

### Introduction

#### **DEFINITION**

**Transportation problem** is a special kind of **Linear Programming Problem** (**LPP**) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized. It is also sometimes called as *Hitchcock problem*.



#### Introduction

Basic structure of transportation problem:

 $Supply(s_i)$ 

0

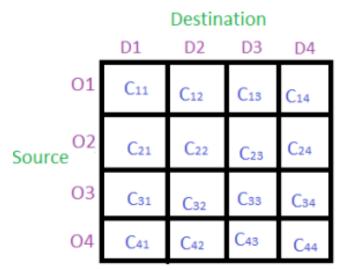
50

S1

S<sub>2</sub>

S3

**S**4



Demand (d <sub>j</sub> ): <sub>d1</sub>	d <sub>2</sub>	dз	d
	-	43	

# frohim conduints: $X_{11} + X_{12} + X_{13} \leq 700$ $X_{21} + X_{12} + X_{23} \leq 500$ $X_{11} + X_{21} = 200$ $X_{12} + X_{22} = 300$ $X_{13} + X_{13} = 250$

					1
	$\mathcal{D}_{I}$	0,	03	Supply	
boston	5	6	4	300	
torodo	6	3	7	590	
Domarin	200	300	250	750 80	0
			1	'	

Decision Variables:

### Types of Transportation problems:

- 1. *Balanced*: When both supplies and demands are equal then the problem is said to be a balanced transportation problem.
- 2. *Unbalanced*: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.



### 2 Phases in Solving Transportation Problem

First Phase: Finding the initial basic feasible solution.

Second Phase: Optimization of the initial basic feasible solution that was obtained in the first phase.

Methods used in finding the initial basic feasible solution:

- NorthWest Corner Cell Method.
- Least Cost Cell Method.
- 3. Vogel's Approximation Method (VAM). 🗸

Methods used in optimizing the basic feasible solution:

- 1. Stepping Stone Method
- 2. MODI method − UV method ✓



### Vogel's Approximation Method (VAM)

- -is one of the methods used to calculate the initial basic feasible solution to a transportation problem.
- -is an iterative procedure such that in each step, we should find the penalties for each available row and column by taking the least cost and second least cost.
- -is the best method of computing the initial basic feasible solution to a transportation problem. As it provided better results when compared with other methods.

#### Steps:

- **Step 1**: Identify the two lowest costs in each row and column of the given cost matrix and then write the absolute row and column difference. These differences are called penalties.
- **Step 2**: Identify the row or column with the maximum penalty and assign the corresponding cell's min(supply, demand). If two or more columns or rows have the same maximum penalty, then we can choose one among them as per our convenience.
- **Step 3**: If the assignment in the previous satisfies the supply at the origin, delete the corresponding row. If it satisfies the demand at that destination, delete the corresponding column.
- **Step 4**: Stop the procedure if supply at each origin is 0, i.e., every supply is exhausted, and demand at each destination is 0, i.e., every demand is satisfying. If not, repeat the above steps, i.e., from step 1.



#### MODI Method – UV method

- -the modified distribution method, also known as MODI method or U-V method, provides a minimum cost solution to the transportation problems.
- -is an improvement over stepping stone method.

#### Steps:

Step 1: Find an initial basic feasible solution using any one of the three methods NWCM, LCM or (VAM)

Step 2: Find  $u_i$  and  $v_j$  for rows and columns. To start

a. assign 0 to  $u_i$  or  $v_j$  where maximum number of allocation in a row or column respectively. b. Calculate other  $u_i$ 's and  $v_j$ 's using  $c_{ij} = u_i + v_j$ , for all occupied cells. Step 3: For all unoccupied cells, calculate  $d_{ij} = c_{ij} - (u_i + v_j)$ , .

Step 4: Check the sign of  $d_{ii}$ 

a. If  $d_{ij} > 0$ , then current basic feasible solution is optimal and stop this procedure.

b. If  $d_{ij} = 0$  then alternative solution exists, with different set allocation and same transportation cost. Now stop this procedure.

c. If  $d_{ij} < 0$ , then the given solution is not an optimal solution and further improvement in the solution is possible.

- Step 5: Select the unoccupied cell with the largest negative value of  $d_{ij}$ , and included in the next solution.
- Step 6: Draw a closed path (or loop) from the unoccupied cell (selected in the previous step). The right angle turn in this path is allowed only at occupied cells and at the original unoccupied cell. Mark (+) and (-) sign alternatively at each corner, starting from the original unoccupied cell.
- Step 7: 1. Select the minimum value from cells marked with (-) sign of the closed path.
  - 2. Assign this value to selected unoccupied cell (So unoccupied cell becomes occupied cell).
  - 3. Add this value to the other occupied cells marked with (+) sign.
  - 4. Subtract this value to the other occupied cells marked with (-) sign.
- Step 8: Repeat Step 2 to step 7 until optimal solution is obtained. This procedure stops when all  $d_{ij} \ge 0$  for unoccupied cells.



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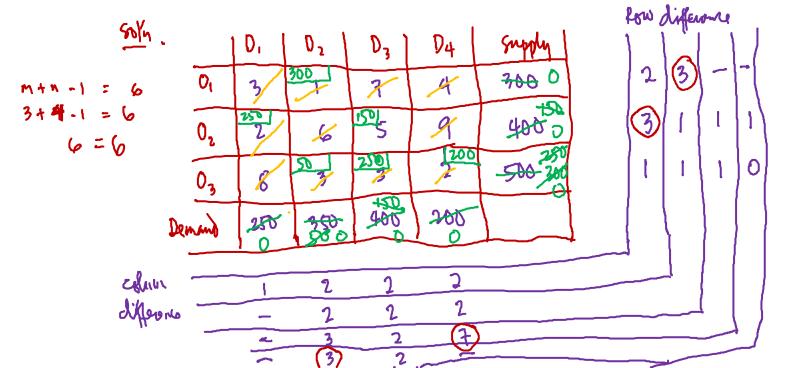


#### Example 1

Solve the given transportation problem using Vogel's approximation method.

	1	1				
			Destination			
		D1	D2	D3	D4	Supply
	01	3	1	7	4	300
Source	O2	2	6	5	9	400
	О3	8	3	3	2	500
	Demand	250	350	400	200	1200

-balanced Transportation Problem



$$Cost = (300 \times 1) + (250 \times 2) + (150 \times 5)$$

$$+ (50 \times 3) + (250 \times 3) + (200 \times 2)$$

$$= 2850$$

