Causal Data Science

Lecture 5:2 Backdoor adjustment

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UvA - Spring 2023

Identification strategies for causal effects

- Given a causal graph G, an **identification strategy** is a formula to estimate an interventional distribution from a combination of observational ones
- **Backdoor criterion (this class), Adjustment criterion (next class)**

$$p(x_j \mid \mathrm{do}(x_i)) = \int_{x_{\mathbf{Z}}} p(x_j \mid x_i, x_{\mathbf{Z}}) p(x_{\mathbf{Z}}) dx_{\mathbf{Z}}$$

- **Frontdoor criterion (next class)**

$$p(x_j \mid \mathrm{do}(x_i')) = \int_{x_{\mathbf{M}}} p(x_{\mathbf{M}} \mid x_i') \int_{x_i} p(x_j \mid x_{\mathbf{M}}, x_i') p(x_i) dx_i$$

- **Instrumental variables (next class)**

Valid adjustment sets

- Given a causal Bayesian network (G, p) with DAG $G = (\mathbf{V}, \mathbf{E})$

- We call **(valid) adjustment sets** for the total causal effect of the **treatment** X_i on the **outcome** X_j with $i \neq j$, the sets $\mathbf{Z} \subseteq \mathbf{V} \backslash \{i, j\}$ such that:

$$p(x_j | \mathrm{do}(x_i)) = \int_{x_\mathbf{Z}} p(x_j | x_i, x_\mathbf{Z}) p(x_\mathbf{Z}) dx_\mathbf{Z}$$

$$P(X_j | \mathrm{do}(X_i = x_i)) = \sum_{x_\mathbf{Z}} P(X_j | X_i = x_i, X_\mathbf{Z} = x_\mathbf{Z}) P(X_\mathbf{Z} = x_\mathbf{Z})$$

(ADJUSTMENT FORMULA)

Additional assumptions for adjustment: Positivity

- Adjustment formula for estimating the causal effect of X_i on X_j with adjustment sets $\mathbf{Z} \subseteq \mathbf{V} \setminus \{i, j\}$:

$$p(x_j \mid \mathrm{do}(x_i)) = \int_{x_\mathbf{Z}} p(x_j \mid x_i, x_\mathbf{Z}) p(x_\mathbf{Z}) dx_\mathbf{Z}$$

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- Values in interventional regime **should be also possible** in the observational regime, specifically: $p(x_i, x_{\mathbf{Z}}) > 0 \text{ for } x_i \in \mathcal{X}_i, x_{\mathbf{Z}} \in \mathcal{X}_{\mathbf{Z}}$

- If all women are treated and men are untreated, we cannot adjust for gender

Valid adjustment sets

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We will see two graphical criteria for identifying these sets directly from the graph: backdoor and adjustment criterion.

Sometimes less is more

- Adjusting for all possible covariates doesn’t always help:

$lm(X_4 \sim X_1 + X_2)$

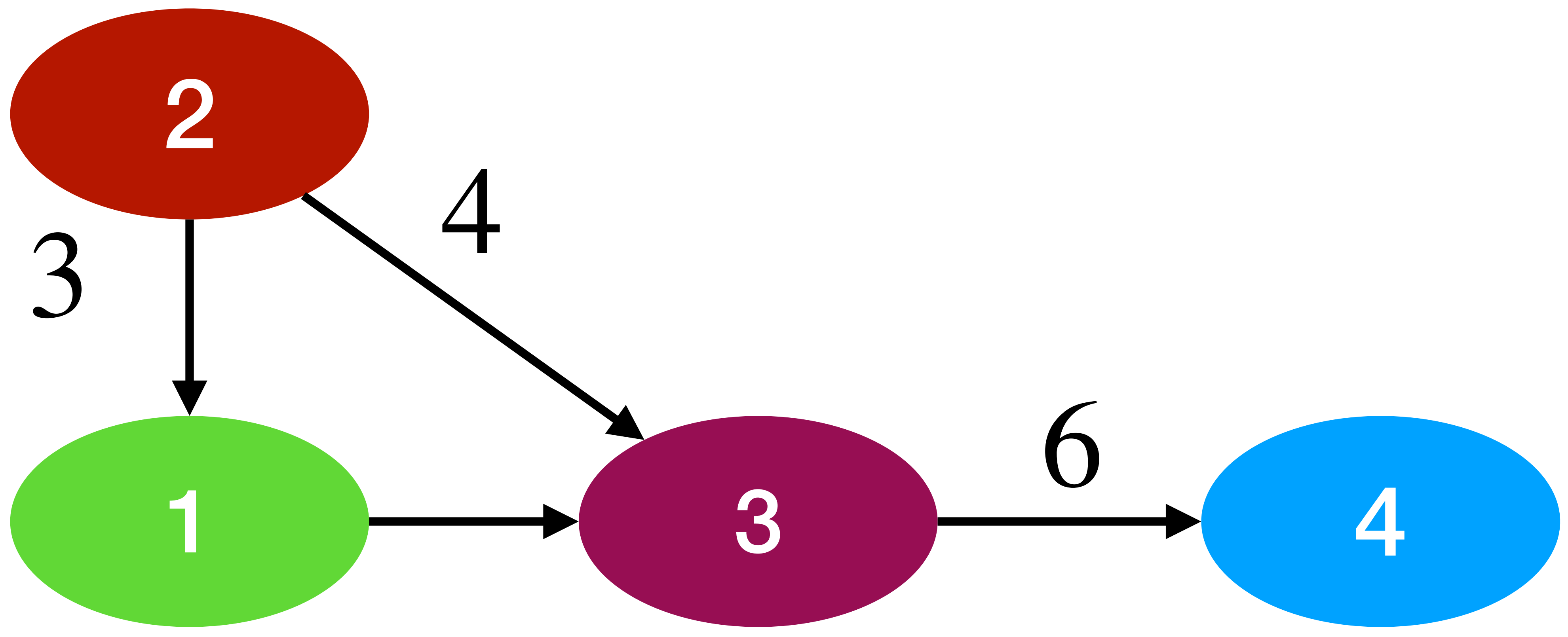
```
linear_regressorX12 = LinearRegression()
linear_regressorX12.fit(X21, Y)
linear_regressorX12.coef_[:,1]
```

```
array([29.87150906])
```

$lm(X_4 \sim X_1 + X_2 + X_3)$

```
linear_regressorX123 = LinearRegression()
linear_regressorX123.fit(X, Y)
linear_regressorX123.coef_[:,1]
```

```
array([0.15806091])
```



$$E[X_4 \mid do(X_1 = 1)] - E[X_4 \mid do(X_1 = 0)] = 30$$

Special case: no causal parents for treatment

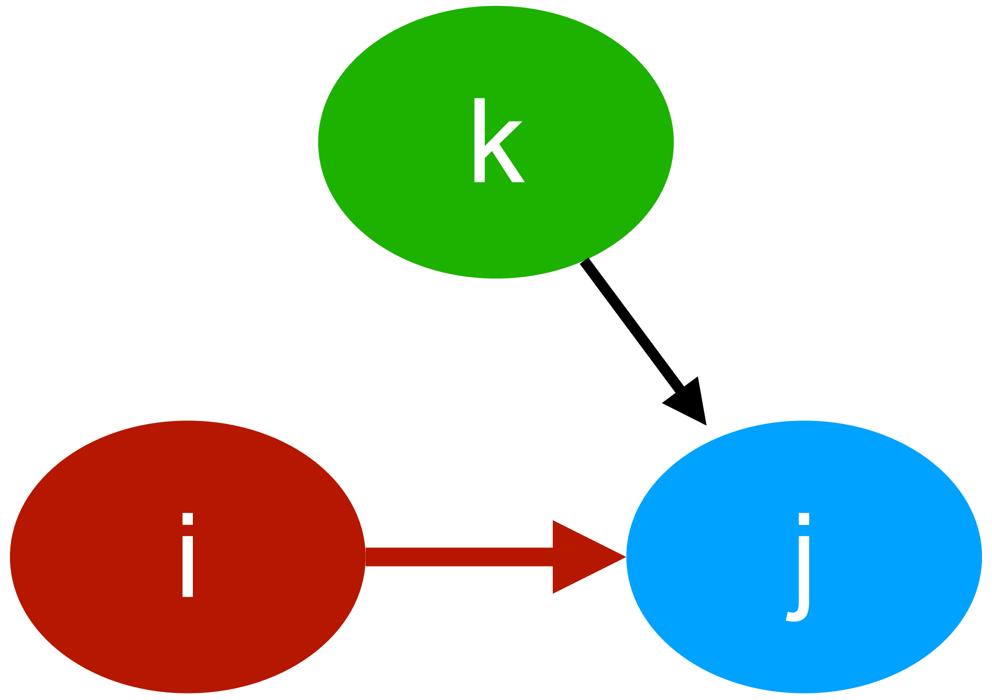
- If treatment X_i has no causal parents, then the interventional distribution is the same as the observational distribution conditioned on $X_i = x_i$:

$$P(X_{\bar V} \mid do(X_i = x_i)) = \prod_{j \in \bar V \setminus \{i\}} P(x_j \mid X_{pa(j)}) \cdot P(x_i \mid \phi)$$

$$= \quad \text{— || —} \quad \mathbb{1}(x_i = x_i)$$

$$= \prod_{j \in V} p(x_j \mid X_{pa(j)}) \Big|_{X_i = x_i}$$

eval. in $X_i = x_i$

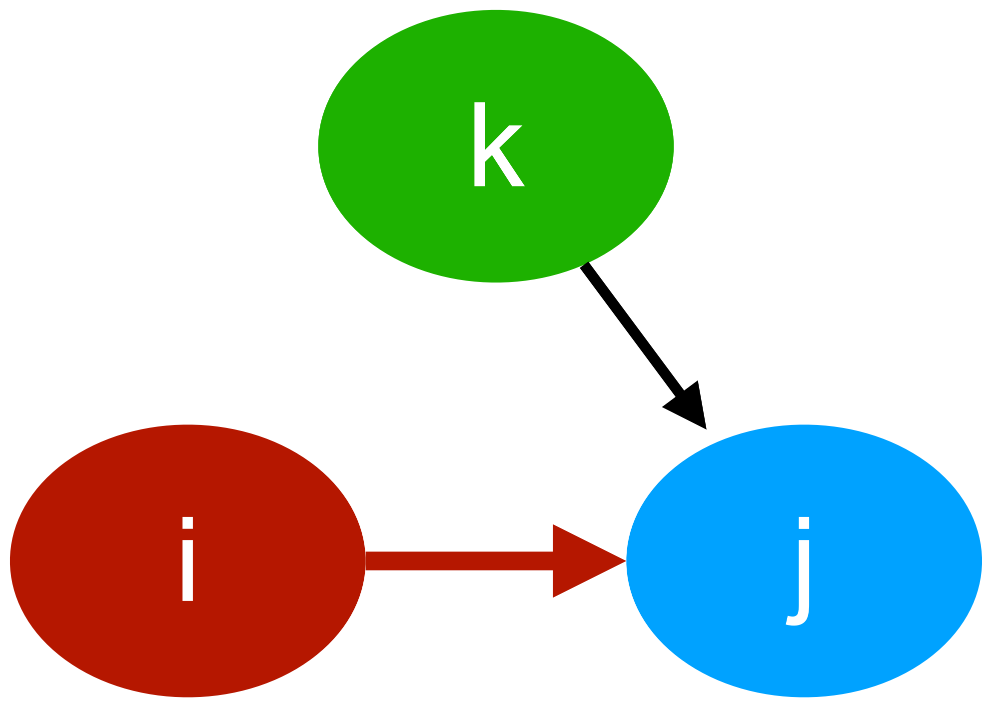


Special case: no causal parents for treatment

- If treatment X_i has no causal parents, then the interventional distribution is the same as the observational distribution conditioned on $X_i = x_i$:

