Chapter 1

Contest

Template.py

```python
import sys
from collections import *
from itertools import permutations #No repeated
    elements
sys.setrecursionlimit(10**5)
itr = (line for line in sys.stdin.read().strip().split(
    '\n'))
INP = lambda: next(itr)
def ni(): return int(INP())
def nl(): return [int(_) for _ in INP().split()]



def solve(n,a):
    pass


t = ni()
for case in range(t):
    n = ni()
    a = nl()
    solve(n,a)
```

Troubleshooting: Pre-submit: Write a few simple test cases if sample is not enough. Are time limits close? If so, generate max cases. Is the memory usage fine? Could anything overflow? Make sure to submit the right file.

Wrong answer: Print your solution! Print debug output, as well. Are you clearing all data structures between test cases? Can your algorithm handle the whole range of input? Read the full problem statement again. Do you handle all corner cases correctly? Have you understood the problem correctly? Any uninitialized variables? Any overflows? Confusing N and M, i and j, etc.? Are you sure your algorithm works? What special cases have you not thought of? Are you sure the STL functions you use work as you think? Add some assertions, maybe resubmit. Create some testcases to run your algorithm on. Go through the algorithm for a simple case. Go through this list again. Explain your algorithm to a teammate. Ask the teammate to look at your code. Go for a small walk, e.g. to the toilet. Is your output format correct? (including whitespace) Rewrite your solution from the start or let a teammate do it.

Runtime error: Have you tested all corner cases locally? Any uninitialized variables? Are you reading or writing outside the range of any vector? Any assertions that might fail? Any possible division by 0? (mod 0 for example) Any possible infinite recursion? Invalidated pointers or iterators? Are you using too much memory? Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded: Do you have any possible infinite loops? What is the complexity of your algorithm? Are you copying a lot of unnecessary data? (References) How big is the input and output? (consider buffering output) What do your teammates think about your algorithm?

Memory limit exceeded: What is the max amount of memory your algorithm should need? Are you clearing all data structures between test cases?

Chapter 2

Mathematics

2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by $x = -b/2a$.

$$\begin{aligned} ax + by = e \\ cx + dy = f \end{aligned} \Rightarrow \begin{aligned} x &= \frac{ed - bf}{ad - bc} \\ y &= \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation $Ax = b$, the solution to a variable x_i is given by

$$x_i = \frac{\det A_i'}{\det A}$$

where A_i' is A with the i'th column replaced by b.

2.2 Recurrences

If $a_n = c_1 a_{n-1} + \cdots + c_k a_{n-k}$, and $r_1, \ldots, r_k$ are distinct roots of $x^k + c_1 x^{k-1} + \cdots + c_k$, there are $d_1, \ldots, d_k$ s.t.

$$a_n = d_1 r_1^n + \cdots + d_k r_k^n.$$

Non-distinct roots r become polynomial factors, e.g. $a_n = (d_1 n + d_2) r^n$.

2.3 Trigonometry

$$\sin(v + w) = \sin v \cos w + \cos v \sin w$$
$$\cos(v + w) = \cos v \cos w - \sin v \sin w$$

$$\tan(v + w) = \frac{\tan v + \tan w}{1 - \tan v \tan w}$$
$$\sin v + \sin w = 2 \sin \frac{v + w}{2} \cos \frac{v - w}{2}$$
$$\cos v + \cos w = 2 \cos \frac{v + w}{2} \cos \frac{v - w}{2}$$

$$(V + W) \tan(v - w)/2 = (V - W) \tan(v + w)/2$$

where V, W are lengths of sides opposite angles v, w.

$$a \cos x + b \sin x = r \cos(x - \phi)$$
$$a \sin x + b \cos x = r \sin(x + \phi)$$

where $r = \sqrt{a^2 + b^2}, \phi = \operatorname{atan2}(b, a)$.

