In-class exercises

Week 2023-02

Relational algebra exercises

Given these sample tables:

car

id	*make*	*model*	*year*	*mileage*	*fuel*	*type*	*price*	*dealer_id*
1	Volkswagen	Passat	2017	97805	diesel	station wagon	425000	Bdf
2	Mazda	CX-3	2019	19777	petrol	suv	378900	Gjvk
3	Volkswagen	UP!	2017	16551	electric	hatchback	125000	Bdf
4	Toyota	RAV4	2019	39661	hybrid	suv	428900	Hmr
5	Mercedes-Benz	C Class	2004	301204	diesel	sedan	31707	Hmr
6	Audi	Q3	2020	18516	diesel	suv	624900	Hrst
7	Toyota	Corolla	2020	8738	hybrid	station wagon	354900	Hmr
8	Mazda	CX-3	2019	23100	petrol	suv	289900	Gjvk
9	Volkswagen	Passat	2019	43162	diesel	station wagon	375000	Hrst

dealer

id	*city*	*county_no*
Bdf	Bardufoss	54
Bo	Bodø	18
Elv	Elverum	34
Frst	Fredrikstad	30
Gjvk	Gjøvik	34
Hmr	Hamar	34
Hrst	Harstad	54
Jsh	Jessheim	30
Ksvg	Kongsvinger	34
Lhr	Lillehammer	34
Mo	Mo i Rana	18
Ms	Moss	30
Ot	Otta	34
Sarp	Sarpsborg	30
Trms	Tromsø	54
Trd	Trondheim	50
Vrd	Verdal	50

county

no	*name*
30	Viken
3	Oslo
34	Innlandet
38	Vestfold og Telemark
42	Agder
11	Rogaland
46	Vestland
15	Møre og Romsdal
50	Trøndelag
18	Nordland
54	Troms og Finnmark

Q1:

- What is the result of: $\Pi_{\texttt{name}}(\texttt{county})$?

Solution:

name
Viken
Oslo
Innlandet
Vestfold og Telemark
Agder
Rogaland
Vestland
Møre og Romsdal
Trøndelag
Nordland
Troms og Finnmark

Q2:

- What is the result of: $\Pi_{\texttt{id, city}}(\texttt{dealer})$?

Solution:

id	***city***
Bdf	Bardufoss
Bo	Bodø
Elv	Elverum
Frst	Fredrikstad
Gjvk	Gjøvik
Hmr	Hamar
Hrst	Harstad
Jsh	Jessheim
Ksvg	Kongsvinger
Lhr	Lillehammer
Mo	Mo i Rana
Ms	Moss
Ot	Otta
Sarp	Sarpsborg
Trms	Tromsø
Trd	Trondheim
Vrd	Verdal

Q3:

- What is the result of: $\Pi_{\texttt{id, make, model, year}}(\texttt{car})$?

Solution:

id	***make***	***model***	***year***
1	Volkswagen	Passat	2017
2	Mazda	CX-3	2019
3	Volkswagen	UP!	2017
4	Toyota	RAV4	2019
5	Mercedes-Benz	C Class	2004
6	Audi	Q3	2020
7	Toyota	Corolla	2020
8	Mazda	CX-3	2019
9	Volkswagen	Passat	2019

Q4:

- What relational algebra expression defines a relation containing names of dealer cities in the `dealer` relation?

Solution:
$\Pi_{\texttt{city}}(\texttt{dealer})$

Q5:

- What relation algebra expression defines a relation containing the id, make, type, year, mileage, and price of cars in the `car` relation?

Solution:
$\Pi_{\texttt{id, make, type, year, mileage, price}}(\texttt{car})$

Q6:

- What is the result of: $\sigma_{\texttt{year < 2010 OR mileage > 50000}}(\texttt{car})$?

Solution:

id	***make***	***model***	***year***	***mileage***	***fuel***	***type***	***price***	***dealer_id***
1	Volkswagen	Passat	2017	97805	diesel	station wagon	425000	Bdf
5	Mercedes-Benz	C Class	2004	301204	diesel	sedan	31707	Hmr

Q7:

- What is the result of: $\sigma_{\texttt{fuel <> 'diesel'}}(\texttt{car})$?