$$P(X_V \mid do(X_i = x_i)) \quad \prod_{j \in V} p(X_j \mid X_{Pa(j)}) \Big|_{X_i = x_i} = \frac{P(X_V)}{P(X_i \mid X_{Pa(i)})} \Big|_{X_i = x_i}$$

$$= P(X_V \mid X_i = x_i)$$



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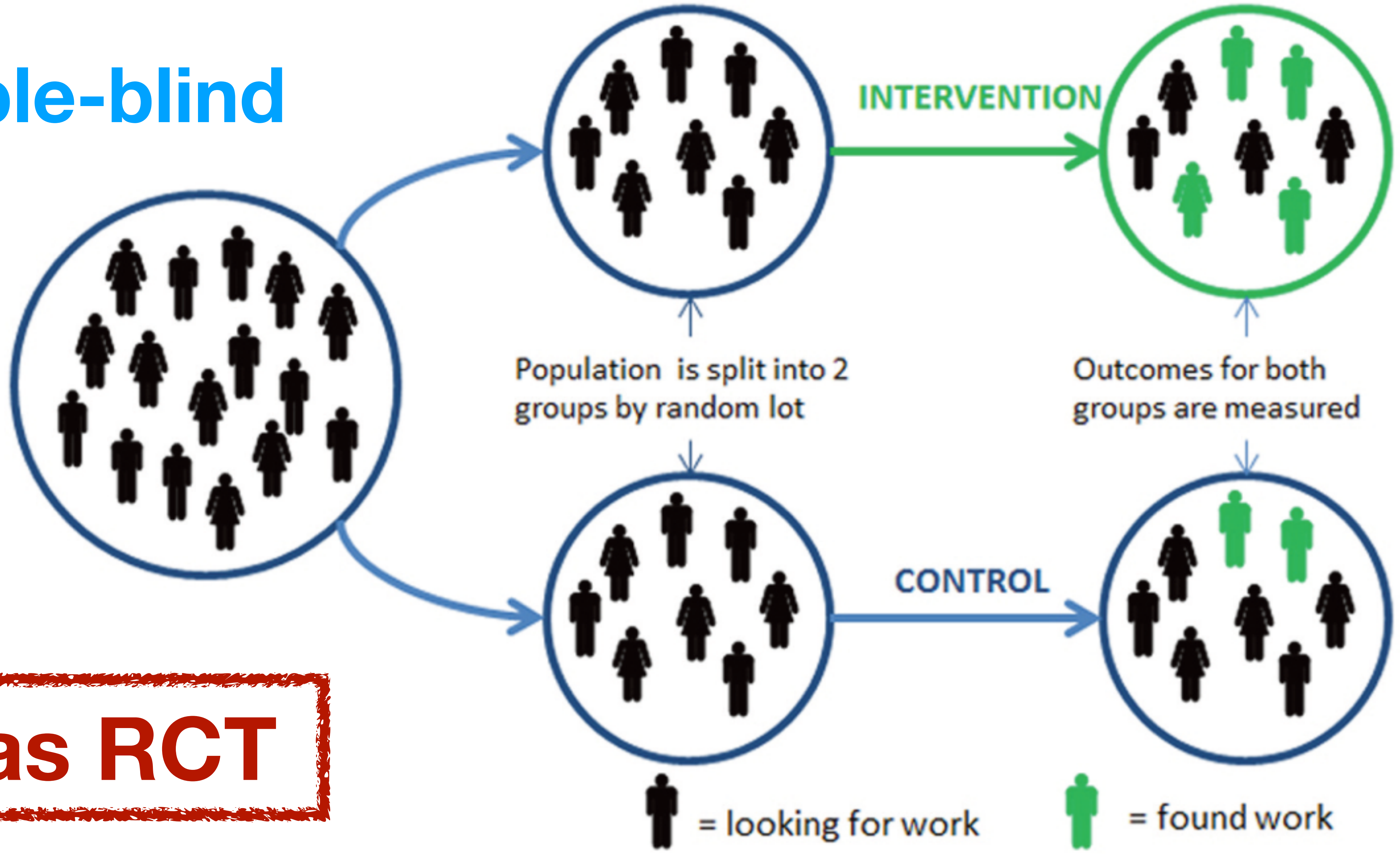
Double-blind



Treatment group

Control group

Same setting as RCT

https://www.researchgate.net/publication/256031307_Test_Learn_Adapt_Developing_Public_Policy_with_Randomised_Controlled_Trials

Special case: using the parents of the treatment

- **The set of causal parents** of the treatment allow us to estimate the interventional distribution:

$$p(X_{\mathbf{V}} \mid \mathrm{do}(X_i = x_i)) = \prod_{j \in \mathbf{V} \setminus \{i\}} p(X_j \mid X_{\mathrm{Pa}(j)}) \mid_{X_i = x_i}$$

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$$= \frac{p(X_{\mathbf{V}})}{p(X_i \mid X_{\mathrm{Pa}(i)})} \Bigg|_{X_i = x_i}$$

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$$= \frac{P(X_{\bar{V}})}{P(X_i \mid X_{Pa(i)})}\Bigg|_{X_i = x_i} = \frac{P(X_{\bar{V}}) \cdot P(X_{Pa(i)})}{P(X_i, X_{Pa(i)})}\Bigg|_{X_i = x_i}$$

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$$= P(X_{\bar{V} \setminus \{i, Pa(i)\}} \mid X_i, X_{Pa(i)}) \cdot P(X_{Pa(i)})\Big|_{X_i = x_i}$$

Special case: using the parents of the treatment

- **The set of causal parents** of the treatment allow us to estimate the interventional distribution:

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$$= P(X_{V \setminus \{i, pa(i)\}} \mid X_i, X_{pa(i)}) \cdot P(X_{pa(i)}) \Big|_{X_i = x_i}$$

$$= p(X_{\mathbf{V}\setminus\{i,\mathrm{Pa}(i)\}} \mid X_i = x_i, X_{\mathrm{Pa}(i)}) p(X_{\mathrm{Pa}(i)})$$

Special case: using the parents of the treatment

- **The set of causal parents** of the treatment allow us to estimate the interventional distribution:

$$p(X_{\mathbf{V}} \,|\, \mathrm{do}(X_i = x_i)) = p(X_{\mathbf{V}\setminus\{i,\mathrm{Pa}(i)\}} \,|\, X_i = x_i, X_{\mathrm{Pa}(i)})p(X_{\mathrm{Pa}(i)})$$

- We can marginalise out the other variables which are not X_i, X_j or $X_{\mathrm{Pa}(i)}$

$$p(X_j \,|\, \mathrm{do}(X_i = x_i)) = \int_{x_{\mathrm{Pa(i)}}} p(X_j \,|\, X_i = x_i, X_{\mathrm{Pa}(i)})p(X_{\mathrm{Pa}(i)})dx_{\mathrm{Pa(i)}}$$

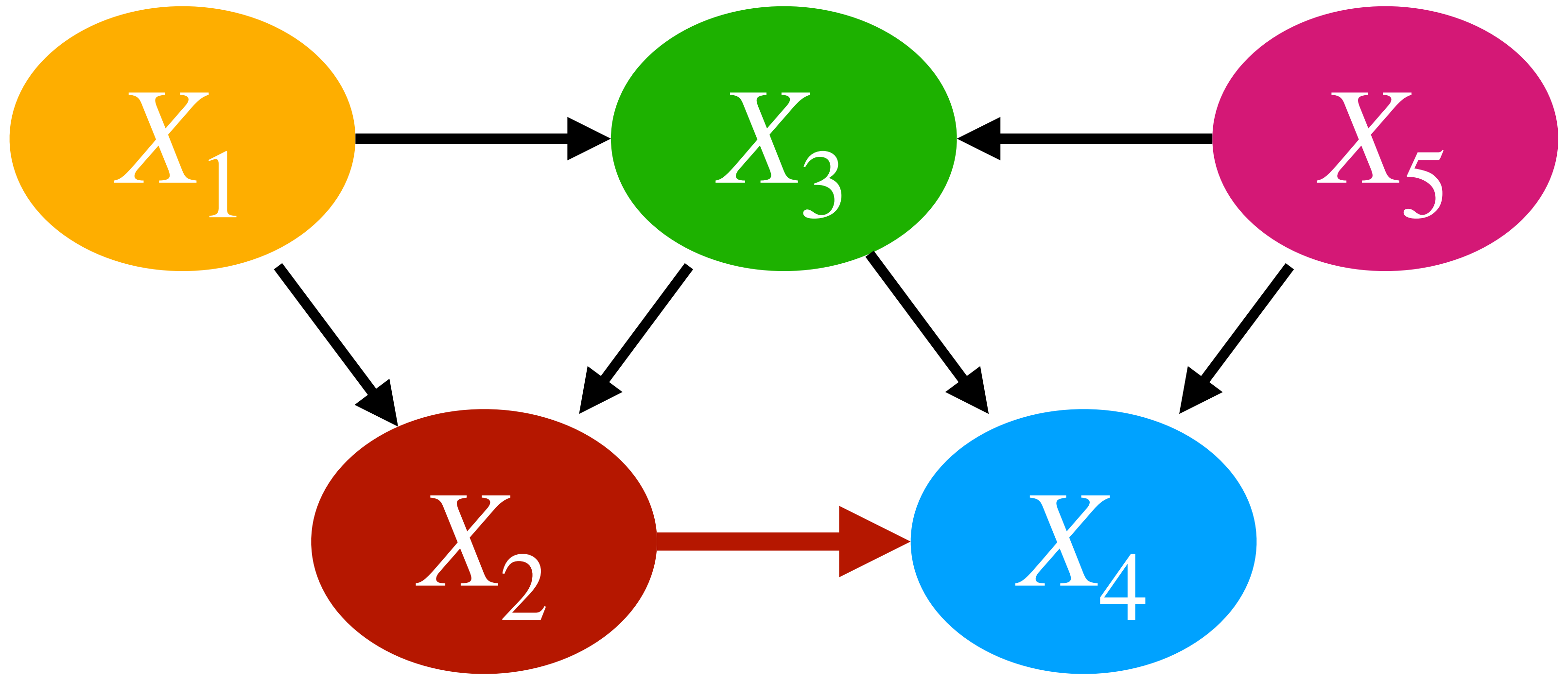
$$p(x_j \,|\, \mathrm{do}(x_i)) = \int_{x_{\mathbf{Z}}} p(x_j \,|\, x_i, x_{\mathbf{Z}})p(x_{\mathbf{Z}})dx_{\mathbf{Z}}$$

