

Your PRINTED name is: \_\_\_\_\_ 1.

Your recitation number is \_\_\_\_\_ 2.

3.

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1. (40 points) Suppose  $u$  is a unit vector in  $R^n$ , so  $u^T u = 1$ . This problem is about the  $n$  by  $n$  symmetric matrix  $H = I - 2u u^T$ .

(a) Show directly that  $H^2 = I$ . Since  $H = H^T$ , we now know that  $H$  is not only symmetric but also \_\_\_\_\_.

(b) One eigenvector of  $H$  is  $u$  itself. Find the corresponding eigenvalue.

(c) If  $v$  is any vector perpendicular to  $u$ , show that  $v$  is an eigenvector of  $H$  and **find the eigenvalue**. With all these eigenvectors  $v$ , that eigenvalue must be repeated how many times? Is  $H$  **diagonalizable**? Why or why not?

(d) Find the diagonal entries  $H_{11}$  and  $H_{ii}$  in terms of  $u_1, \dots, u_n$ . Add up  $H_{11} + \dots + H_{nn}$  and separately add up the eigenvalues of  $H$ .

2. **(30 points)** Suppose  $A$  is a positive definite symmetric  $n$  by  $n$  matrix.

(a) How do you know that  $A^{-1}$  is also positive definite? (We know  $A^{-1}$  is symmetric. I just had an e-mail from the International Monetary Fund with this question.)

(b) Suppose  $Q$  is any **orthogonal**  $n$  by  $n$  matrix. How do you know that  $Q A Q^T = Q A Q^{-1}$  is positive definite? Write down which test you are using.

(c) Show that the block matrix

$$B = \begin{bmatrix} A & A \\ A & A \end{bmatrix}$$

is positive **semidefinite**. How do you know  $B$  is not positive definite?

3. **(30 points)** This question is about the matrix

$$A = \begin{bmatrix} 0 & -1 \\ 4 & 0 \end{bmatrix}$$

(a) Find its eigenvalues and eigenvectors.

Write the vector  $u(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$  as a combination of those eigenvectors.

(b) Solve the equation  $\frac{du}{dt} = Au$  starting with the same vector  $u(0)$  at time  $t = 0$ .

In other words: the solution  $u(t)$  is what combination of the eigenvectors of  $A$ ?

(c) Find the 3 matrices in the Singular Value Decomposition  $A = U \Sigma V^T$  in two steps.

–First, compute  $V$  and  $\Sigma$  using the matrix  $A^T A$ .

–Second, find the (orthonormal) columns of  $U$ .

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## 18.06 Linear Algebra

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