Problem 2: Two-Dimensional Ising Lattice

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1. Write a program to simulate the two-dimensional Ising model ($E_\nu = -\sum'_{ij} J s_i s_j - \mu H \sum_i s_i$ with external field $H = 0$, and setting $J = 1$) with periodic boundary conditions. The system length L (total number of spins is L^2) should be an easily modifiable parameter as should the temperature. Run your simulation as follows: 10^5 Monte Carlo Steps (MCSs) to thermalize the system, 3×10^5 measurement MCS measuring every 10 sweeps. These choices give good statistical accuracy. Do the simulation from $T = 0.015$ to $T = 4.5$ in steps of 0.015 and for L = 10, 20, 30.

2. For each of these temperatures and system sizes, measure the average total energy $E = \langle E_\nu \rangle$, calculate the specific heat C using (just setting $k = 1$)

$$C = \frac{1}{NkT^2} \left\langle (\delta E)^2 \right\rangle = \frac{1}{NkT^2} \left(\left\langle H^2 \right\rangle - \langle H \rangle^2 \right)$$

Also calculate the magnetization per site, $M = \langle \sum_i s_i \rangle$, and the susceptibility (note: there may be different definition of χ in literatures)

$$\chi = \frac{1}{N} \left(\frac{\partial M}{\partial (\beta H)} \right)_\beta = \frac{1}{N} \left\langle (\delta M)^2 \right\rangle = \frac{1}{N} \left(\left\langle M^2 \right\rangle - \langle M \rangle^2 \right).$$

3. Make a plot of E (Fig.1) and M(Fig.2) as functions of T for all the three system size. Comment on what you observe for $2 \leq T \leq 3$.

4. Plot the susceptibility χ(Fig.3) and specific heat C(Fig.4) as functions of T for the systems simulated. There would be peaks. Note the peak positions and heights changes with system size. The peak position gives estimate of the critical temperature T_c. Plot T_c as a function of system size L(Fig.5). Compare the value you obtain for the thermodynamic critical temperature, T_c, with the exact value Tc = 2.269.