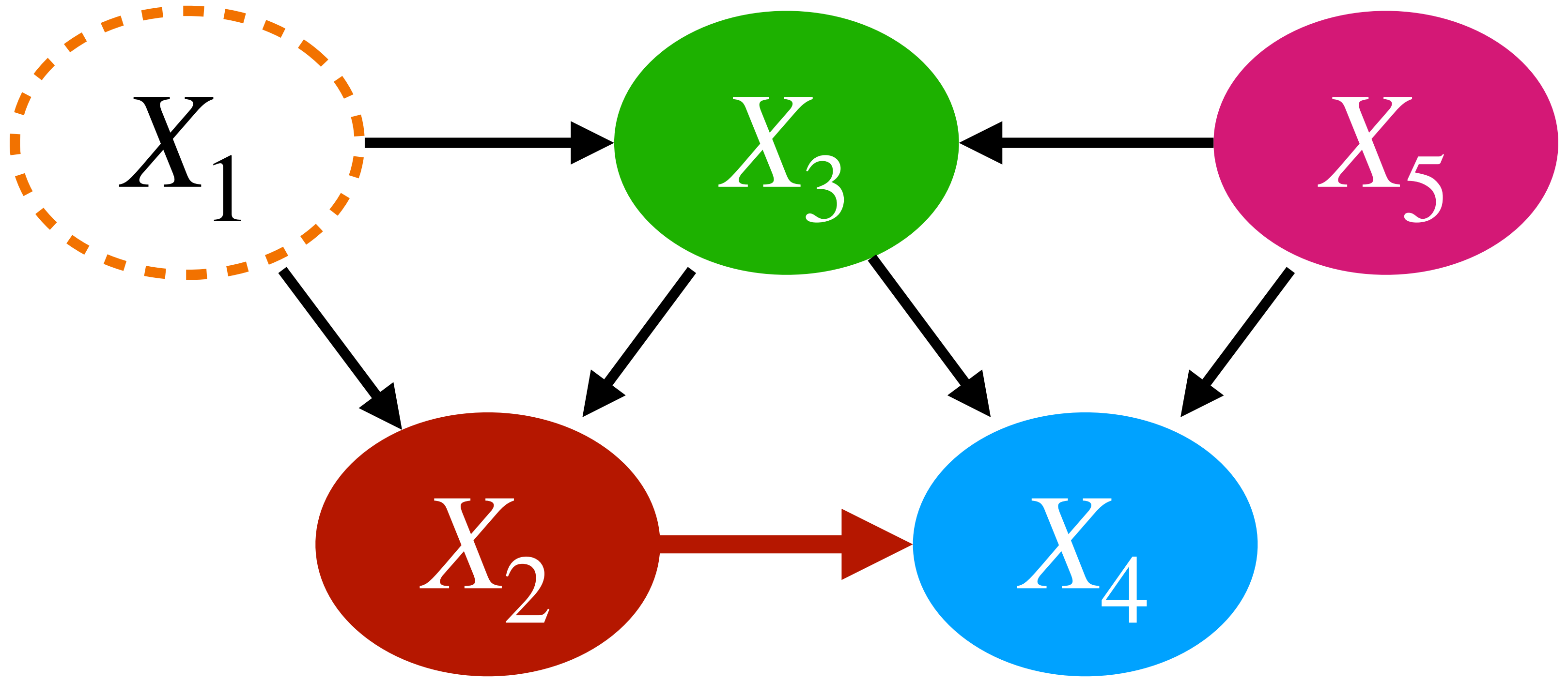
The parents of the treatment are a valid adjustment set

Special case: using the parents of the treatment

- The causal parents of the treatment are a valid adjustment set
- Sometimes some of the parents might not be observed
- We want to list **all valid adjustment sets**

$P(X_4 \mid do(X_2)) = ?$

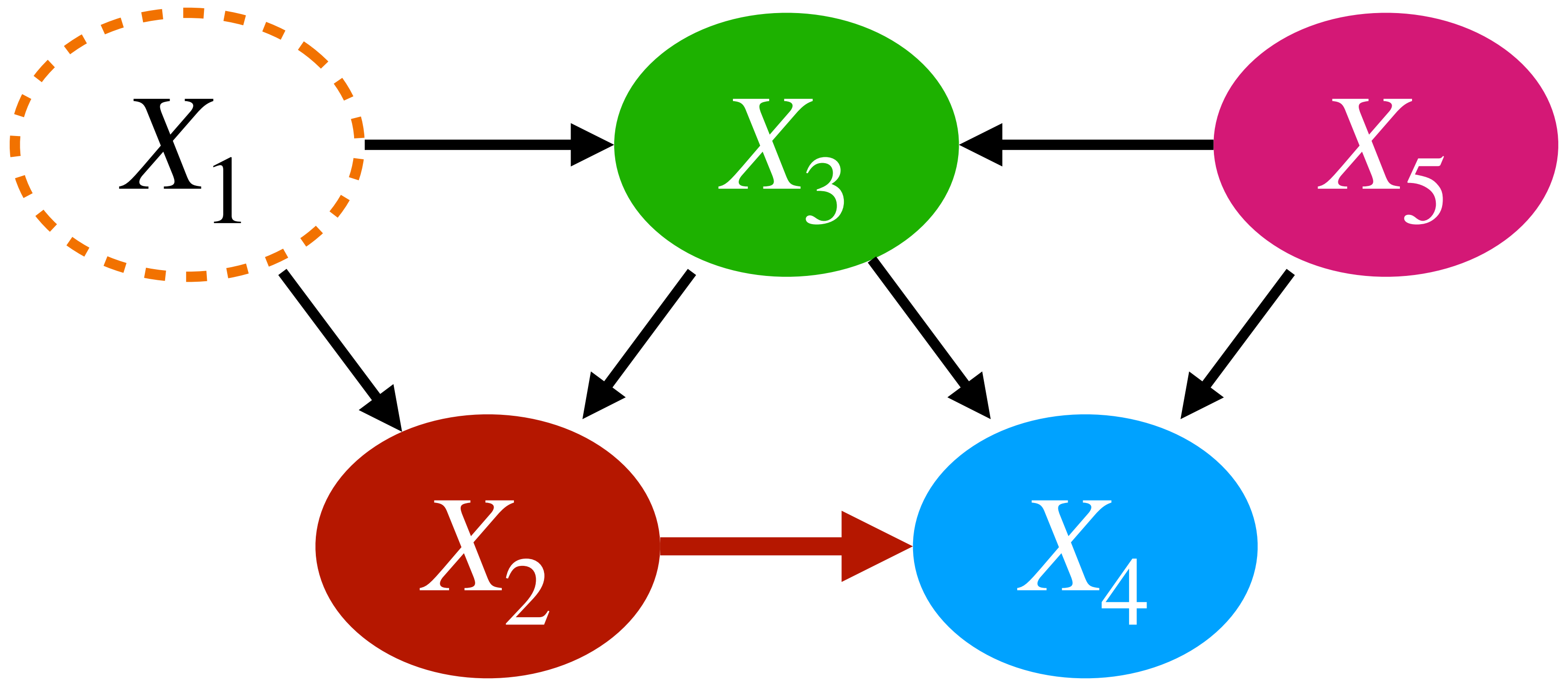




Identifiability of an interventional distribution

- We say an interventional distribution $p(X_i \mid \mathrm{do}(X_j = x_j))$ is **identifiable**, if we can rewrite it using observational distributions
  - For example if a **valid adjustment set** exists
  - But also, front door criterion, instrumental variables

$p(X_4 \mid do(X_2)) = ?$



Backdoor criterion intuition

- Directed path from i to j are **causal (both directly and indirectly)**
- **Backdoor paths** (all paths that start with an arrow into $i \leftarrow \ldots\ j$) induce **spurious associations**
- If we block the spurious associations, we only get the causal effect

Backdoor criterion intuition

- Directed path from i to j are **causal (both directly and indirectly)**
- **Backdoor paths** (all paths that start with an arrow into $i \leftarrow \ldots\ j$) induce **spurious associations**
- If we block the spurious associations, we only get the causal effect
- Including **descendants of i that are also ancestors of j** (*mediators*) would **block causal paths**
- Including **descendants of i that are also descendants of j** would create **collider bias**

Backdoor criterion [Pearl 2009]

- Given a CBN (G, p) with $G = (\mathbf{V}, \mathbf{E})$, a set $\mathbf{Z} \subseteq \mathbf{V} \backslash \{i, j\}$ satisfies the **backdoor criterion** for estimating the causal effect of X_i on X_j with $i \neq j$:
  - $\mathbf{Z}$ does **not contain any descendant of i,** $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**

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The backdoor criterion finds **some (not necessarily all)** valid adjustment sets

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The adjustment criterion is an extension of backdoor that finds all of them - next class

The backdoor criterion finds **some (not necessarily all)** valid adjustment sets

Backdoor criterion example:

- ~~$\mathbf{Z}$ does **not contain any descendant of** i, $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**~~
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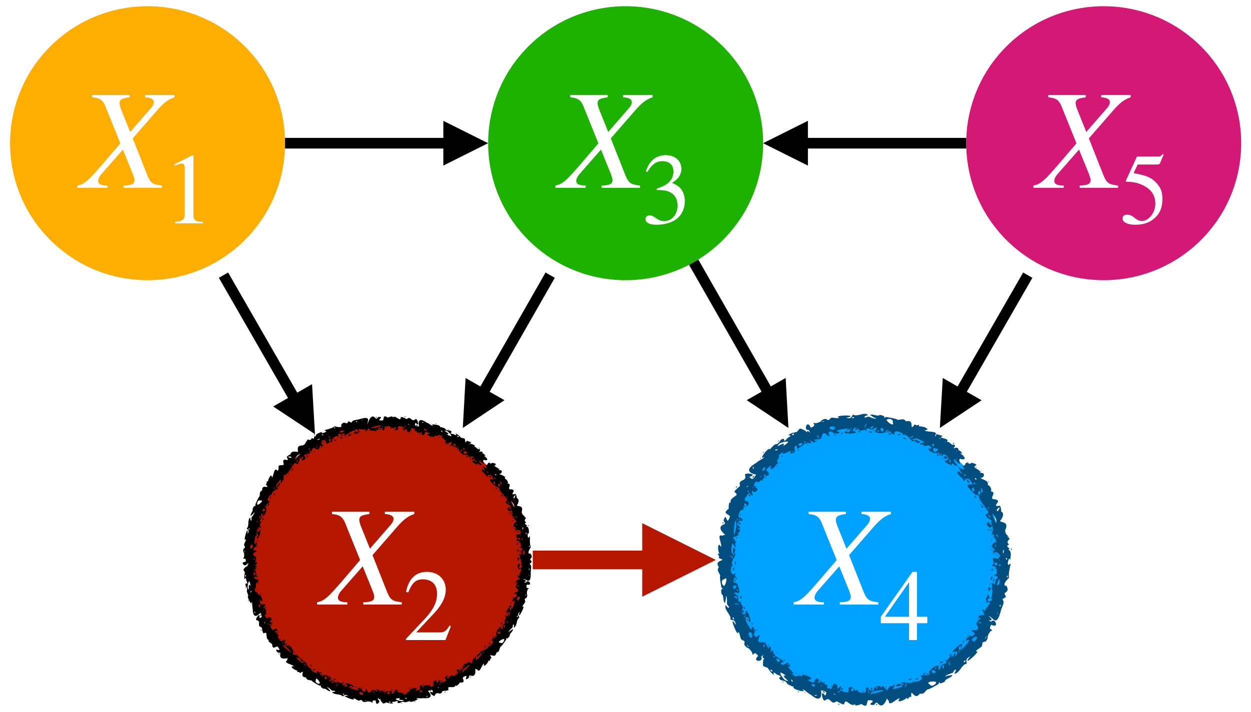

