MATH50003
Numerical Analysis

I.2 Divided Differences

Dr Sheehan Olver

Part I

Calculus on a Computer

1. Rectangular rules for integration
2. Divided differences for differentiation
3. Dual numbers for differentiation

(Right-sided) divided differences

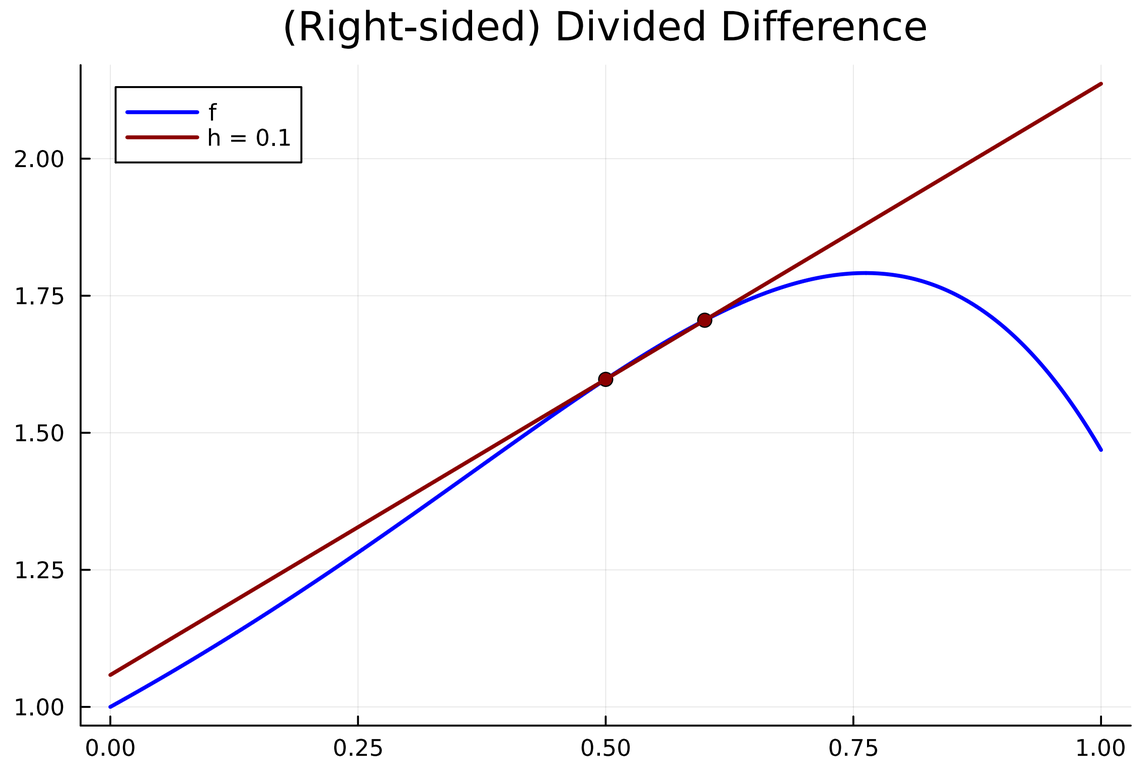
Approximating derivatives from function values

- Start with the definition of a derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Idea: make h small and use

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$



Divided differences is slope of line and approximates derivative

Proposition 1 (divided differences error)**.** *Suppose that f is twice differrentable on the interval $(x, x+h)$. The error in approximating the derivative using divided differences is*

$$f'(x) = \frac{f(x+h) - f(x)}{h} + \delta$$

where $|\delta| \leq Mh/2$ for $M = \sup_{x \leq t \leq x+h} |f''(t)|$.

Other approximations to derivatives

Explored in Problem Sheets/Lab

Central differences

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

Second-order divided differences

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

Now for implementation

Can we get an idea of what goes wrong with h very small?