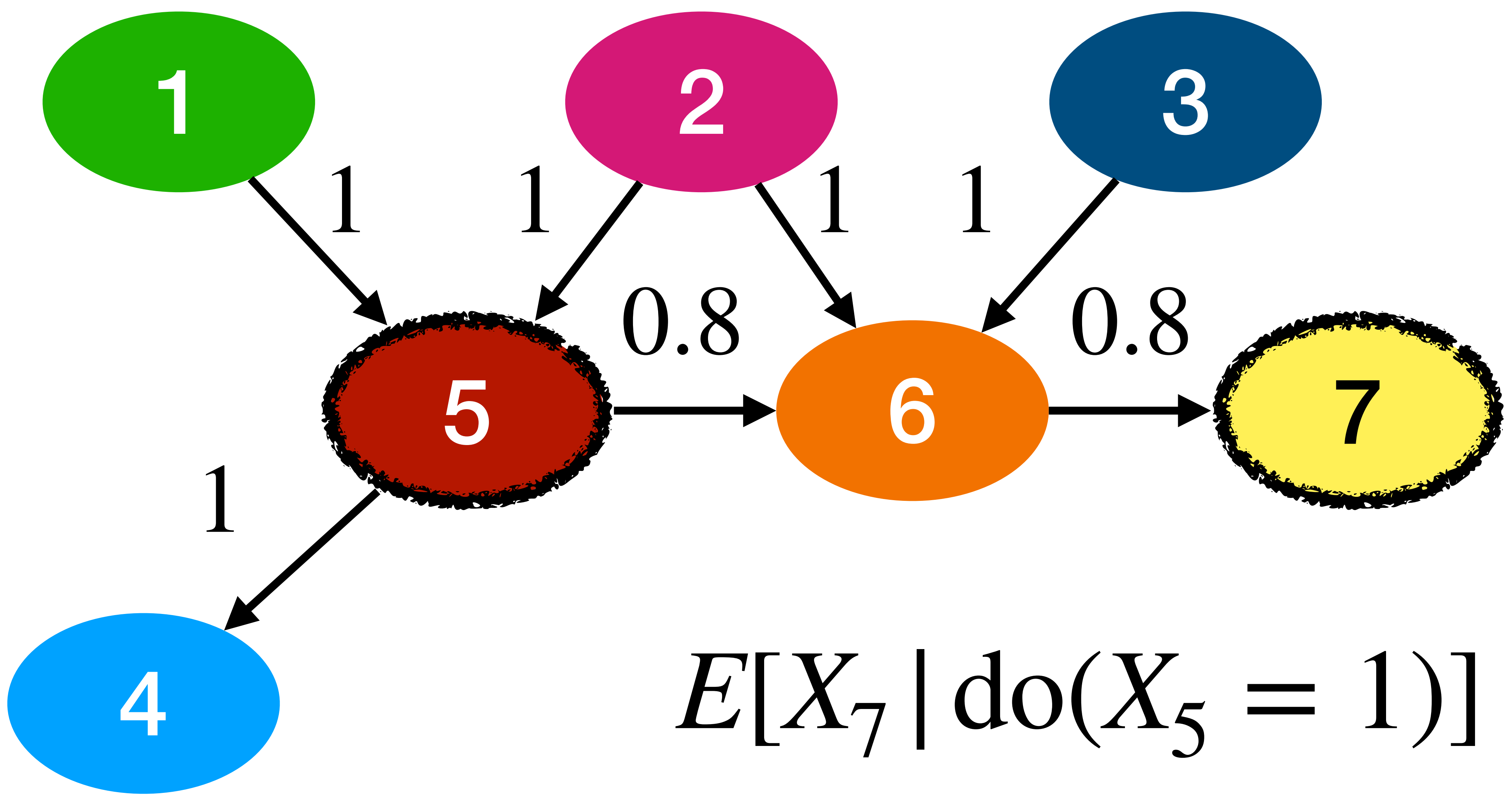
- Usually we have many adjustment sets, but not all have the same asymptotic variance