- X_2 doesn't have descendants except X_4

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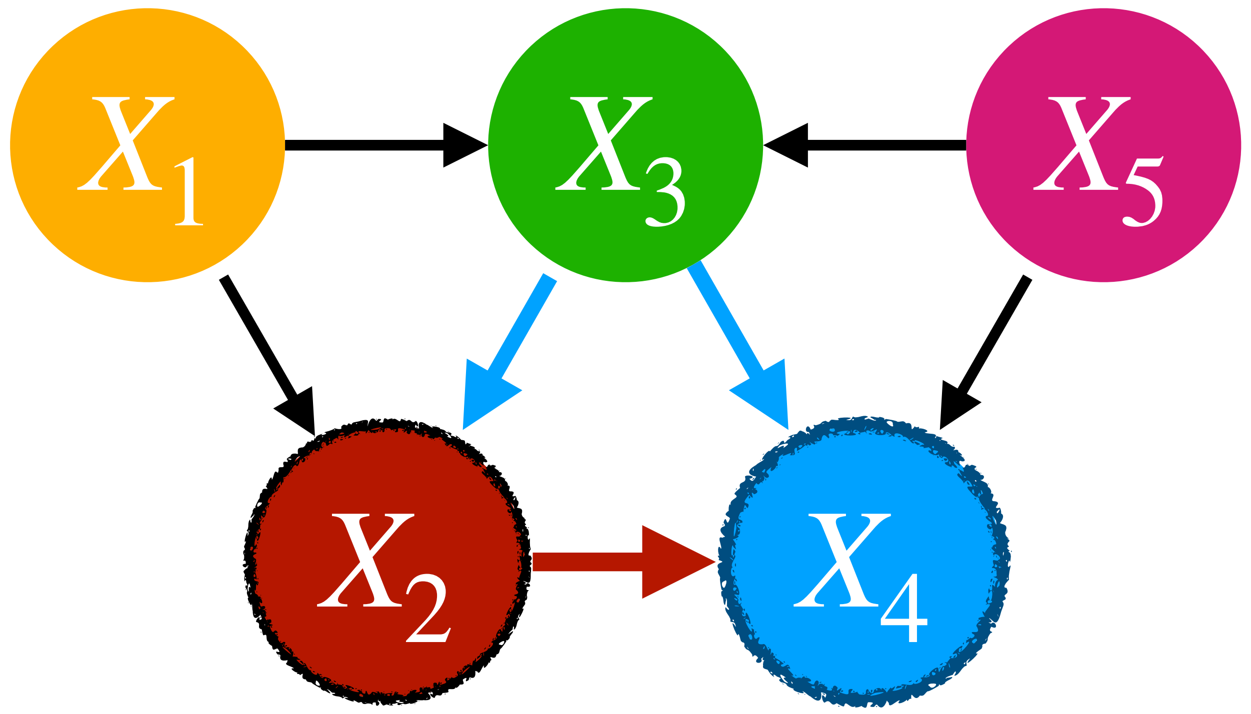

$\mathbf{Z} = \{X_3\}$

- A **path** between **i and j** is **blocked by** $\mathbf{A} \subseteq \mathbf{V} \setminus \{i, j\}$ **if** at least one condition holds:
  - There is a **non-collider** on the path that is in $\mathbf{A}$**, or**
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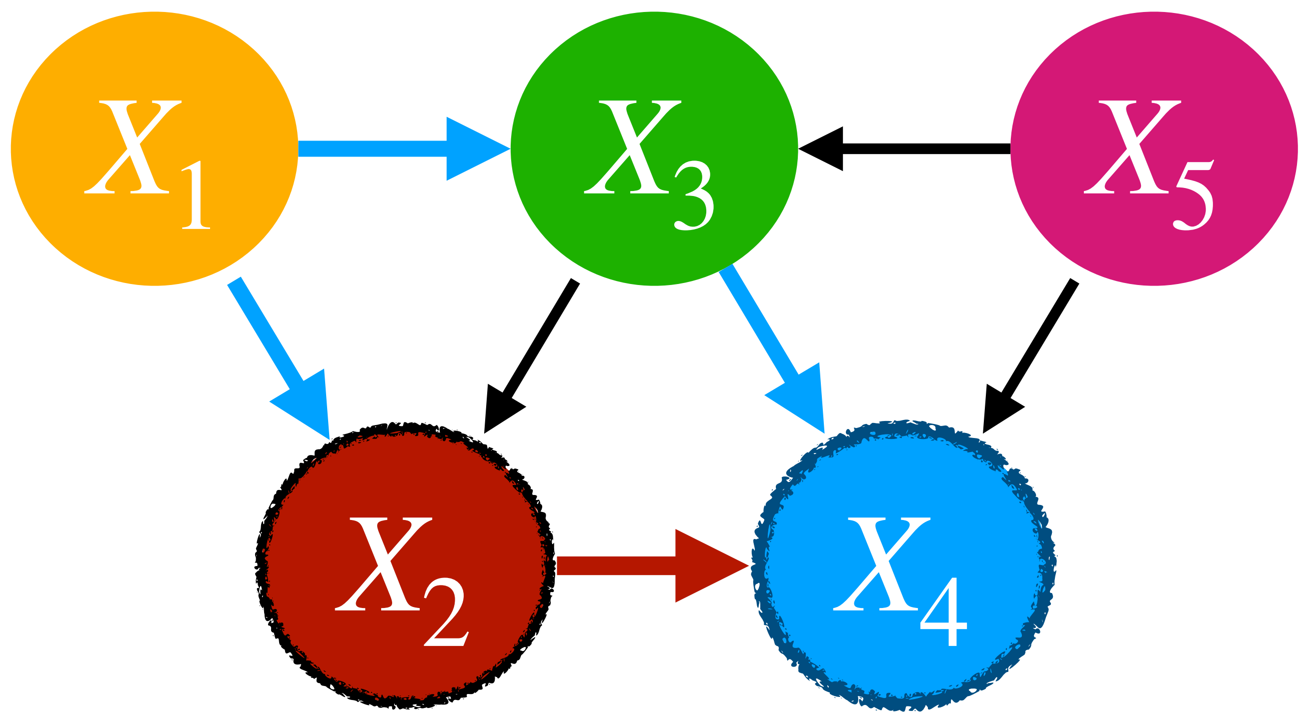

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Backdoor criterion example:

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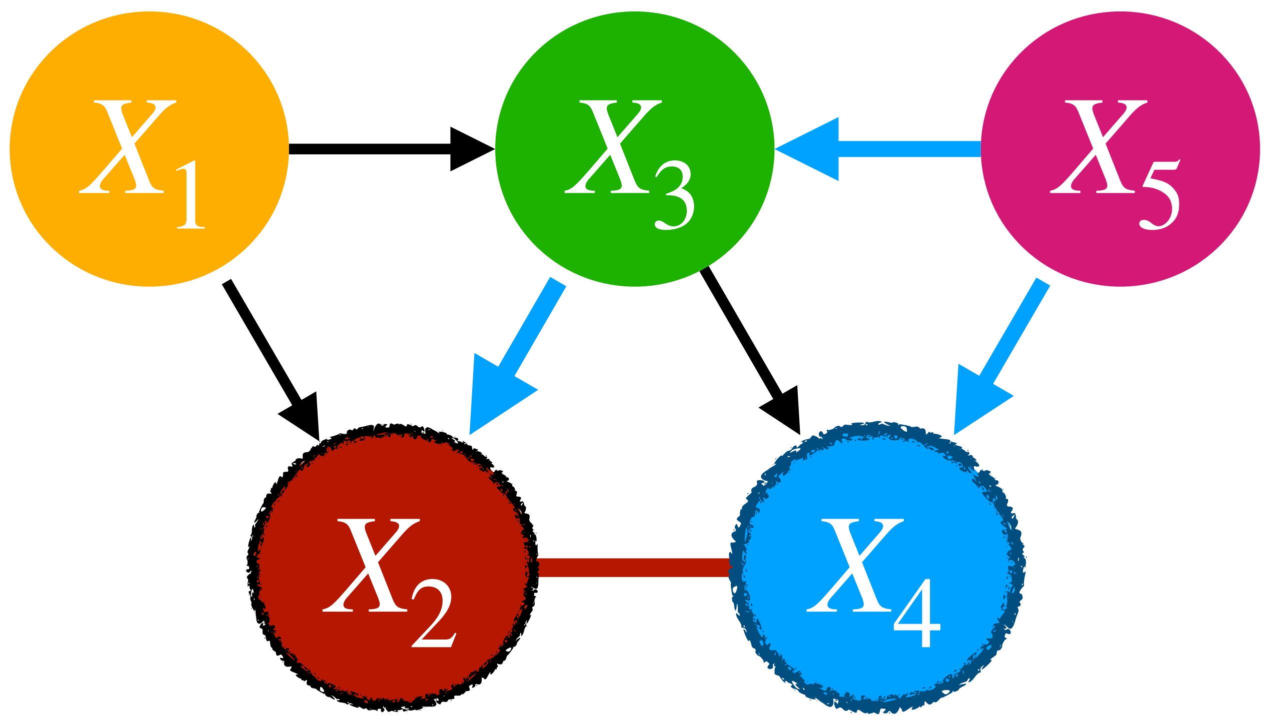

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Backdoor criterion example:

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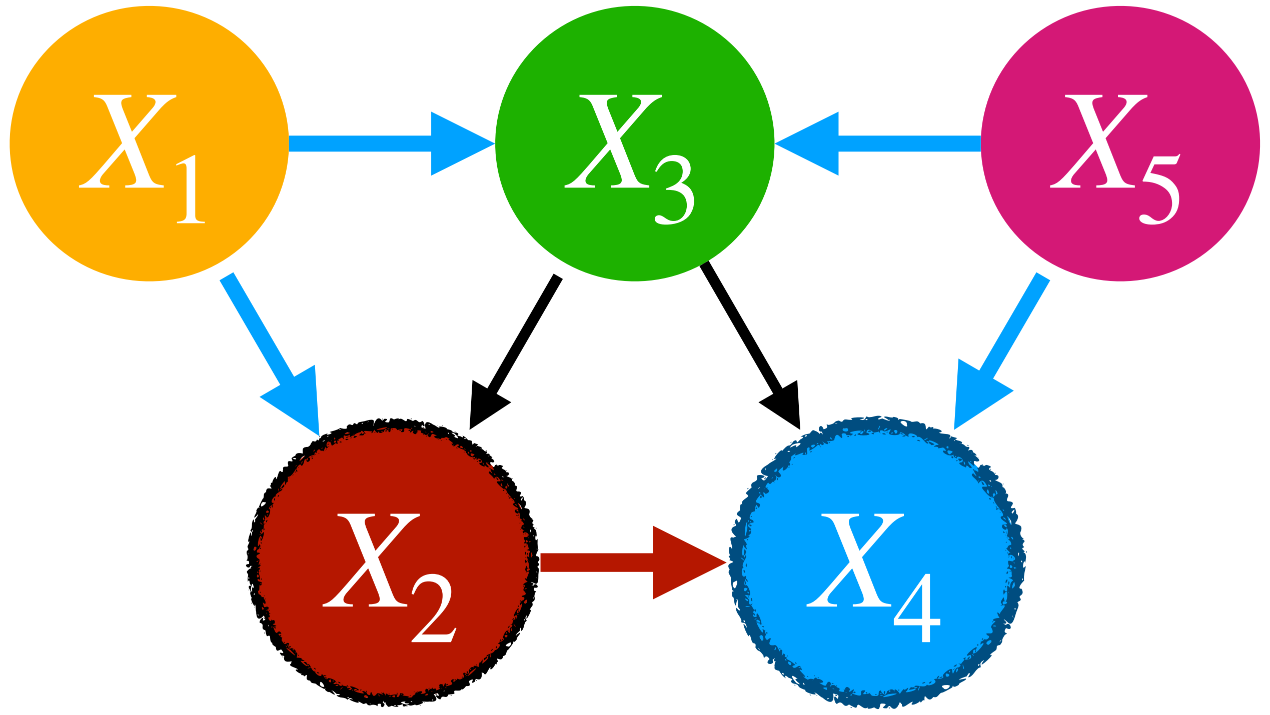

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Backdoor criterion example:

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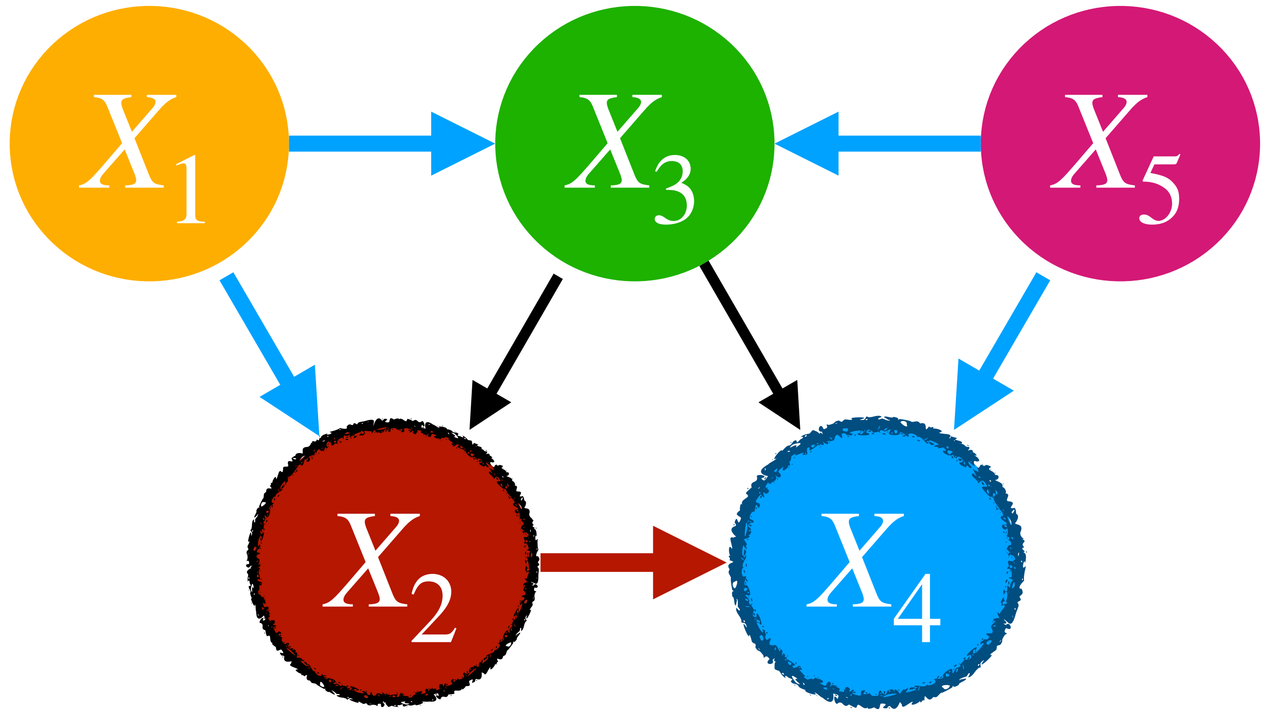

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$\bar{Z} = \{X_3, X_1\}$ OR
$\bar{Z} = \{X_3, X_5\}$

Backdoor criterion example:

- ~~$\mathbf{Z}$ does **not contain any descendant of i,** $\text{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**~~
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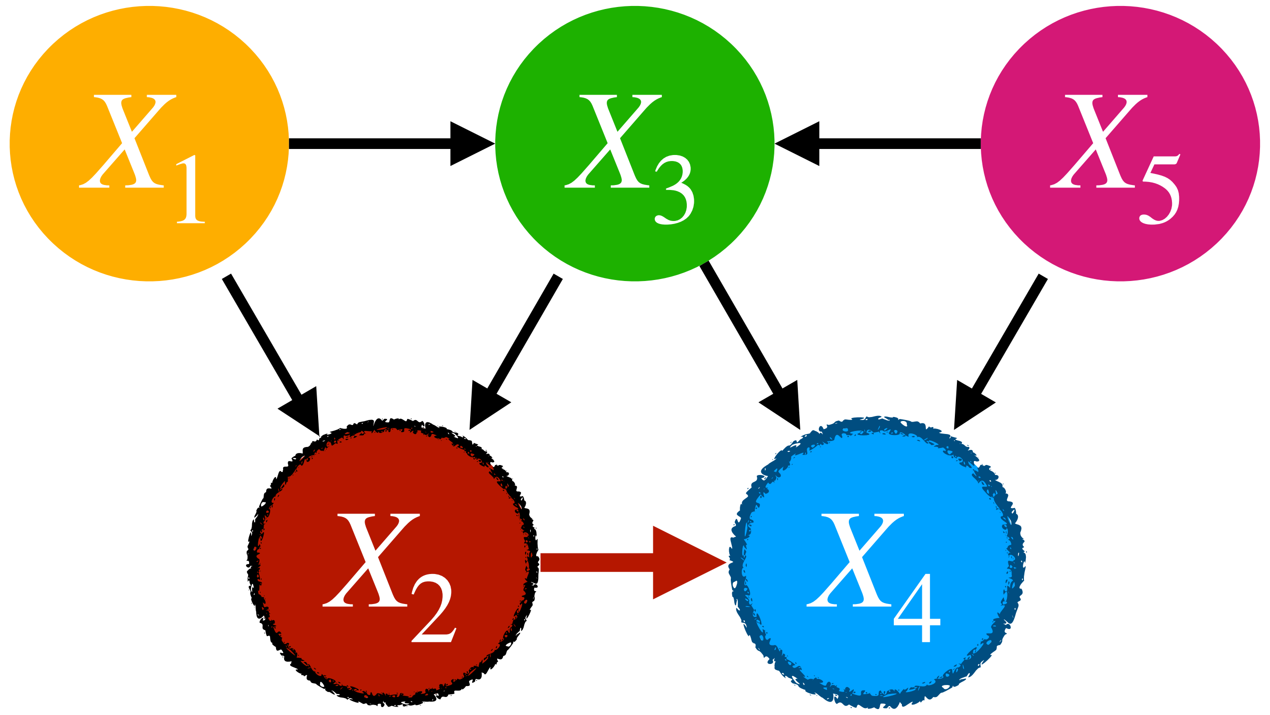

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Is $Z = \{X_1, X_3\}$ an adjustment set that satisfies the backdoor criterion?

Backdoor criterion example:

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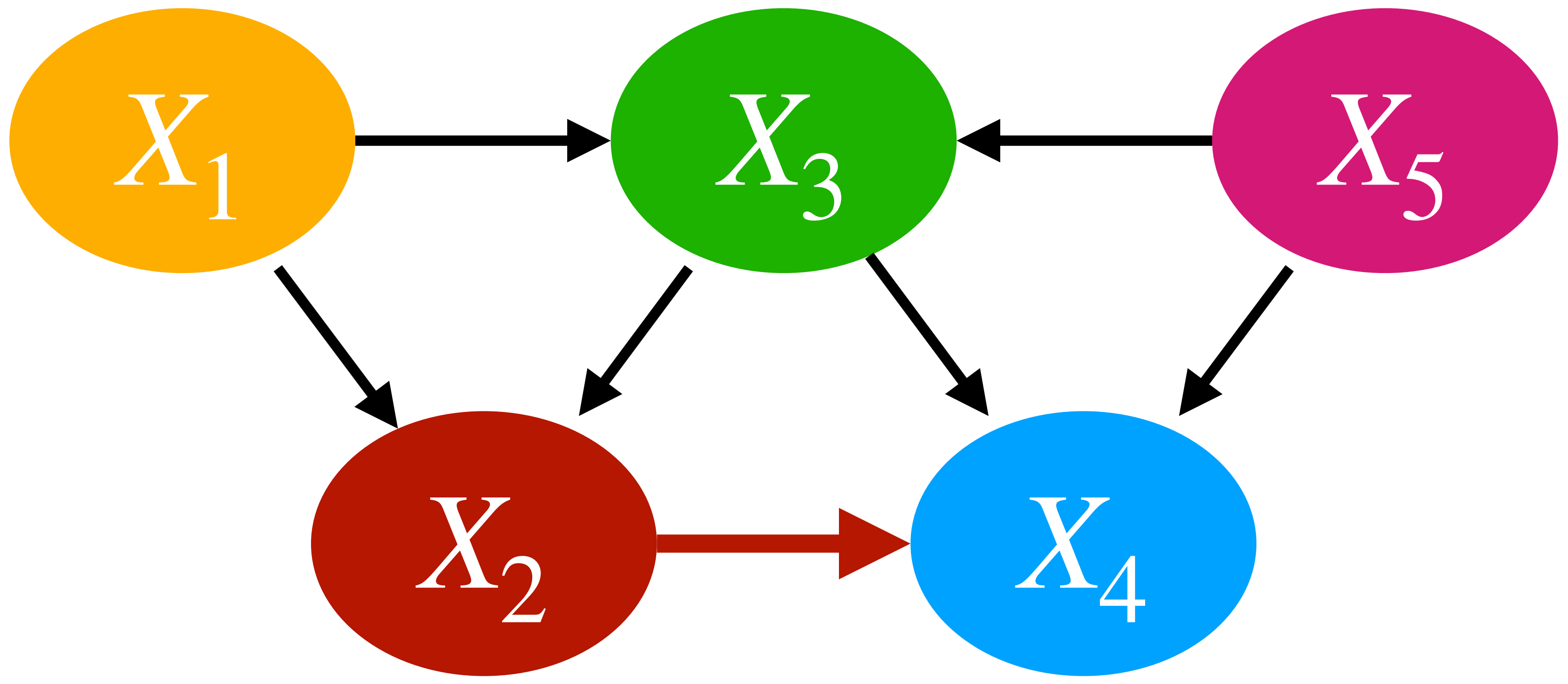