Solution:

id	*make*	*model*	*year*	*mileage*	*fuel*	*type*	*price*	*dealer_id*
2	Mazda	CX-3	2019	19777	petrol	suv	378900	Gjvk
3	Volkswagen	UP!	2017	16551	electric	hatchback	125000	Bdf
4	Toyota	RAV4	2019	39661	hybrid	suv	428900	Hmr
7	Toyota	Corolla	2020	8738	hybrid	station wagon	354900	Hmr
8	Mazda	CX-3	2019	23100	petrol	suv	289900	Gjvk

Q8:

- What is the result of: $\sigma_{\texttt{make IN ('Mercedes-Benz', 'Audi')}}(\texttt{car})$?

Solution:

id	*make*	*model*	*year*	*mileage*	*fuel*	*type*	*price*	*dealer_id*
5	Mercedes-Benz	C Class	2004	301204	diesel	sedan	31707	Hmr
6	Audi	Q3	2020	18516	diesel	suv	624900	Hrst

Q9:

- What relational algebra expression defines a relation containing rows from the `car` relation where the car is either a station wagon or costs no more than 360,000?

Solution:

$\sigma_{\texttt{type = 'station wagon' OR price <= 360000}}(\texttt{car})$

Q10:

- What relational algebra expression defines a relation containing rows from the `car` relation where the car is not an Audi or a Volkswagen

Solution:

$\sigma_{\texttt{make NOT IN ('Audi', 'Volkswagen')}}(\texttt{car})$

The predicate could also be:

```
make <> 'Audi' AND make <> 'Volkswagen')
```

Q11:

- Decompose this expression: $\Pi_{\texttt{city}}(\sigma_{\texttt{county\_no = 34}}(\texttt{dealer}))$?
- What is the result?

Solution:

$R \leftarrow \sigma_{\texttt{county\_no = 34}}(\texttt{dealer})$

$\Pi_{\texttt{city}}(R)$

Q12:

- Decompose this expression:
  $\sigma_{\texttt{dealer\_id IN ('Gjvk', 'Hmr', 'Lhr')}}(\Pi_{\texttt{id, make, year, dealer\_id}}(\texttt{car}))$?
- What is the result?

Solution:

$R \leftarrow \Pi_{\texttt{id, make, year, dealer\_id}}(\texttt{car})$

$\sigma_{\texttt{dealer\_id IN ('Gjvk', 'Hmr', 'Lhr')}}(R)$

Q13:

- What relational algebra expression defines a relation containing the ids from the `dealer` relation where the dealer resides in the county with county number 18 or county number 50?

Solution:

$\Pi_{\texttt{id}}(\sigma_{\texttt{county\_no IN (18, 50)}}(\texttt{dealer}))$

The predicate could also be:

```
conty_no = 18 OR county_no = 50
```

Q14:

- What relational algebra expression defines a relation containing the id, make, model, type, and price from the `car` relation where the car is either a station wagon or a suv and the price is between 300,000 and 450,000?

Solution:

$\Pi_{\texttt{id, make, model, type, price}}(\sigma_{\texttt{type IN ('station wagon', 'suv') AND price BETWEEN 300000 AND 450000}}(\texttt{car}))$

The predicate could also be:

```
(type = 'station wagon' OR type = 'suv') AND price BETWEEN 300000 AND 450000
```

Q15:

- What relational algebra expression defines a relation containing id, make, model, and type from the car relation where the car is not an SUV?

Solution:

$\Pi_{\texttt{id, make, model, type}}(\sigma_{\texttt{type <> 'suv'}}(\texttt{car}))$

Q16:

- What is the result of:
  $\Pi_{\texttt{city, make, model}}(\sigma_{\texttt{fuel <> 'diesel'}}(\texttt{car} \bowtie_{\texttt{dealer\_id = dealer.id}} \texttt{dealer}))$?