$$E[X_7 \,|\, do(X_5 = 1)] - E[X_7 \,|\, do(X_5 = 0)]?$$

Optimal adjustment sets in terms of asymptotic variance

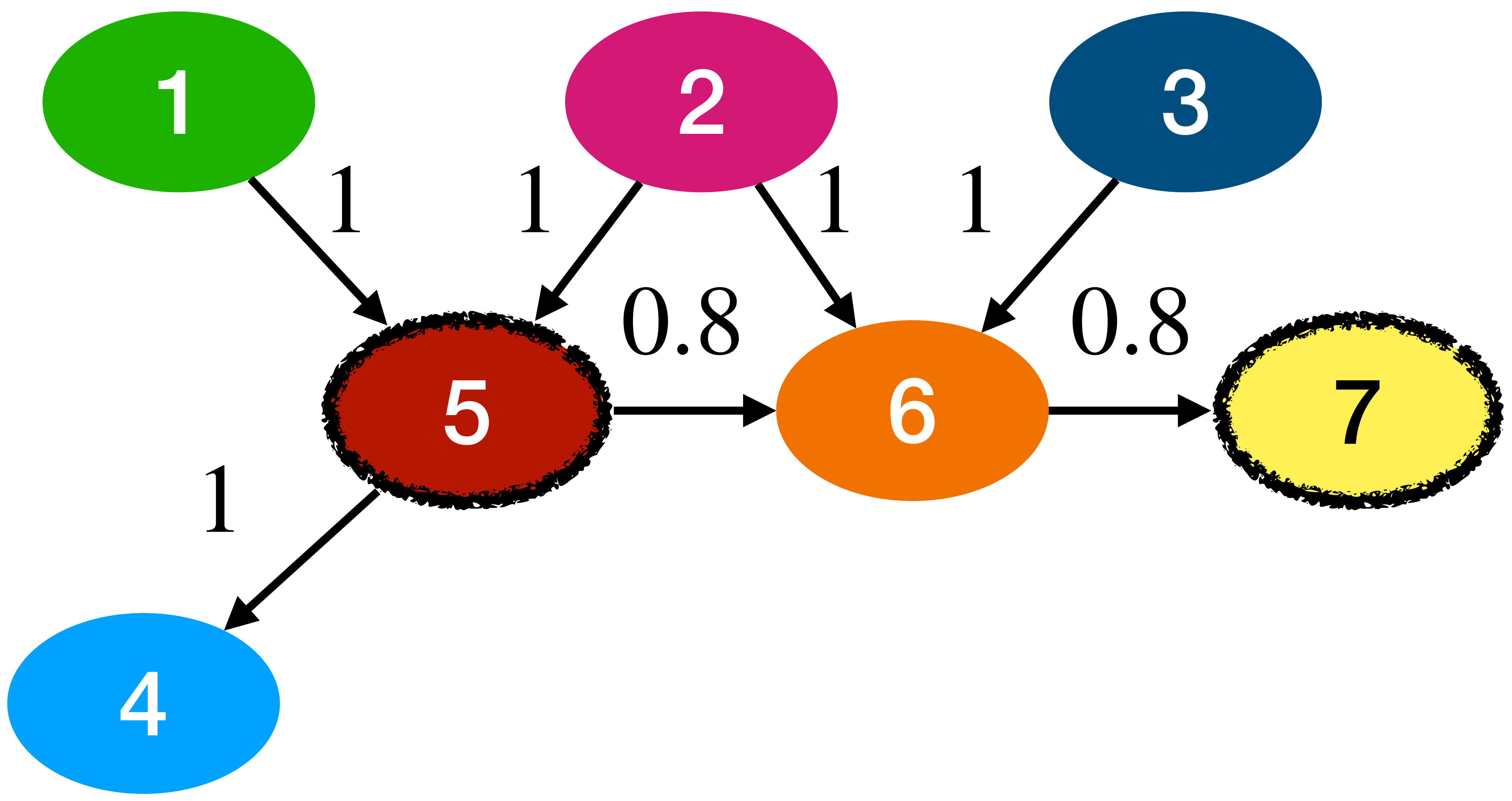
- Usually we have many adjustment sets, but not all have the same asymptotic variance



$$E[X_7 \,|\, do(X_5 = 1)] - E[X_7 \,|\, do(X_5 = 0)] = 0.8^2 = 0.64$$

Optimal adjustment sets in terms of asymptotic variance

- Usually we have many adjustment sets, but not all have the same asymptotic variance

