

Prove that each of the following problems is NP-hard.

- 1 Given an undirected graph  $G$ , does  $G$  contain a simple path that visits all but 374 vertices?
- 2 Given an undirected graph  $G$ , does  $G$  have a spanning tree with at most 374 leaves?
- 3 Recall that a 5-coloring of a graph  $G$  is a function that assigns each vertex of  $G$  a “color” from the set  $\{0,1,2,3,4\}$ , such that for any edge  $uv$ , vertices  $u$  and  $v$  are assigned different “colors”. A 5-coloring is **careful** if the colors assigned to adjacent vertices are not only distinct, but differ by more than 1 ( $\text{mod } 5$ ). Prove that deciding whether a given graph has a careful 5-coloring is NP-hard. (**Hint:** Reduce from the standard 5COLOR problem.)

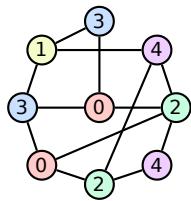


Figure 1: A careful 5-coloring.