2.4 Geometry

2.4.1 Triangles

Side lengths: a, b, c

Semiperimeter: $p = \dfrac{a + b + c}{2}$

Area: $A = \sqrt{p(p - a)(p - b)(p - c)}$

Circumradius: $R = \dfrac{abc}{4A}$

Inradius: $r = \dfrac{A}{p}$

Length of median (divides triangle into two equal-area triangles): $m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$

Length of bisector (divides angles in two): $s_a = \sqrt{bc\left[1 - \left(\dfrac{a}{b + c}\right)^2\right]}$

Law of sines: $\dfrac{\sin \alpha}{a} = \dfrac{\sin \beta}{b} = \dfrac{\sin \gamma}{c} = \dfrac{1}{2R}$

Law of cosines: $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents: $\dfrac{a + b}{a - b} = \dfrac{\tan \frac{\alpha + \beta}{2}}{\tan \frac{\alpha - \beta}{2}}$

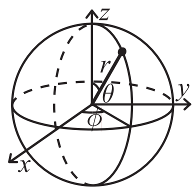
2.4.2 Quadrilaterals

With side lengths a, b, c, d, diagonals e, f, diagonals angle θ, area A and magic flux $F = b^2 + d^2 - a^2 - c^2$:

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is $180°$, $ef = ac + bd$, and $A = \sqrt{(p - a)(p - b)(p - c)(p - d)}$.

2.4.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \operatorname{acos}(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \operatorname{atan2}(y, x) \end{aligned}$$

2.5 Derivatives/Integrals

$$\frac{d}{dx} \arcsin x = \frac{1}{\sqrt{1 - x^2}} \qquad \frac{d}{dx} \arccos x = -\frac{1}{\sqrt{1 - x^2}}$$
$$\frac{d}{dx} \tan x = 1 + \tan^2 x \qquad \frac{d}{dx} \arctan x = \frac{1}{1 + x^2}$$
$$\int \tan ax = -\frac{\ln|\cos ax|}{a} \qquad \int x \sin ax = \frac{\sin ax - ax \cos ax}{a^2}$$
$$\int e^{-x^2} = \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) \qquad \int x e^{ax} dx = \frac{e^{ax}}{a^2}(ax - 1)$$

Integration by parts:

$$\int_a^b f(x) g(x) dx = [F(x) g(x)]_a^b - \int_a^b F(x) g'(x) dx$$

2.6 Sums

$$c^a + c^{a+1} + \cdots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(2n+1)(n+1)}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$
$$1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$$

2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots, \ (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots, \ (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \ldots, \ (-1 \leq x \leq 1)$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots, \ (-\infty < x < \infty)$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots, \ (-\infty < x < \infty)$$

2.8 Probability theory

Let X be a discrete random variable with probability $p_X(x)$ of assuming the value x. It will then have an expected value (mean) $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$ and variance $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$ where σ is the standard deviation. If X is instead continuous it will have a probability density function $f_X(x)$ and the sums above will instead be integrals with $p_X(x)$ replaced by $f_X(x)$.

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent X and Y,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

2.8.1 Discrete distributions

Binomial distribution

The number of successes in n independent yes/no experiments, each which yields success with probability p is $\mathrm{Bin}(n,p)$, $n = 1, 2, \ldots$, $0 \leq p \leq 1$.

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \ \sigma^2 = np(1-p)$$

$\mathrm{Bin}(n,p)$ is approximately $\mathrm{Po}(np)$ for small p.

First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each wich yields success with probability p is $\mathrm{Fs}(p)$, $0 \leq p \leq 1$.

$$p(k) = p(1-p)^{k-1}, \ k = 1, 2, \ldots$$

$$\mu = \frac{1}{p}, \ \sigma^2 = \frac{1-p}{p^2}$$

Poisson distribution

The number of events occurring in a fixed period of time t if these events occur with a known average rate κ and independently of the time since the last event is $\mathrm{Po}(\lambda)$, $\lambda = t\kappa$.