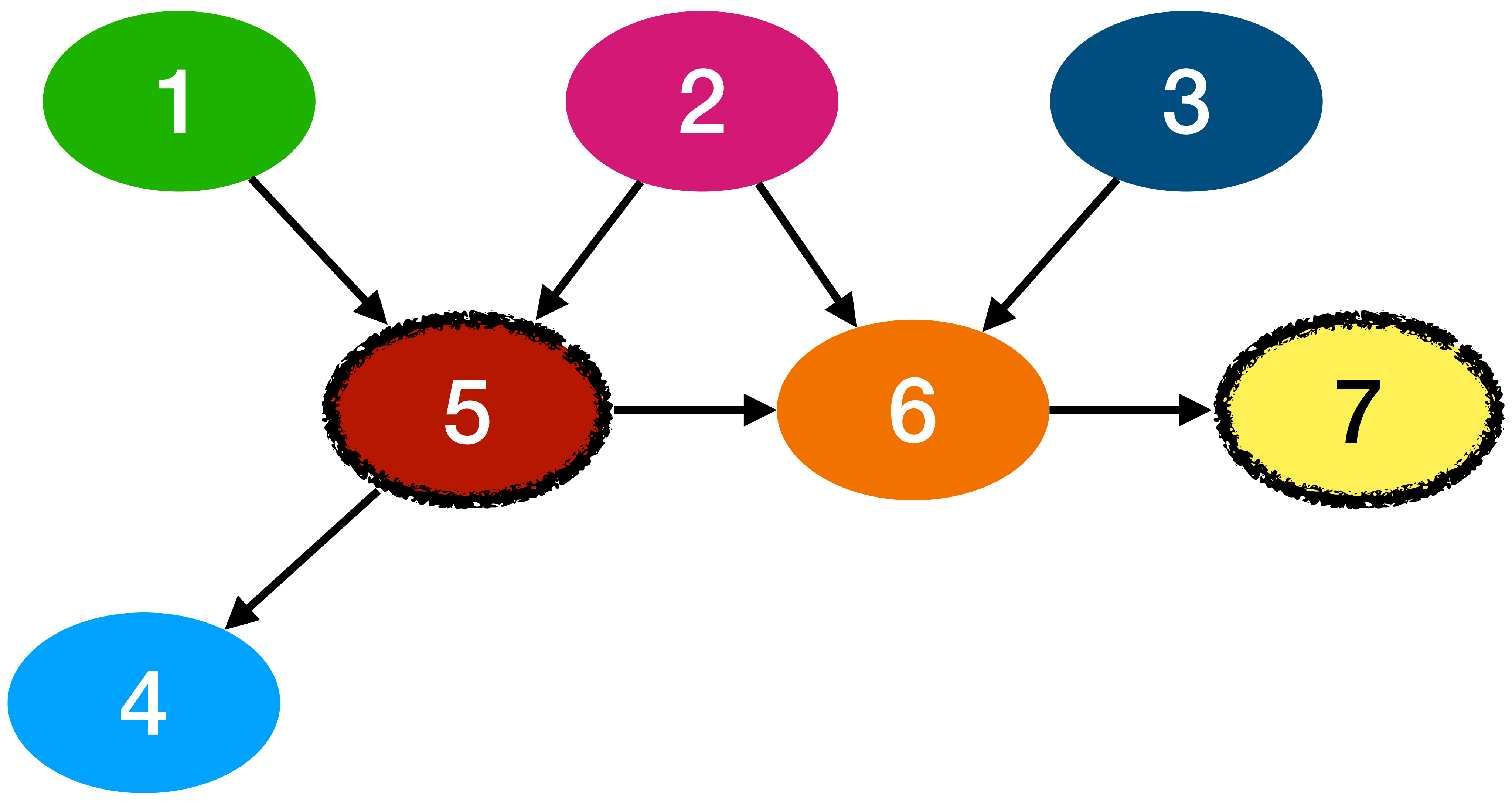


See Jupyter notebook, the variance of the estimator changes across different adjustment sets
We will not discuss this in class

- Recent work with optimality criterion for linear SCMs: https://arxiv.org/abs/1907.02435, see also extension https://arxiv.org/pdf/2002.06825.pdf

