

This is a review of context-free grammars from the lecture on Thursday; in each example, the grammar itself is on the left; the explanation for each non-terminal is on the right.

- Properly nested strings of parentheses.

$$S \rightarrow \epsilon \mid S(S) \quad \text{properly nested parentheses}$$

Here is a different grammar for the same language:

$$S \rightarrow \epsilon \mid (S) \mid SS \quad \text{properly nested parentheses}$$

- $\{\mathbf{0}^m\mathbf{1}^n \mid m \neq n\}$. This is the set of all binary strings composed of some number of **0**s followed by a different number of **1**s.

$S \rightarrow A \mid B$	$\{\mathbf{0}^m\mathbf{1}^n \mid m \neq n\}$
$A \rightarrow \mathbf{0}A \mid \mathbf{0}C$	$\{\mathbf{0}^m\mathbf{1}^n \mid m > n\}$
$B \rightarrow B\mathbf{1} \mid C\mathbf{1}$	$\{\mathbf{0}^m\mathbf{1}^n \mid m < n\}$
$C \rightarrow \epsilon \mid \mathbf{0}C\mathbf{1}$	$\{\mathbf{0}^m\mathbf{1}^n \mid m = n\}$

Give context-free grammars for each of the following languages. For each grammar, describe *in English* the language for each non-terminal, and in the examples above. As usual, we won't get to all of these in section.

1 $\{\mathbf{0}^{2n}\mathbf{1}^n \mid n \geq 0\}$

2 $\{\mathbf{0}^m\mathbf{1}^n \mid m \neq 2n\}$

(Hint: If $m \neq 2n$, then either $m < 2n$ or $m > 2n$. Extend the previous grammar, but pay attention to parity. This language contains the string **01**.)

3 $\{\mathbf{0}, \mathbf{1}\}^* \setminus \{\mathbf{0}^{2n}\mathbf{1}^n \mid n \geq 0\}$

(Hint: Extend the previous grammar. What is missing?)

Work on these later:

4 $\{w \in \{\mathbf{0}, \mathbf{1}\}^* \mid \#(\mathbf{0}, w) = 2 \cdot \#(\mathbf{1}, w)\}$ – Binary strings where the number of **0**s is exactly twice the number of **1**s.

5 $\{\mathbf{0}, \mathbf{1}\}^* \setminus \{ww \mid w \in \{\mathbf{0}, \mathbf{1}\}^*\}$.

[Anti-hint: The language $\{ww \mid w \in \{\mathbf{0}, \mathbf{1}\}^*\}$ is **not** context-free. Thus, the complement of a context-free language is not necessarily context-free!]