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \ldots$$

$$\mu = \lambda, \ \sigma^2 = \lambda$$

2.8.2 Continuous distributions

Uniform distribution

If the probability density function is constant between a and b and 0 elsewhere it is $\mathrm{U}(a,b)$, $a < b$.

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \ \sigma^2 = \frac{(b-a)^2}{12}$$

Exponential distribution

The time between events in a Poisson process is $\mathrm{Exp}(\lambda)$, $\lambda > 0$.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \ \sigma^2 = \frac{1}{\lambda^2}$$

Normal distribution

Most real random values with mean μ and variance σ^2 are well described by $\mathcal{N}(\mu, \sigma^2)$, $\sigma > 0$.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$ and $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$ then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let $X_1, X_2, \ldots$ be a sequence of random variables generated by the Markov process. Then there is a transition matrix $\mathbf{P} = (p_{ij})$, with $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$, and $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$ is the probability distribution for X_n (i.e., $p_i^{(n)} = \Pr(X_n = i)$), where $\mathbf{p}^{(0)}$ is the initial distribution.

π is a stationary distribution if $\pi = \pi\mathbf{P}$. If the Markov chain is *irreducible* (it is possible to get to any state from any state), then $\pi_i = \frac{1}{\mathbb{E}(T_i)}$ where $\mathbb{E}(T_i)$ is the expected time between two visits in state i. π_j / π_i is the expected number of visits in state j between two visits in state i.

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors, π_i is proportional to node i's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1). $\lim_{k\to\infty} \mathbf{P}^k = \mathbf{1}\pi$.

A Markov chain is an A-chain if the states can be partitioned into two sets $\mathbf{A}$ and $\mathbf{G}$, such that all states in $\mathbf{A}$ are absorbing ($p_{ii} = 1$), and all states in $\mathbf{G}$ leads to an absorbing state in $\mathbf{A}$. The probability for absorption in state $i \in \mathbf{A}$, when the initial state is j, is $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$. The expected time until absorption, when the initial state is i, is $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ki} t_k$.

Chapter 3

Data Structures

Segment tree:

```python
# Tested on: https://open.kattis.com/problems/supercomputer
class SegmentTree:
    def __init__(self, arr, func=min):
        self.sz = len(arr)
        assert self.sz > 0
        self.func = func
        sz4 = self.sz*4
        self.L, self.R = [None]*sz4, [None]*sz4
        self.value = [None]*sz4
        def setup(i, lo, hi):
            self.L[i], self.R[i] = lo, hi
            if lo == hi:
                self.value[i] = arr[lo]
                return
            mid = (lo + hi)//2
            setup(2*i, lo, mid)
            setup(2*i + 1, mid+1, hi)
            self._fix(i)
        setup(1, 0, self.sz-1)
    def _fix(self, i):
        self.value[i] = self.func(self.value[2*i], self.value[2*i+1])

    def _combine(self, a, b):
        if a is None: return b
        if b is None: return a
        return self.func(a, b)

    def query(self, lo, hi):
        assert 0 <= lo <= hi < self.sz
        return self.__query(1, lo, hi)

    def __query(self, i, lo, hi):
        l, r = self.L[i], self.R[i]
        if r < lo or hi < l:
            return None
        if lo <= l <= r <= hi:
            return self.value[i]
        return self._combine(
            self.__query(i*2, lo, hi),
            self.__query(i*2 + 1, lo, hi)
            )

    def assign(self, pos, value):
        assert 0 <= pos < self.sz
        return self.__assign(1, pos, value)

    def __assign(self, i, pos, value):
        l, r = self.L[i], self.R[i]
        if pos < l or r < pos: return
        if pos == l == r:
            self.value[i] = value
            return
        self.__assign(i*2, pos, value)
        self.__assign(i*2 + 1, pos, value)
        self._fix(i)

    def inc(self, pos, delta):
        assert 0 <= pos < self.sz
        self.__inc(1, pos, delta)

    def __inc(self, i, pos, delta):
        l, r = self.L[i], self.R[i]
        if pos < l or r < pos: return
        if pos == l == r:
            self.value[i] += delta
            return
        self.__inc(i*2, pos, delta)
        self.__inc(i*2 + 1, pos, delta)
        self._fix(i)

    # for indexing - nice to have but not required
    def __setitem__(self, i, v):
        self.assign(i, v)
    def __fixslice__(self, k):
        return slice(k.start or 0, self.sz if k.stop == None else k.stop)
    def __getitem__(self, k):
        if type(k) == slice:
            k = self.__fixslice__(k)
            return self.query(k.start, k.stop - 1)
        elif type(k) == int:
            return self.query(k, k)
```

