Mathematics Year 1, Calculus and Applications I, 2022

Autumn Take-home Assessment

The equation

$$y' = 1 + y^2, \qquad y(0) = 0, \qquad (**)$$

can be solved using separation of variables and integration to find $y = \tan x$.

(a) Find a power series solution of (**) and hence show that

$$\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \dots$$

[Note: you will need to use the earlier formulas for multiplication of two infinite power series.]

(b) Now get the result above by repeated differentiation of (**) and use of the formula $a_n = \frac{f^{(n)}(0)}{n!}$.