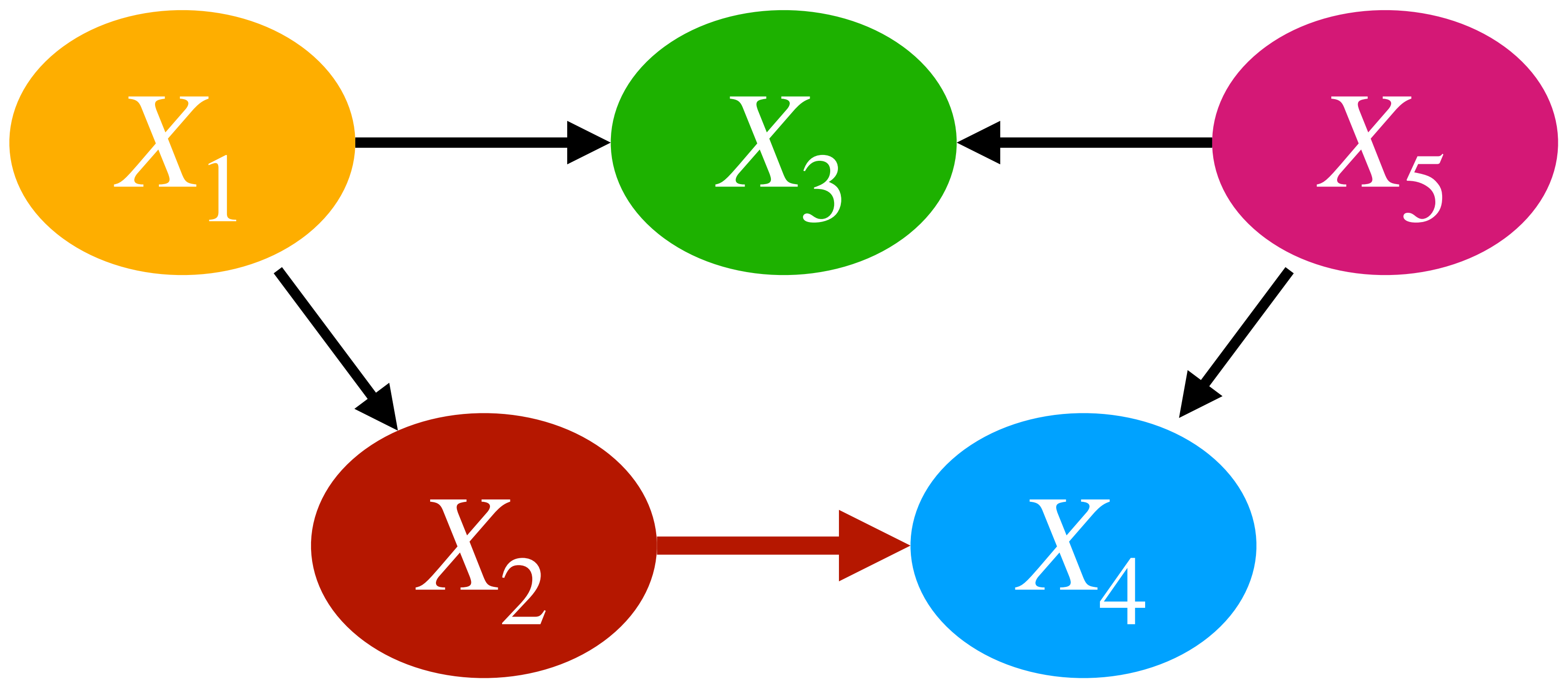
$Z = \{X_3, X_1\}$

$Z = \{X_3, X_5\}$

$Z = \{X_3, X_5, X_1\}$

Backdoor criterion example 2:

- ~~$\mathbf{Z}$ does **not contain any descendant of** i, $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**~~
- $\mathbf{Z}$ blocks all **backdoor paths** from i to j, i.e. $i \leftarrow \dots\ j$



Does $Z = \emptyset$ satisfy the backdoor criterion for $P(X_4 \mid do(X_2))$?

Backdoor criterion example 2:

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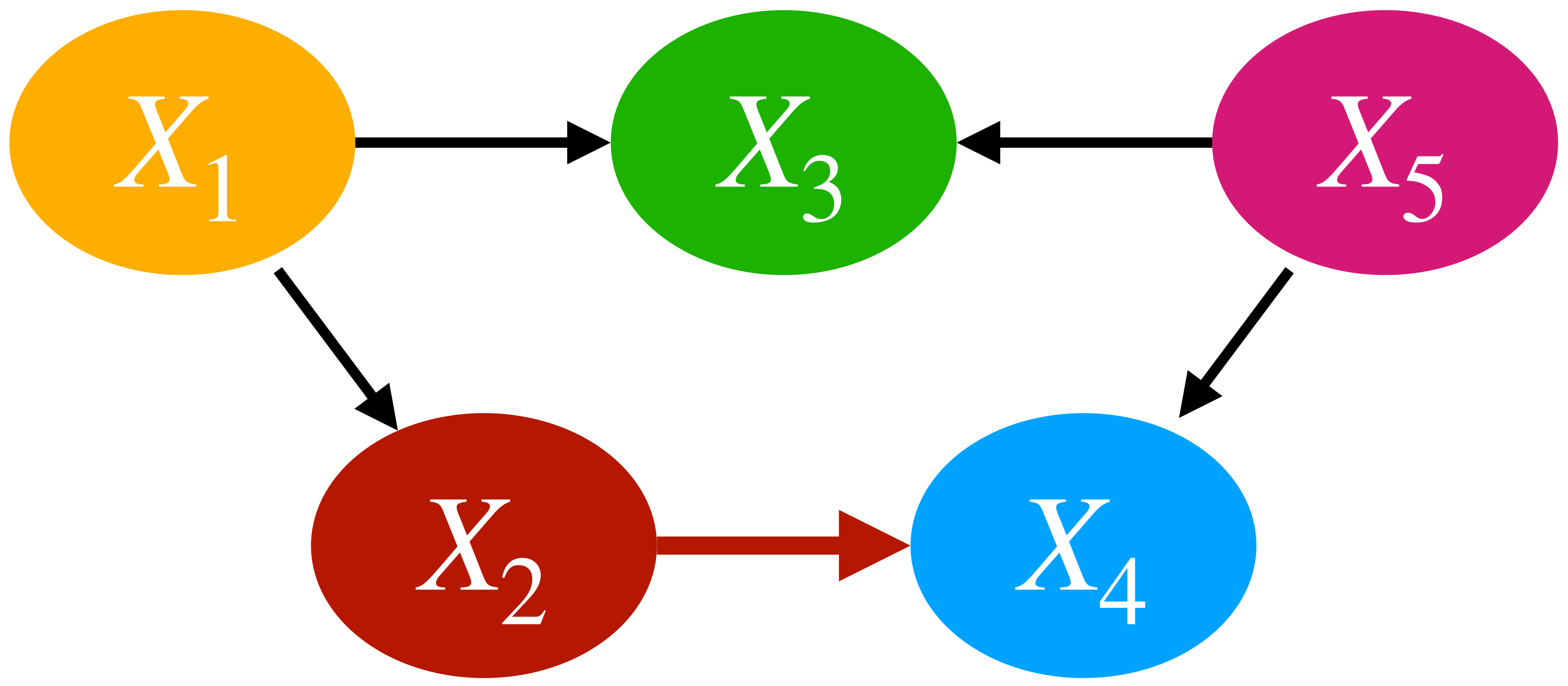

$\mathbf{Z} = \emptyset$

$$P(X_4 \mid \mathrm{do}(X_2 = x_2)) = \sum_{x_\mathbf{Z}} P(X_4 \mid X_2 = x_2, X_z = x_\mathbf{Z}) P(X_z = x_\mathbf{Z}) = P(X_4 \mid X_2 = x_2)$$

Backdoor criterion example 2:

- ~~$\mathbf{Z}$ does **not contain any descendant of i**, $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**~~
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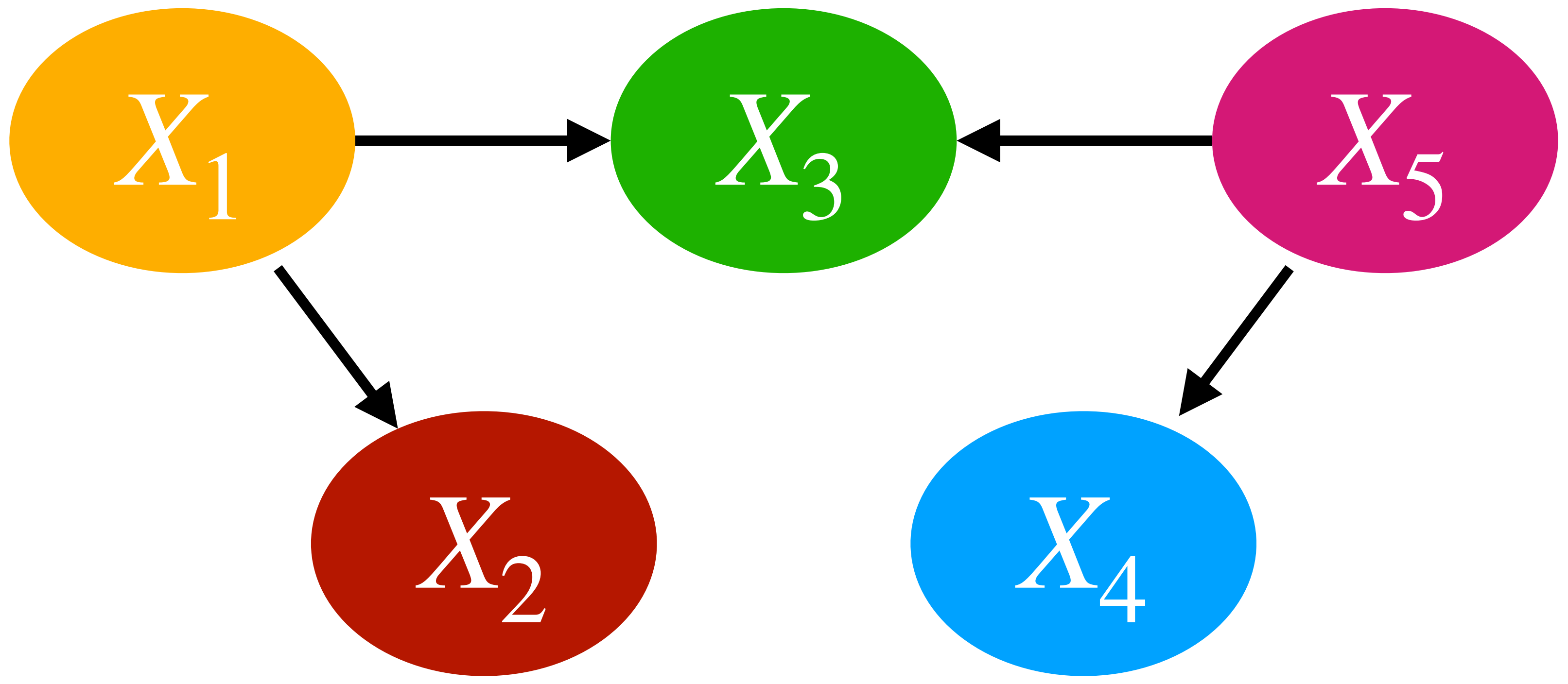