Fenwick Tree:

```python
# Tested on: https://open.kattis.com/problems/froshweek
class FenwickTree: # zero indexed calls!
    # Give array or size!
    def __init__(self, blob):
        if type(blob) == int:
            self.sz = blob
            self.data = [0]*(blob+1)
        elif type(blob) == list:
            A = blob
            self.sz = len(A)
            self.data = [0]*(self.sz + 1)
            for i, a in enumerate(A):
                self.inc(i, a)
    # A[i] = v
    def assign(self, i, v):
        currV = self.query(i, i)
        self.inc(i, v - currV)
    # A[i] += delta
    # this method is ~3x faster than doing A[i] += delta
    def inc(self, i, delta):
        i += 1 # (to 1 indexing)
        while i <= self.sz:
            self.data[i] += delta
            i += i&-i # lowest oneBit
    # sum(A[:i+1])
    def sum(self, i):
        i += 1 # (to 1 indexing)
        S = 0
        while i > 0:
            S += self.data[i]
            i -= i&-i
        return S
    # return sum(A[lo:hi+1])
    def query(self, lo, hi):
        return self.sum(hi) - self.sum(lo-1)

    # for indexing - nice to have but not required
    def __fixslice__(self, k):
        return slice(k.start or 0, self.sz if k.stop == None else k.stop)
    def __setitem__(self, i, v):
        self.assign(i, v)
    def __getitem__(self, k):
        if type(k) == slice:
            k = self.__fixslice__(k)
            return self.query(k.start, k.stop - 1)
        elif type(k) == int:
            return self.query(k, k)
```

RMQ:

```python
import math
class RMQ:
    def __init__(self, arr, func=min):
        self.sz = len(arr)
        self.func = func
        MAXN = self.sz
        LOGMAXN = int(math.ceil(math.log(MAXN + 1, 2)))
        self.data = [[0]*LOGMAXN for _ in range(MAXN)]
        for i in range(MAXN):
            self.data[i][0] = arr[i]
        for j in range(1, LOGMAXN):
            for i in range(MAXN - (1<<j)+1):
                self.data[i][j] = func(
                    self.data[i][j-1],
```

```
                    self.data[i + (1<<(j-1))][j-1])

    def query(self, a, b):
        if a > b:
            # some default value when query is empty
            return 1
        d = b - a + 1
        k = int(math.log(d, 2))
        return self.func(self.data[a][k], self.data[b
    -(1<<k)+1][k])
```

Uniion Find:

```
class UnionFind:
    def __init__(self, N):
        self.parent = [i for i in range(N)]
        self.sz = [1]*N
    def find(self, i):
        path = []
        while i != self.parent[i]:
            path.append(i)
            i = self.parent[i]
        for u in path: self.parent[u] = i
        return i
    def union(self, u, v):
        uR, vR = map(self.find, (u, v))
        if uR == vR: return False
        if self.sz[uR] < self.sz[vR]:
            self.parent[uR] = vR
            self.sz[vR] += self.sz[uR]
        else:
            self.parent[vR] = uR
            self.sz[uR] += self.sz[vR]
        return True
```