Exercise in Canvas - adjustment criterion

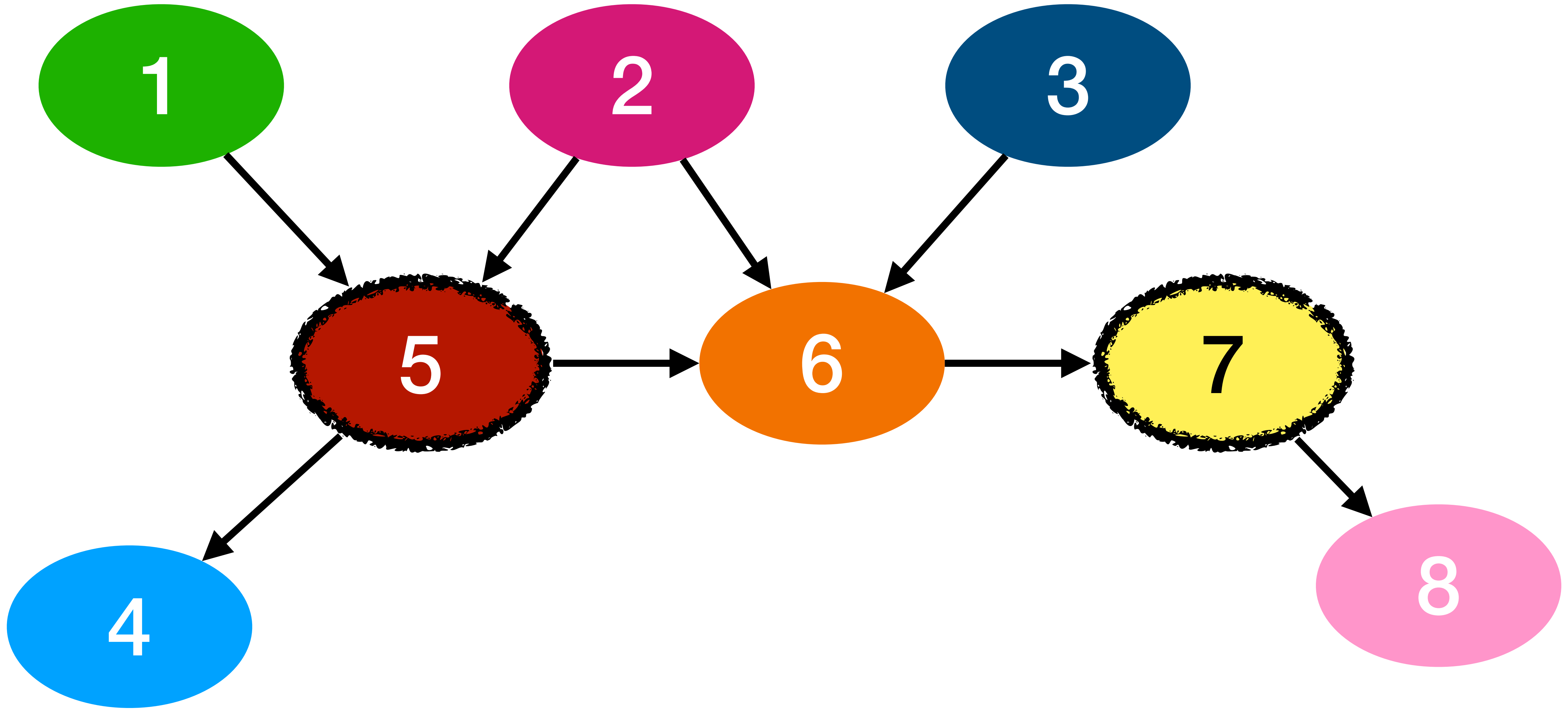
- $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j**
- $\mathbf{Z}$ blocks all paths from i to j that are **not directed paths from i to j**



$$P(X_7 \mid \mathrm{do}(X_5)) = ?$$

Question: what about descendants of j?

- $\mathbf{Z}$ does not contain any descendant of **nodes $r \neq i$ on a directed path from i to j** **(this includes descendants of r that are not on directed paths between i and j)**



$\{6,8\} \notin Z$