$X_2 \perp\!\!\!\perp X_4$

$$P(X_4 \mid \mathrm{do}(X_2 = x_2)) = P(X_4 \mid X_2 = x_2) = P(X_4)$$

Backdoor criterion example 4:

- $\mathbf{Z}$ does **not contain any descendant of i,** $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, **and**
- $\mathbf{Z}$ blocks all **backdoor paths** from i to j, i.e. $i \leftarrow \dots\ j$

$lm(X_4 \sim X_1 + X_2)$

```
linear_regressorX12 = LinearRegression()
linear_regressorX12.fit(X21, Y)
linear_regressorX12.coef_[:,1]
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array([29.87150906])
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$lm(X_4 \sim X_1 + X_2 + X_3)$

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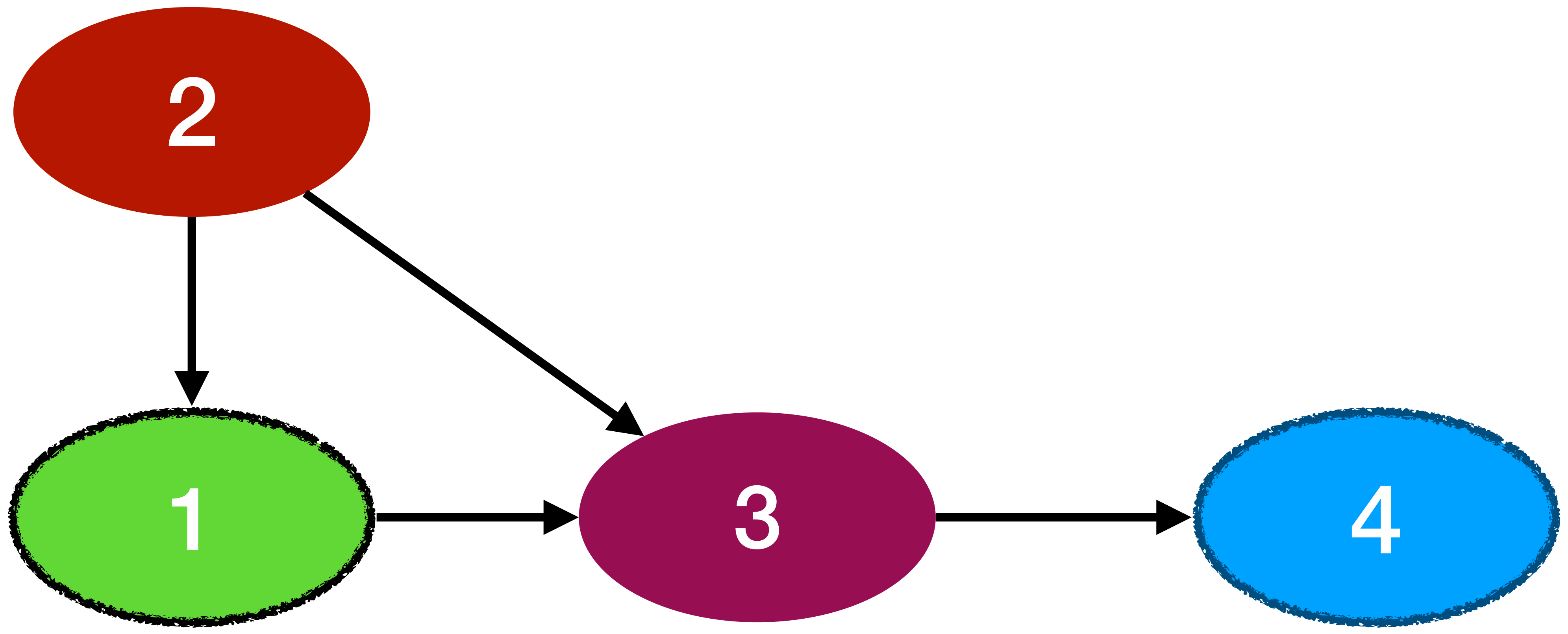

$X_3 \notin \bar{Z}$ $\bar{Z} = \{X_2\}$

Linear SCMs - adjustment can be done with regression

- One can prove that in linear SCMs (not just Gaussian) the formula:

$$p(x_j \mid \mathrm{do}(x_i)) = \int_{x_{\mathbf{Z}}} p(x_j \mid x_i, x_{\mathbf{Z}}) p(x_{\mathbf{Z}}) dx_{\mathbf{Z}}$$

is equivalent to the **coefficient of X_i for the linear regression of X_j on $X_i, X_{\mathbf{Z}}$**

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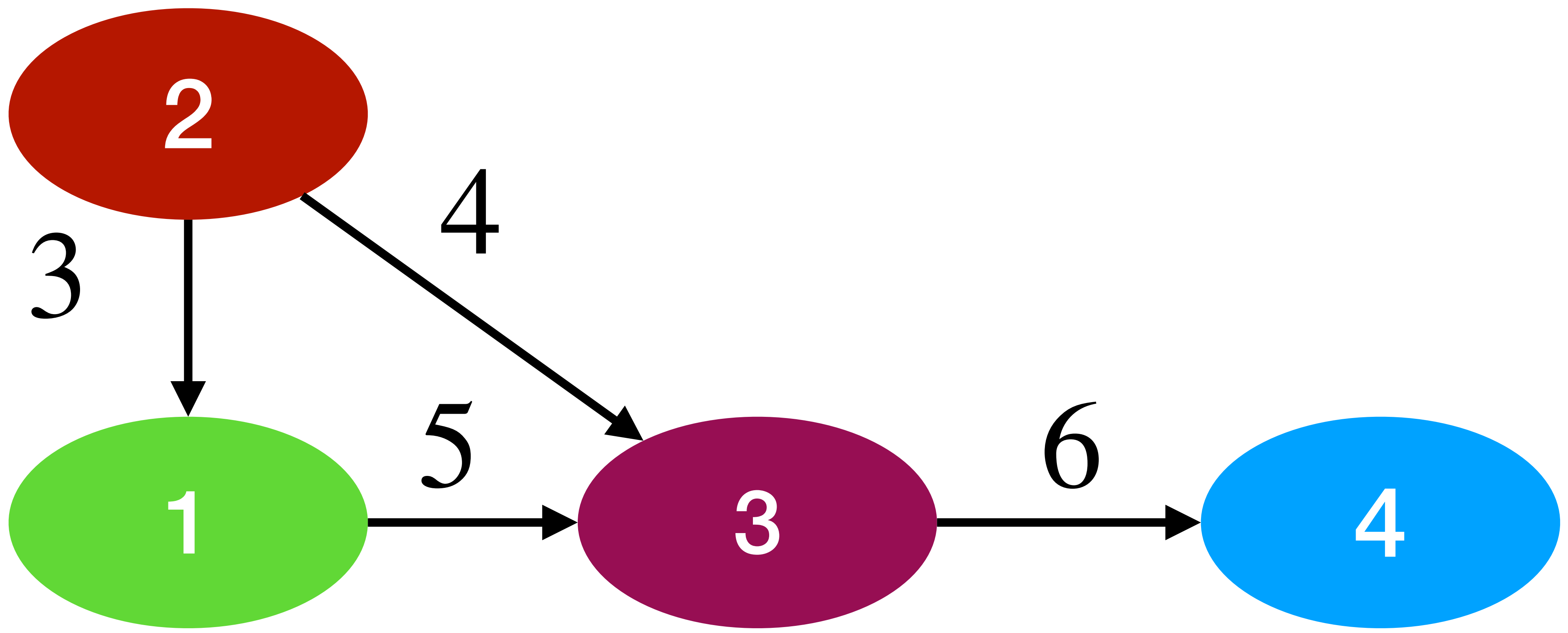
$lm(X_4 \sim X_1 + X_2)$

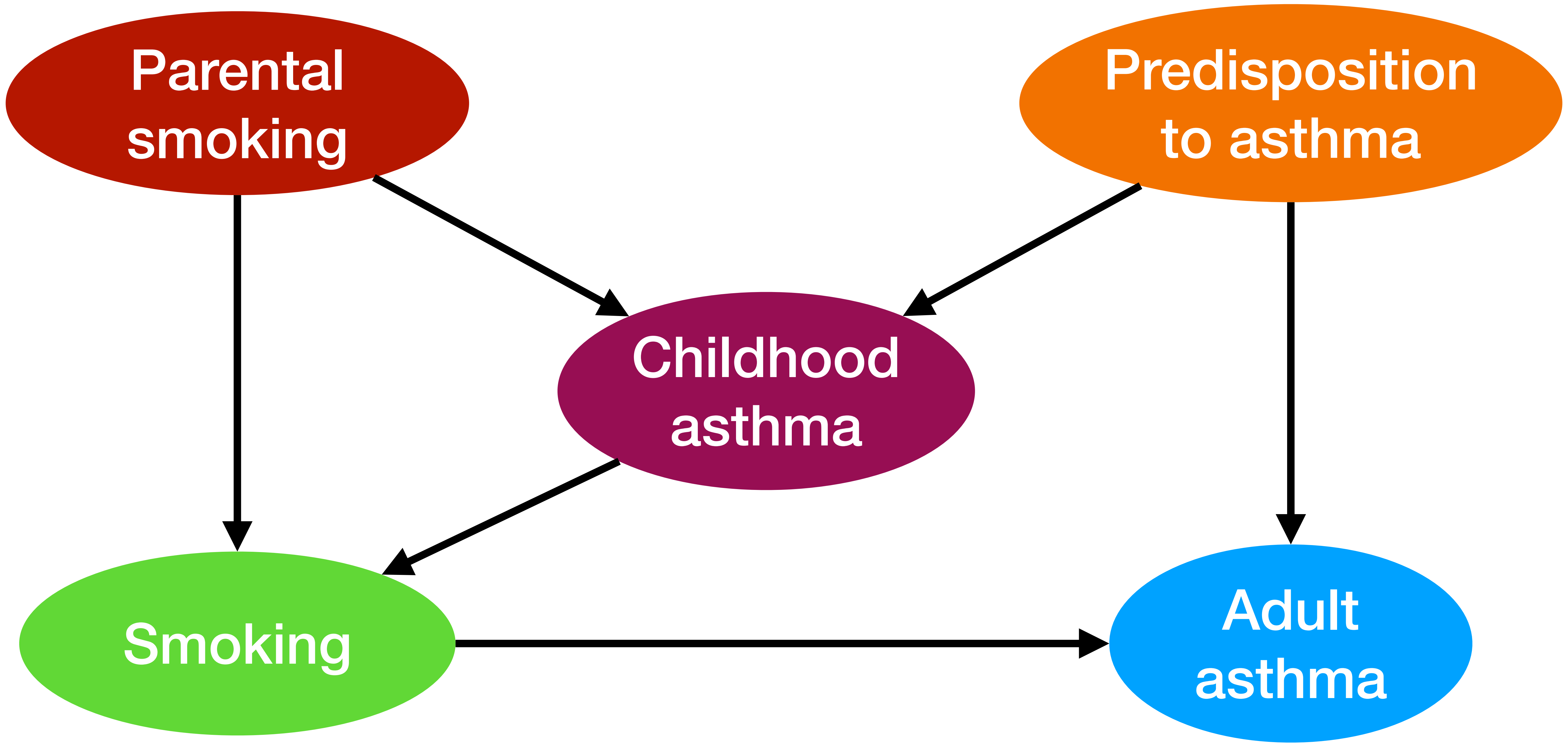
```
linear_regressorX12 = LinearRegression()
linear_regressorX12.fit(X21, Y)
linear_regressorX12.coef_[:,1]
```

