MATH50003
Numerical Analysis

https://github.com/Imperial-MATH50003/MATH50003NumericalAnalysis

Office Hour: Monday 11am, Huxley 6M40

Dr Sheehan Olver

What is Numerical Analysis?

Algorithms for continuous problems

Implementation in software

Analysis of convergence and stability

What is Numerical Analysis for?

- Applied mathematics
    - Simulating solutions to differential equations underlies most of modern applied mathematics
    - It is important to understand errors in algorithms to rely on computations
- Pure mathematics
    - Computer-assisted proofs built on numerical methods are increasingly important
    - Famous examples include Kepler conjecture, stability of Lorenz system, and verifying the Riemann Hypothesis
- Statistics / Machine Learning
    - Numerical linear algebra underlies principle component analysis
    - Machine Learning is all built on numerical algorithms like stochastic steepest descent

Who am I?

Dr Sheehan Olver

- PhD in Cambridge followed by Junior Research Fellow at St. John's, Oxford
- Imperial since 2016
- Researcher in numerical analysis / scientific computing studying:
    - Complex analysis
    - Random matrix theory
    - Partial differential equations
    - Fractional differential equations
- Work combines pure and applied analysis and algebra
- Won the Adam's Prize in 2012 for numerical methods for Riemann–Hilbert problems

Course content

I. Calculus on a Computer
   - Integration, differentiation, root finding

II. Representing Numbers
   - Modular arithmetic, floating point numbers, bounding errors

III. Numerical Linear Algebra
   - Data regression, differential equations, least squares

IV. Approximation Theory
   - Fourier series, orthogonal polynomials, Gaussian quadrature

ASSESSMENT

Computer-based

- Labs
- Practice Computer-based Exam
- Computer-based Exam

Pen-and-paper

- Problem sheets
- Final Exam

Julia is a programming language designed by MIT for Scientific Computing, Numerical Analysis and Machine Learning

Compiled: generates efficient high performance code and allows us to see what the computer is actually doing

Easy to add custom types to understand mathematical concepts

Course website

https://github.com/Imperial-MATH50003/MATH50003NumericalAnalysis

Part I

Calculus on a Computer

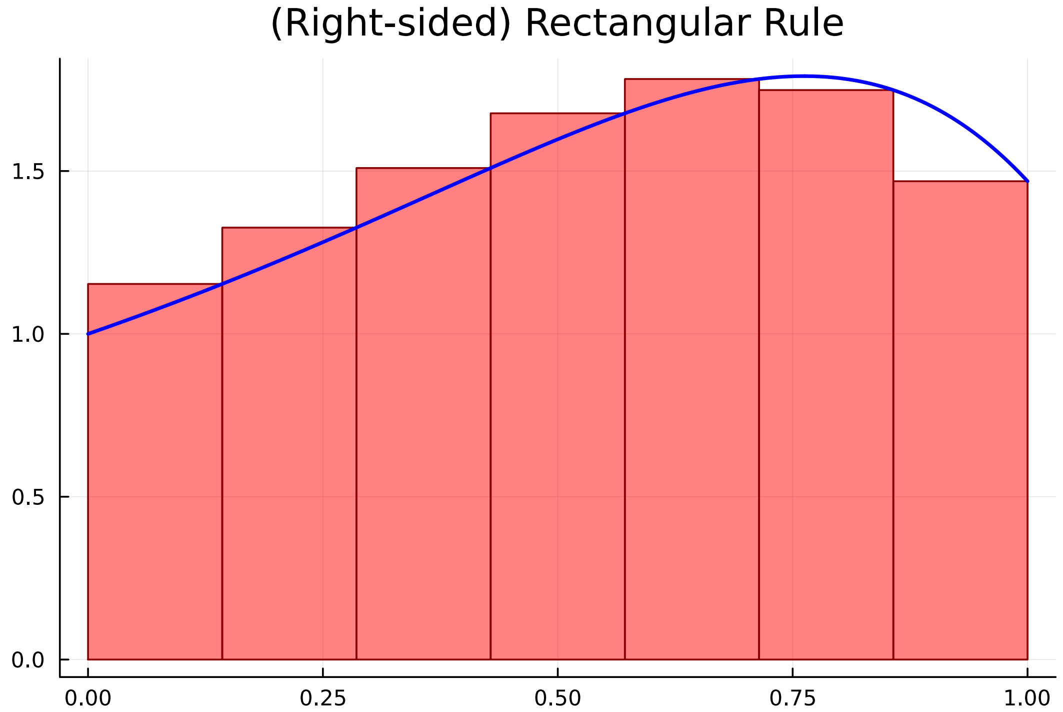
1. Rectangular rules for integration
2. Divided differences for differentiation
3. Dual numbers for differentiation