Solution:

city	***make***	***model***
Gjøvik	Mazda	CX-3
Bardufoss	Volkswagen	UP!
Hamar	Toyota	RAV4
Hamar	Toyota	Corolla
Gjøvik	Mazda	CX-3

Q17:

- Which of these expressions are equivalent – if any:
  1. $car \times dealer$
  2. $\sigma_{dealer\_id = dealer.id}(car \times dealer)$
  3. $car \bowtie_{dealer\_id = dealer.id} dealer$

Solution:
2 and 3 are equivalent: Selecting the rows where $dealer\_id = dealer.id$ *from the cartesian product is exactly what the equijon on the same predicate produces.*

Q18:

- What relational algebra expression defines a relation containing county name and city names for dealers in the counties named Trøndelag or Nordland?

Solution:

$\Pi_{name,\ city}(\sigma_{name\ IN\ ('Trøndelag',\ 'Nordland')}(county \bowtie_{county\_no = no} dealer))$

The predicate could also be:

```
name = 'Trøndelag' OR name = 'Nordland'
```

Q19:

- What relational algebra expression defines a relation containing city name, maker name for cars on sale in the county named Innlandet?

Solution:

$$\Pi_{name,\ make}(\sigma_{name = 'Innlandet'}(county \bowtie_{county\_no = no} dealer \bowtie_{dealer\_id = dealer.id} car))$$

Or:

$$\Pi_{name,\ make}(\sigma_{name = 'Innlandet'}(car \bowtie_{dealer\_id = dealer.id} dealer \bowtie_{county\_no = no} county))$$

Decomposed version:

$R \leftarrow county \bowtie_{county\_no = no} dealer$

$S \leftarrow R \bowtie_{dealer\_id = dealer.id} car$

(Or

$R \leftarrow car \bowtie_{dealer\_id = dealer.id} dealer$

$S \leftarrow R \bowtie_{county\_no = no} county$

)

$T \leftarrow \sigma_{name = 'Innlandet'}(S)$

$\Pi_{name,\ make}(T)$

Q20:

- What relational algebra expression defines a relation containing county name, make, and year for cars of type suv costing less than 400,000?

Solution:

$$\Pi_{name,\ make,\ year}(\sigma_{type = 'suv'\ AND\ price < 400000}(county \bowtie_{county\_no = no} dealer \bowtie_{dealer\_id = dealer.id} car))$$

Q21:

- What relational algebra expression defines a relation containing city name, make, year, and price for hybrid or diesel cars on sales by dealers in the cities of Gjøvik, Hamar or Lillehammer?

Solution:

$$\Pi_{\texttt{city, make, year, price}}(\sigma_{\texttt{city IN ('Gjøvik', 'Hamar', 'Lillehammer') AND fuel IN ('hybrid', 'diesel')}}(\texttt{dealer} \bowtie_{\texttt{dealer\_id = dealer.id}} \texttt{car}))$$

The predicate could also be:

```
(city = 'Gjøvik' OR city = 'Hamar' OR city = 'Lillehammer') AND (fuel = 'hybrid' OR fuel = 'diesel')
```

Q22:

- What is the result of: $\Pi_{\texttt{city, make, model}}(\texttt{car} \bowtie_{\texttt{dealer\_id = dealer.id}} \texttt{dealer})$?

Solution:

city	***make***	***model***
Bardufoss	Volkswagen	Passat
Gjøvik	Mazda	CX-3
Bardufoss	Volkswagen	UP!
Hamar	Toyota	RAV4
Hamar	Mercedes-Benz	C Class
Harstad	Audi	Q3
Hamar	Toyota	Corolla
Gjøvik	Mazda	CX-3
Harstad	Volkswagen	Passat
Bodø	NULL	NULL
Elverum	NULL	NULL
Fredrikstad	NULL	NULL
Jessheim	NULL	NULL
Kongsvinger	NULL	NULL
Lillehammer	NULL	NULL
Mo i Rana	NULL	NULL
Moss	NULL	NULL
Otta	NULL	NULL
Sarpsborg	NULL	NULL
Tromsø	NULL	NULL
Trondheim	NULL	NULL
Verdal	NULL	NULL

Q22:

- Which of these expressions are equivalent – if any:

1. $car \bowtie_{dealer\_id = dealer.id} dealer$
2. $car \rtimes_{dealer\_id = dealer.id} dealer$
3. $car \ltimes_{dealer\_id = dealer.id} dealer$

Solution:
1 and 2 are equivalent because there are no cars that are not related to a dealer so the INNER and the LEFT OUTER JOIN are the same.
2 and 3 would be equivalent if 2 looked like:

2. $dealer \rtimes_{dealer\_id = dealer.id} car$

Q23:

- What relational algebra expression defines a relation containing the city name and the make name for 2019 and 2020 cars on sale in all the cities (even those that do not have such cars on sale).

Solution:

$\Pi_{city,\ make}(dealer \rtimes_{dealer\_id = dealer.id} \sigma_{year\ IN\ (2019,\ 2020)}(car)$

Or:

$\Pi_{city,\ make}(\sigma_{year\ IN\ (2019,\ 2020)}(car) \ltimes_{dealer\_id = dealer.id} dealer))$

The predicate could also be:

```
year = 2019 OR year = 2020
```

Or:

```
year BETWEEN 2019 AND 2020
```

Q24:

- What is the result of: $\gamma_{COUNT\ id}(\sigma_{fuel = 'petrol'}(car))$

Solution:

COUNT id
2

Q25:

- What is the result of:

$\gamma_{MIN\ mileage,\ MAX\ mileage}(car \bowtie_{dealer\_id = dealer.id} dealer \bowtie_{county\_no = no} \sigma_{name = 'Innlandet'}(county))$

Solution:

MIN mileage	***MAX mileage***
8738	301204

Q26:

What is the result of:

$$\gamma_{\text{COUNT dealer.id}}(\sigma_{\text{car.id IS NULL}}(\text{car} \ ⟕_{\text{dealer\_id = dealer.id}} \ \text{dealer}))$$

Solution:

COUNT dealer.id
13

Q27:

- What relational algebra expression defines a relation that holds the year of the oldest car for sale.

Solution:

$$\gamma_{\text{MIN year}}(\text{car})$$

Q28:

- What relational algebra expression defines a relation counting the number of Volkswagen Passat cars for sale in the cities of Harstad and Bardufoss combined.

Solution:

$$\gamma_{\text{COUNT id}}(\sigma_{\text{city IN ('Harstad', 'Bardufoss') AND make = 'Volkswagen' AND model = 'Passat'}}(\text{car} \bowtie_{\text{dealer\_id = dealer.id}} \text{dealer}))$$

Q29

- What is the result of:

$$\gamma_{\text{city SUM price}}(\text{car} \bowtie_{\text{dealer\_id = dealer.id}} \text{dealer})$$

Solution:

city	***SUM price***
Bardufoss	550000
Gjøvik	668800
Hamar	815507
Harstad	999900

Q30

- What is the result of:

$$_{\text{city, make SUM price}}(\text{car} \bowtie_{\text{dealer\_id = dealer.id}} \text{dealer})$$

Solution:

city	***make***	***SUM price***
Bardufoss	Volkswagen	550000
Gjøvik	Volkswagen	668800
Hamar	Toyota	783800
Hamar	Mercedes-Benz	31707
Harstad	Audi	624900
Harstad	Volkswagen	375000

Q30

- What is the result of:

$$\sigma_{\text{SUM price > 750000}}(_{\text{city, make SUM price}}(\text{car} \bowtie_{\text{dealer\_id = dealer.id}} \text{dealer}))$$

Solution:

city	***make***	***SUM price***
Hamar	Toyota	783800

Q31:

- What relational algebra expression defines a relation that holds the average mileage for each car model year.

Solution:

$$_{\text{year, model AVG mileage}}(\text{car})$$

Q32:

- What relational algebra expression defines that holds the number of cars per type per county.

Solution:

$$_{\text{name, type COUNT car.id}}(\text{car} \bowtie_{\text{dealer\_id = dealer.id}} \text{dealer} \bowtie_{\text{county\_no = county.no}} \text{county})$$