```
array([29.87150906])
```



Example - effect of smoking on adult asthma?

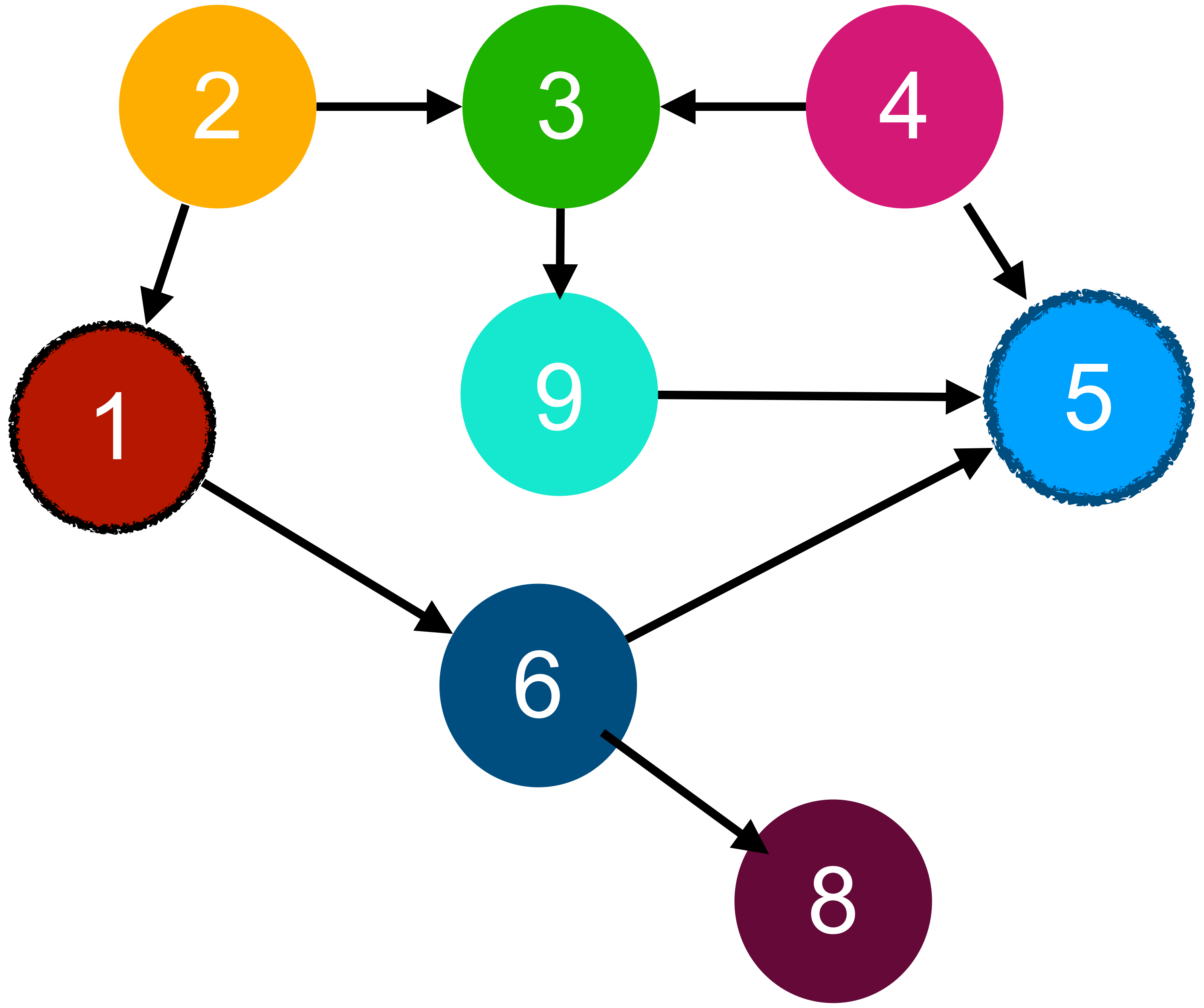
- Pretreatment variables are not always ok to adjust for
- Childhood asthma unblocks backdoor path
  - M-bias
  - Cannot adjust only on that



Example by Christina Heinze

Backdoor criterion exercise in Canvas

- $\mathbf{Z}$ does **not contain any descendant of i,** i.e. $\mathrm{Desc}(i) \cap \mathbf{Z} = \emptyset$, and
- $\mathbf{Z}$ blocks all **backdoor paths** from i to j, i.e. $i \leftarrow \dots\ j$



$P(X_5 | do(X_1)) = ?$

Identification strategies for causal effects

- Given a causal graph G, an **identification strategy** is a formula to estimate an interventional distribution from a combination of observational ones
- **Backdoor criterion (this class), Adjustment criterion (next class)**

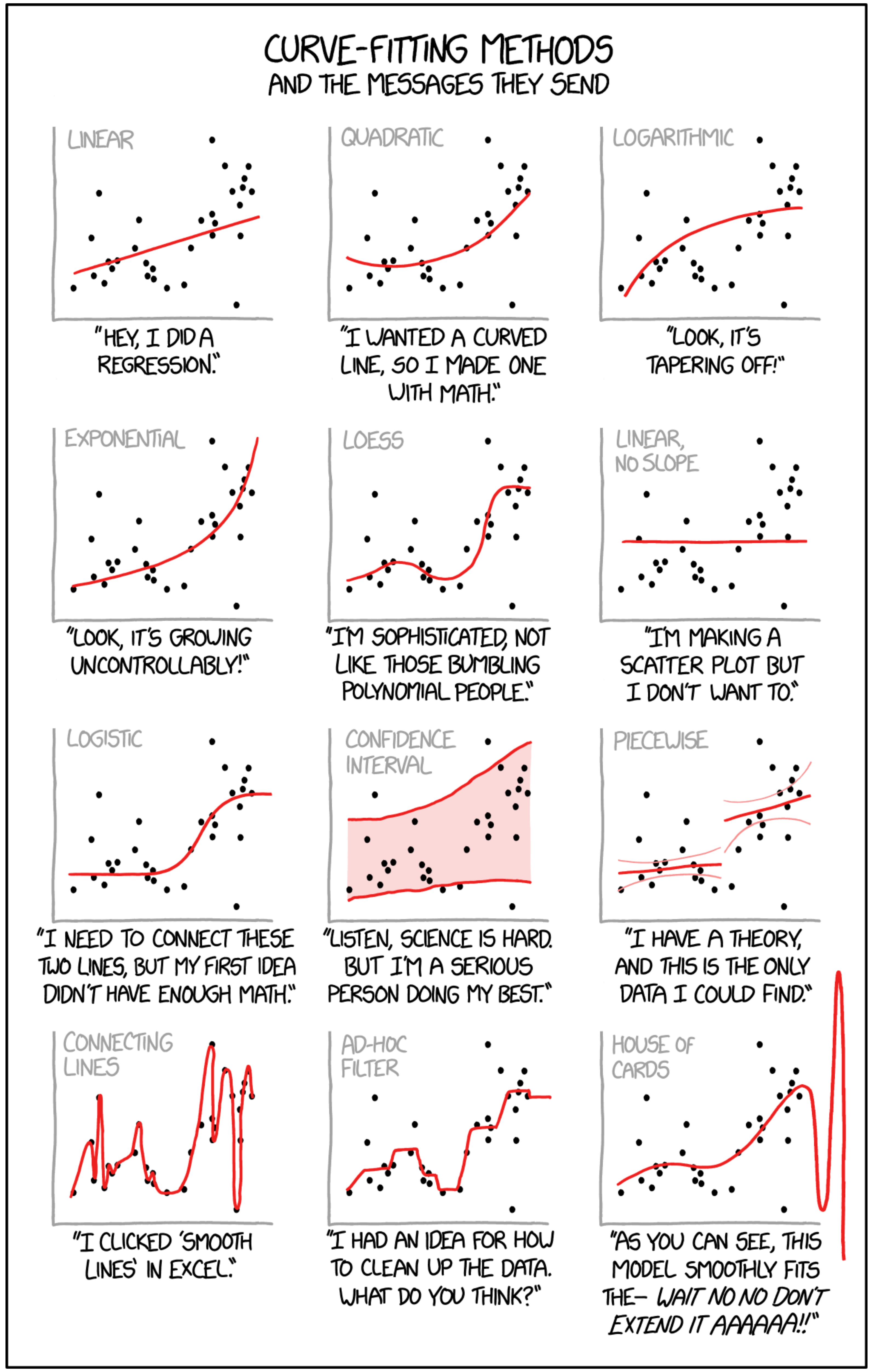
$$p(x_j \mid \mathrm{do}(x_i)) = \int_{x_{\mathbf{Z}}} p(x_j \mid x_i, x_{\mathbf{Z}}) p(x_{\mathbf{Z}}) dx_{\mathbf{Z}}$$

- **Frontdoor criterion (next class)**

$$p(x_j \mid \mathrm{do}(x_i')) = \int_{x_{\mathbf{M}}} p(x_{\mathbf{M}} \mid x_i') \int_{x_i} p(x_j \mid x_{\mathbf{M}}, x_i') p(x_i) dx_i$$

- **Instrumental variables (next class)**

Questions?



https://xkcd.com/2048/