Problems for CW3

Analysis 1 - Spring 2023

Choose TWO of the following problems for your CW3 due on 17 Mar 2023 by 1PM.

1. Let A and B be non-empty subsets of $\mathbb{R}$ such that $a \leq b$ for all $a \in A$ and $b \in B$. Show that $\sup A \leq \inf B$ and that the equality holds if and only if for all $\varepsilon > 0$, there are $a \in A$ and $b \in B$ such that $b - a < \varepsilon$.

2. Using lower and upper sums, show that the function $t \mapsto t^2$ is integrable on $[0, x]$ for all $x > 0$ and that $\int_0^x t^2 dt = \frac{x^3}{3}$.

3. Using lower and upper sums, show that if f is integrable on $[a, b]$ then $|f|$ is also integrable on $[a, b]$ and $|\int_a^b f(x)dx| \leq \int_a^b |f(x)|dx$.