Pricing an Asian Option by Applying the Random Numbers Generated from the Halton Sequence

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1 Monte-Carlo methods

1.1 Introduction

Monte Carlo methods are a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results.The modern version of the Markov Chain Monte Carlo method was invented in the late 1940s by Stanislaw Ulam.Their essential idea is using randomness to solve problems that might be deterministic in principle. In principle, Monte Carlo methods can be used to solve any problem having a probabilistic interpretation. By the law of large numbers, integrals described by the expected value of some random variable can be approximated by taking the empirical mean (a.k.a. the sample mean) of independent samples of the variable.

1.2 Monte Carlo Methods for Option Pricing

A three-step approach to pricing an option by Monte Carlo method is :

- Generate a large number of possible (but random) price paths for the underlying (or underlyings) via simulation,
- Calculate the associated exercise value (i.e. "payoff") of the option for each path.
- Discount the payoffs back to today and average them to determine the expected price

2 Halton Sequence

2.1 Introduction

In statistics, Halton sequences are sequences used to generate points in space for numerical methods such as Monte Carlo simulations.Halton's sequence is one of several available low discrepancy sequences. If n is an integer in base 10 (i.e. decimal notation) then it may be written in base b as

$$n = \sum_{k=0}^{m} d_k b^k, \quad where \quad d_k = 1 \quad or \quad 0.$$

Then the n^{th} number in the Halton sequence of base b is given by

$$h(n, b) = \sum_{k=0}^{m} d_k b^{-(k+1)}, \quad where \quad d_k = 1 \quad or \quad 0.$$

2.2 Example

Example 1 : Generate the Halton Sequence of base 2.

We start by dividing the interval (0,1) in half, then in fourths, eighths, etc., which generates

$$\frac{1}{2}, \quad \frac{1}{4}, \quad \frac{3}{4}, \quad \frac{1}{8}, \quad \frac{5}{8}, \quad \frac{3}{8}, \quad \frac{7}{8}, \quad \frac{1}{16}, \quad \frac{9}{16}, \quad \cdots$$

3 Asian Options

3.1 Definition of Asian Option

Definition:An Asian option (or average value option) is one of the basic forms of exotic options. For Asian options, the payoff is determined by the average underlying price over some pre-set period of time rather than the price of the underlying instrument at exercise.

3.2 Pricing Models

Asian options are priced according to the following models:

- Fixed strike Asian call/put payoff

$$\begin{aligned} C(T) &= \max(A(0,T) - K, 0) \\ P(T) &= \max(K - A(0,T), 0) \end{aligned}$$

- Floating strike Asian call/put option

$$\begin{aligned} C(T) &= \max(S(T) - kA(0,T), 0); \\ P(T) &= \max(kA(0,T) - S(T), 0) \end{aligned}$$

where, *A* means Average price at [0, T];*K* means Strike price; *S(T)* means Price at maturity; *k* means Weighting.

Further more, we can compute average price A using the following models:

- The continuous case

$$A(0,T) = \frac{1}{T}\int_0^T S(t)dt$$

- The discrete monitoring case

$$A(0,T) = \frac{1}{n}\sum_{i=0}^{n-1} S(t_i)$$

- The geometric average in the continuous case

$$A(0,T) = \exp\left(\frac{1}{T}\int_0^T \ln(S(t))dt\right)$$

3.3 A Specific Case

Consider an Asian Option with a payoff given by:

$$Payoff_{call} = \max(S - X, 0)$$

$$Payoff_{put} = \max(X - S, 0)$$

where S is the average value of the asset price over the life of the option and X is the strike. **Then how to price the above Asian option?**

4 Application & Results

4.1 Procedures

Procedures to Pricing an Asian Option Using Halton Sequence:

Step 1: Generate the Halton sequence.

Step 2: Transform the Halton sequence that might appear to be derived from a uniform distribution to a normal distribution series by using the Box-Muller transformation. Assume X and Y are two independent random samples on the interval (0,1] then,

$$P = R\cos\Theta$$

$$Q = R\sin\Theta;$$

where,

$$R = -2\ln(X)$$

$$\Theta = 2\pi Y$$

may be treated as samples from a normal distribution.

