TRANSPORTATION PROBLEM

Introduction

DEFINITION

Transportation problem is a special kind of **Linear Programming Problem (LPP)** in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as *Hitchcock problem*.

Introduction

Basic structure of transportation problem:

		Destination				
		D1	D2	D3	D4	Supply(s_i)
Source	O1	C_{11}	C_{12}	C_{13}	C_{14}	s_1
	O2	C_{21}	C_{22}	C_{23}	C_{24}	s_2
	O3	C_{31}	C_{32}	C_{33}	C_{34}	s_3
	O4	C_{41}	C_{42}	C_{43}	C_{44}	s_4
Demand (d_j):		d_1	d_2	d_3	d_4	

D_4
0
0
50

e.g.

	D_1	D_2	D_3	Supply
Boston	5	6	4	300
Toronto	6	3	7	500
Demand	200	300	250	750 800

Decision Variables:

x_{ij}
- X_{11} - # of units shipped from Boston to D_1
- X_{12} - . . .
- X_{13} - . .
- X_{21} - # of units shipped from Toronto to D_1
- X_{22} - . . . .
- X_{23} .

Objective function:

$$\min Z = 5X_{11} + 6X_{12} + 4X_{13} + 6X_{21} + 3X_{22} + 7X_{23}$$

Functional constraints:

$$X_{11} + X_{12} + X_{13} \le 300$$
$$X_{21} + X_{22} + X_{23} \le 500$$
$$X_{11} + X_{21} = 200$$
$$X_{12} + X_{22} = 300$$
$$X_{13} + X_{23} = 250$$

and $x_{ij} \ge 0$

Transportation Problem

Types of Transportation problems:

1. *Balanced*: When both supplies and demands are equal then the problem is said to be a balanced transportation problem.

2. *Unbalanced*: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.

Transportation Problem

2 Phases in Solving Transportation Problem

First Phase: Finding the initial basic feasible solution.

Second Phase: Optimization of the initial basic feasible solution that was obtained in the first phase.

Methods used in finding the initial basic feasible solution:

1. NorthWest Corner Cell Method.
2. Least Cost Cell Method.
3. Vogel's Approximation Method (VAM). ✓

Methods used in optimizing the basic feasible solution:

1. Stepping Stone Method
2. MODI method – UV method ✓

Transportation Problem

Vogel's Approximation Method (VAM)

-is one of the methods used to calculate the initial basic feasible solution to a transportation problem.

-is an iterative procedure such that in each step, we should find the penalties for each available row and column by taking the least cost and second least cost.

-is the best method of computing the initial basic feasible solution to a transportation problem. As it provided better results when compared with other methods.

Steps:

Step 1: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.

Step 2: Identify the row or column with the maximum penalty and assign the corresponding cell's min(supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.

Step 3: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.

Step 4: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.

Transportation Problem

MODI Method – UV method

-the modified distribution method, also known as MODI method or U-V method, provides a minimum cost solution to the transportation problems.

-is an improvement over stepping stone method.

Steps:

Step 1: Find an initial basic feasible solution using any one of the three methods NWCM, LCM or VAM.

Step 2: Find u_i and v_j for rows and columns. To start

a. assign 0 to u_i or v_j where maximum number of allocation in a row or column respectively.

allocated cells

b. Calculate other u_i's and v_j's using $c_{ij} = u_i + v_j$, for all occupied cells.

unallocated cells

Step 3: For all unoccupied cells, calculate $d_{ij} = c_{ij} - (u_i + v_j)$, .

Step 4: Check the sign of d_{ij}

a. If $d_{ij} > 0$, then current basic feasible solution is optimal and stop this procedure.

b. If $d_{ij} = 0$ then alternative solution exists, with different set allocation and same transportation cost. Now stop this procedure.

c. If $d_{ij} < 0$, then the given solution is not an optimal solution and further improvement in the solution is possible.

Transportation Problem

Step 5: Select the unoccupied cell with the largest negative value of d_{ij}, and included in the next solution.

Step 6: Draw a closed path (or loop) from the unoccupied cell (selected in the previous step). The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.

Step 7: 1. Select the minimum value from cells marked with (-) sign of the closed path.

2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell).

3. Add this value to the other occupied cells marked with (+) sign.

4. Subtract this value to the other occupied cells marked with (-) sign.

Step 8: Repeat *Step 2* to *step 7* until optimal solution is obtained. This procedure stops when all $d_{ij} \geq 0$ for unoccupied cells.

Transportation Problem

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Transportation Problem

Example 1

Solve the given transportation problem using Vogel's approximation method.

		Destination				
		D1	D2	D3	D4	Supply
	O1	3	1	7	4	300
Source	O2	2	6	5	9	400
	O3	8	3	3	2	500
	Demand	250	350	400	200	1200 / 1200

- balanced Transportation Problem

Soln.

$m+n-1 = 6$

$3+4-1 = 6$

$6 = 6$

	D_1	D_2	D_3	D_4	Supply	Row difference			
O_1	3	$\boxed{300}$ 1	7	4	~~300~~ 0	2	3	–	–
O_2	$\boxed{250}$ 2	6	$\boxed{150}$ 5	9	~~400~~ ~~150~~ 0	3	1	1	1
O_3	8	$\boxed{50}$ 3	$\boxed{250}$ 3	$\boxed{200}$ 2	~~500~~ ~~300~~ ~~250~~ 0	1	1	1	0
Demand	~~250~~ 0	~~350~~ ~~50~~ 0	~~400~~ ~~150~~ 0	~~200~~ 0					
column difference	1	2	2	2					
	–	2	2	2					
	–	3	2	7					
	–	3	2	–					

$$\text{Cost} = (300\times 1) + (250\times 2) + (150\times 5) + (50\times 3) + (250\times 3) + (200\times 2)$$

$$= 2850$$

Transportation Problem