$$\int_a^b f(x)\mathrm{d}x = \lim_{n\to\infty} h \sum_{j=1}^{n} f(x_j)$$

where

$$h = \frac{b-a}{n}$$

$$x_j = a + jh$$



Lemma 1 ((Right-sided) Rectangular Rule error on one panel)**.** *Assuming f is differentiable we have*

$$\int_a^b f(x)\mathrm{d}x = (b-a)f(b) + \delta$$

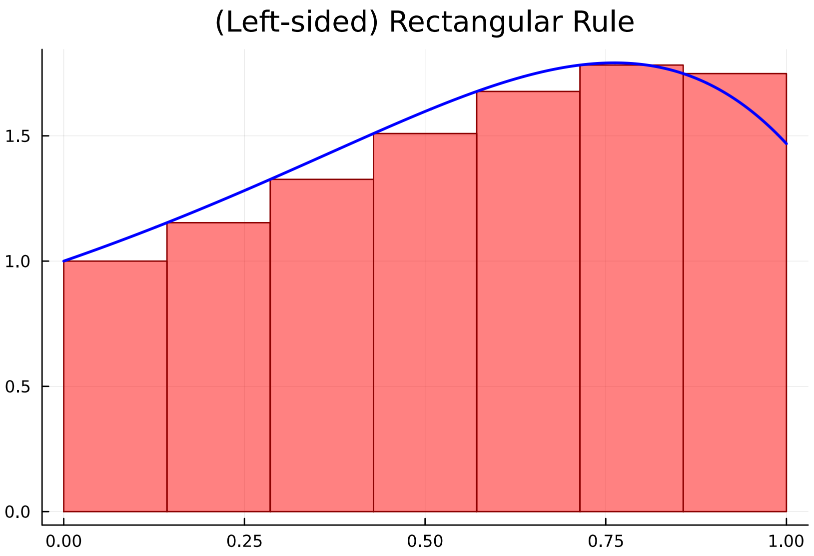
where $|\delta| \leq M(b-a)^2$ *for* $M = \sup_{a \leq x \leq b} |f'(x)|$.

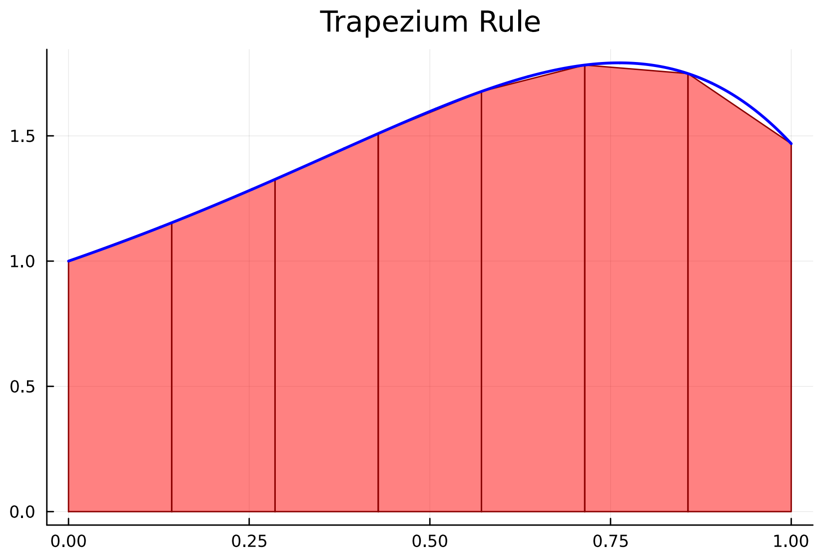
Theorem 1 (Rectangular Rule error). *Assuming f is differentiable we have*

$$\int_a^b f(x)\mathrm{d}x = h\sum_{j=1}^{n} f(x_j) + \delta$$

where $|\delta| \leq M(b-a)h$ *for* $M = \sup_{a\leq x\leq b} |f'(x)|$, $h = (b-a)/n$ *and* $x_j = a + jh$.

Other Approximations





$$h\sum_{j=0}^{n-1} f(x_j)$$

$$h\left[\frac{f(x_0)}{2} + \sum_{j=1}^{n-1} f(x_j) + \frac{f(x_n)}{2}\right]$$

How to do it in practice?

Three setup steps

1. Download julia

2. Download course content on Git from
https://github.com/Imperial-MATH50003/MATH50003NumericalAnalysis

3. Open Lab 1 in Jupyter