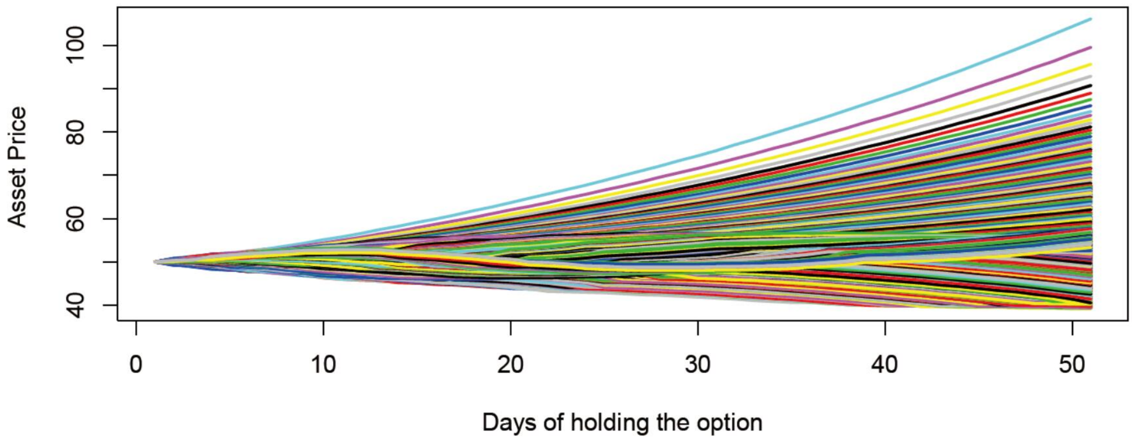

Figure 1: SFM10Halton_priAsiopt

Step 3: Simulating Asset Paths. Assuming the asset follows the standard geometric Brownian motion model,

$$S(\Delta t) = S(0)\exp[(\mu - \frac{\sigma^2}{2})\Delta t + (\sigma\sqrt{\Delta t})\varepsilon] \tag{1}$$

where,

- $S(0)$: The stock price today;
- $S(\Delta t)$: The stock price at a (small) time into the future;
- Δt: A small increment of time;
- μ: The expected return;
- σ: The expected volatility;
- ε: A (random) number sampled from a standard normal distribution.

Repeated use of (1) allows multiple potential future asset paths (between now and expiry) to be generated .An example of 1000 such paths is given in Figure 1:

Step 4: Averaging the asset price for each of the simulated paths and applying the appropriate formula of (2) and (3).

$$Payoff_{call} = \max(S - X, 0) \tag{2}$$

$$Payoff_{put} = \max(X - S, 0) \tag{3}$$

Step 5: Averaging the payoffs for all paths and discounting the result to time 0.

4.2 Example

Example: we used the above five steps to price the Asian Call and Put options with the following parameters: $X = 50$, $\mu = 0.04$, $\sigma = 0.1$, $r = 0.03$, $dt = 1/365$, $steps = 50$, $T = dt * steps$, $nsims = 1000$. Results are give by: https://github.com/SFMWISE2016/SFM10Halton_priAsiopt

5 Further Reading

Books:

- Peter Jaeckel (2002). *Monte Carlo Methods in Finance*. John Wiley and Sons. ISBN 0-471-49741-X.
- Bruno Dupire (1998). *Monte Carlo: Methodologies and Applications for Pricing and Risk Management. Risk.*
- Don L. McLeish (2005). *Monte Carlo Simulation & Finance.* ISBN 0-471-67778-7.

Journal:

- *“Monte Carlo Simulation"*. Palisade Corporation. 2010. Retrieved 2010-09-24.
- Boyle, Phelim P. *“Options: A Monte Carlo Approach"*. Journal of Financial Economics, Volume (Year): 4(1977), Issue (Month): 3(May). pp. 323 − 338. Retrieved 2010-09-24.ąć