Prove that each of the following languages is ***not*** regular.

1 $\left\{0^{2^n} \mid n \geq 0\right\}$.

2 $\left\{0^{2n}1^n \mid n \geq 0\right\}$

3 $\left\{0^m1^n \mid m \neq 2n\right\}$

4 Strings over $\{0, 1\}$ where the number of 0s is exactly twice the number of 1s.

5 Strings of properly nested parentheses $()$, brackets $[]$, and braces $\{\}$. For example, the string $([]) \{\}$ is in this language, but the string $([)]$ is not, because the left and right delimiters don't match.

6 Strings of the form $w_1 \# w_2 \# \cdots \# w_n$ for some $n \geq 2$, where each substring w_i is a string in $\{0, 1\}^*$, and some pair of substrings w_i and w_j are equal.

Extra problems

7 $\{w \in (0+1)^* \mid w \text{ is the binary representation of a perfect square}\}$